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The Price of Decentralization in Cooperative Coverage Problems with Energy-Constrained Agents

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***Abstract*— A multi-agent coverage problem is considered with energy-constrained agents, where a charging station is used to replenish an agent's energy as it becomes depleted while performing the coverage task. The objective of this paper is to compare the coverage performance between centralized and decentralized approaches. To this end, a centralized coverage control method is developed to switch agents between an optimal coverage formation and an optimal charging formation. We design a controller for agent trajectories that include dwell times at the optimal coverage locations and charging times at the charging station to maximize a coverage metric over a finite time interval. Our controller guarantees that at any time there is at most one agent leaving the team for energy repletion. We also derive a tight bound which allows us to quantify the gap between the coverage performance of the proposed strategy and the unknown globally optimal coverage performance.**

***Index Terms*— Centralized control, Energy efficiency, Multi-agent systems, Trajectory optimization**

I. INTRODUCTION

Systems consisting of cooperating mobile agents are often used to perform tasks such as coverage [1]–[5], surveillance [6], monitoring and sweeping [7]. In coverage tasks, agents interact with the mission space through their sensing capabilities which are normally dependent upon their physical distance from an event location. Outside its sensing range, an agent has no ability to detect events. The objective is to cooperatively maximize a measure of coverage over a given mission space [8] usually defined through the joint detection probability of random events [9]. Multi-agent coverage problems can be classified into static [10] and dynamic [1]. The static coverage problem concerns finding fixed locations for agents such that a performance metric for the area under coverage is maximized. Widely used approaches for solving this problem include Voronoi-partition-based gradient-descent algorithms [11]–[14] and submodularity-based [15] gradient-descent algorithms [16]. The dynamic coverage problem concerns planning trajectories for agents to maximize the coverage performance over time. This problem arises due to agents with limited sensing range [17], [18] or changing conditions in the mission space [1].

In most existing coverage problem settings, agents are assumed to have unlimited on-board energy to perform the coverage task. However, in practice, battery-powered agents can only work for a limited time in the field [19]. Developing distributed algorithms for multi-agent systems with energy constraints is considered in [20]–[22]. Unlike other multi-agent energy-aware algorithms in the aforementioned references whose purpose is to reduce energy cost, we assume that a charging station is available for agents to replenish their energy according to some policy. We take into account such energy constraints thus adding another dimension to the traditional static and dynamic coverage problems. The objective is to maximize an overall environment coverage measure by controlling the movement of all agents while also guaranteeing that no agents run out of energy in the mission space. Along these lines, closely related to our work is [23], the major difference being the need to manage contention between agents when competing for the charging station, whereas in [23] each agent has a dedicated station for charging.

A *decentralized* feasible solution to this problem is proposed in [24] via a hybrid system approach. Due to the decentralized nature of this algorithm, agents have limited local information. Therefore, the performance is degraded due to the information inaccessibility. This raises the question: what would be the "best" performance when all information is available? This motivates us to study the coverage problem via a centralized approach. The objectives of this paper are to find a *centralized* solution to multi-agent coverage problems, to quantify its performance gap to the globally optimal coverage performance,

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and to characterize the "price of decentralization" with respect to the decentralized algorithm proposed in [24]. However, the decentralized algorithm proposed in [24] assumes that the energy consumption of sensing is zero. To include the energy costs of sensing, a modified algorithm is proposed based on [24], which allows agents turning off their sensing to preserve energy. To this end, we assume that the environment to be monitored is completely known. Then, the *optimal coverage* (OCV) locations of the agents while none of them needs recharging can be found through the distributed gradient-based algorithm [1]. When the battery level of an agent is low, the agent will head to the charging station. If the agent still performs the coverage task at the charging station, the optimal locations for the remaining agents can be found using the aforementioned approach. The optimal locations for all agents in this case are referred to as "*optimal charging* (OCH) formation". Therefore, every agent's behavior is to switch between the OCV formation and the OCH formation. However, finding the optimal trajectories of all agents during transitions between two different formations turns out to be a challenging task. To reduce the transient time between switches, a Traveling Salesman Problem (TSP) is solved to find the shortest total distance if an agent traverses all locations in both the OCV and OCH formations. The solution from the TSP dictates the order of locations being visited by any agent. Next, when the switching times of all agents are synchronized, that is, all agents leave the OCV formation at the same time and arrive at the OCH formation at the same time, the objective becomes minimizing the transient time and the energy cost during that time. Therefore, the transient time is determined by the agent which travels the longest distance. The speeds of other agents can be determined by the transient time and the travel distance. A performance gap is obtained to characterize how good the proposed algorithm is when compared with the globally optimal performance which is generally unknown and very challenging to determine. Our simulation results show that the centralized approach improves the coverage performance compared to the decentralized one in [24].

Compared with the preliminary version of this work in [25], the contributions of this paper are as follows:

- A modified decentralized coverage algorithm based on [24] is described in Section III.
- The shortest path problem in Section IV-B is mathematically formulated and the solution is provided by using the optimization toolbox in MATLAB.
- Theorem 1 in Section IV-C is established so as to apply to a general energy depletion model.
- Theorem 2 in Section V derives a bound for the globally optimal solution and formally shows that the performance of the proposed algorithm is guaranteed to be within a certain fraction of the optimal performance.

II. Problem Formulation

Consider a bounded mission space $\mathcal{S} \in \mathbb{R}^2$. The value of a point $(x, y) \in \mathcal{S}$ in the mission space is characterized by a reward function $R(x, y)$, where $R(x, y) \geq 0$ and $\int\int_{\mathcal{S}} R(x, y)dxdy < \infty$. The value of $R(x, y)$ is monotonically increasing in the importance associated with the point (x, y). If all points in $\mathcal{S}$ are treated indistinguishably, $R(x, y) = \sigma$ for any $(x, y) \in \mathcal{S}$, where $\sigma > 0$ is a constant. A team of mobile agents labeled by $\mathcal{V} = \{1, 2, \ldots, N\}$ is deployed in the mission space to collect the rewards. Each agent has an isotropic sensing system with range δ_i, that is, an agent located at (x_i, y_i) is able to collect all rewards of the points in its sensing range

$$\Omega_i(x_i, y_i) = \left\{(x, y) \,|\, (x - x_i)^2 + (y - y_i)^2 \leq \delta_i^2\right\}.$$

The ability of an agent covering a point (x, y) within its sensing range $\Omega_i(x_i, y_i)$ is characterized by the sensing function $p_i(x, y, x_i, y_i) \in [0, 1]$, and it depends on the distance between the agent location (x_i, y_i) and the point (x, y). In particular, it is monotonically decreasing in the distance between (x_i, y_i) and (x, y) and if a point (x, y) is out of the sensing range of agent i, that is, $(x, y) \notin \Omega_i(x_i, y_i)$, then $p_i(x, y, x_i, y_i) = 0$. For any given point (x, y) in the sensing range of multiple agents, the joint sensing capability is given by [1]

$$P(x, y, \mathbf{s}) = 1 - \prod_{i=1}^{N} \left[1 - p_i(x, y, x_i, y_i)\right]. \tag{1}$$

The form of the function $p_i(x, y, x_i, y_i)$ does not affect our subsequent analysis.

