Transportation Research Part B 146 (2021) 289–313

Contents lists available at ScienceDirect

Transportation Research Part B

journal homepage: www.elsevier.com/locate/trb

Optimal contract design for ride-sourcing services under dual sourcing

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ARTICLE INFO

Article history:
Received 17 July 2020
Revised 22 January 2021
Accepted 29 January 2021
Available online 12 March 2021

Keywords:
Ride-sourcing market
Dual sourcing
Contract design
Freelancers
Contractors

ABSTRACT

To cope with the uncertainty of labor supply from freelance/self-scheduling drivers, some ride-sourcing platforms recruit contractual drivers, who are paid a fixed salary for pre-specified work schedules. This paper develops an aggregate modeling framework to examine the practicability of such a dual-sourcing strategy. We investigate the optimal contract design of dual sourcing under demand uncertainty, varying price sensitivity of freelancers, and heterogeneity in drivers' risk attitude. Our results uncover the conditions under which dual sourcing benefits both the platform and drivers. We show that the platform's staffing and pricing decisions are most responsive to freelancers' price sensitivity. When the price sensitivity stays adequately low, both the platform and drivers can be better off under dual sourcing compared to the self-scheduling counterpart. On the contrary, with moderate price sensitivity, freelancers will be made worse off by dual sourcing. The dual-sourcing contracts are most effective in markets where drivers are risk-averse.

1. Introduction

Ride-sourcing services offered by companies like Uber, Lyft, and Didi Chuxing are becoming one of the mainstay travel modes in many cities. These companies provide mobile applications that efficiently connect customers and participating drivers who drive their own vehicles to provide the ride-for-hire service (Zha et al., 2016). Ride-sourcing drivers enjoy great flexibility in scheduling their work hours, which may help them achieve better work-life balance (Hall and Krueger, 2018). However, such a flexibility poses a significant challenge to the platforms for managing their workforce. Drivers' imperfect information on market conditions can render considerable uncertainty to their labor supply, often causing a supply-demand imbalance in the market, i.e., platforms encounter a shortage of supply under excessive travel demand and vice versa. As drivers' preferences and availability are heterogeneous and unobservable (Chen et al., 2017), it has been a challenge for the platforms to guide or incentivize drivers to provide services when and where they are most needed.

Ride-sourcing platforms often use price to address the supply-demand imbalance. Among various pricing strategies, surge pricing, which allows platforms to raise service prices at peak periods and locations, has drawn considerable attention in

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https://doi.org/10.1016/j.trb.2021.01.014
0191-2615/

the literature (Cachon et al., 2017; Zha et al., 2018a; 2018b; Hu et al., 2019; Guda and Subramanian, 2019; Lu et al., 2018; Besbes et al., 2020; Nourinejad and Ramezani, 2020; Yang et al., 2020). As a short-term regulatory tool, surge pricing can be effective in suppressing demand, but not necessarily in attracting drivers to get online or reposition themselves towards areas with a supply shortage, at least in the short time interval during the surge (Chen et al., 2015; Hu et al., 2019). Besides, surge pricing itself is controversial. Some customers are frustrated after being charged significantly higher fares, particularly during non-recurring congestion caused by adverse weather or emergency. Others expressed concern that platforms may use surge pricing to collect more revenue under the current percentage commission structure.

To ensure more reliable labor supply while retaining flexible scheduling for those freelancers, this paper investigates dual sourcing, a strategy widely implemented in supply chain and inventory management (Minner, 2003; Song et al., 2020). Dual sourcing, in a conventional way, means that a producer/buyer places orders from two suppliers differentiating in their reliability, price or other factors to mitigate supply risks (Tomlin and Wang, 2005; Yu et al., 2009). In our context, dual sourcing suggests that in addition to existing freelancers, platforms can hire a certain number of contractual drivers who are paid a fixed salary and required to work at specific hours. These contractors supply reliable labor hours and will enable platforms to hedge against the labor supply uncertainty. Because contractors will have a more restricted work schedule but are guaranteed a competitive income, a dual sourcing contract differentiates drivers with different risk attitude and scheduling constraints. Risk-averse drivers with more foreseeable or less scheduling constraints may opt-in the contract while others can remain as freelancers and enjoy the scheduling flexibility.

This paper develops analytical models and conducts numerical experiments to investigate optimal contract design under the above dual sourcing strategy and examine its impacts on all stakeholders in the ride-sourcing market. Although the concept of dual sourcing for ride-sourcing services is similar to that in supply chain and inventory control, previous results obtained in those domains are not readily applicable due to distinct characteristics of the ride-sourcing market. For ride sourcing, extensive efforts have been made to optimize pricing, matching or dispatching decisions of platforms or analyze the market to derive insights for public policies and regulations. See, e.g., Wang and Yang (2019), for a recent review of these studies. In contrast, the investigation on dual sourcing is rather limited. The current literature on the contract design for ride-sourcing services primarily considers single sourcing. For example, by assuming only price-sensitive demand, Cachon et al. (2017) analyzed drivers' participation behavior and concluded that the fixed ratio contract is near optimal. Hu and Zhou (2019) adopted the same demand assumption and investigated the optimal driver compensation under different contract schemes for stochastic demand. They showed that a fixed commission contract is quite robust in terms of a platform's profit with varying supply and demand conditions. Recognizing that ride-sourcing demand features highly delay sensitive, Taylor (2018) examined how delay sensitivity and agent independence impact the platform's optimal per-service price and wage. Bai et al. (2019) developed an analytical model to understand the platform's optimal pricing decision considering both time-sensitive customers and earning-sensitive providers. Among a few exceptions on dual sourcing, Dong and Ibrahim (2020) formulated a queuing-theoretic framework to study the cost-minimizing staffing level of flexible and fixed agents for on-demand services with uncertain agents' arrival. For time-varying demand, they found that blending workforce could benefit customers in low-demand periods. Zhong et al. (2019) leveraged a static model to compare the system performance under different sourcing structures. Without considering market frictions and customers' delay sensitivity, they showed that dual sourcing in a monopoly market could benefit the platform and customers but hurt drivers. Both studies treat the staffing cost of fixed agents as an exogenous parameter and thus the contract design problem is not fully addressed. In addition, these two studies implicitly assume that freelancers and contractors are sourced from separate pools and ignore drivers' choices of being a freelancer or contractor. Considering these two key features in contract design, i.e., pricing and drivers' choices, this paper intends to answer the following questions:

1. What is the optimal dual-sourcing contract that a profit-maximizing platform would provide?
2. What are the impacts of dual sourcing on various stakeholders in the ride-sourcing market?

To our best knowledge, this framework is one of the first attempts to consider dual sourcing contract design for the ride-sourcing market. It provides a tractable way to analyze the contract design problem under dual sourcing and examine its potential impacts on various stakeholders. The proposed model not only sheds light on the staffing policy for ride-sourcing services, but can be further customized to analyze other emerging urban mobility services, e.g., autonomous on-demand mobility services and crowd-sourced goods delivery.

The remaining paper is organized as follows. Section 2 presents our base model tailored for a ride-sourcing market where a platform controls the supply of drivers via dual-sourcing contracts. The market equilibrium is investigated and comparative statics are presented. Section 3 applies the equilibrium framework to a special case for which the properties of the optimal contract are examined both analytically and numerically. Section 4 conducts a sensitivity analysis to identify key factors that impact the optimal contract design. The paper concludes with a summary of research findings and avenues for future research.

2. Base model

2.1. Model settings

This section first develops a base model for an isotropic ride-sourcing market, where a monopolistic platform recruits drivers while providing them two service options: they can either work with complete scheduling flexibility and get paid as

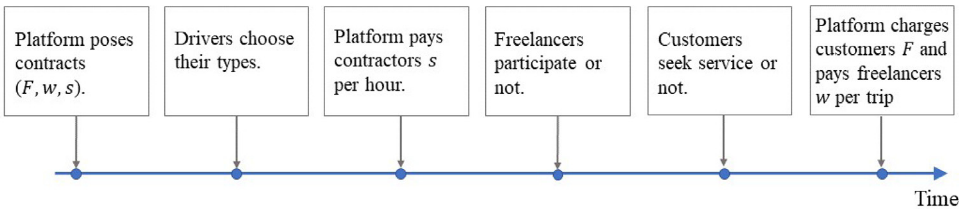


Fig. 1. Timeline of events.

per the number of orders fulfilled, or commit with certain work schedules in exchange for fixed wage rates. For simplicity, we refer drivers who take the first option as freelancers and those with the second option as contractors. Assume the platform has the full bargaining power and makes a take-it-or-leave-it offer to drivers, while drivers make one-shot decisions to become freelancers or contractors based on their preferences. Both types of drivers are insured with equal treatment in the process of customer matching.

We assume that the potential demand profile is common knowledge to the platform and drivers. Denote F as the per-trip fare paid by customers, w as the earnings received by freelancers per trip completed, and s as the fixed hourly wage rate specified for contractors. In this paper, we treat the fare F and earning w as a prior commitment of the platform that does not change with the market realizations. Then, the contract design can be viewed as a leader-follower game, where the platform acts as the leader who releases the contractual specifications while customers and drivers follow with their choices. The events of this game proceed in the following sequence (see Fig. 1 for a graphical presentation):

1. Before the market realizes, the platform releases to drivers the trip fare F, the trip earning w for freelancers, and the wage rate s for contractors. Then, based on their risk attitude and knowledge of the market, drivers decide whether to sign up as contractors.
2. Once the contractual decisions are made, freelancers adapt their work schedule in accordance to the daily realizations of their opportunity costs, while contractors work as per the pre-specified schedule.
3. Customers decide whether to use the ride-sourcing service after they observe the real-time service availability and trip fare. The trip fare F will be charged to customers once their requests get fulfilled.

2.2. Model components

This section details the base model with three interdependent components, including customer demand, contractual choices of drivers, and labor supply of freelancers. A glossary of notations is provided in Table A.3 in Appendix A.

2.2.1. Uncertain demand

Assume the potential demand Q^0 is uncertain and follows a discrete distribution over the set of scenarios $J = \{1, 2, .n\}$. Each scenario $j \in J$ occurs with a probability p_j, with Q^0 being $Q_j^0 > 0$. The realized demand q_j of scenario j is assumed to be a function of the potential demand Q_j^0, trip fare F, customers' average waiting time t_j^c, and in-vehicle time l, written as follows:

$$q_j = Q_j^0 \cdot f_q(F + \alpha \cdot t_j^c + \tau \cdot l), \quad \forall j \in J,$$

where f_q characterizes a decreasing function in total trip cost, i.e., $f_q' < 0$ and satisfies $\lim_{t_j^c \to \infty} f_q = 0$; α and τ denotes customers' value of unit waiting time and in-vehicle time. Note that in-vehicle travel time of riders is assumed to be a constant across scenarios.

As ride-sourcing platforms typically provide quick matching for customers, we consider no waiting time for matching and thus the waiting time t_j^c only consists of the time awaiting pickup (Castillo et al., 2017). The waiting time can then be specified as follows:

$$t_j^c = f_t(N_j^v), \quad \forall j \in J,$$

where N_j^v denotes the total number of idle drivers. Intuitively, the waiting time t_j^c decreases with an increasing number of idle drivers N_j^v, i.e., $f_t' < 0$. We further assume that the marginal return of idle drivers is diminishing, i.e., $f_t'' > 0$.

2.2.2. Contractual choices of drivers

We assume that drivers' contractual choices are ex-ante. More specifically, drivers choose to sign up as a freelancer or contractor according to market conditions before the introduction of dual sourcing. In our model setting, contractors commit to providing services each day and receive a risk-free salary s, while freelancers decide whether to work at a particular day and earn an uncertain income. Let Δs denote the disutility or cost associated with the loss of scheduling flexibility when a driver signing up as a contractor. Assuming each driver is a rational utility maximizer, we obtain the following condition of becoming a contractor:

$$s - \Delta s \geq \mathbb{E}[r_0] + \gamma \cdot \sigma[r_0], \tag{1}$$

where $\mathbb{E}[r_0]$ and $\sigma[r_0]$ respectively represent the mean and standard deviation of freelancers' earnings before the introduction of dual sourcing, and γ is a parameter representing the driver's risk attitude with $\gamma < 0$ being risk-averse and $\gamma > 0$ being risk-seeking. Here, both $\mathbb{E}[r_0]$ and $\sigma[r_0]$ are exogenous parameters. In the above, the left side spells the utility of becoming a contractor while the right side represents the utility of serving as a freelancer, which follows a mean-variance utility approach with $\gamma \cdot \sigma[r_0]$ standing for a risk premium.

Suppose there are in total N_0 drivers in the labor market with N_c of them being contractors. We assume that drivers' risk attitude parameter γ follows a cumulative distribution $G(\cdot)$. Then, applying the above condition (1) over the whole driver population yields the following relationship regarding the number of contractors,

$$N_c = N_0 \cdot G\left(\frac{s - \Delta s - \mathbb{E}[r_0]}{\sigma[r_0]}\right).$$

Without loss of generality, we assume that it is increasingly more expensive to recruit less risk-averse contractors, i.e., $G''(\cdot) \leq 0$. Note that to highlight the impact of risk attitude, we assume all drivers sharing the same inflexibility cost Δs. However, it is mathematically tractable to incorporate a distributional cost into the function above. With N_c, the number of freelancers N_f^0 should satisfy:

$$N_f^0 = N_0 - N_c.$$

2.2.3. Labor supply of freelancers

It is assumed that freelancers are aware of the average market performance, but cannot foresee the exact demand realization and the effective wage on each day before they participate. The freelancers decide whether to provide the service by comparing the expected service revenue $\mathbb{E}[r]$ with their reservation wage, i.e., the lowest wage rate they can accept for working, which is assumed to be a random variable following a concave cumulative distribution function of $T(\cdot)$, i.e., $T''(\cdot) \leq 0$. Then, the number of in-service freelancers N_f can be determined by the following equation:

$$N_f = N_f^0 \cdot T(\mathbb{E}[r]).$$

Note that the above equation implies positive price elasticity of freelancers, an assumption commonly made in the literature (Bai et al., 2019; Cachon et al., 2017; Hu and Zhou, 2019), following the neoclassical theory of labor supply (see, e.g., Farber, 2015). Although negative price elasticity resulting from drivers' income-target behavior was reported for the taxi market (see, e.g., Camerer et al., 1997), relevant empirical results on ride-sourcing drivers, to our knowledge, mostly evidence the neoclassical behavior thus far (Angrist et al., 2017; Chen and Sheldon, 2016; Sheldon, 2016; Sun et al., 2019; Xu et al., 2020a).

