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Noise Figure of an Active Antenna Array and Receiver System

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***Abstract*—Progress has been made in extending traditional antenna terms to complex phased array receivers, but a satisfactory definition for the noise figure of an active antenna and receiver system has not yet been given. This is particularly so for the case of a complex phased array with electronic gains in each signal path and beams formed using digital signal processing. We provide a simple definition for active antenna and receiver noise figure that is consistent with existing active antenna terms. The active antenna noise figure can be measured using the antenna Y factor technique.**

***Index Terms*—Active antenna noise figure, antenna Y factor method, radiation efficiency, receiving efficiency**

I. Introduction

Defining and measuring antenna parameters and figures of merit for high-sensitivity receivers is important in satellite communications, radio astronomy, and remote sensing applications. In recent years, antenna terms have been extended and placed on a rigorous foundation for active, nonreciprocal, and mutually coupled array receivers. For transmitting antennas, gain is defined in terms of the radiated power density integrated over a sphere around the antenna. For receivers, the equivalent role is played by the system's response to an isotropic external noise distribution [1]. The isotropic noise response is used to define figures of merit for active antenna systems, including active antenna available gain, receiving efficiency, effective area, and antenna noise temperature.

A remaining open question is how to define the noise figure of an active antenna and receiver system. In previous work, the noise figure of a phased array antenna has been given in terms of loss factors in the individual array signal paths [2], [3]. This leads to complex formulas for noise figure and it is unclear how the resulting noise figure relates to other system parameters such as G/T or sensitivity. We give a definition of noise figure based on the isotropic noise response framework and show that this definition is consistent with the noise-based active antenna terms found in the IEEE standard for antenna terms [4]. The proposed active antenna noise figure can be measured with the antenna Y factor method that is described in the IEEE Standard Test Procedures for Antennas [5].

II. Active Antenna Terms and Isotropic Noise Response

Before defining active antenna noise figure, we will briefly review key formulas for active antennas from [6]. The isotropic

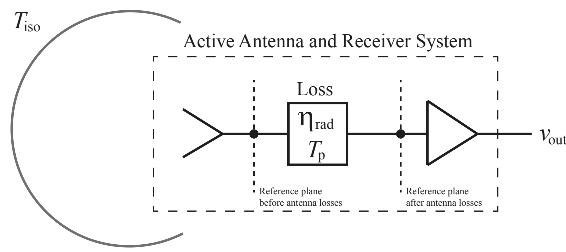

Fig. 1. Active antenna and receiver system diagram. The external environment is at temperature T_{iso}. The physical temperature of the antenna is T_{p} and the radiation efficiency is η_{rad}. Receiver noise is introduced by electronics after the antenna, represented by the amplifier in the diagram. A single antenna is shown, but the method is applicable to antenna arrays of arbitrary complexity.

noise response is the thermal noise power at the system output due to a thermal environment with brightness temperature T_{iso} with noiseless receiver electronics and with the antenna in thermal equilibrium with the environment,

$$P_{\text{t,iso}} = P_{\text{ext,iso}} + P_{\text{loss}} \tag{1}$$

where $P_{\text{ext,iso}}$ is the contribution from the external environment and P_{loss} is the noise power due to antenna losses. The active antenna system and the isotropic external thermal noise distribution are shown in Fig. 1.

Noise added by antenna losses is quantified by the receiving efficiency

$$\eta_{\text{rec}} = \frac{P_{\text{ext,iso}}}{P_{\text{t,iso}}} \tag{2}$$

For a single-port, passive antenna, receiving efficiency is equal to the radiation efficiency, so that $\eta_{\text{rec}} = \eta_{\text{rad}}$ [6]. For an active array with nonreciprocal components in the signal paths, it may not be possible to define or measure the radiation efficiency directly. In the single antenna case, either receiving efficiency or radiation efficiency can be used as a measure of the antenna losses.

In terms of these parameters, the equivalent system noise temperature is

$$\begin{aligned} T_{\text{sys}} &= \eta_{\text{rad}} T_{\text{ext}} + (1-\eta_{\text{rad}}) T_{\text{p}} + T_{\text{rec}} \\ &= T'_{\text{ext}} + T_{\text{loss}} + T_{\text{rec}} \end{aligned} \tag{3}$$

where T_{ext} is the brightness temperature distribution of the external noise environment integrated over the antenna directivity pattern, T_{p} is the physical temperature of the antenna array elements, and T_{rec} is the equivalent receiver noise temperature contributed by electronic components in the signal paths after the antenna or array elements. T'_{ext} is the external noise temperature at the reference plane after antenna losses. These quantities in general depend on beam scan angle. By

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convention in IEEE Std 145, noise temperatures are defined at the reference plane after antenna losses (i.e., at the antenna terminals) shown in Fig. 1.