Remark 1: The sensing functions

$$p_i(x, y, x_i, y_i) = 1 - \frac{(x - x_i)^2 + (y - y_i)^2}{\delta_i^2}, \tag{2}$$

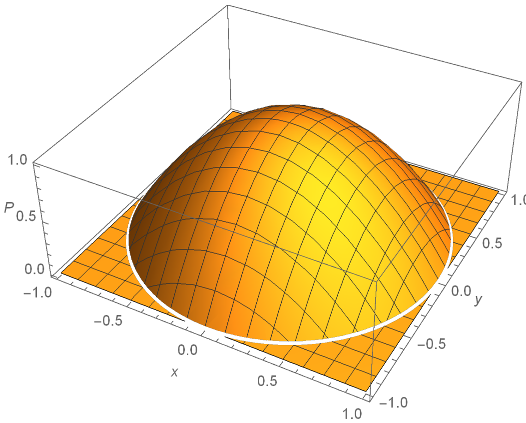
and $p_i(x, y, x_i, y_i) = \alpha_i \exp\left[-\beta_i \sqrt{(x - x_i)^2 + (y - y_i)^2}\right]$ where $0 < \alpha_i \leq 1$ and $\beta_i > 0$ are sensing parameters, were used in [24] and [1], respectively, for all $(x, y) \in \Omega_i$. For illustration purposes, Fig. 1 depicts the sensing capabilities of a single agent (Fig. 1a) and two agents with overlapping sensing range (Fig. 1b) for the sensing function (2). Here the sensing range of agents is set to $\delta_i = 1$ for all is.

Finally, the coverage performance of the mobile agent team over the area $\mathcal{S}$ is defined as

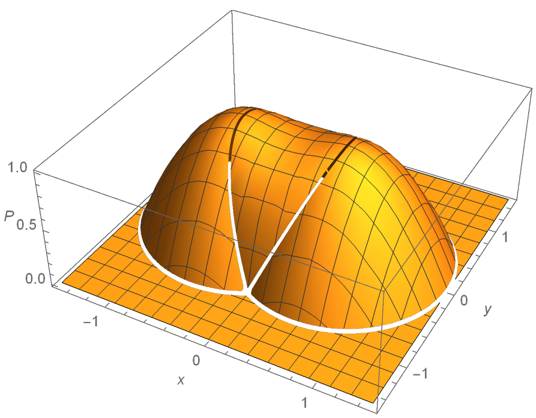
$$H(\mathbf{s}) = \int\int_{\mathcal{S}} R(x, y) P(x, y, \mathbf{s}) dxdy, \tag{3}$$

where $\mathbf{s} = (s_1, \ldots, s_N) \in \mathbb{R}^{2N}$ with $s_i = (x_i, y_i)$ contains all agent positions. Note that $H(\mathbf{s})$ is a function mapping $\mathbf{s} \in \mathbb{R}^{2N}$ into $\mathbb{R}$.

To find the optimal locations of all agents is a static optimization problem, which has been extensively studied [1], [11], [16]. Here we are interested in a *dynamic* coverage control problem with energy constraints, where each agent is associated with two state variables: location variable $s_i(t)$ and state-of-charge (SOC) variable $0 \leq q_i(t) \leq 1$, which captures the fraction of available energy (battery level) at time t. The agents' sensing and motion activities are all powered by batteries. The binary variable $b_i(t) \in \{0, 1\}$ controls the sensing of agent i, where $b_i(t) = 1$ and $b_i(t) = 0$ indicate "on" and "off" of the sensing functionality, respectively. There is a charging station at $(0, 0)$ for an agent to replenish its energy. The binary variable $I_i(t) \in \{0, 1\}$ indicates that agent i is in-charging or not, where $I_i(t) = 1$ and $I_i(t) = 0$ means that the agent is in *charging mode* and *energy depletion mode*,



(a) A single agent at (0,0)



(b) Two agents at (0.5,0) and (-0.5, 0)

Fig. 1: Sensing capability over an area with one and two agents performing coverage.

respectively. We assume that there is only one outlet in the charging station. In other words, only one agent can be served at any time. The agent's motion is described by the following kinematic equations:

$$\dot{x}_i(t) = v_i(t)\cos[\theta_i(t)], \quad \dot{y}_i(t) = v_i(t)\sin[\theta_i(t)] \tag{4}$$

where $v_i(t)$ and $\theta_i(t)$ denoting the instantaneous speed and heading of agent i at time t, respectively, are two independent control inputs. The speed $v_i(t) \in [0, \bar{v}]$, where $\bar{v}$ is the maximum speed of an agent.

Assume that all agents are of the same type. The SOC state satisfies the following dynamical equation:

$$\begin{aligned}\dot{q}_i(t) =& I_i(t) f(q_i(t), b_i(t)) \\ &+ (1 - I_i(t)) g(q_i(t), v_i(t), b_i(t))\end{aligned} \tag{5}$$

where $f(q_i(t), b_i(t)) \geq 0$, and $g(q_i(t), v_i(t), b_i(t)) \leq 0$. Moreover, $g(q_i(t), v_i(t), b_i(t)) = 0$ when $v_i(t) = 0$ and $b_i(t) = 0$. In other words, an agent is in energy conservation mode if there is neither motion nor sensing. Let us assume $q_i(t') = q$ and $b_i(t) = 1$ for $t \in [t', t' + \tau]$. The solutions to (5) when $I_i(t) = 1$ for $t \in [t', t' + \tau]$ can be parameterized as

$$q_i(t' + \tau) = q + \kappa(q, \tau); \tag{6}$$

The solutions to (5) when $I_i(t) = 0$ for $t \in [t', t' + \tau]$ can be parameterized as

$$q_i(t' + \tau) = q - h(q, \tau, d), \tag{7}$$

where d is the travel distance.