Since the matching algorithm will not differentiate freelancers and contractors, the effective wage rate of each in-service freelancer r can be estimated below for a realized demand q:

$$r = \frac{w \cdot q}{N_f + N_c}. \tag{2}$$

Let N be the total number of drivers in service, i.e. $N_c + N_f$. At a steady state, the following fleet conservation condition holds for each scenario $j \in J$:

$$N = N_j^v + q_j t_j^c + q_j l, \quad \forall j \in J,$$

where the terms represent the number of drivers being idle, picking up customers, and delivering them to their final destination respectively.

2.3. Comparative statics analysis

This part proves the existence of market equilibrium and derives comparative statics under market equilibrium to uncover how the platform's decisions impact market equilibrium.

The ride-sourcing market equilibrium under dual sourcing can be described by the following system:

$$\mathbb{E}[r] = w \cdot \mathbb{E}\left[\frac{q}{N}\right] \tag{3}$$

$$N_c = N_0 \cdot G\left(\frac{s - \Delta s - \mathbb{E}[r_0]}{\sigma[r_0]}\right) \tag{4}$$

$$q_j = Q_j^0 \cdot f_q\left(F + \alpha \cdot f_t(N_j^v) + \tau \cdot l\right) \quad \forall j \in J \tag{5}$$

$$N = N_0 \cdot T(\mathbb{E}[r]) + N_c \cdot (1 - T(\mathbb{E}[r])) \tag{6}$$

$$N = N_j^v + q_j l + q_j \cdot f_t(N_j^v) \quad \forall j \in J \tag{7}$$

The above system consists of $2|J| + 3$ equations and $2|J| + 6$ variables with three degrees of freedom. Specifically, with contract variables (F, w, s), the unknowns N, N_c, $\mathbb{E}[r]$ are variables staying the same across all scenarios while q_j, N_j^v are two set of scenario-specific variables. With this system, we obtain the following proposition for the existence of market equilibrium:

Proposition 1 (Existence of market equilibrium)**.** *Given a feasible contract, i.e., trip fare F, per-trip earning for freelancers w, and salary for contractors s, there exists market equilibrium.*

The proof of Proposition 1 is presented in Appendix B, which also includes proofs for other propositions, theorems and lemmas stated later in the paper.

In the above system, Eqs. (3) and (6) couple all scenarios via freelancers' expected wage rate $\mathbb{E}[r]$. Thus, directly treating (F, w, s) as external control variables would result in a "chicken-egg" relationship between the supply N and the expected effective wage $\mathbb{E}[r]$, which obstructs analytical examination of the market properties. To facilitate the analysis and derivation, we temporarily drop Eq. (6) and then treat $(F, N, N_c, \mathbb{E}[r])$ as the decision variables to accommodate the additional degree of freedom released. Meanwhile, to retain the system equilibrium, we specify Eq. (6) later as an auxiliary equality constraint in the contract design optimization problem. In essence, below we analyze the market equilibrium without considering the labor supply behavior of freelancers. With this simplification, the trip fare F and the number of in-service drivers N dictate the market equilibrium. To look into the comparative statics, some derivatives with respect to F and N are presented as follows:

$$\frac{\partial q_j}{\partial F} = \frac{Q_j^0 f_q' \cdot (1 + q_j f_t')}{(1 + q_j f_t') + \alpha Q_j^0 f_q' f_t' \cdot (t_j^c + l)}, \quad \forall j \in J,$$

$$\frac{\partial \mathbb{E}[q]}{\partial F} = \sum_j \left(p_j Q_j^0 f_q' \cdot \frac{\prod_{J/j} \Delta_j}{\prod_J \Delta_j} \right) + \sum_j \left(p_j q_j Q_j^0 f_q' f_t' \cdot \frac{\prod_{J/j} \Delta_j}{\prod_J \Delta_j} \right),$$

$$\frac{\partial N_j^v}{\partial F} = -\frac{Q_j^0 f_q' \cdot (t_j^c + l)}{(1 + q_j f_t') + \alpha Q_j^0 f_q' f_t' \cdot (t_j^c + l)}, \quad \forall j \in J,$$

$$\frac{\partial q_j}{\partial N} = \frac{\alpha Q_j^0 f_q' f_t'}{(1 + q_j f_t') + \alpha Q_j^0 f_q' f_t' \cdot (t_j^c + l)}, \quad \forall j \in J,$$

$$\frac{\partial \mathbb{E}[q]}{\partial N} = \sum_j \left(\alpha p_j Q_j^0 f_q' f_t' \cdot \frac{\prod_{J/j} \Delta_j}{\prod_J \Delta_j} \right),$$

$$\frac{\partial N_j^v}{\partial N} = \frac{1}{(1 + q_j f_t') + \alpha Q_j^0 f_q' f_t' \cdot (t_j^c + l)}, \quad \forall j \in J,$$

where $\Delta_j = (1 + q_j f_t') + \alpha Q_j^0 f_q' f_t' \cdot (t_j^c + l)$ for $j \in J$. Noting that the sign $(1 + q_j f_t')$ is undetermined. When $(1 + q_j f_t' > 0)$, the market operates in an efficient regime and we thus have $\frac{\partial q_j}{\partial F} < 0$, $\frac{\partial \mathbb{E}[q]}{\partial F} < 0$, $\frac{\partial q_j}{\partial N} > 0$, $\frac{\partial \mathbb{E}[q]}{\partial N} > 0$ and $\frac{\partial N_j^v}{\partial N} > 0$. It implies that the realized demand increases with system supply and decrease with trip fare. The increased supply also yields a shorter waiting time for customers since more idle drivers are available. If $(1 + q_j f_t' < 0)$, wild-goose-chases (WGC) arise in the market (Castillo et al., 2017). When the market equilibrium is in this WGC regime, a price reduction or supply increase can yield a lower demand and higher waiting time for customers.

The other two variables, the staffing level of contractors N_c and the expected wage $\mathbb{E}[r]$, are directly related to the contract variables (s, w), with the following derivatives:

$$\frac{ds}{dN_c} = \frac{\sigma[r_0]}{N_0} \cdot \left(G^{-1}\left(\frac{N_c}{N_0}\right) \right)' > 0, \qquad \frac{\partial w}{\partial \mathbb{E}[r]} = \frac{N}{\mathbb{E}[q]} > 0.$$

The second inequality is trivial while the first implies that the salary s increases with number of contractors N_c and a higher variation of the market will magnify the increasing rate.

2.4. Optimal dual sourcing contract

To take the full advantage of dual sourcing, the platform needs to carefully design the contract and redistribute the workforce suitably into freelancers and contractors. This section thus introduces the contract design problem for dual sourcing, where the platform determines both trip fare F, system supply N, the expected effective wage $\mathbb{E}[r]$ and staffing level of contractors N_c to maximize its own expected profit. In formulating the problem, we consider all the other variables are functions of these four decision variables implicitly defined via the market equilibrium system of (3)–(7) except (6), which will be treated as a constraint. The formulation thus reads:

$$\max \quad z(F,\mathbb{E}[r],N,N_c) = F\cdot\mathbb{E}[q] - N_0\cdot T(\mathbb{E}[r])\cdot\mathbb{E}[r] + N_c\cdot(T(\mathbb{E}[r])\cdot\mathbb{E}[r] - s(N_c)) \tag{8}$$
$$\text{s.t.} \quad N = N_0\cdot T(\mathbb{E}[r]) + N_c\cdot(1-T(\mathbb{E}[r]))$$
$$0\le N_c\le N_0, \quad \underline{r}\le\mathbb{E}[r]\le\bar{r}$$

where $\underline{r}$ and $\bar{r}$ specify the minimum and maximum reservation wage of freelancers respectively. Indeed, by setting $N_c=0$, the above problem reduces to the contract design problem with freelancers only. Thus, under the same market condition, the proposed optimal dual sourcing strategy is always no less favorable than the purely self-scheduling counterpart and allows the platform to earn higher or equal profit.

Assuming the inequality constraints are not binding, the optimality conditions of problem (8) yield the following equations:

$$F^* = -\frac{\mathbb{E}[q]^*}{|J|}\cdot\frac{1}{\hat{E}_{f1}\left(p_jQ_j^0f_q'\right)} + \left(\mathbb{E}[r]^* + \frac{T\left(\mathbb{E}[r]^*\right)}{T'\left(\mathbb{E}[r]^*\right)}\right)\cdot\frac{1}{|J|}\cdot\hat{E}_{f2}\left(\frac{t_j^c+l}{p_j}\right), \tag{9}$$

$$\mathbb{E}[r]^* + \frac{T(\mathbb{E}[r]^*)}{T'(\mathbb{E}[r]^*)} = -\mathbb{E}[q]^*\cdot\hat{E}_c\left(\frac{\alpha f_t'}{1+q_jf_t'}\right), \tag{10}$$

$$s^* + N_c^*\cdot\frac{\sigma[r_0]}{N_0}\cdot\left(G^{-1}\left(\frac{N_c^*}{N_0}\right)\right)' = \mathbb{E}[r]^* + \frac{T(\mathbb{E}[r]^*)}{T'(\mathbb{E}[r]^*)} - \frac{T^2(\mathbb{E}[r]^*)}{T'(\mathbb{E}[r]^*)}. \tag{11}$$

where $\hat{E}_{f1}(p_jQ_j^0f_q')$, $\hat{E}_{f2}(\frac{t_j^c+l}{p_j})$ and $\hat{E}_c(\frac{\alpha f_t'}{1+q_jf_t'})$ denote the following weighted average values:

$$\hat{E}_{f1}(p_jQ_j^0f_q') = \frac{\sum_j p_jQ_j^0f_q'\cdot\left((1+q_jf_t')\cdot\prod_{J/j}\Delta_j\right)}{\sum_j\left((1+q_jf_t')\cdot\prod_{J/j}\Delta_j\right)},$$
$$\hat{E}_{f2}\left(\frac{t_j^c+l}{p_j}\right) = \frac{\sum_j(t_j^c+l)\cdot\left(Q_j^0f_q'f_t'\cdot\prod_{J/j}\Delta_j\right)}{\sum_j\left(p_jQ_j^0f_q'f_t'\cdot\prod_{J/j}\Delta_j\right)},$$
$$\hat{E}_c\left(\frac{\alpha f_t'}{1+q_jf_t'}\right) = \frac{\sum_j\frac{\alpha f_t'}{1+q_jf_t'}\cdot\left(p_jQ_j^0f_q'\cdot(1+q_jf_t')\cdot\prod_{J/j}\Delta_j\right)}{\sum_j\left(p_jQ_j^0f_q'\cdot(1+q_jf_t')\cdot\prod_{J/j}\Delta_j\right)}.$$

Eq. (9) specifies the monopoly pricing formula for the ride-sourcing market under demand uncertainty. Indeed, setting $|J|=1$, Eq. (9) reduces to the monopoly pricing for the ride-sourcing market with deterministic demand (Castillo et al., 2017). The price formula (9) maintains the structure of the Lerner formula (Lerner, 1995). The right-hand-side (RHS) of Eq. (9) consists of two terms: the weighted average monopoly markup $-\frac{\mathbb{E}[q]^*}{|J|}\cdot\frac{1}{\hat{E}_{f1}(p_jQ_j^0f_q')}$ and the weighted marginal cost for recruiting a driver, i.e., $\left(\mathbb{E}[r]^*+\frac{T(\mathbb{E}[r]^*)}{T'(\mathbb{E}[r]^*)}\right)\frac{1}{|J|}\cdot\hat{E}_{f2}(\frac{t_j^c+l}{p_j})$.

The left-hand-side (LHS) of Eq. (10) represents the marginal cost of attracting an additional freelancer, i.e., $\mathbb{E}[r]^*+\frac{T(\mathbb{E}[r]^*)}{T'(\mathbb{E}[r]^*)}$ while the RHS refers to the marginal benefit of customers, i.e., $-\mathbb{E}[q]^*\cdot\hat{E}_c(\frac{\alpha f_t'}{1+q_jf_t'})$.

Eq. (11) clarifies the relationship of marginal recruiting costs of two labor sources at optimality. The LHS of Eq. (11) represents the marginal cost of recruiting one additional contractor, which depends on drivers' risk attitude distribution $G(\cdot)$ and increases with standard deviation $\sigma[r_0]$. The RHS of Eq. (11) describes the marginal cost of recruiting a freelancer with an addition term $-\frac{T^2(\mathbb{E}[r]^*)}{T'(\mathbb{E}[r]^*)}$. This term sheds light on the 'pooling-choice' effect under dual sourcing, as we consider contractors and freelancers are from the same driver pool and form two labor sources via their contractual choice. Thus, transforming a driver into a contractor will impact the size of freelancers, thereby impacting the marginal costs of two labor sources simultaneously. The term $-\frac{T^2(\mathbb{E}[r]^*)}{T'(\mathbb{E}[r]^*)}$ captures such a connection. If freelancers and contractors source from separate pools (Zhong et al., 2019; Dong and Ibrahim, 2020), this 'pooling-choice' term is absent and marginal costs of two labor sources will be equal at optimality. Therefore, the separate pooling assumption will overestimate the salary for contractors for the same level of expected wage $\mathbb{E}[r]$.

We also obtain the following necessary boundary conditions for the platform's labor sourcing choice:

- Contract with freelancers only ($N_c^*=0, N^*\in(0,N_0]$):

$$\Delta s + \mathbb{E}[r_0] + \sigma[r_0]\cdot G^{-1}(0) \ge \mathbb{E}[r]^* + \frac{N^*}{N_0}\cdot\frac{1}{T'(\mathbb{E}[r]^*)} - \left(\frac{N^*}{N_0}\right)^2\cdot\frac{1}{T'(\mathbb{E}[r]^*)}, \tag{12}$$

- Contract with contractors only ($N_c^* = N^*, N^* \in (0, N_0)$):

$$\underline{r} \geq \Delta s + \mathbb{E}[r_0] + \sigma[r_0] \cdot G^{-1}\left(\frac{N_c^*}{N_0}\right) + \sigma[r_0] \cdot \frac{N_c^*}{N_0} \cdot \left(G^{-1}\left(\frac{N_c^*}{N_0}\right)\right)', \tag{13}$$

- Dual sourcing contract ($0 < N_c^* < N^*, N^* \in (0, N_0)$): Eq. (11).

Let $b_g = \Delta s + \mathbb{E}[r_0] + \sigma[r_0] \cdot G^{-1}(0)$ as the minimum salary for contractors, i.e, the salary that the most risk-averse driver can accept. In both (12) and (13), the LHS specifies the minimum wage of one labor source while the RHS describes the marginal cost of the other labor source considering the 'pooling-choice' effect. Thus, the above conditions suggest that the platform should adopt dual sourcing contracts if the minimum wage of one labor source is less than the marginal cost of the other labor source with the 'pooling-choice' effect taken into consideration.

Lastly, it is trivial to point out that the optimal dual sourcing contract will depend on the market conditions and drivers' characteristics. The former is characterized by the distribution of market potential demand and profitability, i.e., Q_j^0, $\mathbb{E}[r_0]$ and $\sigma[r_0]$ while the latter includes the minimum wage of each labor source, freelancers' reservation wage distribution $T(\cdot)$, drivers' risk attitude distribution $G(\cdot)$, and the inflexibility cost Δs.

