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Pluto's Resonant Orbit Visualized in 4D

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ABSTRACT

Orbital resonance phenomena are notoriously difficult to communicate in words due to the complex dynamics arising from the interplay of gravity and orbital angular momentum. A well known example is Pluto's 3:2 mean motion resonance with Neptune. We have developed a python software tool to visualize the full three dimensional aspects of Pluto's resonant orbital dynamics over time. The visualizations include still images and animated movies. By contrasting Pluto's resonant dynamics with the dynamics of a nearby non-resonant orbit, this tool enables better understanding and exploration of complex planetary dynamics phenomena.

MEAN MOTION RESONANCE

Resonant phenomena are most simply understood in the case of mechanical motion in which a system is driven at one of its natural frequencies by an external force. In an orbital mean motion resonance (MMR), two objects orbiting the same central object "drive" one another resonantly with their mutual gravitational force if their orbital periods are close to a ratio of two small integers. When one of the orbiting objects is of negligible mass, the more massive object "drives" the nearly-massless object. Examples of MMRs abound in the solar system and in exo-planet systems. A well known example is the 3:2 MMR between Neptune and Pluto: Neptune completes three revolutions around the Sun in the same amount of time Pluto takes to complete two revolutions. In exoplanet dynamics, MMRs are important in measuring planet masses and modeling their evolution (Zhu & Dong 2021)

To determine whether two objects are in a MMR relationship, planetary dynamicists usually examine the time evolution of the "critical resonant argument", a combination of angular orbital elements, often denoted with ϕ. For many prominent MMRs, the critical resonant argument takes the form $\phi = p\lambda_1 - q\lambda_2 - (p-q)\varpi$, where λ is the mean longitude, ϖ is the longitude of pericenter of either the inner or the outer orbit, the subscripts 1 and 2 refer to the inner and outer orbit, and p and q are the integers from the MMR ratio. If $\phi(t)$ is found to oscillate around a central value, then the two objects are said to be in MMR. When $\phi(t)$ is found to rotate through all values 0–2π, it indicates absence of that resonant condition. For Pluto and Neptune, the critical resonant argument, $\phi = 3\lambda_2 - 2\lambda_1 - \varpi_2$, oscillates around 180° with an amplitude of about 80° and a period of about 20,000 years; this period is about 80 times Pluto's orbital period and about 120 times Neptune's orbital period. Examining the behavior of $\phi(t)$ in numerical simulations is useful for identifying MMRs, however, it is not readily translated into a physical picture for the dynamics of the system.

The mechanics and physics of an MMR are notoriously difficult to communicate due to the non-intuitive complex dynamics arising from the interplay of gravity and orbital angular momentum. One of the greatest obstacles students face when trying to explore MMRs and other types of orbital resonances in planetary dynamics is in visualizing these phenomena that take place over many thousands to millions of years. The lengthy analytical developments of the equations of planetary dynamics — and their visualizations limited to non-intuitive parameters such as the "critical resonant argument" — make it difficult to easily grasp the consequences of complex resonant orbits. To help improve understanding of MMRs, we set out to build a python tool to create visualizations and animations of the Pluto-Neptune MMR in the restricted three body model. Our python tool produces graphics to visualize the

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resonant dynamics in 2D and 3D real-space plots as well as animations of Pluto's resonant dynamics over time; the time evolution in the 3D plots can be called 4D visualizations.

Plots and animations of Pluto's resonant orbit projected in the ecliptic plane (such as the top left panel in Fig. 1, below) have appeared from time to time in the previous literature (e.g. Malhotra & Williams 1997), but, to our knowledge, a tool for the visualization of its orbit in 3+1 dimensions (as well as a contrasting nearby non-resonant case), is published here for the first time.

VISUALIZING PLUTO'S ORBIT

In our PYTHON tool, we first establish the 3-body system of the Sun, Neptune, and Pluto with orbital data in the J2000 coordinate system from JPL-Horizons (https://ssd.jpl.nasa.gov/?horizons). The orbits are then propagated from their initial conditions (under Newtonian gravity) for 20,000 years, and positional information saved every five years. Our Python tool uses the public domain WHFAST orbit integrator built into REBOUND (Rein & Tamayo 2015), and uses REBOUND's facility for direct access to the JPL–Horizons database to read in data for Solar System objects. A moderately advanced user may also manually enter their own input data to explore other MMRs for any 3-body system.

Pluto's MMR libration is best visualized in a frame rotating with Neptune's mean angular velocity about the solar system barycenter. After propagating the three body system for 20,000 years, we compute Neptune's average angular speed over the time span of the orbit integration. (Note that because Neptune's orbit is slightly elliptical, its orbital angular velocity is not constant over its orbital period.) We use Neptune's average angular speed to calculate the angle of rotation between Neptune's initial position and each output time step. Then, at each time step, we rotate with this angle all bodies' coordinates about the z-axis. In this way we have the positions with time of all bodies in a frame rotating with the mean angular velocity of Neptune. The results are shown in Figure 1. The three left panels plot the traces of the Sun, Neptune and Pluto's motion in the rotating frame in two 2D projections, $(x, y), (y, z)$, as well as in 3D (x, y, z) space; all projections are in the rotating frame described above, and over the 20,000 year time span of the orbit integration. In the (x, y) projection, we observe that Neptune traces a small oval (due to its small nonzero eccentricity) centered on the x-axis at a heliocentric distance of about 30 au, and we observe that Pluto traces out two loop structures as it cycles through perihelion during two orbits. We can also observe the extent of the libration of Pluto's perihelion loops relative to Neptune's longitudinal position. In the (y, z) projection, we see that Pluto's perihelion loops occur at a high latitude. And in the (x, y, z) space we see the full 3D complex structure of Pluto's motion over time. To contrast with Pluto's resonant dynamics, the three panels on the right show the behavior of a non-resonant orbit in proximity to the resonant orbit of Pluto; the initial conditions for this case were created by manually inputting the orbital elements and masses for all three bodies, except that Pluto's semimajor axis was increased by 1%, sufficient to shift it out of the 3:2 MMR with Neptune. In these plots we see how a non-resonant orbit precesses in three dimensions (x, y, z), its projection in the (x, y) plane nearly fills up an annular zone, and in the (y, z) projection it fills up an approximately trapezoidal zone, whereas the resonant orbit is more constrained and patterned in all projections. Animated movies of both the resonant and nearby non-resonant cases are available at https://github.com/renumalhotra/2021-Pluto-s-Resonant-Dynamics-Visualized-in-4D.