Our objective is to maximize the coverage of the mission space $\mathcal{S} \in \mathbb{R}^2$ over a time interval $[0, T]$, and at the same time to keep all agents alive, that is, $q_i(t) \geq 0$ for all $t \in [0, T]$. The case $q_i(t) = 0$ can occur only at the charging station $(0, 0)$. The problem of multi-agent coverage with energy-constrained agents (MACECA) can be mathematically formulated as the following optimization problem:

$$\max_{v(t),\, \theta(t),\, b(t)} \frac{1}{T} \int_0^T H(\mathbf{s}(t))\, dt \tag{8}$$

$$\text{s.t.} \quad (4) \text{ and } (5) \tag{9}$$

$$I_i(t) = 1 - \text{sgn}(\|s_i(t)\|), \tag{10}$$

$$0 \leq v_i(t) \leq \bar{v}, \; 0 \leq q_i(t) \leq 1 \tag{11}$$

$$b_i(t) \in \{0, 1\}, i = 1, \ldots, N, \tag{12}$$

$$0 \leq \sum_{i=1}^{N} I_i(t) \leq 1, \tag{13}$$

where $v(t) = [v_1(t), \ldots, v_N(t)]^T$, $\theta(t) = [\theta_1(t), \ldots, \theta_N(t)]^T$, $b(t) = [b_1(t), \ldots, b_N(t)]^T$, $\|s_i(t)\| = \sqrt{x_i^2(t) + y_i^2(t)}$, and the coverage metric $H(\mathbf{s}(t))$ is defined in (3). The constraints (10) indicate that an agent is in charging mode whenever it arrives at the charging station; (13) ensures that only one agent can be served at the charging station at any time.

III. Review of the Decentralized Solution in [24]

A decentralized modeling and control approach was proposed in [24] to solve the MACECA problem. Here we modify the algorithm proposed in [24] in order to consider the sensing cost which was ignored.

For any agent, we define three different modes: coverage (Mode 1), to-charging (Mode 2) and in-charging (Mode 3). A hybrid system is constructed to model the transitions between different modes of each agent: Mode 1→Mode 2→Mode 3→Mode 1 as shown in Fig. 2, and a transition to a different mode occurs when the guard conditions labeling each arrow are satisfied. The details will be given below.

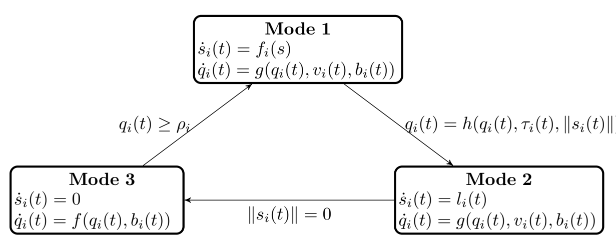


Fig. 2: A hybrid system model

At Mode 1, $v_i(t) = \bar{v}\,\text{sgn}\,\|\nabla_i H(\mathbf{s}(t))\|$, where the gradient $\nabla_i H(\mathbf{s}(t))$ with respect to s_i is a column vector defined as

$$\nabla_i H(\mathbf{s}(t)) = [\partial H(\mathbf{s}(t))/\partial x_i(t) \quad \partial H(\mathbf{s}(t))/\partial y_i(t)]^T.$$

Detailed expressions of $\nabla H_i(\mathbf{s}(t))$ can be found in [1]. To ease notation, $f_i(\mathbf{s})$ in Fig. 2 is given by $f_i(\mathbf{s}) = [v_i(t)\cos\theta_i(t) \quad v_i(t)\sin\theta_i(t)]^T$, and

$$\theta_i = \arctan\left(\frac{\partial H(\mathbf{s}(t))/\partial y_i(t)}{\partial H(\mathbf{s}(t))/\partial x_i(t)}\right).$$

In other words, agent i travels at the maximum speed when the gradient is nonzero, and the heading direction follows the gradient direction of the coverage metric with respect to agent i's location. Once the gradient is zero, the instantaneous velocity of agent i is set to zero. The SOC of the battery monotonically decreases with rate $g(q_i(t), v_i(t), b_i(t))$ and when it drops to a certain value, the agent switches to Mode 2.

A transition from Mode 1 to Mode 2 occurs when the SOC satisfies the following equality $q_i(t) = h\left(q_i(t), \tau(t), \|s_i(t)\|\right)$, where $\tau_i(t) = \|s_i(t)\|/\bar{v}$. The function $h(q_i(t), \tau_i(t), \|s_i(t)\|)$ defined in (7) is the minimum energy requirement for agent i with the SOC $q_i(t)$ to arrive at the charging state from agent i's current location $s_i(t)$ using the maximum speed $\bar{v}$. At Mode 2, an agent may need to turn off its sensing and motion functionalities to conserve energy. This is determined by the scheduling algorithm used to assign the priority of an agent to the charging station. The heading direction is constant and determined by the location of agent i at the time of switching from Mode 1 to Mode 2, say τ_2. Then, $l_i(t)$ in Fig. 2 is defined as $l_i(t) = -\bar{v}\left[x_i(\tau_2)/\|s_i(\tau_2)\| \quad y_i(\tau_2)/\|s_i(\tau_2)\|\right]^T$.

A transition from Mode 2 to Mode 3 occurs when agent i arrives at the charging station, that is, $\|s_i(t)\| = 0$. At Mode 3, an agent remains at rest at the charging station. Therefore, the motion dynamics satisfy $\dot{x}_i(t) = 0$, $\dot{y}_i(t) = 0$. While the agent is in the charging mode, the SOC dynamics are given by $\dot{q}_i(t) = f(q_i(t), b_i(t))$.

Finally, a transition from Mode 3 to Mode 1 occurs when $q_i(t) = \rho_i$, where $\rho_i \in (0, 1]$ is a controllable threshold parameter indicating the desired SOC at which the agent may stop its recharging process. The result in [24] indicates that fully charging $\rho_i = 1$ is the optimal policy.

Since the charging station can only serve one agent at a time, a scheduling algorithm is needed to resolve conflicts among agents competing over access to it. Here, we use the First-Request-First-Serve (FRFS) scheduling policy as an example. Other scheduling policies are also possible. Suppose that when agent i sends a charging request at τ_r^i, the charging station is not reserved. Then, agent i will use the maximum speed $\bar{v}$ to reach the charging station. If another agent j sends a charging request at $\tau_r^j > \tau_r^i$, the arrival time of agent j will be scheduled at $\max\{\tau_f^i, \tau_a^j\}$, where τ_f^i is the time when agent i finishes charging, and τ_a^j is the arrival time of agent j at the charging station using the maximum speed. There are two different cases: $\tau_f^i \leq \tau_a^j$ and $\tau_f^i > \tau_a^j$. For the former case, there are no conflicts between agents i and j. This is because when agent j arrives at the charging station using the maximum speed, agent i has already left the charging station. For the latter case, agent j needs to turn off its sensing and motion functionalities, and resume its sensing and motion functionalities at $\tau_f^i - \left\|s_j(\tau_r^j)\right\|/\bar{v}$. Therefore, agent j will arrive at the charging station right after agent i finishes charging. It is straightforward to extend the case of two agents to the case of multiple competing agents.