2.5. Reformulation of contract design problem

The above analytical insights derived from the first-order optimality conditions are rather limited due to the complexity of the functional forms of F^* and $\mathbb{E}[r]^*$. In this section, we transform the contract design problem into a one-dimensional problem, which enables us to reveal more properties of the optimal contract.

In doing so, notice that the realized demand q_j can be determined if the trip fare F and the number of in-service drivers N are given (see the proof of Proposition 1 for details). Thus, the revenue of the platform can be specified for given F and N. On the supply side, we express the expected wage $\mathbb{E}[r]$ as a function of N and N_c and relates the labor cost to the platform's labor sourcing choice. Thus, treating F, N, N_c as decision variables, we reformulate the contract design problem as

$$\begin{aligned} \max \quad & z(F, N, N_c) = R(F, N) - C(N, N_c), \\ \text{s.t.} \quad & 0 \leq N_c \leq N, \\ & 0 \leq N \leq N_0. \end{aligned} \tag{14}$$

where R and C denote the platform's revenue and staffing cost, respectively. For given probability, we then write the platform's expected revenue as:

$$R(F, N) = F \cdot \sum_j p_j \cdot q_j(F, N).$$

Given the total supply N, we maximize the platform's revenue $R(F, N)$ by optimizing the trip fare F. For brevity, we refer $\max_F R(F, N)$ to $R_p(N)$ parameterized by N.

In the analysis of system supply, the staffing cost is given by

$$C(N, N_c) = \begin{cases} (N - N_c) \cdot T^{-1}\left(\frac{N-N_c}{N_0-N_c}\right) + s(N_c) \cdot N_c & \text{if } N_c < N_0, \\ s(N_c) \cdot N_c & \text{Otherwise.} \end{cases}.$$

For a given system supply N, $C(N, N_c)$ is strictly convex on $N_c < N_0$ (see the proof of Proposition 2). Similarly, denote $C_s(N) = \min_{N_c} C(N, N_c)$.

Subsequently, the contract design problem can be converted into the following one-dimensional staffing problem

$$N^* = \arg\max_N \{R_p(N) - C_s(N) | 0 \leq N \leq N_0\}. \tag{15}$$

The optimal contract can be determined via a single-dimensional line search of N^*. The contract variables, i.e., the salary for contract drivers s^*, optimal trip fare F^*, and per-trip earning for freelancers w^* can then be determined.

With the above formulation, we derive the following properties of the dual-sourcing contract design problem:

Proposition 2 (Relationship between staffing cost and supply)**.** *The staffing cost $C_s(N)$ is a convex increasing function regardless of the staffing strategy. For the same supply level, the staffing cost under the optimal dual sourcing is no greater than that under its self-scheduling counterpart.*

The comparison of labor cost in Proposition 2 is straightforward, as the self-scheduling contract characterizes a special case of dual sourcing. Proposition 2 suggests that to sustain the same supply level, the platform can reduce labor cost by implementing the optimal dual-sourcing contracts. The following proposition further points out that such a contract will maintain or increase the number of drivers in service.

Proposition 3. *Optimal dual-sourcing contracts either maintain or increase the number of drivers in service, as compared with the self-scheduling counterpart.*

The above two propositions imply that dual sourcing enables the platform to increase its labor supply without necessarily increasing the staffing cost.

3. Impacts of optimal contracts

To generate more managerial insights, this section considers a specific, stylized instance of the optimal contract design problem. We will first specify the model and then analyze the impacts of optimal dual-sourcing contracts on various stakeholders in the market.

3.1. Model specification

We consider a case where the potential demand Q^0 is either low or high, i.e., $J = \{L, H\}$, with a given probability of occurrence p_j. To facilitate the analysis of dual-sourcing contracts, we further pose the following specifications and assumptions for the proposed base model.

- Specification 1: The realized demand linearly decreases with the full trip cost as described below:

$$q_j = Q_j^0 \cdot \left(1 - k \cdot (F + \alpha \cdot t_j^c + \tau \cdot l)\right), \quad \forall j \in J, \tag{16}$$

  where parameter k measures customers' sensitivity to the full trip cost $(F + \alpha \cdot t_j^c + \tau \cdot l)$ and $F + \alpha \cdot t_j^c + \tau \cdot l \leq 1/k$. Since the in-vehicle time stays the same across all scenarios, we set $\tau = 0$ for analysis simplification.
- Specification 2: The average waiting time of customers in a given scenario is specified as follows (Daganzo, 1978):

$$t_j^c = f_t(N_j^v) = \frac{\beta}{(N_j^v)^\eta}, \quad \forall j \in J, \tag{17}$$

  where β and η are positive parameters. Without the loss of generality, we set $\eta = 1$ for simplicity.
- Specification 3: The reservation wages uniformly distribute between 0 and a constant $\bar{r}$. Define $k_t = 1/\bar{r}$, which represents the price sensitivity of freelancers. The number of in-service freelancers is then specified as

$$N_f = N_f^0 \cdot \min\{k_t \cdot \mathbb{E}[r], 1\}. \tag{18}$$

- Specification 4: Drivers' risk attitude parameter γ uniformly distributes over $[\underline{\gamma}, \bar{\gamma}]$. Then, the percentage of drivers joining as contractors $G(\cdot)$ can be expressed to be the following function:

$$G\left(\frac{s - \Delta s - \mathbb{E}[r_0]}{\sigma[r_0]}\right) = \frac{1}{\bar{\gamma} - \underline{\gamma}}\left(\frac{s - \Delta s - \mathbb{E}[r_0]}{\sigma[r_0]} - \underline{\gamma}\right),$$

  which is equivalent to the following linear inverse supply function for N_c:

$$s = k_g N_c + b_g, \tag{19}$$

  where

$$k_g = \frac{\bar{\gamma} - \underline{\gamma}}{N_0}\sigma[r_0], \quad b_g = \mathbb{E}[r_0] + \underline{\gamma}\sigma[r_0] + \Delta s$$

  The parameter b_g is the aforementioned minimum salary for contractors.
- Assumption 1: We consider $r = w \cdot q/N + \epsilon$, where ϵ is a random variable introduced to smooth the distribution of freelancers' effective wage. The term ϵ can be interpreted as a random income due to unanticipated effects, e.g., tips for drivers. We assume that ϵ is independent of market conditions and the platform's decision, i.e., $\mathbb{E}[\epsilon] = 0$ with $\sigma[\epsilon] > 0$.
- Assumption 2: The ride-sourcing market operates in the non-WGC regime, considering that the platform could leverage pricing and matching to promote efficient operations (Castillo et al., 2017; Xu et al., 2020b).

With the above specifications and assumptions, Lemma 1 describes the freelancers' effective wage changes with respect to the supply condition:

Lemma 1 (Effective wage with supply)**.** *Given the trip fare F and trip earning w, for each demand scenario, the effective wage of freelancers first increases then decreases with the total number of drivers in-service N in the non-WGC regime.*

The above suggests that given pricing decision (F, w), the impact of system supply on freelancers' wage rate is indeterminate. Recall that dual sourcing affects system equilibrium by directly impacting system supply N. Therefore, Lemma 1 and Proposition 3 together suggest that dual sourcing has an indeterminate impact on freelancers' expected effective wage.

3.2. Impacts of optimal contracts

Based on the one-dimensional optimization problem in Section 2.5, we analyze the impacts of optimal dual-sourcing contracts and identify the prerequisite market conditions for practicing them.

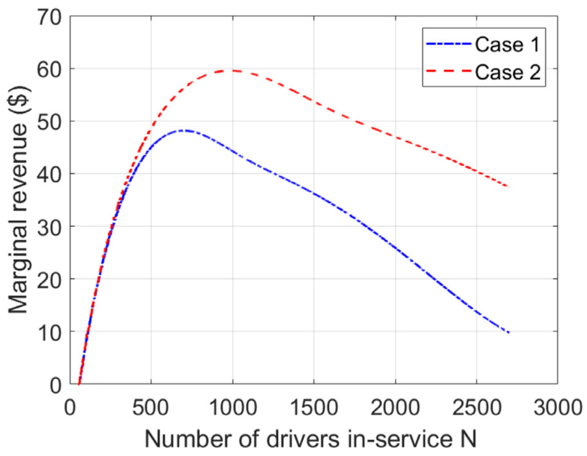


Fig. 2. Ride-sourcing platform's marginal revenue with dual sourcing .

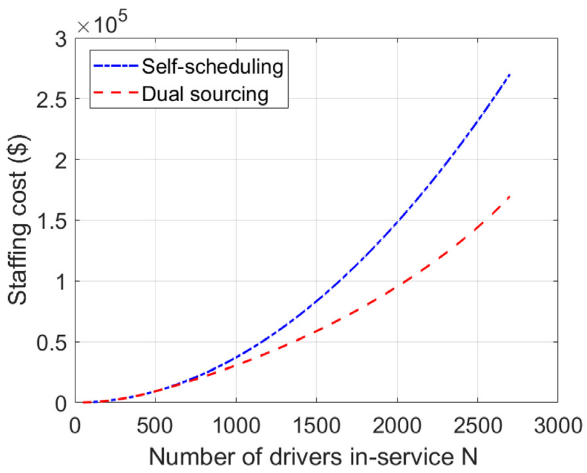


Fig. 3. Ride-sourcing platform's staffing cost with dual sourcing ($k_t = 0.01,\ \sigma[r_0] = 2,\ \mathbb{E}[r_0] = 30$).

3.2.1. Platform's revenue and cost

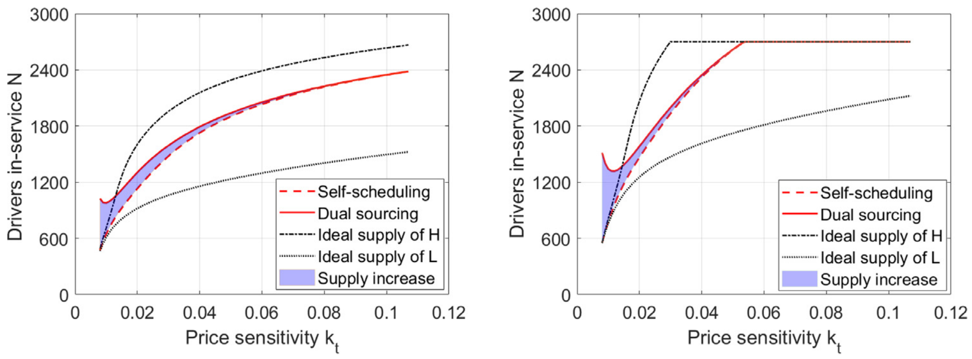
The following theorem details the properties of the platform's revenue with dual-sourcing contracts.

Theorem 1 (Marginal revenue of the platform)**.** *In the non-WGC regime, there exists two threshold values $\underline{N}$ and $\overline{N}$. When the supply is less than $\underline{N}$, the marginal revenue increases with system supply. When the supply is greater than $\overline{N}$, the marginal revenue decreases with system supply.*

Note that the two thresholds $\underline{N}$ and $\overline{N}$ often overlap (e.g., see Fig. 2). According to Theorem 1, there are at least two regimes in the platform's expected revenue $R_p(N)$. In the former regime with supply shortage and long time of waiting for customers, enlarging the supply will significantly reduce the customers' waiting time while considerably boosting the revenue of the platform. For the latter regime with adequate supply, additional supply becomes marginal in improving the service level but intensify the competition among drivers for customers. This two-regime property was also discovered by Benjaafar et al. (2018) with deterministic demand setting, and is consistent with Lemma 1.

To highlight the impacts of different demand-supply levels on optimal contracts, we discuss two distinct cases of potential demand through a numerical experiment. Case 1 characterizes low demand scenarios, where the potential demands Q_L^0 and Q_H^0 are relatively small, while Case 2 features high demand scenarios, where the potential demands are higher. The parameters of the numerical experiment are summarized in Table C.4 in Appendix C. Unless specified otherwise, the figures presented later in this section are all based on these parameter settings.

Fig. 2 presents how the platform's marginal revenue changes with the total supply in the two cases. It verifies the two different market regimes as in Theorem 1. Clearly, the marginal revenue increases for a wider range of supply in Case 2, where each additional driver brings higher value to the platform.



(a) Total supply change in Case 1 (b) Total supply change in Case 2

Fig. 4. System supply under self-scheduling and dual sourcing.

Fig. 3 exemplifies the staffing cost under optimal self-scheduling and dual-sourcing contracts for a given supply. The staffing cost convexly increases with the system supply under both staffing strategies and is lower under dual sourcing, which is consistent with Proposition 2 in Section 2.5.

3.2.2. Impact on system supply

Recall that Proposition 3 suggests that the optimal dual sourcing contracts either maintain or increase the number of drivers in service. Fig. 4 provides a verification of this proposition. The shaded areas in Fig. 4 represent the supply increase resulting from dual sourcing. Fig. 4a describes the case when the service demand is relatively low. In this case, optimal dual sourcing contracts strictly increase the system supply. In contrast, when the demand becomes high as the case shown by Fig. 4b, there appear situations that all drivers participate in service. In those cases, the system supply stays the same regardless of the staffing strategy.

In the figure, the "ideal" supply of H (or L) represents the optimal system supply under self-scheduling when the scenario H (or L) is realized. The difference between the ideal supply and realized supply indicates the supply shortage/excess resulting from drivers' imperfect information on market conditions. As we can see from the figures, dual sourcing could alleviate the supply shortage caused by drivers' imperfect information when the service demand is relatively high. The dual sourcing strategy is more effective for supply management when freelancers feature relatively low price sensitivity. In this situation, more supply is made available by dual sourcing in response to the high potential demand. With higher price sensitivity, freelancers are more responsive and thus become more reliable to the service, thereby diminishing the value of dual sourcing in supply enhancement.