III. Noise Figure for an Active Antenna

Based on these known results from the theory of active antennas, we will now formulate a new definition for the noise figure of an active antenna receiver system, or more concisely, the active antenna noise figure.

To find noise figure from its definition in terms of SNR, input and output signal powers are required. The directivity of a formed beam in the direction of the signal source is $D = k_B T_{\text{iso}} B P_{\text{sig}}/(S^{\text{sig}} P_{\text{ext,iso}})$, where S^{sig} is the incident signal power density, P_{sig} is the signal power at the system output, and k_B is Boltzmann's constant [6]. The available signal power before antenna losses is

$$P_{\text{sig,in}} = \frac{\lambda^2}{4\pi} D S^{\text{sig}} \tag{4}$$

From the effective receiving area $A_e = \lambda^2 G/(4\pi)$, where $G = \eta_{\text{rad}} D$, the available signal power after antenna losses is $P_{\text{sig,in}}\eta_{\text{rad}}$. Amplifier gains, component losses, beamformer coefficients, and other scale factors after the antenna ports are quantified by the active antenna available gain [1], [4],

$$G_{\text{av}} = \frac{P_{\text{t,iso}}}{k_B T_{\text{iso}} B} \tag{5}$$

The available signal power scaled by the active antenna available gain yields $P_{\text{sig}} = P_{\text{sig,in}}\eta_{\text{rad}} G_{\text{av}}$ for the received signal power.

By convention for noise figure, the input noise power is $P_{\text{n,in}} = k_B T_0 B$. The noise power at the system output is $P_{\text{n}} = k_B T_{\text{sys}} B G_{\text{av}}$. The active antenna and receiver noise figure is then

$$F = \frac{\text{SNR}_{\text{in}}}{\text{SNR}_{\text{out}}} = \frac{P_{\text{sig,in}}/P_{\text{n,in}}}{P_{\text{sig}}/P_{\text{n}}} = 1 + \frac{T_{\text{loss}} + T_{\text{rec}}}{\eta_{\text{rad}} T_0} \tag{6}$$

where $T_{\text{loss}} = (1-\eta_{\text{rad}})T_{\text{p}}$ is the equivalent antenna loss noise temperature. The equivalent temperature corresponding to the noise figure is

$$T_{\text{eq}} = (F-1)T_0 = \frac{T_{\text{loss}} + T_{\text{rec}}}{\eta_{\text{rad}}} \tag{7}$$

This is the antenna array loss and receiver noise referred to an equivalent external isotropic noise temperature before antenna losses (i.e., to an equivalent sky temperature).

The same result can be obtained by cascading the antenna losses with the receiver. The noise figure of all components after the array element ports (receiver electronics including amplifiers and other components and the beamformer) is

$$F_{\text{rec}} = 1 + \frac{T_{\text{rec}}}{T_0} \tag{8}$$

The array noise figure is

$$F_{\text{ant}} = 1 + \frac{T_{\text{loss}}/\eta_{\text{rad}}}{T_0} \tag{9}$$

Cascading the antenna and receiver using the antenna loss factor η_{rad} leads to

$$F = F_{\text{ant}} + \frac{F_{\text{rec}} - 1}{\eta_{\text{rad}}} = 1 + \frac{T_{\text{loss}} + T_{\text{rec}}}{\eta_{\text{rad}} T_0} \tag{10}$$

which agrees with (6). The agreement shows that the proposed definition for active antenna noise figure is consistent with other active antenna terms and with the noise figure cascade formula.

IV. Antenna Y Factor Measurement Technique

Figures of merit for active antennas and arrays can be measured using the antenna Y factor method. We will show that when noise figure for an active antenna receiver system is properly defined using (6), the antenna Y factor method provides a direct measurement of the active antenna noise figure.

The antenna Y factor method has been used for many years and is well established [1], [7]–[11]. When measuring the noise figure of a microwave component, known hot and cold noise power levels are introduced at the system input. The ratio in the output noise power level is the Y factor. The Y factor can be used to calculate the the system noise figure and equivalent noise temperature. The antenna Y factor method extends the benchtop technique to antennas using free space loads that produce a hot and cold brightness temperature distribution in the scene observed by the antenna and has been added to the most recent update of the IEEE Standard Test Procedures for Antennas [5].