FUTURE WORK

The PYTHON tool we have developed is currently limited to visualizing the orbital dynamics in the three body model, specifically that of the Sun, Neptune and Pluto. Future extensions would include the other planets for a more realistic model of the Solar System, as well as enable exploration of other MMR systems. On multi-mega-year timescales, Pluto undergoes the Lidov-Kozai oscillation (also known as the von Zeipel-Lidov-Kozai cycles, Ito & Ohtsuka (2019)); animations to visualize this additional libration would require further algorithm and code development to address the data size and graphics issues associated with the multi-timescale phenomena of the MMR libration and the vzLK cycles over the much longer time span.

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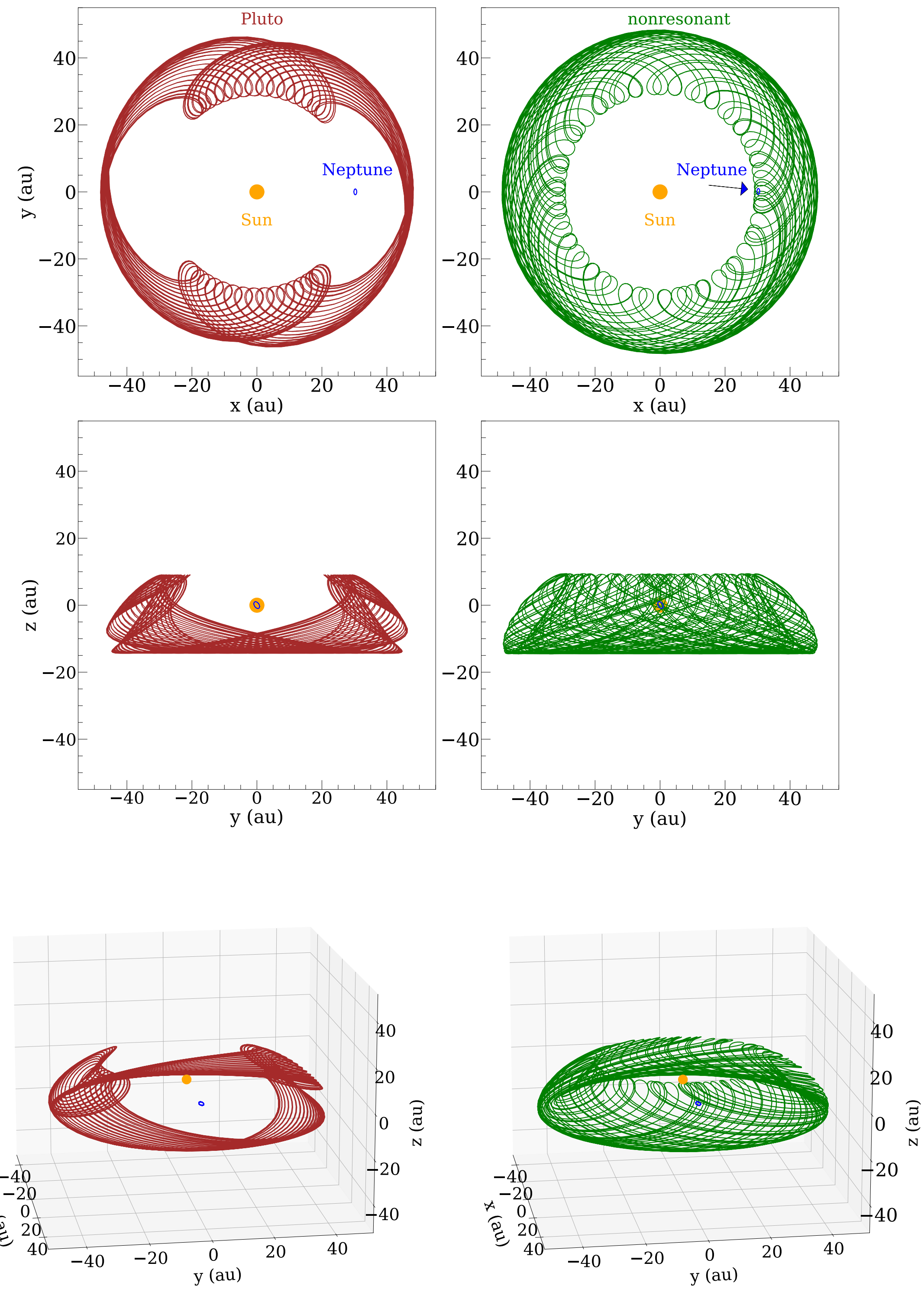

Figure 1. Visualization of Pluto's orbital dynamics over 20,000 years. The panels on the left (in brown) show the trace of Pluto's motion in the rotating frame in (x,y), (y,z), and (x,y,z) projections, respectively. The panels on the right (in green) show a nearby non-resonant case visualized in the same rotating frame. In all six panels, the Sun is represented with an orange dot near the origin and Neptune's location is traced in blue. Animated versions of these graphics are available.

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