3.2.3. Prerequisite market condition for dual sourcing

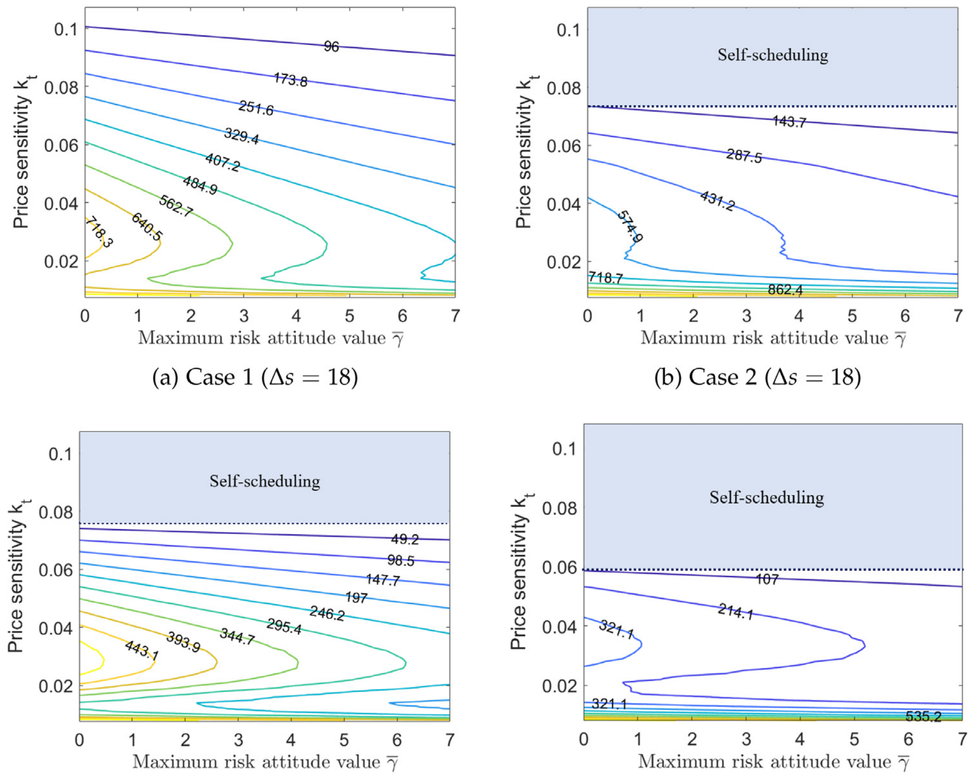
To assist with the platform's staffing decision, it is crucial to know when dual sourcing would be a more preferable choice. The following theorem specifies the condition.

Theorem 2 (Prerequisite condition for dual sourcing)**.** *When freelancers' price sensitivity k_t is greater than $\frac{2\bar{N}}{N_0} \cdot \left(\frac{dR_p(\bar{N})}{dN}\right)^{-1}$, the dual sourcing strategy increases the platform's expected profit only if the minimum salary for contractors is less than $\mathbb{E}[r_0] \cdot (2 - k_t \mathbb{E}[r_0])$.*

Theorem 2 points out that to the platform dual sourcing is not always better than self-scheduling amid the uncertain market, which is consistent with the boundary condition (12). Under the condition that even the most risk-averse drivers require high salaries to get contracted, the platform in fear of the unbearable labor cost will abandon driver contraction and recruit only self-scheduling freelancers. However, as per Theorem 2, this can happen only when the freelancers' price sensitivity is high enough (mathematically, when $k_t \geq \frac{2\bar{N}}{N_0}(\frac{dR_p(\bar{N})}{dN})^{-1}$. When drivers are insensitive to the price, dual sourcing could still be workable even if the minimum salary for contractors is set higher than $\mathbb{E}[r_0] \cdot (2 - k_t \mathbb{E}[r_0])$. From Theorem 2, we can also see that the prerequisite condition does not depend on the heterogeneity degree of drivers' risk attitude k_g.

In numerical experiments, we take the staffing level of contractors as an indicator of the superiority of dual sourcing. The self-scheduling strategy should be the better choice if there are no drivers participating as contractors, i.e., $N_c^* = 0$. We first focus on the minimum salary b_g and drivers' risk attitude k_g by varying Δs and $\bar{\gamma}$. For demonstration purposes, this section selects two typical values of Δs and presents in Fig. 5 the optimal number of contractors for different combinations of k_t and $\bar{\gamma}$. Results corresponding to more comprehensive parametric settings are left to the sensitivity analysis in Section 4.

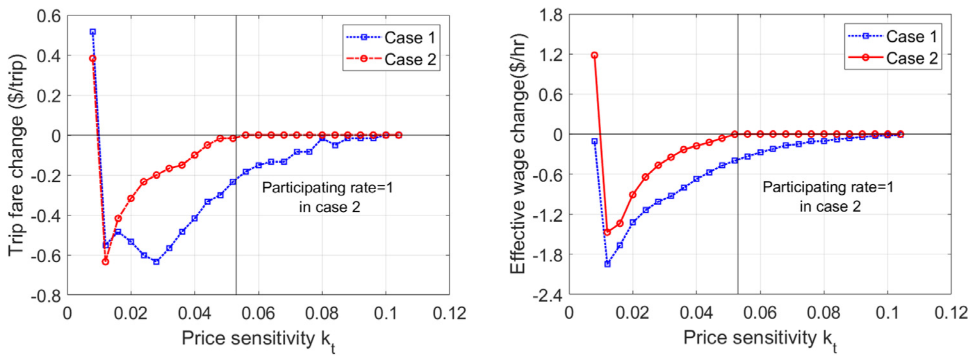
The results adhere well to Theorem 2. For both Case 1 and 2, the region where dual sourcing dominates shrinks as $b_g(\Delta s)$ increases but remains insensitive to $k_g(\bar{\gamma})$. Although the prerequisite condition for dual sourcing is independent of heterogeneity degree of drivers' risk attitude, the value of k_g does impact the staffing level of contractors. Note that a



(a) Case 1 ($\Delta s = 18$) (b) Case 2 ($\Delta s = 18$)

(c) Case 1 ($\Delta s = 23$) (d) Case 2 ($\Delta s = 23$)

Fig. 5. Staffing level of contractors.



(a) Trip fare change (b) Expected effective wage change

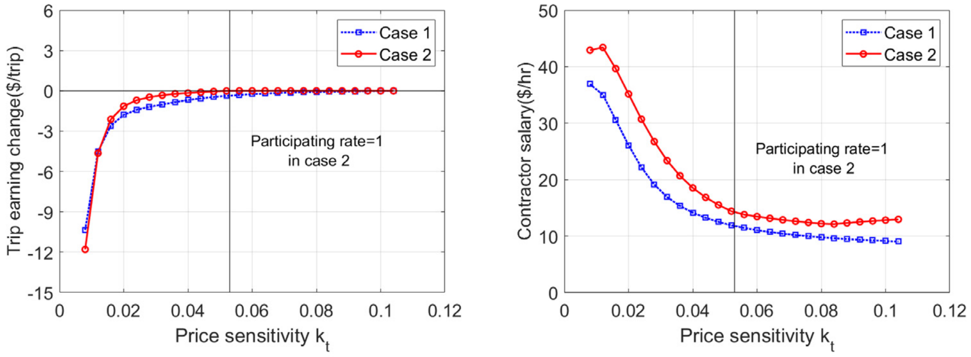
Fig. 6. Changes in trip fare and effective wage.

larger $\bar{\gamma}$ means a smaller proportion of risk-averse drivers. With the fixed price sensitivity of freelancers, the number of contractors decreases with $\bar{\gamma}$ in both cases. Fig. 5 suggests that the platform should recruit more contractors given more risk-averse drivers. To conclude, the dual sourcing contract does allow the platform to capitalize on drivers' risk-taking attitude.

Further, the numerical experiments indicate that the relationship between the number of contractors and the freelancers' sensitivity could be non-monotonic. Taking Case 1 as an example, the staffing level of contractors first increases then decreases with k_t (see Fig. 5a and c). It suggests that, under certain circumstances, the platform may prefer fewer contractors even when freelancers are less reliable.

3.2.4. Impact on the two-sided pricing decision

In the two-sided ride-sourcing market, effective supply management inevitably involves pricing for customers. Fig. 6a delineates the relationships between changes in optimal trip fare and staffing strategies. One interesting observation is that,



(a) Trip earning change (b) Salary for contractors

Fig. 7. Changes in trip earning and contractor salary.

yielding an increase in supply, optimal dual sourcing contracts do not necessarily reduce the trip fare for customers. This is different from previous studies showing that a lower trip fare is expected after dual sourcing or increasing supply when there is a shortage of supply (Zhong et al., 2019; Hu and Zhou, 2019). The market friction and customers' delay sensitivity play a critical role in this interplay. For maintaining the same level of realized demand, the platform may rather reduce the matching delay for customers while keeping the same price level. In the figure, "Participating rate=1" denotes the situation where all drivers stay online and provide trip services, i.e., $N^* = N_0$.

A natural concern about dual sourcing is that the introduction of contractors may damage the profitability of freelancers in the ride-sourcing market. The following proposition examines the impact of dual sourcing on freelancers:

Proposition 4 (Impact on effective wage)**.** *When the price sensitivity of freelancers k_t is greater than $\frac{2\bar{N}}{N_0} \cdot \left(\frac{dR_p(\bar{N})}{dN}\right)^{-1}$, optimal dual sourcing reduces freelancers' expected effective wage. Otherwise, the introduction of dual sourcing will not necessarily harm freelancers' expected effective wage.*

Proposition 4 suggests that freelancers could also benefit from the optimal dual sourcing strategy. When the scheduling decisions of freelancers are insensitive to their wage, the freelance labor supply becomes unresponsive to the market variations. Contractors complement freelancers to sustain the service provision in this situation, under which freelancers earn higher effective wages. For the opposite condition when the price sensitivity of freelancers is large, introducing dual sourcing intensifies the competition among drivers. However, when freelancers' price sensitivity stays sufficiently high, the wage reduction due to the adoption of dual sourcing becomes marginal.

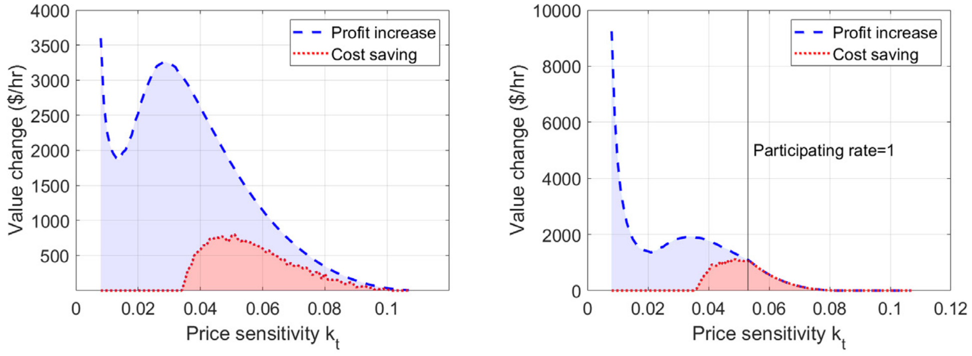
Our numerical results support the above proposition. Fig. 6b illustrates the changes of freelancers' expected effective wages with respect to the freelancers' price sensitivity under dual sourcing. We observed that dual sourcing can increase freelancers' expected effective wage when they are insensitive to the price. High level of service demand are more likely to render freelancers benefit from dual-sourcing when they are insensitive to price and suffer less when they are price sensitive.

We also examine freelancers' per-trip earning and contractors' salary under dual sourcing. First, Fig. 7a shows that optimal dual sourcing contracts lower the per-trip earning for freelancers, although they could receive higher expected effective wage. Second, dual sourcing contracts raise the salary for contractors when the potential demand is high in Fig. 7b. More reliable freelancers lessen the salary paid to contractors.

3.2.5. Impact on profit and welfare

Impact on the profit of the platform The ride-sourcing platform benefits from dual sourcing in two way: increasing the expected revenue and reducing the staffing cost. The expected revenue increases mainly because a higher realized demand from the increased supply, as the reason mentioned in Section 3.2.1. The labor cost saving stems from the lower marginal staffing cost. The following analysis confirms the platform's motivation via numerical experiments in Fig. 8.

The dominating factor for profit growth varies with drivers' price sensitivity k_t. When drivers are insensitive to price, the complement effect between workforce dominates. The increased supply resulting from dual sourcing boosts the platforms' revenue considerably. Fig. 8a shows how much the incremental revenue contributes to the platform's profit. When drivers are high price-sensitive, drivers' competition brings down the marginal revenue generated by the blended workforce. The labor cost-saving becomes the dominating factor by attracting risk-averse contractors. Fig. 8b shows that labor saving is the only source of the profit increase for high potential demand and drivers' sensitivity to price. Besides, we observe that dual sourcing can save labor costs even when all drivers participate in service (see Fig. 8b, Case 2). Risk-averse drivers are reluctant to undertake the consequences of market uncertainty and would be willing to be contracted for a reduced guaranteed salary. This enables the platform to sustain the labor supply while lowering the staffing costs.



(a) Case 1 (b) Case 2

Fig. 8. Profit change and labor cost saving.

Table 1
Change of the platform profit.

Price sensitivity k_t	0.02	0.03	0.04	0.05	0.06	0.07
Profit increase Δz(C1,%)	14.05	11.44	7.23	4.27	2.44	1.31
Profit increase Δz(C2,%)	4.96	4.05	2.97	1.77	0.72	0.18

Notes: 'C1' means Case 1; 'C2' means Case 2.

We evaluate the profit margin that the dual sourcing contract contributes to. To warrant an enlightening evaluation, the profit margins are compared across different levels of price sensitivity sampled from its prevailing value range. According to the recent statistics, over 97% of the US Uber drivers received less than $30 per hour (Campbell, 2020) and only very few drivers could earn more than $40 per hour (Iqbal, 2020). This suggests a rough price sensitivity range of (0.025,0.033) in the US market. The comparison in Table 1 draws over a slightly wider range of (0.02,0.07). In general, the result confirms the huge potential of dual sourcing for ride-sourcing platforms, as they can possibly enjoy more than 10% profit boosts under the prevailing price sensitivity of freelancers. But the profit margins decline significantly as the price sensitivity rises. The sharp decline alerts ride-sourcing platforms of the necessity to carefully measure the freelancers' price sensitivity before implementing the dual sourcing strategy.

Impact on the welfare of customers and drivers Since the dual sourcing strategy can potentially increase the platform's overall profit, one may be concerned about whether it actually exploits customers or drivers. The following numerical analyses show the change of customers' and drivers' welfare incurred by dual sourcing.