IV. Main Results

The decentralized approach in [24] solves the MACECA problem from an individual agent point of view. Without a centralized coordination, agents may need to turn off their sensing, that is, $b_i(t) = 0$, when they compete for the charging station. However, a centralized approach can solve this problem from a team point of view. It can ensure that agents never stop sensing the environment to perform the coverage task, that is, $b_i(t) = 1$ for any $t \geq 0$, relying on centralized coordination. This shows one aspect of the "price of decentralization".

We would ultimately like to maximize the coverage level in (8) and minimize transient times that occur between the OCV and OCH formations. This comes down to solving the following problems:

1) In Section IV-A, find the OCV and OCH locations for all agents.
2) In Section IV-B, solve a TSP to get an optimal path connecting all OCV and OCH locations found in Section IV-A.
3) In Section IV-C, solve for the optimal speed problem over transient intervals.
4) In Section IV-D, establish problem schedulability.
5) In Section IV-E, maximize the coverage performance by optimizing dwell and charging times based on the optimal speed profile found in Section IV-C and the schedulability condition found in Section IV-D.

A. Optimal Locations

Let us assume that the environment is known, that is, $R(x, y)$ is known. The OCV locations can be obtained by solving the static optimal coverage problem

$$\max_{\mathbf{s}} H(\mathbf{s}),$$

in which the optimal solution can be determined by using the standard gradient algorithm [1]. Let us denote the OCV locations of N agents as $\Phi = (\phi_1, \ldots, \phi_N)$ in the mission space $\mathcal{S}$.

By assuming that an agent also performs the coverage task while resting at the charging station because we set $b_i(t) = 1$ for all $t \geq 0$, we can calculate the OCH locations of the remaining $N-1$ agents by constraining one agent at point $(0, 0)$. The OCH locations can be found by solving the following optimization problem

$$\max_{\mathbf{s}} H(\mathbf{s}) \tag{14}$$

$$\text{s.t.} \quad s_1 = (0, 0), \tag{15}$$

where s_1 are constrained to the origin. Again, the OCH locations can be found using the gradient method proposed in [1]. Let $\Psi = (\psi_1, \ldots, \psi_{N-1}, \psi_N)$ be the OCH locations with $\psi_1 = (0, 0)$. When all agents have enough energy, the optimal choice is to occupy all locations at Φ. When an agent is at the charging station, the optimal choice for all other agents is at the locations specified by Ψ. Whenever an agent leaves or re-joins the team, the agents switch between Φ and Ψ. The reader can refer to Fig. 3 for an example with $N=3$.

A reasonable question to ask is whether switching between the OCV and OCH formations is preferable compared to any policy which prevents such a switch because of the extra cost of agent motion involved. It is hard to answer this question a priori since it depends on many factors, such as the transient time, the SOC dynamics, the sensing capability function, the reward function, etc. A posteriori evaluation of the switching policy can be done after operating the system for some time and observing its performance. Here we adopt the switching policy and note that its performance is ensured by the performance gap established in Section V. Roughly speaking, the switching policy is preferable when the charging process is slow and the resulting coverage performance of the OCH formation $H(\Psi)$ is significantly better than any policy which prevents switching.

B. Shortest Path

Recall that the scheduling algorithm is used in the decentralized approach in [24] to solve the conflicts among agents competing for the charging station. Here, we take a conflict-free scheduling approach by letting agents take turns to visit the charging station, which shows another benefit of centralized coordination, that is, the price of decentralization. We define a cycle as all agents returning to their original OCV locations after $2N$ switches between the OCV and OCH formations, and each agent visiting the charging station exactly once. To minimize the total travel distance of all agents is equivalent to solve $2N$ coupled balanced assignment problems with the constraint that all agents visit the charging station exactly once during $2N$ switches. A naive solution is to check all possibilities, and calculate the cost of each one. The time complexity of this solution is $\mathcal{O}\big((N-1)!^N N!^{N+1}\big)$. Given this computational complexity, a complete globally optimal solution is intractable. Instead, to bypass this complexity, we limit ourselves to a subset which conforms to a TSP. In this way, the time complexity is reduced to $\mathcal{O}\left(N!^2\right)$. Even though the TSP solution may not be optimal, note that we are able to quantify the overall performance gap in Section V. The TSP solution corresponds to the case of two fixed and bijection mappings between the OCV and OCH formations. Thus, we can use a bipartite graph to model such constraints. The definitions of bipartite graphs and complete bipartite graphs can be found in [26]. As the agents switch between the formations, we need to minimize the total traveled distance during transient times. Let the bipartite graph $\mathcal{K}_N^2=(\mathcal{V},\mathcal{E})$ denote the underlying topology, where the vertex set $\mathcal{V}$ can be partitioned into two sets: $\mathcal{V}_1=\Phi$ and $\mathcal{V}_2=\Psi$ such that $\mathcal{V}_1\cup\mathcal{V}_2=\mathcal{V}$, $\mathcal{V}_1\cap\mathcal{V}_2=\emptyset$ and $|\mathcal{V}_1|=|\mathcal{V}_2|=N$. Every edge in $\mathcal{V}$ has one end in $\mathcal{V}_1$ and the other end in $\mathcal{V}_2$ and vertices in the same set are not adjacent. The reader can refer to Fig. 3 for the sets $\mathcal{V}_1$ and $\mathcal{V}_2$. In addition, $\mathcal{K}_N^2$ is complete, that is, every two vertices from different sets are adjacent. The weight of every edge is the distance between the two vertices. Let us use $d_{s_i}^{s_j}$ to denote the distance from location s_i to location s_j.

Finding the shortest transitional distance is equivalent to finding the shortest path in the graph $\mathcal{K}_N^2$. Define $c_{ij}=1$ if there is a link from location s_i to location s_j; otherwise $c_{ij}=0$. Then, the TSP can be written as the following integer linear programming problem:

$$\begin{aligned}
\min_{c_{ij}}\ & \sum_{s_i\in\mathcal{V}_1}\sum_{s_j\in\mathcal{V}_2} c_{ij}d_{s_i}^{s_j}+\sum_{s_j\in\mathcal{V}_2}\sum_{s_i\in\mathcal{V}_1} c_{ji}d_{s_j}^{s_i}\\
\text{s.t.}\ \ & c_{ij}\in\{0,1\}, c_{ji}\in\{0,1\}, s_i\in\mathcal{V}_1, s_j\in\mathcal{V}_2\\
& \sum_{s_i\in\mathcal{V}_1} c_{ij}=1, s_j\in\mathcal{V}_2 && (16)\\
& \sum_{s_j\in\mathcal{V}_2} c_{ji}=1, s_i\in\mathcal{V}_1 && (17)\\
& \sum_{s_i\in\mathcal{W}_1}\sum_{s_j\in\mathcal{W}_2} c_{ij}\le|\mathcal{W}_1|+|\mathcal{W}_2|-1, && (18)\\
& \sum_{s_j\in\mathcal{W}_2}\sum_{s_i\in\mathcal{W}_1} c_{ji}\le|\mathcal{W}_1|+|\mathcal{W}_2|-1, && (19)\\
& \forall\mathcal{W}_1\subsetneq\mathcal{V}_1,\mathcal{W}_2\subsetneq\mathcal{V}_2.
\end{aligned}$$