The Y factor is obtained by changing the external brightness temperature T_{iso} in Fig. 1 respectively to two known values, T_{hot} and T_{cold}. The hot and cold loads are typically cold sky and ambient temperature microwave absorber foam with a conductive shield to block ground noise. In some cases, absorber infused with liquid nitrogen or even a light bulb have been used [12]. The receiver output powers P_{hot} and P_{cold} are measured. In the case of a phased array system, the powers are taken at the output of an analog or digital beamformer. From the Y factor $P_{\text{hot}}/P_{\text{cold}}$, the external isotropic noise response can be determined. The antenna and receiver equivalent noise temperature is

$$T_{\text{eq}} = \frac{T_{\text{hot}} - Y T_{\text{cold}}}{Y - 1} \tag{11}$$

While this measurement technique has been widely used, several questions have remained unanswered. How does the equivalent temperature obtained from the antenna Y factor method in (11) relate to the system noise temperature? How does it depend on antenna losses and antenna efficiency? Is there a connection between this quantity and the active antenna and receiver noise figure, particularly for a complex phased array receiver system?

To answer these questions, we will relate the antenna Y factor to the noise-based antenna terms reviewed in Sec. II and the new definition for noise figure given in Sec. III. The antenna Y factor can be be expressed using (3) as

$$Y = \frac{\eta_{\text{rad}} T_{\text{hot}} + T_{\text{loss}} + T_{\text{rec}}}{\eta_{\text{rad}} T_{\text{cold}} + T_{\text{loss}} + T_{\text{rec}}} \tag{12}$$

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Using this relationship in (11), we find that

$$T_{\text{eq}} = \frac{T_{\text{loss}} + T_{\text{rec}}}{\eta_{\text{rad}}} \tag{13}$$

By comparing this expression to (3), we see that (13) is the antenna loss and receiver noise temperature referred to the reference plane before antenna losses shown in Fig. 1, or in other words, to an equivalent sky temperature. This result shows that the equivalent noise temperature (11) provided by the antenna Y factor method is the antenna loss and receiver noise equivalent temperature at the reference plane before antenna losses.

We will now show that the antenna Y factor method provides a direct measurement of the active antenna noise figure. Combining (6) and (13) allows us to express the array and receiver noise figure in terms of the measured Y factor as

$$F = \frac{T_{\text{hot}} - T_0 + Y(T_0 - T_{\text{cold}})}{(Y-1)T_0} \tag{14}$$

This result shows that the antenna Y factor method can be used to measure the noise figure of an active antenna and receiver system. For a single antenna, this is a fixed quantity that depends on the antenna losses and noise figure of the receiver electronics. For an antenna array, the noise response, receiving efficiency, and other parameters in general depend on analog or digital beamformer weights, which means that the active antenna noise figure (14) is scan angle dependent.

As a side note, with an additional receiver noise measurement it is possible in principle to determine the antenna radiation efficiency. Using (13) with $T_{\text{loss}} = (1 - \eta_{\text{rad}})T_{\text{p}}$ leads to

$$\eta_{\text{rad}} = \frac{(T_{\text{p}} + T_{\text{rec}})(Y-1)}{T_{\text{hot}} - T_{\text{p}} + Y(T_{\text{p}} - T_{\text{cold}})} \tag{15}$$

For a single antenna system, the receiver noise temperature T_{rec} can be measured using the standard benchtop Y factor method with a connectorized noise source at the receiver input in place of the antenna. For an active array antenna with low noise amplifiers connected to each element, the receiver noise contributed by each front-end amplifier is determined by beam-dependent active impedances [13]. The receiver noise can be modeled using the noise parameters of the amplifiers and the antenna array mutual impedances. Once the receiver noise contribution is known, the antenna radiation efficiency can be computed using (15).

V. Conclusion

We have shown that the noise-based antenna terms in IEEE Standard 145 can be used to generalize well-known formulas for noise figure to active array antenna systems of arbitrary complexity. The noise figure of an active antenna and receiver can be measured using the well-established antenna Y factor method. The result provides a simple, self-consistent and measurable definition for noise figure for active antenna receiver systems. Future work might focus on quantifying the effect of antenna losses on the system noise figure by isolating the receiving or radiation efficiency of the antenna from other system noise contributions.

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