The welfare of customers and drivers are measured by the customers' surplus and the drivers' surplus defined as follows:

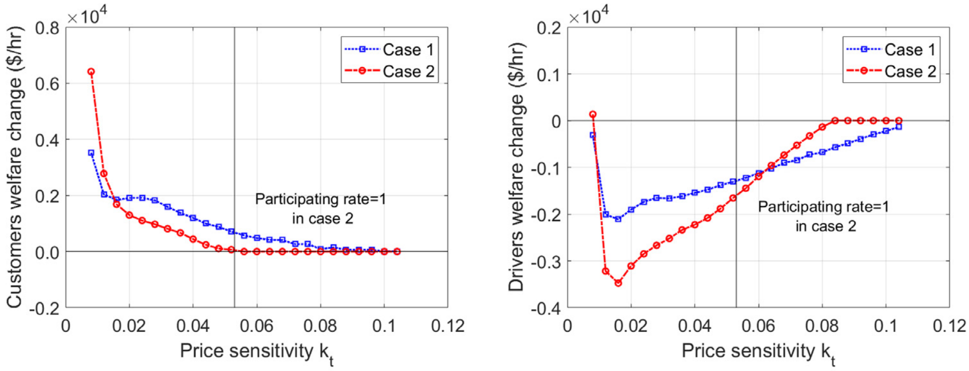
$$C_w = \sum_{j\in J} p_j \cdot \left(\int_0^{q_j^*} f_q^{-1}(x)\mathrm{d}x - q_j^* \cdot (F^* + \alpha \cdot t_j^{c*} + \tau \cdot l) \right), \tag{20}$$

$$P_w = N_f^* \cdot (\mathbb{E}[r]^*) - \int_0^{N_f^*} T^{-1}\left(\frac{x}{N_f^{0*}}\right)\mathrm{d}x + N_c^* \cdot (s^* - \Delta s - \mathbb{E}[r_0]) - \sigma[r_0] \cdot \int_0^{N_c^*} G^{-1}\left(\frac{x}{N_0}\right)\mathrm{d}x,$$

where the consumers' surplus is equal to their willingness to pay for the service subtracting the costs they actually paid across different scenarios; for the drivers' surplus, the first two terms and last two terms represent drivers' surplus as freelancers and contractors respectively.

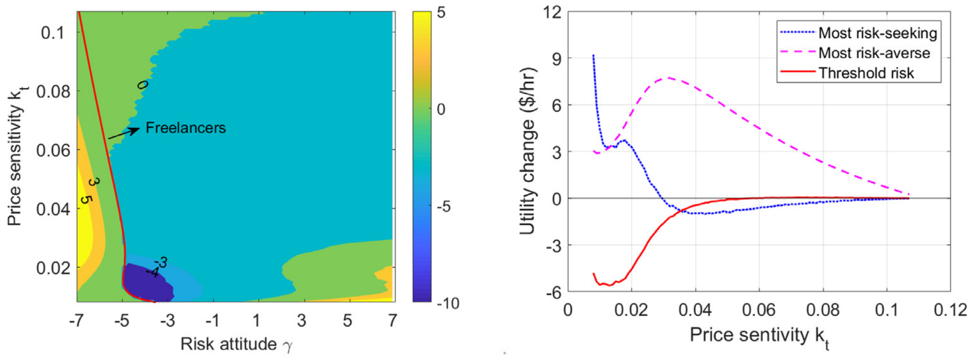
Fig. 9 shows how the customers' and drivers' welfare change incurred by dual sourcing as compared with self-scheduling. We observe that customers consistently enjoy higher welfare under the optimal dual sourcing than that under self-scheduling. Although we hypothesize that in the non-WGC regime, the welfare of customers under the optimal dual sourcing contract is no less than that under self-scheduling, its theoretical proof remains an open question. On the other hand, the impacts of dual sourcing on drivers' welfare are mixed. With high level of demand, drivers' welfare increases when freelancers feature small price sensitivity (Case 2). The increased staffing level of drivers and expected effective wage under dual sourcing are the driving forces for this positive change. With increasing price sensitivity of freelancers, drivers as a group experience lower welfare in both numerical cases. Specifically, within the prevailing price sensitivity range, i.e., $k_t \in (0.025, 0.033)$, the ride-sourcing market realizes in the competition regime, where customers' welfare rises while drivers' drops with freelancers' price sensitivity.

Distributional impact on drivers Figs. 10 and 11 show the numerical results on how dual sourcing impacts drivers with different risk attitude for the two cases in the parametric space (k_t, γ). The red line represents the critical line that separates contractors and freelancers.



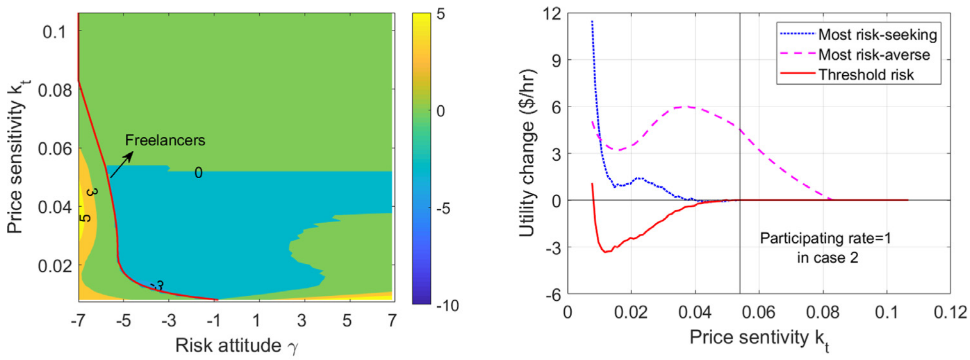
(a) Changes in welfare of customers (b) Changes in welfare of drivers

Fig. 9. Changes in welfare.



(a) Individual driver utility change (b) Specific driver utility change

Fig. 10. Changes in driver utility of Case 1.

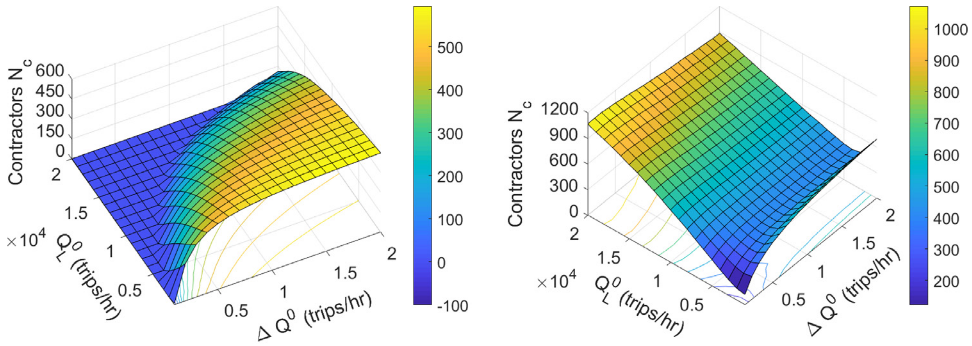


(a) Individual driver utility change (b) Specific driver utility change

Fig. 11. Changes in driver utility of Case 2.

Compared with purely self-scheduling contract, dual sourcing contracts benefit both the highly risk-averse and risk-seeking drivers when freelancers feature small price sensitivity. Those risk-averse drivers would sign up as contractors to hedge against the risk disutility. They obtain an improvement in their utility for their contractual choices. Those highly risk-seeking drivers remain as freelancers. Their utility goes up since dual sourcing contracts magnify the variance of market profitability and even increase their expected effective wage (Case 2). A low potential demand will make freelancers suffer from more utility loss after dual sourcing, compared with high potential demand. This effect is more significant for those who are mildly risk-averse (see Figs. 10a and 11a).

In addition, we observe drivers' losses because of making ex-ante contractual choices as assumed in our model. For example, although freelancers who are indifferent in signing up as either type of drivers expect the same level of utility as



(a) The number of contractors ($k_t = 0.04$) (b) The number of contractors ($k_t = 0.012$)

Fig. 12. Platform staffing decision.

before under dual sourcing, they actually see a decrease in their utility (Fig. 10b). However, a high potential demand can offset the negative impact from short-sighted choices (Fig. 11b).

In summary, under dual sourcing, the ride-sourcing market could operate in two possible regimes, the complement regime and competition regime. Drivers exert positive effect on each other in the former while competing for customers with each other in the latter. In both regimes, the optimal dual sourcing will maintain or enhance the system supply. The changes in optimal pricing decisions and welfare under the optimal dual sourcing strategy depend on which market regimes the market belongs to.

4. Sensitivity analysis

We carry out sensitivity analyses of the platform's optimal dual-sourcing staffing and pricing decisions with respect to the market demand and drivers' characteristics.

4.1. Demand uncertainty

Demand uncertainty is the intrinsic reason for adopting dual sourcing contracts. Therefore, we study how its characteristics affects the platform's staffing decision. The setting of this numerical analysis is as follows. On the demand side, we define $\Delta Q^0 = Q_H^0 - Q_L^0$ as a measure for the level of demand uncertainty. More specifically, with the probabilities of occurrence fixed, the demand uncertainty increases with ΔQ^0. On the supply side, we test two values of freelancers' price sensitivity ($k_t = 0.04$ and $k_t = 0.012$) that reflect two market regimes discussed above. The former value of k_t replicates the competition market regime while the latter is set for the complement regime. Other parameters remain the same as those in Table C.4 in Appendix C. Fig. 12 shows the numerical results over $(Q_L^0, \Delta Q^0)$.

When the competition effect dominates (Fig. 12a), we observe that the number of contractors increases with demand uncertainty and decreases with potential demand. Given potential demand Q_L^0, higher uncertainty of demand forces contractors to accept a low salary for their revenue security. In response, the platform employs a higher level of contractors for labor cost reduction. With fixed demand uncertainty, the market stays much stable and the labor cost advantage from drivers' risk attitude is diminishing. With relative low marginal revenue under competition effect, platform hire fewer contractors even when the total potential demand is high. In sum, using the staffing level of contractors as an indicator, dual sourcing contracts are better than its self-scheduling counterpart for low potential demand and high demand uncertainty.

When the complement effect dominates (Fig. 12b), the analysis is more complex as the number of contractors is not monotonic with potential demand. Given demand uncertainty ΔQ^0, the number of contractors first decreases and then increases with Q_L^0. The trade-off of labor cost and marginal revenue contributes to this non-monitonicity. Since freelancers are insensitive to price, the platform can expect a considerable increase in its total revenue from dual sourcing. The incremental revenue dominates the additional labor cost with larger potential demand. Hence,the platform hire more contractors for profit when potential demand is high.

In sum, depending on drivers' price sensitivity, the extraneous demand uncertainty has different impact on the staffing level of contractors. When freelancers are price sensitive, the optimal number of contractors decreases with the potential demand; when freelancers are very insensitive to price, the platform should hire more contractors when the potential demand is extremely high.

4.2. Supply price sensitivity and value of flexibility

On the supply side, the sensitivity analysis focuses on how the platform's pricing and contractual choices change regarding drivers' characteristics, respectively. The drivers' characteristics are captured by drivers' price sensitivity k_t, value of flexibility Δs, and their risk attitude γ.

Table 2
Characteristics of different parameter regions.

Region	Dual sourcing	Effective wage	trip fare	Participation rate
R1	no	_	_	≤ 1
R2	yes	↑	↑	< 1
R3	yes	↑	↓	< 1
R4	yes	↓	↓	< 1
R5	yes	_	_	$= 1$

Notes: '↑' means increase; '↓' means decrease; ' - ' means no changes.

The setting of the numerical experiments is as follows. Varying $\underline{\gamma}$ and $\overline{\gamma}$, we change drivers' risk attitude distribution for γ. A smaller value of $\underline{\gamma}$ represents a higher degree of risk aversion. We categorize the degree of drivers' risk aversion to four conditions: "all risk-averse", "high risk-averse", "low risk-averse", and "all risk-seeking ". All drivers are risk-averse in the first condition and risk-seeking in the fourth condition. The second and the third condition differ in proportion of risk-averse drivers and risk-aversion-degree. Drivers are more risk-averse in the second than in the third condition. We compute the optimal solutions for different combinations of $(k_t, \Delta s)$ in each condition and compare the results in Case 2 for demonstration purposes. Other parameters remain the same as those in Table C.4 in Appendix C.

In terms of the platform's pricing sensitivity and contractual choice, five regions of $(k_t, \Delta s)$ are identified. Table 2 summarizes the characteristics of different regions:

- R1: Purely self-scheduling. R1 features relatively high value of flexibility and price sensitivity. High labor costs will be expected by recruiting contractors.
- R2: Dual sourcing. R2 is characterized by drivers' price insensitivity. This region represents a win-win situation where freelancers see a higher expected wage and the platform gain higher expected profit compared with that under purely self-scheduling. However, customers are charged with higher trip fares. In this region, complement effect dominates and freelancers benefit from contractors' high value of flexibility.
- R3: Dual sourcing. A win-win-win situation is achieved. Apart from the platform, freelancers are better off in their expected effective wages and customers are charged a lower trip fare under optimal dual sourcing than before.
- R4: Dual sourcing. Freelancers are very sensitive to their effective wage. Competition among drivers dominates the market, thus, freelancers receive a lower expected effective wage. The platform reduces its trip fare to attract more customers.
- R5: Dual sourcing. All drivers participate in service and the pool size of drivers limits the supply level.

We observe that the dual sourcing strategy is beneficial under relatively low freelancers' price sensitivity and value of flexibility (Fig. 13). Otherwise, the platform should only hire self-scheduling freelancers even though the system's demand is uncertain. This is consistent with the conclusion in Section 3.2.3.

The choice of contract (purely self-scheduling or dual sourcing contracts) is sensitive to drivers' risk-averse degree. The platform is more likely to use dual sourcing if drivers are more risk-averse. For example, Fig. 13a shows a high potential for dual sourcing if all drivers are risk-averse. When the risk-aversion is lower, the feasibility region of dual sourcing shrinks (see Fig. 13b–d).

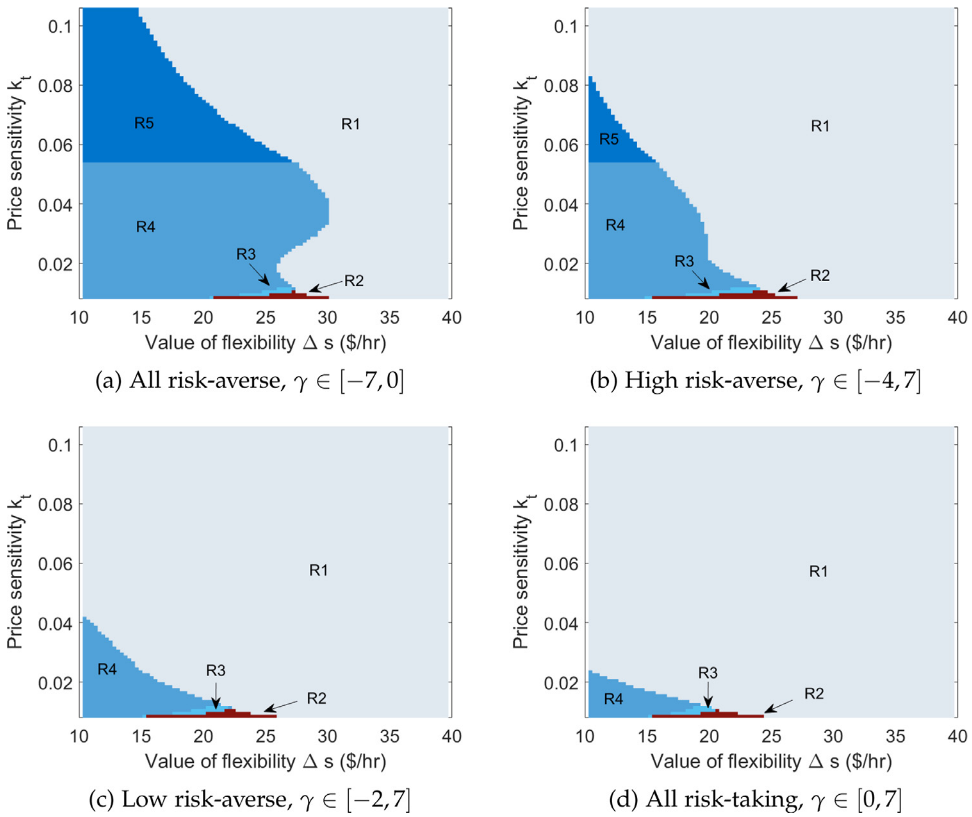
Finally, we identify the potential of dual sourcing contract that enables the platform to take advantage of drivers' disutility for risk. For example, Fig. 13d shows that the dual sourcing strategy can still beat self-scheduling even though all drivers are risk-seeking. Nevertheless, this happens only when the potential demand is high enough and the realized demand is excessive.