Here c_{ij} and c_{ji} are binary variables. The equality constraints (16) and (17) enforce that for every location $s_i\in\mathcal{V}_1$ or $s_j\in\mathcal{V}_2$, there is only one path departing from it. The inequality constraints (18) and (19) ensure that there are no subtours. The TSP is an NP-hard problem in combinatorial optimization. We use the binary integer programming `intlinprog` solver with `round-diving` heuristics in the MATLAB optimization toolbox for the above optimization problem. We note that there are several other solvers for such binary integer programming problems, such as IBM CPLEX Optimizer, Gurobi Optimizer, `LPSolve` in Maple, just to name a few.

C. Optimal Speed

Here we will derive the most energy efficient speed profile parameterized by the travel time and distance of a transitional segment of an agent trajectory determined in the previous section. Therefore, we assume that the travel time τ, the starting location $\underline{s}$ and final location $\bar{s}$ are given. Then, the following optimization problem is formulated:

$$\begin{aligned}
\min_{v_i(t),\,\theta_i(t)}\ & \int_{t_0}^{t_0+\tau} -\dot{q}_i(t)dt && (20)\\
\text{s.t.}\ \ & (4) \text{ and } (5) && (21)\\
& 0\le v_i(t)\le\bar{v} && (22)\\
& s_i\left(t_0\right)=\underline{s},\quad s_i\left(t_0+\tau\right)=\bar{s},\quad q_i\left(t_0\right)=\underline{q}, && (23)
\end{aligned}$$

where $\underline{q}$ is the initial SOC of agent i. Let us assume that the optimal speed problem is feasible, that is, $\|\underline{s}-\bar{s}\|\le\tau\bar{v}$. When the detailed form of (5) is given, a numerical solution of the above optimization problem can be calculated. In the following, we show a case where the analytical solution is available for a particular class of energy depletion models in which the function g in (5) is independent of the SOC q_i.

Theorem 1: Assume that the energy depletion model in (5) satisfies

$$\frac{\partial g(q_i,v_i,1)}{\partial q_i}=0. \tag{24}$$

Then, the optimal solutions to the above optimization problem are $v^*(t) = \|\bar{s} - \underline{s}\|/\tau$ and $\theta^*(t) = \angle\bar{s} - \underline{s}$ for $t \in [t_0, t_0 + \tau)$, where $\angle\bar{s} - \underline{s}$ is the heading from $\underline{s}$ to $\bar{s}$.

Proof: The Hamiltonian function and Lagrangian function are defined as

$$\begin{aligned}\mathcal{H}(s_i, v_i, \theta_i, q_i, t) = &-\dot{q}_i + \lambda_x v_i \cos(\theta_i) + \lambda_y v_i \sin(\theta_i)\\ &+ \lambda_q g(q_i, v_i, 1),\end{aligned}$$

and

$$\mathcal{L}(s_i, v_i, \theta_i, q_i, t) = \mathcal{H}(s_i, v_i, \theta_i, q_i, t) - \eta_1 v_i + \eta_2(v_i - \bar{v}),$$

where $\eta_1 \geq 0$, $\eta_2 \geq 0$, and $-\eta_1 v_i + \eta_2(v_i - \bar{v}) = 0$. We have the co-state equations: $-\dot{\lambda}_x = \partial\mathcal{L}/\partial x_i = 0$, and $-\dot{\lambda}_y = \partial\mathcal{L}/\partial y_i = 0$.

Therefore, we know that λ_x and λ_y are two constants. From the stationarity condition, we have

$$\frac{\partial\mathcal{L}}{\partial\theta_i} = -\lambda_x v_i \sin\theta_i + \lambda_y v_i \cos\theta_i = 0.$$

Then, we know that θ_i is also a constant determined by the initial and final positions. Let $\lambda_x = \lambda_\theta \cos\theta_i$ and $\lambda_y = \lambda_\theta \sin\theta_i$ with a constant λ_θ. Thus, the Hamiltonian function becomes

$$\mathcal{H}(s_i, v_i, \theta_i, q_i, t) = (\lambda_q - 1)g(q_i, v_i, 1) + \lambda_\theta v_i.$$

Then, we have $-\dot{\lambda}_q = \partial\mathcal{H}/\partial q_i = (\lambda_q - 1)\partial g(q_i, v_i, 1)/\partial q_i$. Based on the condition (24), we know that λ_q is a constant. Since $q_i(t_0 + \tau)$ is not fixed, it is required that $\lambda_q = 0$. Since $\mathcal{H}$ is not an explicit function of time t, we have $\mathcal{H} = 0$. Then, taking the time derivative of $\mathcal{H}$ yields

$$\dot{\mathcal{H}} = -\frac{\partial g}{\partial v_i}\dot{v}_i + \lambda_\theta \dot{v}_i = \left(\lambda_\theta - \frac{\partial g}{\partial v_i}\right)\dot{v}_i, \tag{25}$$

which leads to $\dot{v}_i(t) = 0$. Therefore, v_i is a constant determined by the distance between the initial and final positions and the travel time τ. ■

The energy cost can be determined once a detailed energy depletion model is given. Consider the energy depletion model $g(q_i(t), v_i(t), 1) = -\alpha v_i(t) - \beta$ used in [25], where $\alpha > 0$ and $\beta > 0$ are two constants. These two terms can be regarded as the motion and sensing energy costs, respectively. In this case, the energy cost of traveling a distance d in time τ is $\alpha d + \beta\tau$. Therefore, to reduce the transient time, it is always optimal for agents who travel the longest distance during the transient times to use the maximum speed when the energy consumption model is a linear function of the speed. For the alternative energy depletion model $g(q_i(t), v_i(t), 1) = -\alpha v_i^2(t) - \beta$ used in [24], the least energy cost of traveling a distance of d in time τ is $\alpha d^2/\tau + \beta\tau$. From the energy efficient point of view, the optimal traveling time is $d\sqrt{\alpha/\beta}$ with $v^* = \sqrt{\beta/\alpha}$ if $\bar{v} \geq \sqrt{\beta/\alpha}$. If $\bar{v} < \sqrt{\beta/\alpha}$, the maximum speed is optimal.