In sum, drivers' characteristics determines under which regime the market operates. Thus, drivers' characteristics have substantial impact on the platform's pricing decision and contract choice. The platform is more likely to choose dual sourcing over purely self-scheduling when drivers are more risk-averse and less price sensitive. Drivers' value of flexibility has a mixed effect on freelancers' effective wages.

4.3. Result summary

We summarize our main findings in numerical experiments as follows:

- Optimal dual sourcing contracts can maintain or increase system labor supply. However, the supply increase is marginal when freelancers become more price sensitive.
- Optimal dual sourcing contracts can increase the ride-sourcing platform's expected profit. When freelancers are sensitive to effective wage, the profit growth mainly stems from the labor cost saving. When freelancers are insensitive to effective wage, the revenue increase contributes much to the profit growth. Dual sourcing can save labor cost even when all drivers participate in service.
- Dual sourcing is profitable for the platform only if the minimum salary for contractors is small enough. The staffing level of contractors increases with the percentage of risk-averse drivers.



(a) All risk-averse, $\gamma \in [-7, 0]$ (b) High risk-averse, $\gamma \in [-4, 7]$ (c) Low risk-averse, $\gamma \in [-2, 7]$ (d) All risk-taking, $\gamma \in [0, 7]$

Fig. 13. Changes in platform decision.

- The ride-sourcing market could operate in two possible regimes, the complement regime and competition regime, based on drivers' price sensitivity. The changes in optimal pricing decisions and drivers' welfare under dual sourcing depend on in which market regime the market operates.
- Arguing against intuition, it is possible that optimal dual sourcing contracts increase freelancers' expected effective wage. This is true when the market operates in the complement regime, creating a win-win situation for freelancers and the platform. Otherwise, with a moderate price sensitivity, dual sourcing compromises freelancers' expected effective wage. In the complement regime, freelancers could benefit from contractors' high value of flexibility in terms of their expected wage.
- Customers' welfare increases with employing the optimal dual sourcing contracts. Those risk-neutral freelancers are likely to be worse off in utility because of their short-sighted choices.
- Dual sourcing sees a high potential of implementation when drivers feature high risk aversion and low price sensitivity.

5. Conclusion and future work

This paper examines optimal contract design for ride-sourcing services, where the reliability of labor supply from freelancers poses a great challenge for supply management in face of demand uncertainty. We resort to the idea of dual sourcing with a blended workforce consisting of contractors and freelancers. The effectiveness of dual sourcing is confirmed via analytical models and numerical experiments. We find that optimal dual sourcing contracts can increase the platform's profit and raise customers' welfare. Based on freelancers' price sensitivity, the market can operate in either a complement regime or a competition regime. Freelancers benefit in the former while suffering a decline in their effective wage in the latter. To explore the impacts of demand uncertainty and drivers' characteristics on the platform's staffing and pricing decision, we conduct a sensitivity analysis. It is found that the platform's pricing decision is most sensitive to freelancers' price sensitivity parameter, and the staffing decision is closely related to drivers' risk attitude and minimum contractual wage.

The proposed modeling framework is flexible and can be modified to model other types of shared mobility services, e.g., crowd-sourced urban delivery. It is also applicable to a future scenario where a ride-sourcing platform owns and operates a fleet of automated vehicles in addition to regular/automated vehicles driven/owned by freelancers. Our future efforts will be devoted to the following avenues. First, this paper does not differentiate contractors and freelancers in terms of their working duration and assumes that contractors and freelancers' marginal revenue are equal. However, as the platform has authority over contractors' movements across the network, contractors render a higher marginal value to the platform under efficient empty-car routing instructions. Besides, freelancers and contractors would be subject to different work scheduling constraints, leading to context-dependent contribution margin. It is thus meaningful to explore how the heterogeneity among drivers could potentially impact the system performance both spatially and temporally. Second, our discussion is limited to demand uncertainty. The uncertainty on the supply side could also result in the supply-demand mismatch. Future studies may aim to replicate the analysis under both demand and supply uncertainty. Third, we assume the platform treats freelancers and contractors equally when they are matched with trip requests. However, the prepaid mechanism for staffing contractual drivers could motivate a platform to give contractors priority in the matching procedure. As a result, freelancers may feel discouraged and choose to exit the market. Thus, it would be meaningful to include the matching strategies into the model and explore implementable dual-source contracts with different matching strategies. Fourth, dual sourcing can improve drivers' loyalty toward a platform and strengthen the platform's competitiveness in a duopoly or oligopoly market. Thus, another potential direction is to explore the platforms' staffing and managing strategies under various market contexts. Last but not least, with strategic drivers, it is meaningful to examine the optimal contract and market equilibrium in a dynamic framework when drivers' contractual choices are endogenously determined by the actual market equilibrium under dual sourcing.

Acknowledgements

The work described in this paper was partly supported by research grants from the National Science Foundation (CMMI-1854684; CMMI-1904575) and DiDi Chuxing. Tingting Dong appreciates the financial support provided by China Scholarship Council (CSC) for sponsoring her two-year visit at University of Michigan. Jian Wang thanks the support of National Nature Science Foundation of China (No. 91846301).

Appendix A. Summary of Notations

Table A1
Notation list of variables, sets, functions and parameters.

Notation	Description	Notation	Description
Variables			
F	per-trip fare (\$/trip)	w	earning per completed trip (\$/trip)
s	salary for contractors (\$/hr)	z	profit of the platform (\$/hr)
r	effective wage of freelancers (\$/hr)	r_j	effective wage of freelancers in scenario j (\$/hr)
q	realized demand (trip/hr)	q_j	realized demand in scenario j (trip/hr)
t^c	customers' average waiting time (hr)	t_j^c	customers' average waiting time in scenario j (hr)
N	system supply or number of in-service drivers	N_c	number of contractors
N_f^0	number of freelancers	N_f	number of freelancers in-service
R	platform's revenue (\$/hr)	C	platform's staffing cost (\$/hr)
R_p	platform's optimal revenue given N (\$/hr)	C_s	platform's optimal staffing cost given N (\$/hr)
P_w	drivers' surplus (\$/hr)	C_w	customers' surplus (\$/hr)
N_j^v	number of idle drivers in scenario j		
Sets			
J	set of all scenarios for potential demand		
Functions			
$f_q(.)$	percentage of served customers	$f_t(.)$	customers' waiting time function
$T(.)$	cumulative distribution of drivers' reservation wage	$G(.)$	cumulative distribution of drivers' risk attitude
Parameters			
Q^0	potential demand (trips/hr)	Q_j^0	potential demand in scenario j (trips/hr)
ΔQ^0	difference of potential demand (trips/hr)	N_0	driver pool size
p_j	probability of scenario j	l	average in-vehicle time (hr)
α	customers' value of unit waiting time (\$/hr)	β	calibrated parameter in waiting time function
η	calibrated parameter in waiting time function	γ	risk attitude of drivers
$\overline{\gamma}$	risk attitude of the most risk-seeking driver	$\underline{\gamma}$	risk attitude of the most risk-averse driver
Δs	drivers' inflexibility cost or value of flexibility (\$/hr)	k	customers' sensitivity to full trip price (/\$)
k_t	freelancers' sensitivity to effective wage	k_g	slope of contractors' inverse supply function
b_g	minimum salary for contractors (\$/hr)	ϵ	random revenue gains of drivers (\$/hr)
r_0	optimal effective wage under self-scheduling (\$/hr)	$\bar{r}$	maximum reservation wage of freelancers (\$/hr)
$\underline{r}$	minimum reservation wage of drivers (\$/hr)	τ	customers' value of unit in-vehicle time (\$/hr)

Appendix B

Proposition

Proof of Proposition 1

Proposition 1 (Existence of market equilibrium)**.** *Given a feasible contract, i.e., trip fare F, per-trip earning for freelancers w, and salary for contractors s, there exists market equilibrium.*

Proof. To facilitate the proof, we specify $\underline{r}$ and $\bar{r}$ as the minimum and maximum reservation wage of freelancers respectively. Let $\underline{\gamma}$ and $\overline{\gamma}$ denote the minimum and maximum value of drivers' risk attitude γ. Note that N_c can be uniquely determined by the salary s. Thus, we next prove the existence of market equilibrium by the value of s:

Case 1: $s \geq \overline{\gamma}\sigma[r_0] + \mathbb{E}[r_0] + \Delta s$

All drivers sign up as contractors where $N_c = N = N_0$. For any feasible value of w, Eqs. (3), (4), (6) are satisfied automatically. By Eqs. (5) (7), proving Proposition 1 is equivalent to finding a solution of N_j^v satisfying

$$\frac{N - N_j^v}{f_t(N_j^v) + l} = Q_j^0 \cdot f_q\left(F + \alpha \cdot f_t(N_j^v) + \tau \cdot l\right) \quad \forall j \in J. \tag{B.1}$$

The right-hand side (RHS) of Eq. (B.1) increases with N_j^v since $f_q' \cdot f_t' > 0$. As $N_j^v \to 0$, then $q_j = 0$. As $N_j^v = N$, then $q_j \geq 0$. For the left-hand side (LHS) of Eq. (B.1), (Castillo et al., 2017) has proved that it will first increase and decrease with $f_t(N_j^v)$. Since $f_t' < 0$, we conclude that the LHS of Eq. (B.1) will also first increase and then decrease with N_j^v for $N_j^v \in (0, N]$. As $N_j^v \to 0$, it has been proved $\frac{N_0 - N_j^v}{f_t(N_j^v)+l} = O(\frac{1}{f_t(N_j^v)}) > 0$ (Castillo et al., 2017). As $N_j^v = N$, then the LHS is equal to 0. Thus, we have the LHS is greater than the RHS of Eq. (B.1) as $N_j \to 0$. The opposite relationship can be obtained as $N_j^v = N$. Since both sides of Eq. (B.1) are continuous function of N_j^v, we conclude that there must exist at least one solution $N_j^v \in (0, N]$ satisfying Eq. (B.1) as per the intermediate value theorem. Note that since $\lim_{t_j^c \to \infty} f_q = 0$, when N is smaller than a certain threshold $\hat{N}$, $\mathbb{E}[q]$ could be zero at equilibrium.

Case 2: $\underline{\gamma}\sigma[r_0] + \mathbb{E}[r_0] + \Delta s < s < \overline{\gamma}\sigma[r_0] + \mathbb{E}[r_0] + \Delta s$

Part of drivers sign up as contractors, whose number is uniquely determined by Eq. (4). The system Eqs. (3),(5),(6),(7) lead to (B.1) and the following equation:

$$N = (N_0 - N_c) \cdot T\left(w \cdot \frac{\mathbb{E}[q]}{N}\right) + N_c. \tag{B.2}$$

Rewriting (B.1) to be

$$q_j = \frac{N - N_j^v}{f_t(N_j^v) + l} \quad \forall j \in J. \tag{B.3}$$

Proving the existence of market equilibrium is equivalent to finding a solution (N, q_j, N_j^v) satisfying Eq. (5), (B.2) and (B.3). Note that using an inverse demand function, Eq. (5) can be rewritten to express N_j^v to be a function of q_j. Therefore, Eq. (5), (B.2) and (B.3) essentially define a mapping $H(\cdot)$ that maps (N, q_j, N_j^v) to itself. This self-map is continuous and the feasible set of (N, q_j, N_j^v) is compact and convex. Based on Brouwer's fixed-point theorem, there exists a solution to this fixed-point problem.

Case 3: $s \leq \underline{\gamma}\sigma[r_0] + \mathbb{E}[r_0] + \Delta s$

By Eq. (4), all drivers sign up as freelancers where $N_c = 0$. The existence of market equilibrium can be similarly proved as Case 2. To include the situation where no drivers provide service, we prescribe $\mathbb{E}[q]/0 = 0$ in Eq. (3). With this prescription, Eqs. (3)–(7) suggests that $(q_j, N_j^v, N, N_c, \mathbb{E}[r]) = (0, 0, 0, 0, 0)$ can be a trivial solution to the system. □

Proof of Proposition 2

Proposition 2 (Relationship between staffing cost and supply)**.** *The staffing cost $C_s(N)$ is a convex increasing function regardless of the staffing strategy. For the same supply level, the staffing cost under the optimal dual sourcing is no greater than that under its self-scheduling counterpart.*

Proof. For clarification, we restate the notations in the following proof. Let $C(N, N_c)$ denote the platform's staffing cost for any given (N, N_c), and $C_s(N)$ represent the optimal staffing cost for a fixed system supply N, i.e., $C_s(N) = \min_{N_c} C(N, N_c)$.

We first show that $C(N, N_c)$ is a convex function of N_c for a fixed $N < N_0$. For a given N, the staffing cost is

$$C(N, N_c) = \begin{cases} (N - N_c) \cdot T^{-1}\left(\frac{N - N_c}{N_0 - N_c}\right) + s(N_c) \cdot N_c & \text{if } N_c < N_0, \\ s(N_c) \cdot N_c & \text{Otherwise.} \end{cases}$$

Let T^{-1} denotes $T^{-1}(\frac{N - N_c}{N_0 - N_c})$ for simplicity. For $N_c < N$ and a given N, we have

$$\begin{aligned} \frac{\partial C(N, N_c)}{\partial N_c} &= -T^{-1} - \frac{(N_0 - N) \cdot (N - N_c)}{(N_0 - N_c)^2} (T^{-1})' + N_c \cdot s'(N_c) + s(N_c), \\ \frac{\partial^2 C(N, N_c)}{\partial N_c^2} &= (T^{-1})' \cdot \frac{2(N_0 - N)^2}{(N_0 - N_c)^3} + (T^{-1})'' \cdot \frac{(N_0 - N)^2 \cdot (N - N_c)}{(N_0 - N_c)^4} + 2s'(N_c) + N_c \cdot s''(N_c). \end{aligned}$$

Since both $T(\cdot)$ and $G(\cdot)$ are increasing concave function, we have $(T^{-1})' \geq 0$, $(T^{-1})'' \geq 0$, $s'(N_c) \geq 0$ and $s''(N_c) \geq 0$. Thus, we conclude $\partial^2 C(N, N_c)/\partial N_c^2 \geq 0$. The function $C(N, N_c)$ is a convex function of N_c over $(0, N)$.

For a given N, the optimal number of contractors $N_c^*(N)$ is obtained as follows:

$$N_c^*(N) = \begin{cases} \text{argmin}_{N_c \in (0,N)} C(N, N_c), & \text{if } \frac{\partial C(N,0)}{\partial N_c} < 0, \\ 0, & \text{Otherwise}, \end{cases} \tag{B.4}$$

The above suggests that the platform adopts dual sourcing only when adding contractors will reduce labor cost for a given N. Otherwise, it goes with the self-scheduling contract only where $N_c^*(N) = 0$. If so, the platform's staffing cost is given as :

$$C_s(N) = C(N, 0) = N \cdot T^{-1}\left(\frac{N}{N_0}\right).$$

The above Equation leads to the following properties of the staffing cost function:

$$\frac{dC_s(N)}{dN} = T^{-1}\left(\frac{N}{N_0}\right) + \frac{N}{N_0} \cdot \left(T^{-1}\left(\frac{N}{N_0}\right)\right)' \geq 0, \tag{B.5}$$

$$\frac{d^2C_s(N)}{dN^2} = \frac{2}{N_0} \cdot \left(T^{-1}\left(\frac{N}{N_0}\right)\right)' + \frac{N}{N_0^2} \cdot \left(T^{-1}\left(\frac{N}{N_0}\right)\right)'' \geq 0. \tag{B.6}$$

The above equations show that the staffing cost $C_s(N)$ convexly increases with N under self-scheduling. The marginal labor cost increases with N.

With contractors only where $N_c^* = N$, we have

$$C_s(N) = N \cdot s(N), \quad \frac{dC_s(N)}{dN} = s(N) + Ns'(N) \geq 0, \quad \frac{d^2C_s(N)}{dN^2} = 2s'(N) + Ns''(N) \geq 0.$$

Thus, we have $C_s(N)$ convexly increase with N when there are only contractors in labor market.

With dual sourcing where $N_c^*(N) \in (0, N)$, the envelope theorem leads to the following marginal staffing cost:

$$\frac{dC_s(N)}{dN} = \frac{\partial C(N, N_c^*(N))}{\partial N} = T^{-1} + (T^{-1})' \cdot \frac{N - N_c^*(N)}{N_0 - N_c^*(N)} \geq 0,$$

where the number of contractors $N_c^*(N)$ is obtained by solving the first-order condition for $\min C(N_c, N)$:

$$\frac{\partial C(N, N_c)}{\partial N_c} = -T^{-1} - \frac{(N_0 - N) \cdot (N - N_c)}{(N_0 - N_c)^2}(T^{-1})' + N_c \cdot s'(N_c) + s(N_c) = 0. \tag{B.7}$$

Thus, the staffing cost is an increasing function of N. To prove the convexity of the staffing cost function, we apply the implicit function theorem to Eq. (B.7). Thus, the change of $N_c^*(N)$ with respect to N is given as

$$\frac{\partial N_c^*(N)}{\partial N} = \frac{\psi \cdot (N_0 - N_c^*(N))}{\psi \cdot (N_0 - N) + (2s'(N_c) + N_c \cdot s''(N_c)) \cdot (N_0 - N_c^*(N))} \geq 0,$$

where $\psi = 2 \cdot \frac{N_0 - N}{(N_0 - N_c)^2} \cdot (T^{-1})' + \frac{(N_0 - N)\cdot(N - N_c)}{(N_0 - N_c)^3} \cdot (T^{-1})''$. Let $H = \frac{dC_s(N)}{dN}$, we have

$$\frac{d^2C_s(N)}{dN^2} = \frac{\partial H}{\partial N} + \frac{\partial H}{\partial N_c^*}\frac{\partial N_c^*}{\partial N} = \frac{N_0 - N_c^*}{N_0 - N} \cdot \left(\psi - \frac{\psi^2 \cdot (N_0 - N)}{\psi \cdot (N_0 - N) + 2(s' + N_c^* \cdot s'') \cdot (N_0 - N_c^*)}\right) \geq 0.$$

Thus, we conclude that the staffing cost $C_s(N)$ convexly increases with N under dual sourcing.

Note that both $C_s(N)$ and $\partial C_s(N)/\partial N$ is a continuous function of N. With the above three cases of N_c^*, the staffing cost $C_s(N)$ is a convex increasing function of system supply N regardless of staffing strategy. Since self-scheduling is a special case of dual sourcing, for the same supply level, the staffing cost under the optimal dual sourcing is no greater than that under self-scheduling. □

Proof of Proposition 3

Proposition 3. *Optimal dual-sourcing contracts either maintain or increase the number of drivers in service, as compared with the self-scheduling counterpart.*

Proof. Let $N_c^*(N)$ be the optimal number of contractors a given supply N. For $N_c^*(N^*) = 0$, problem (14) reduces to the contract design problem with only self-scheduling drivers. Proposition 3 is satisfied automatically and the same level of system supply N^* is maintained. Thus, we in the next focus on the case where $N_c^*(N^*) \in (0, N)$.

For a given N, we denote $MC_s^f(N)$ and $MC_s^d(N)$ as the marginal staffing cost under self-scheduling and dual sourcing. Define the function $H(N, N_c) = T^{-1}\left(\frac{N-N_c}{N_0-N_c}\right) + \frac{N-N_c}{N_0-N_c} \cdot \left(T^{-1}\left(\frac{N-N_c}{N_0-N_c}\right)\right)'$. For $N_c = N_c^*(N)$, $H(N, N_c)$ is indeed the marginal staffing cost for a fixed N, i.e., $H(N, N_c^*(N)) = MC_s^d(N)$. For an arbitrary N_c, we have

$$\frac{\partial H(N, N_c)}{\partial N_c} = -2\frac{N_0 - N}{(N_0 - N_c)^2} \cdot \left(T^{-1}\right)' - \frac{(N_0 - N) \cdot (N - N_c)}{(N_0 - N_c)^3} \cdot \left(T^{-1}\right)'' \leq 0,$$

where $\left(T^{-1}\right)'$ and $\left(T^{-1}\right)''$ refer to the first and second derivative of function $T^{-1}(\frac{N-N_c}{N_0-N_c})$. The above equation suggests that $H(N, N_c)$ decreases with N_c for a given N. Since $N_c^*(N) > 0$, we conclude that $H(N, N_c^*(N)) \le H(N, 0))$ for a given N. Thus, the marginal staffing cost under dual sourcing is equal to or less than that under self-scheduling, i.e., $MC_s^f(N) \ge MC_s^d(N)$ for $N_c^*(N) \in (0, N)$.

Let N_s^* denote the optimal system supply under self-scheduling. For a given N, the platform's profit under dual sourcing $z_d(N)$ is given as

$$z_d(N) = \int_0^N \left(\frac{dR_p(N)}{dN} - MC_s^d(N)\right)dN = z_s(N) + \int_0^N \left(MC_s^f(N) - MC_s^d(N)\right)dN,$$

where $z_s(N)$ denotes the platform's profit under self-scheduling for a given N, i.e., $z_s(N) = \int_0^N (\frac{dR_p(N)}{dN} - MC_s^f(N))dN$. Since $z_s(N)$ is maximized at N_s^* and $MC_s^f(N) \ge MC_s^d(N)$, we have $z_d(N) \le z_d(N_s^*)$ for $N \in (0, N_s^*)$. Since $z_d(N^*) > z_d(N_s^*)$, we must have $N^* \in (N_s^*, N_0]$ if $N_s^* < N_0$. For $N_s^* = N_0$, we have $\frac{dR_p(N)}{dN} \ge MC_s^f(N) \ge MC_s^d(N)$. The platform obtains the highest profit at N_0 and the level of supply will be maintained.

To conclude, the number of in-service drivers under the optimal dual-sourcing is equal or greater than that under self-scheduling. □

Lemma

Proof of Lemma 1

Lemma 1 (Effective wage with supply)**.** *Given the trip fare F and trip earning w, for each demand scenario, the effective wage of freelancers first increases then decreases with the total number of drivers in-service N in the non-WGC regime.*

Proof. Under the special case defined in Section 3.1, the realized demand q_j satisfies the following equations:

$$\frac{N_j^v \cdot (N - N_j^v)}{\beta + N_j^v \cdot l} = q_j, \quad q_j = Q_j^0 \cdot \left(1 - k \cdot (F + \frac{\alpha\beta}{N_j^v})\right), \quad \forall j \in J, \tag{B.8}$$

Thus, given trip fare F, the implicit function theorem leads to the following derivative of realized demand q_j with respect to N:

$$\frac{\partial q_j}{\partial N} = \frac{Q_j^0 k\alpha\beta}{Q_j^0 k\alpha\beta \cdot (t_j^c + l) + (N_j^v)^2 - q_j\beta}.$$

In the non-WGC regime, we have $(N_j^v)^2 - q_j\beta > 0$ and $\partial q_j/\partial N > 0$.

Let $r_j = w \cdot q_j/N$ be the effective wage of freelancers in scenario j. Given trip fare F and trip earning w, the derivative of effective wage r_j with respect to N is given as follows:

$$\frac{\partial r_j}{\partial N} = \frac{w}{N} \cdot \left(\frac{\partial q_j}{\partial N} - r_j\right) = \frac{w}{N} \cdot \left(\frac{Q_j^0 k\alpha\beta}{Q_j^0 k\alpha\beta \cdot (t_j^c + l) + (N_j^v)^2 - q_j\beta} - r_j\right), \forall j \in J, \tag{B.9}$$

By the above differential equation of r_j, we first prove that $\partial r_j/\partial N$ is undetermined with system supply N. Note that as $N \to k\alpha\beta/(1-kF)$, Eq. (B.8) indicates $N_j^v \to k\alpha\beta/(1-kF)$, $q_j \to 0$ and $r_j \to 0$, which leads to $\partial q_j/\partial N > r_j$. Thus, by Eq. (B.9), we have $\partial r_j/\partial N > 0$ as $N \to k\alpha\beta/(1-kF)$ in the non-WGC regime. The effective wage will increase with N initially. For the decrease part of r_j with N, we use a contradiction. Assume r_j always increases with N, i.e., $\partial r_j/\partial N > 0$ for $N \in (k\alpha\beta/(1-kF), \infty)$. Then as $N \to \infty$, the boundary condition of q_j, i.e., $q_j \le Q_j^0$ suggests $\partial q_j/\partial N \to 0$ and $r_j > \partial q_j/\partial N$. Thus, Eq. (B.9) leads to $\partial r_j/\partial N < 0$ as $N \to \infty$, which contradicts with the assumption that r_j always increases. Thus, $\partial r_j/\partial N$ is undetermined with system supply N. There must exist at least one supply level $\hat{N}$ such that r_j will first increase and then decrease with N in the neighborhood of $\hat{N}$, i.e., $N \in (\hat{N} - \delta, \hat{N} + \delta)$.

We then complete the proof by showing the uniqueness of the tipping point $\hat{N}$ indicating the decrease of $r_j(N)$. Let $\hat{N}$ be the point that r_j begins to decrease with N for the first time. Since $\partial r_j/\partial N$ is a continuous function of N, at tipping point $\hat{N}$, the effective wage satisfies $\partial r_j/\partial N = 0$. Thus, Eq. (B.9) indicates that $\partial q_j/\partial N = r_j$ and $\partial q_j/\partial N > 0$ at the tipping point $\hat{N}$. Since r_j begins to decrease, i.e., $\partial r_j/\partial N < 0$, Eq. (B.9) leads to $\partial q_j/\partial N < r_j$ at $N = \hat{N} + \delta$. Thus, we have $\partial q_j/\partial N|_{N=\hat{N}+\delta} < r_j|_{N=\hat{N}+\delta} < r_j|_{N=\hat{N}} = \partial q_j/\partial N|_{N=\hat{N}}$. By definition, $\partial q_j/\partial N$ decreases with N at $\hat{N}$, i.e., $\partial^2 q_j/\partial N^2 < 0$. Under the specific instance defined in Section 3.1, $\partial^2 q_j/\partial N^2$ is given as follows:

$$\frac{\partial^2 q_j}{\partial N^2} = -2 \cdot \left(\frac{\partial q_j}{\partial N}\right)^3 \cdot \left(\frac{N_j^v}{Q_j^0 k\alpha\beta}\right)^2 \cdot \left(N_j^v - \frac{Q_j^0 k\alpha\beta^2}{(N_j^v)^2}\right), \quad \forall j \in J.$$

In the above equation, the last term $(N_j^v - Q_j^0 k\alpha\beta^2/(N_j^v)^2)$ is an increasing function of N_j^v. Recall that in the non-WGC regime, we have $\partial q_j/\partial N > 0$ and $\partial N_j^v/\partial N > 0$. Thus, the term $(N_j^v - Q_j^0 k\alpha\beta^2/(N_j^v)^2)$ increases with system supply N while

$\partial^2 q_j/\partial N^2$ decreases with N. Note that $\partial^2 q_j/\partial N^2 < 0$ at $\hat{N}$, the decrease of $\partial^2 q_j/\partial N^2$ with respect to N indicates $\partial^2 q_j/\partial N^2 < 0$ for any $N \in [\hat{N}, \infty)$. Thus, $\partial q_j/\partial N$ continuously decreases with N for $N > \hat{N}$. With the decreasing of $\partial q_j/\partial N$, there is no point greater than $\hat{N}$ can achieve $\partial q_j/\partial N = r_j$ in Eq. (B.9). Otherwise, it implies that r_j also decreases with N at other tipping points, i.e., $\partial r_j/\partial N < 0$, which is a contradiction since $\partial r_j/\partial N = 0$ at tipping points. Thus, the tipping point $\hat{N}$ is unique.

To conclude, in the non-WGC regime, the effective wage of freelancers in scenario j will first increase and then decrease with system supply N. □

Theorem

Proof of Theorem 1

To prove the theorem, we will first introduce the following lemma.

Lemma 2. *Given the system supply N, for a linear demand function, there exists a unique $F^*(N)$ that maximizes the platform's revenue $R(F,N)$ if the system lies in the non-WGC regime.*

Proof. Given potential demand profile and system supply N, the platform's revenue is as follows:

$$R(F,N) = F \cdot \sum_j p_j \cdot q_j(F,N).$$

For a contract to be feasible, the trip fare satisfies $F^* \geq 0$. Thus, Eq. (B.8) suggests that the system supply N satisfies $N \geq k\alpha\beta$. When $N = k\alpha\beta$, there exists only one feasible solution, $F^* = q_j(F^*, N) = 0$. For a given supply $N > k\alpha\beta$, we prove Lemma 2 by showing the platform's revenue $R(F,N)$ is a strictly concave function of trip fare F.