D. Schedulability

Schedulability in this case means that when an agent switches to "to-charge" mode, no other agents will switch to this mode until the agent returns and the OCV formation is attained. We will find a condition to guarantee that this holds at all times. We assume that the behavior of all agents is synchronized, that is, they start and finish the process of switching from Φ to Ψ at the same time, and vice versa (i.e., from Ψ to Φ). The intuition behind this assumption is that the coverage performance depends on the agents' relative distances. Then, the problem reduces to finding four critical times: (1) the charging time τ_c at the charging station, (2) the dwell time τ_d of agents on the OCV locations, (3) the transient time τ_t^{N-1} from the OCV locations to the OCH locations, and (4) the transient time τ_t^N from the OCH locations to the OCV locations. Note that the dwell time of agents on the OCH locations is exactly equal to the charging time at the charging station.

Without loss of generality, let us label the optimal path of the TSP solution to visit all OCV and OCH locations as: $0 \to 1 \to 2 \to 3 \to \cdots 2N - 1 \to 0$, where the nodes with odd numbers belong to the OCV locations, the nodes with even numbers belong to the OCH locations, and node 0 is the charging station. Define $d_i^{i+1 \pmod{2N}}$ as the distance between node i and node $i + 1 \pmod{2N}$ for $i = 0, 1, \ldots, 2N - 1$, where $\bmod$ is the modulo operation. The idea is to reduce the transient time using the maximum speed, which is determined by the agent traveling the longest distance, that is,

$$\tau_t^N = \frac{\bar{d}^N}{\bar{v}}, \qquad \tau_t^{N-1} = \frac{\bar{d}^{N-1}}{\bar{v}}, \tag{26}$$

where

$$\bar{d}^N = \max_{i=0,1,\ldots,N-1} d_{2i}^{2i+1}, \quad \bar{d}^{N-1} = \max_{i=1,\ldots,N} d_{2i-1}^{2i \pmod{2N}}.$$

Let us define μ_i^- and μ_i^+ as the energy when agents arrive at node i and leave node i, respectively, and index each cycle by an integer k. Assume that each cycle starts with 0, and the initial SOC at cycle k is $\mu_0^-(k)$. Clearly, we must have $\mu_0^-(k) \geq 0$ to make the problem schedulable. Let us proceed forward. After the charging time τ_c, the SOC increases to

$$\mu_0^+(k) = \mu_0^-(k) + \kappa(\mu_0^-(k), \tau_c), \tag{27}$$

where κ is defined in (6) with the initial condition $\mu_0^-(k)$. Then, the agent leaving the charging station will be in the energy depletion mode until the next cycle $k + 1$. The SOC decreases to $\mu_1^-(k) = \mu_0^+(k) - h(\mu_0^+(k), \tau_t^N, d_0^1)$ when this agent arrives at location 1; the SOC decreases to

$$\mu_1^+(k) = \mu_1^-(k) - h(\mu_1^-(k), \tau_d, 0),$$

when the agent leaves location 1, where h is defined in (7), and the third argument of h is 0 because the agent does not move during this time. In general, the SOCs before and after location $2i + 1$ are

$$\begin{aligned}\mu_{2i+1}^-(k) &= \mu_{2i}^+(k) - h\left(\mu_{2i}^+(k), \tau_t^N, d_{2i}^{2i+1}\right),\\ \mu_{2i+1}^+(k) &= \mu_{2i+1}^-(k) - h\left(\mu_{2i+1}^-(k), \tau_d, 0\right)\end{aligned} \tag{28}$$

for $0 \leq i \leq N - 1$ and the SOCs before and after location $2i$ are

$$\begin{aligned}\mu_{2i}^-(k) &= \mu_{2i-1}^+(k) - h(\mu_{2i-1}^+(k), \tau_t^{N-1}, d_{2i-1}^{2i})\\ \mu_{2i}^+(k) &= \mu_{2i}^-(k) - h\left(\mu_{2i}^-(k), \tau_c, 0\right)\end{aligned} \tag{29}$$

for $1 \leq i \leq N - 1$.

If this process is repeated recursively until the next cycle $k+1$, the initial SOC of the $k+1$ cycle is

$$\mu_0^-(k+1) = \mu_{2N-1}^+(k) - h\left(\mu_{2N-1}^+, \tau_t^{N-1}, d_{2N-1}^0\right).$$

Using all the above equations, the relationship between $\mu_0^-(k+1)$ and $\mu_0^-(k)$ can be established and can be written in the form

$$\mu_0^-(k+1) = \mathcal{F}\left(\mu_0^-(k), \bar{v}, \tau_d, \tau_c, d_0^1, d_1^2, \ldots, d_{2N-1}^0\right)$$

where $\mathcal{F}$ is a function of d_0^1, $d_1^2, \ldots, d_{2N-1}^0$, $\mu_0^-(k)$, $\bar{v}$, τ_d, and τ_c. To make the problem schedulable, the condition

$$\mu_0^-(k+1) \geq \mu_0^-(k) \geq 0$$

for $k = 0, 1, \ldots,$ must be satisfied.

To find the minimum SOC in the worst case where there are no agents dwelling at the OCV locations, we set $\tau_d = 0$. Given $d_i^{i+1(\bmod 2N)}$ for $i = 0, 1, \ldots, 2N-1$, τ_t^N, τ_t^{N-1}, and $\tau_d = 0$, to find the minimum initial SOC, the following optimization problem can be solved

$$\min_{\tau_c} \quad \mu \tag{30}$$

$$\text{s.t.} \quad \mu \geq \mathcal{F}(\mu, \bar{v}, 0, \tau_c, d_0^1, d_1^2, \ldots, d_{2N-1}^0). \tag{31}$$

This optimization problem aims to find a minimum charging time so that the SOC of an agent does not decrease after one cycle. Only if a solution to (30) and (31) exists we can further maximize the dwell time τ_d. Once the minimum μ is obtained, we can calculate the minimum SOC requirements for all locations.

E. Optimal Dwell Time and Charging Time

Once the problem in (30) and (31) is determined to be feasible, the remaining task is to maximize the average coverage performance. Since the agent trajectories are fixed during transition, the average coverage performance can be determined. The average coverage performance from OCV to OCH, and from OCH to OCV, is denoted by $J_{\Phi\Psi}$ and $J_{\Psi\Phi}$, respectively. Thus, we maximize the average coverage performance during a total cycle defined as $\tau_T = \tau_c + \tau_d + \tau_t^N + \tau_t^{N-1}$ (which provides maximum coverage):

$$\max_{\tau_c, \tau_d} \quad \frac{\tau_d}{\tau_T} H(\Phi) + \frac{\tau_c}{\tau_T} H(\Psi) + \frac{\tau_t^{N-1}}{\tau_T} J_{\Phi\Psi} + \frac{\tau_t^N}{\tau_T} J_{\Psi\Phi} \tag{32}$$

$$\text{s.t.} \quad \mu + \kappa(\mu, \tau_c) \leq 1 \tag{33}$$

$$\mathcal{F}\left(\mu, \bar{v}, \tau_d, \tau_c, d_0^1, d_1^2, \ldots, d_{2N-1}^0\right) \leq \mu. \tag{34}$$

The first constraint requires an agent to leave the charging station once its battery is fully charged. The second constraint ensures that the charging time and the dwell time must satisfy the schedulability constraint.

V. Performance Bound

In this section, we characterize the performance gap between the proposed solution and the optimal solution to the problem (8)-(13) even though the optimal solution still remains unknown.

Theorem 2: The coverage performance of the proposed solution is guaranteed to be within

$$\frac{\tau_d H(\Phi) + \tau_c H(\Psi) + \tau_t^{N-1} J_{\Phi\Psi} + \tau_t^N J_{\Psi\Phi}}{\tau_T H(\Phi)} \tag{35}$$

of the optimal performance J^*.

Proof: The performance (8) can be upper bounded by the performance corresponding to the OCV locations, that is,

$$\frac{1}{T}\int_0^T H(\mathbf{s}(t))dt \leq J^* \leq \frac{1}{T}\int_0^T H(\Phi)dt = H(\Phi)$$

since $H(\Phi)$ is the best possible performance.

Let us use J to denote the coverage performance of our proposed algorithm. Then the following inequalities hold:

$$J \leq J^* \leq H(\Phi).$$

From the above inequalities, we can obtain

$$\frac{J}{H(\Phi)} \leq \frac{J}{J^*} \tag{36}$$

Let us assume that there exists an integer k such that $T = k\tau_T$. The performance of our proposed algorithm is

$$J = \frac{1}{T}\int_0^T H(\mathbf{s}(t))\,dt = \frac{1}{k\tau_T}\sum_{i=0}^{k-1}\int_{i\tau_T}^{i\tau_T+\tau_T} H(\mathbf{s}(t))\,dt.$$

Each cycle can be partitioned into four parts: τ_c, τ_d, τ_t^N, and τ_t^{N-1}. The performance of our proposed algorithm in each cycle can be written as

$$\int\limits_{i\tau_T}^{i\tau_T+\tau_T} H(\mathbf{s}(t))\,dt = \tau_d H(\Phi) + \tau_c H(\Psi) + \tau_t^{N-1} J_{\Phi\Psi} + \tau_t^N J_{\Psi\Phi}.$$

Therefore, we can obtain the fractional performance gap based on the inequality in (36). ■

The fractional performance gap in (36) can be written in the form of

$$\frac{\tau_d H(\Phi) + a}{\tau_d H(\Phi) + b} \tag{37}$$

where $0 < a < b$. From (37), it makes sense to maximize the dwell time during a cycle since the larger τ_d leads to the larger value of (36).

VI. Simulation Examples

Let us consider a 600×500 rectangular mission space, where $R(x, y) = 1$, i.e., all points in the mission space are equally important. All agents have the same sensing function (2) with a sensing range $\delta_i = 220$ for all is. Let us assume that the charging dynamics in (5) have the form $f(q_i(t), 1) = c - \beta$, and the energy depletion dynamics in (5) have the form $g(q_i(t), v_i(t), 1) = -\alpha v_i(t) - \beta$, where α, β and c are three constants. For a properly defined problem, the following constraint should be satisfied $c \geq N(\alpha\bar{v} + \beta)$, where $\bar{v}$ is the maximum allowable speed of all agents. By treating the charging station as a server, the charging rate is c if it is occupied at all times, and the worst case energy depletion rate over N agents is $N(\alpha\bar{v} + \beta)$. Thus, this condition provides a well-posed condition of preventing any agent from running out of energy in the mission space.

In the following, we will discuss the coverage performance of a multi-agent team with three agents. The coverage performance in terms of the number of agents ranging from 3 up to 10 will be summarized as well.

A. Three Agents

Let us consider a small network with 3 agents. By using the gradient approach [1], the OCV locations of all three agents are found to be $\phi_1 = (186.7, 119.3)$, $\phi_2 = (160.3, 371.1)$, and $\phi_3 = (451.4, 290.4)$ shown in blue in Fig. 3, and the OCH locations are $\psi_1 = (0, 0)$, $\psi_2 = (169.3, 320.2)$ and $\psi_3 = (430.6, 185.0)$ shown in red in Fig. 3, where x and y represent the coordinates in the mission space. The charging station is located at ψ_1. By solving the TSP, the shortest path is $\psi_1 \to \phi_2 \to \psi_2 \to \phi_3 \to \psi_3 \to \phi_1 \to \psi_1$. The total traveling distance is 1321.4.

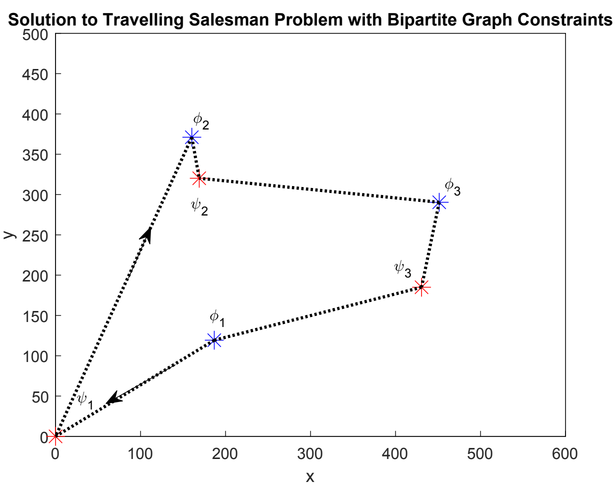


Fig. 3: The shortest path for the TSP

Let us solve the schedulability problem (30)-(31) first. Note that ψ_1 is defined as the charging station in Section IV-D. The SOC when an agent arrives at the charging station is μ_0^-. The distances are $d_{\phi_1}^{\psi_1} = 221.5612, d_{\psi_1}^{\phi_2} = 252.5939, d_{\phi_2}^{\psi_2} = 107.4328, d_{\psi_2}^{\phi_3} = 283.9381, d_{\phi_3}^{\psi_3} = 51.6432, d_{\psi_3}^{\phi_1} = 404.2416$. When the energy depletion model is linear in v_i, it is optimal to choose the shortest transient time. The lower bounds of transient times τ_t^2 and τ_t^3 are determined by the distances and maximum speed. Therefore, we can choose

$$\tau_t^3 = \frac{\max\{d_{\psi_1}^{\phi_2}, d_{\psi_2}^{\phi_3}, d_{\psi_3}^{\phi_1}\}}{\bar{v}} = \frac{404.2416}{\bar{v}}$$
$$\tau_t^2 = \frac{\max\{d_{\phi_1}^{\psi_1}, d_{\phi_2}^{\psi_2}, d_{\phi_3}^{\psi_3}\}}{\bar{v}} = \frac{221.5612}{\bar{v}}.$$