For a given $N > k\alpha\beta$, the first derivative of realized demand q_j with respect to F is given as follows:

$$\frac{\partial q_j}{\partial F} = \frac{Q_j^0 f_q' \cdot (1 + q_j f_t')}{(1 + q_j f_t') + \alpha Q_j^0 f_q' f_t' \cdot (t_j^c + l)}, \quad \forall j \in J,$$

For notation simplicity, let $y_j = \frac{(t_j^c + l)}{1 + q_j f_t'}$ for each scenario $j \in J$. With linear function, i.e., $f_q'' = 0$, we have the following formula:

$$\frac{\partial^2 q_j}{\partial F^2} = -\alpha \cdot \left(\frac{Q_j^0 f_q' \cdot (1 + q_j f_t')}{(1 + q_j f_t') + \alpha Q_j^0 f_q' f_t' \cdot (t_j^c + l)} \right)^2 \cdot (f_t'' \cdot y_j \cdot \frac{\partial N_j^v}{\partial F} + f_t' \cdot \frac{\partial y_j}{\partial F}), \quad \forall j \in J,$$

where $\frac{\partial y_j}{\partial F}$ satisfies

$$\frac{\partial y_j}{\partial F} = \frac{f_t'}{(1 + q_j f_t')} \frac{\partial N_j^v}{\partial F} - \frac{q_j f_t'' \cdot (t_j^c + l)}{(1 + q_j f_t')^2} \cdot \frac{\partial N_j^v}{\partial F} - \frac{f_t' \cdot (t_j^c + l)}{(1 + q_j f_t')^2} \cdot \frac{\partial q_j}{\partial F}, \quad \forall j \in J.$$

When the system lies in the non-WGC regime, inequalities $(1 + q_j f_t') > 0$ and $\frac{\partial N_j^v}{\partial F} > 0$ stand. Besides, by assumptions in the base model, we have $f_t' < 0$, $f_t'' > 0$ and $f_q' < 0$. Thus, we have

$$\frac{\partial y_j}{\partial F} < 0, \quad \frac{\partial q_j}{\partial F} < 0, \quad \frac{\partial^2 q_j}{\partial F^2} < 0, \quad \forall j \in J.$$

With the above inequalities, platform's revenue function $R(F,N)$ satisfies the following formula:

$$\frac{\partial^2 R(F,N)}{\partial F^2} = \sum_j p_j \cdot (2 \cdot \frac{\partial q_j}{\partial F} + F \cdot \frac{\partial^2 q_j}{\partial F^2}) < 0.$$

The above inequality suggests $R(F,N)$ is a strictly concave function of F. When $F \to 0$ or $F \to -1/f_q'$, we have $R(F,N) \to 0$. Thus, for a linear demand function, there exist a unique $F^*(N) \in (0, -1/f_q')$ that gives $R_p(N) = \max_F R(F,N)$ if the system lies in the non-WGC regime. □

Theorem 1 (Marginal revenue of the platform)**.** *In the non-WGC regime, there exists two threshold values $\underline{N}$ and $\overline{N}$. When the supply is less than $\underline{N}$, the marginal revenue increases with system supply. When the supply is greater than $\overline{N}$, the marginal revenue decreases with system supply.*

Proof. As per Lemma 2, $F^*(N)$ is unique under in the non-WGC regime. Thus, by the envelope theorem, we can write the platform's marginal revenue as

$$\frac{dR_p(N)}{dN} = F^*(N) \cdot \left(p_L \cdot \frac{\partial q_L(F,N)}{\partial N} + p_H \cdot \frac{\partial q_H(F,N)}{\partial N} \right) \tag{B.10}$$

where the optimal trip fare is obtained by Eq. (16) and is as follows:

$$F^*(N) = \frac{1}{k} - \frac{1}{k} \cdot \frac{q_j(F^*(N), N)}{Q_L^0} - \frac{\alpha\beta}{N_j^{\nu*}(N)}. \tag{B.11}$$

The derivative $\partial q_j(F, N)/\partial N$ is derived in Section 2.3. For the special case defined in Section 3.1, $\partial q_j(F, N)/\partial N$ is given as:

$$\frac{\partial q_j(F, N)}{\partial N} = \frac{Q_j^0 k\alpha\beta}{Q_j^0 k\alpha\beta \cdot \left(t_j^c + l\right) + \left(N_j^\nu\right)^2 - q_j(F, N) \cdot \beta}, \quad \forall j \in J.$$

With the above equations, the marginal revenue of the platform under demand uncertainty is as follows:

$$\frac{dR_p(N)}{dN} = F^*(N) \cdot \sum_j p_j \cdot \frac{Q_j^0 k\alpha\beta}{Q_j^0 k\alpha\beta \cdot \left(t_j^{c*}(N) + l\right) + \left(N_j^{\nu*}(N)\right)^2 - q_j^*(F^*(N), N) \cdot \beta}. \tag{B.12}$$

We prove Theorem 1 by first analyzing the asymptotic behavior of platform's marginal revenue defined by the above equation. When $N = k\alpha\beta$, Lemma 2 suggests that $F^*(N) = q_j^*(F^*(N), N) = 0$. Thus, we have $dR_p(N)/dN = 0$ as $N = k\alpha\beta$. When $N \to \infty$, the fleet conservation condition (7) and the boundary of q_j, i.e., $q_j \le Q_j^0$ implies $N_j^{\nu*}(N) \to \infty$. By maximizing the platform's revenue $R(F, N)$ over F, we obtain $F^*(N) \to 1/(2k)$ and $q_j^*(F^*(N), N) \to Q_j^0/2$ as $N \to \infty$. Thus, Eq. (B.12) leads to $dR_p(N)/dN \to 0$ when $N \to \infty$.

We then prove that $dR_p(N)/dN > 0$ for $N \in (k\alpha\beta, \infty)$. By Lemma 2, the unique optimal trip fare satisfies $F^*(N) > 0$ for $N \ge k\alpha\beta$. Since system supply N takes a finite value, both optimal number of idle drivers $N_j^{\nu*}$ and vehicle waiting time $t_j^{c*}(N)$ are also finite for a given $N \in (k\alpha\beta, \infty)$. Recall that $\left((N_j^{\nu*}(N))^2 - q_j^*(F^*(N), N)\beta > 0\right)$ in the non-WGC regime, Eq. (B.12) indicates $dR_p(N)/dN > 0$ for $N \in (k\alpha\beta, \infty)$. Thus, for any given $N \in (k\alpha\beta, \infty)$ we have $dR_p(N)/dN > dR_p(N)/dN|_{N=k\alpha\beta}$ and $dR_p(N)/dN > dR_p(N)/dN|_{N\to\infty}$.

We complete the proof by showing that $dR_p(N)/dN$ is a continuous function of N. Given a system supply $N \in [k\alpha\beta, \infty)$, the feasible region of maximization problem $\max_F R(F, N)$ is compact and continuously change with N. Thus, according to Berge's maximum theorem, the optimal solution $F^*(N)$ is a continuous function of N. Thus, both $q_j^*(F^*(N), N)$ and $N_j^{\nu*}(N)$ continuously change with system supply N. By Eq. (B.12), the marginal revenue $dR_p(N)/dN$ is also a continuous function of N. For a continuous function satisfying the aforementioned property, i.e., $dR_p(N)/dN > dR_p(N)/dN|_{N=k\alpha\beta} = 0$ and $dR_p(N)/dN > dR_p(N)/dN|_{N\to\infty} \to 0$ holding for $N \in (k\alpha\beta, \infty)$, there must exist $\underline{N}$ and $\overline{N}$ such that the marginal revenue $dR_p(N)/dN$ increases with N when $N \le \underline{N}$ and decreases with N when $N \ge \overline{N}$. □

Proof of Theorem 2

Theorem 2 (Prerequisite condition for dual sourcing)**.** *When freelancers' price sensitivity k_t is greater than $\frac{2\overline{N}}{N_0} \cdot (\frac{dR_p(\overline{N})}{dN})^{-1}$, the dual sourcing strategy increases the platform's expected profit only if the minimum salary for contractors is less than $\mathbb{E}[r_0] \cdot (2 - k_t\mathbb{E}[r_0])$.*

Proof. For the dual sourcing contract design problem (14), proving the prerequisite condition of dual sourcing is equivalent to obtain conditions such that $N_c^*(N^*) > 0$.

For problem (14), the optimal system supply N^* satisfies $\frac{dR_p(N^*)}{dN} = \frac{dC_s(N^*)}{dN}$ for $N^* < N_0$. Thus, under self-scheduling, the optimal system supply is given as:

$$N_s^* = \begin{cases} \frac{N_0}{2} \cdot \frac{dR_p(N_s^*)}{dN} \cdot k_t & \text{if } N_s^* < N_0, \\ N_0 & \text{Otherwise}, \end{cases}$$

where we use N_s^* denote the optimal system supply under self-scheduling for differentiation purpose.

We first examine the case where $N_s^* < N_0$. Note that for the same N, the marginal labor cost under the optimal dual sourcing is not greater than that under self-scheduling. Thus, the optimal system supply under optimal dual sourcing $N^* \ge N_s^*$ (see Proposition 3). For $k_t \ge \frac{2\overline{N}}{N_0} \cdot (\frac{dR_p(\overline{N})}{dN})^{-1}$, N_s^* satisfies $N_s^* \ge \overline{N}$. According to Theorem 1, the marginal revenue of platform decreases with system supply at N_s^*. Thus, $N_c^*(N^*) > 0$ only if $N_c^*(N_s^*) > 0$, which is equivalent to:

$$\frac{\partial C(N_s^*, 0)}{\partial N_c} = \frac{N_s^* \cdot (N_s^* - 2N_0)}{k_t \cdot N_0^2} + b_g < 0, \tag{B.13}$$

where the above inequality results from (B.4) in Proposition 2. By using the supply function of for freelancers, i.e., (18), the above condition is rewritten as:

$$b_g < \mathbb{E}[r_0](2 - k_t\mathbb{E}[r_0]). \tag{B.14}$$

For the case where $N_s^* = N_0$, $N^* = N_0$ and $N_c^*(N^*) > 0$. Hence, (B.13) is equivalent to $b_g < \frac{1}{k_t}$. Since $\mathbb{E}[r_0] = \frac{1}{k_t}$ for $N_s^* = N_0$, the prerequisite condition (B.14) still holds. □

Proof of Proposition 4

Proposition 4 (Impact on effective wage)**.** *When the price sensitivity of freelancers k_t is greater than $\frac{2\overline{N}}{N_0} \cdot (\frac{dR_p(\overline{N})}{dN})^{-1}$, optimal dual sourcing reduces freelancers' expected effective wage. Otherwise, the introduction of dual sourcing will not necessarily harm freelancers' expected effective wage.*

Proof. Let N_s^* denote the optimal system supply under self-scheduling. With the case where $N_s^* = N_0$ or $N_c^*(N^*) = 0$, Proposition 4 stands automatically since the effective wage of freelancers remains the same. Thus, we focus on the case where $N_s^* < N_0$ and $N_c^*(N^*) > 0$ in the following proof.

For the optimal dual sourcing contract, freelancers' expected effective wage is given as

$$\mathbb{E}[r]^* = \min\left\{\frac{F^*}{2} \cdot \frac{\partial \mathbb{E}[q]}{\partial N},\ \bar{r}\right\} = \min\left\{\frac{1}{2} \cdot \frac{dR_p(N^*)}{dN},\ \bar{r}\right\},$$

where the above equation comes from Eqs. (18), and (B.10). When $k_t \geq \frac{2\overline{N}}{N_0} \cdot (\frac{dR_p(\overline{N})}{dN})^{-1}$, we have $N_s^* \geq \overline{N}$ with self-scheduling. Thus, the marginal revenue will decrease with system supply N (see Theorem 1) and we have:

$$\frac{dR_p(N_s^*)}{dN} > \frac{dR_p(N^*)}{dN}, \quad \mathbb{E}[r]^* < \mathbb{E}[r_0],$$

where the first inequality results from $N_c^*(N^*) > N_s^*$ according to Proposition 3. Thus, the optimal dual sourcing will reduce freelancers' expected effective wage for the case where $k_t \geq \frac{2\overline{N}}{N_0} \cdot (\frac{dR_p(\overline{N})}{dN})^{-1}$, $N_s^* < N_0$, and $N_c^*(N^*) > 0$.

For $k_t < \frac{2\overline{N}}{N_0} \cdot (\frac{dR_p(\overline{N})}{dN})^{-1}$, to prove the proposition, it is suffice to show that there exists (k_t, k_g, b_g) such that $\mathbb{E}[r]^* \geq \mathbb{E}[r_0]$. Thus, we take a special case where $k_t < 2\frac{\underline{N}}{N_0} \cdot \frac{dR_p(\underline{N})}{dN}$ such that $N_s^* < \underline{N}$. Since $\frac{dR_p(N)}{dN}$ is a continuous function of N, Theorem 1 indicates that there must exist $\hat{N} > N_s^*$ such that $\frac{dR_p(\hat{N})}{dN} = \frac{dR_p(N_s^*)}{dN}$ and $\frac{dR_p(N)}{dN} > \frac{dR_p(N_s^*)}{dN}$ for $N \in (N_s^*, \hat{N})$. Let $b_g \leq \mathbb{E}[r_0](2 - k_t\mathbb{E}[r_0])$ which ensures $N_c^*(N^*) \geq 0$. Under the optimal dual sourcing contract specified by (14), N^* satisfies $N^* \to \hat{N}$ if $k_g \to 0$ and $N^* \to N_s^*$ if $k_g \to \infty$. Since N^* is a continuous function of k_g, there must exist k_g such that $N^* = \underline{N}$. Since the marginal revenue increases with N for $N < \underline{N}$, we have:

$$\frac{dR_p(N_s^*)}{dN} < \frac{dR_p(N^*)}{dN}, \quad \mathbb{E}[r]^* > \mathbb{E}[r_0].$$

□

Appendix C. Numerical Experiments Parameters

Table C1
Parameters of numerical example.

Parameters	Values	Parameters	Values
k	0.02	α	30
β	100	η	1
l	0.3	N_0	2700
$\underline{\gamma}$	−7	$\overline{\gamma}$	7
τ	0	Δs	18
$\mathbb{E}[\epsilon]$	0	$\sigma[\epsilon]$	0.5
p_L	0.5	p_H	0.5
Q_L^0 (C1)	0.5×10^4	Q_H^0 (C1)	1×10^4
Q_L^0 (C2)	1×10^4	Q_H^0 (C2)	1.8×10^4

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