After charging for τ_c, the SOC increases to $\mu_0^- + \tau_c(c - \beta)$. Then, the agent heads to ϕ_2, and its SOC decreases to $\mu_0^- + \tau_c(c - \beta) - \alpha d_{\psi_1}^{\phi_2} - \beta\tau_t^3$, where the third term and the last term correspond to the energy cost of motion and sensing, respectively. To solve the schedulability problem, we set the dwell time at the OCV locations as zero. After one cycle, when an agent returns to the charging station, its SOC becomes

$$\mu_0^- + \tau_c c - 1321.4\alpha - 3\beta\tau_t^3 - 3\beta\tau_t^2 - 3\beta\tau_c.$$

and we require:

$$\mu_0^- + \tau_c c - 1321.4\alpha - 3\beta\tau_t^3 - 3\beta\tau_t^2 - 3\beta\tau_c \geq \mu_0^-.$$

Therefore, in this case it is possible that $\mu_0^- = 0$, and the minimum charging time is

$$\tau_c = \frac{1321.4\alpha + 3\beta(\tau_t^3 + \tau_t^2)}{c - 3\beta}$$

Based on $\mu_6^- = 0$, τ_c, τ_t^2, and τ_t^3, we are able to calculate the minimum SOC for all 3 OCV locations as shown at the end of Section IV-D.

If an agent stays at the charging station more than the minimum τ_c, then the dwell time τ_d will not be zero. Therefore, we need to solve the optimization problem (32)-(34):

$$\max_{\tau_c, \tau_d} \quad \frac{\tau_d H(\Phi) + \tau_c H(\Psi) + \tau_t^{N-1} J_{\Phi\Psi} + \tau_t^N J_{\Psi\Phi}}{\tau_c + \tau_d + \tau_t^3 + \tau_t^2}$$

subject to

$$\tau_c \leq \frac{1}{c - \beta}$$
$$\tau_c \geq \frac{1321.4\alpha + 3\beta(\tau_t^3 + \tau_t^2 + \tau_d)}{c - 3\beta}$$

To solve the above optimization problem, the optimal solution occurs when the second inequality becomes an equality. Then, we can write the relationship between τ_c and τ_d as $\tau_c = a + b\tau_d$. If we substitute τ_c by $a + b\tau_d$, we know that the larger τ_d leads to better performance. Therefore, the optimal solution for the above problem is to let the agent be fully charged, that is, $\tau_c = 1/(c - \beta)$ and

$$\tau_d = \frac{1 - 1321.4\alpha}{3\beta} - \frac{2}{3(c - \beta)} - \tau_t^3 - \tau_t^2$$

Table I shows the average and minimum coverage performance of both centralized and decentralized approaches for different numbers of agents with different parameters α, β, c. The maximum velocity of all cases is $\bar{v} = 50$ except $\bar{v} = 100$ when $N = 6$.

The coverage performance of the above centralized algorithm and the decentralized approach for $N = 3$is depicted and compared in Fig. 4. The cycles are clearly visualized in the figure, where the top and the bottom horizontal lines correspond to the time when agents are in the OCV formation, and in the OCH formation, respectively. The coverage performance of the decentralized approach is computed using the approach in Section III, which is a modification of the algorithm proposed in [24]. In the decentralized approach, agents may compete for the charging station. When this case occurs, agents with lower priority have to turn off their sensing capability as indicated in the bottom horizontal line of the decentralized approach in Fig. 4. The coverage performance is significantly compromised. The performance lower bound of the centralized approach is determined by the OCH formation. The results show that both the average and the worst performance is

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TABLE I: Performance Comparison Between Centralized and Decentralized Approaches in terms of Average Performance (AP) and Minimum Performance (MP)

N	(α, β, c)		Cen.	Dec.	Imp.
3	$(0.0005, 0.0005, 0.010)$	AP	179632	166917	7.62%
		MP	161600	92010	75.63%
4	$(0.0005, 0.0005, 0.015)$	AP	221840	205085	8.17%
		MP	204400	154100	32.64%
5	$(0.0005, 0.0005, 0.020)$	AP	247721	236649	4.49%
		MP	240400	138100	74.08%
6	$(0.0005, 0.0005, 0.025)$	AP	262874	253278	3.78%
		MP	255000	189700	31.78%
7	$(0.0005, 0.0005, 0.030)$	AP	274219	270320	1.44%
		MP	273500	233600	17.08%
8	$(0.0004, 0.0001, 0.05)$	AP	284016	263400	7.83%
		MP	271052	223270	21.40%
9	$(0.0004, 0.0001, 0.05)$	AP	287418	271300	5.94%
		MP	284280	224758	26.48%
10	$(0.0004, 0.0001, 0.05)$	AP	292028	277929	4.83%
		MP	289477	252056	14.85%

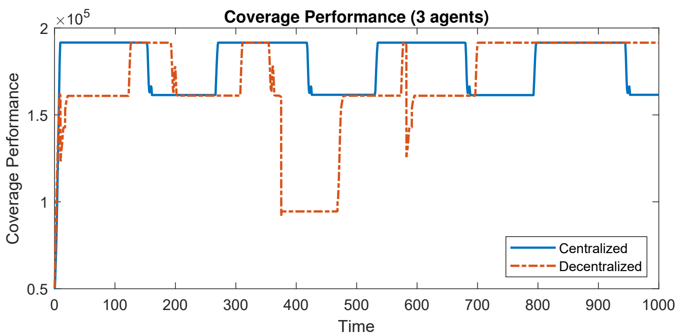

Fig. 4: Performance of Centralized and Decentralized Approaches

significantly improved by the centralized approach. Based on the result in Theorem 2, the proposed solution is within 93.75% of the globally optimal solution.

When the number of agents increases, the centralized approach keeps a minimum coverage performance above $255,000$ for $N = 6$. The coverage performance over time for the centralized approach and decentralized approach is depicted and compared in Fig. 5. However, the performance is critically compromised for the decentralized approach when more agents compete for the charging stations, as shown in Fig. 5. Based on the result in Theorem 2, the proposed solution is within 97.60% of the globally optimal solution. Videos of the decentralized and centralized coverage algorithms for $N = 7$ can be found at `https://www.youtube.com/watch?v=alc9Ndgtygw` and `https://www.youtube.com/watch?v=xGgHoItwDnY`, respectively.

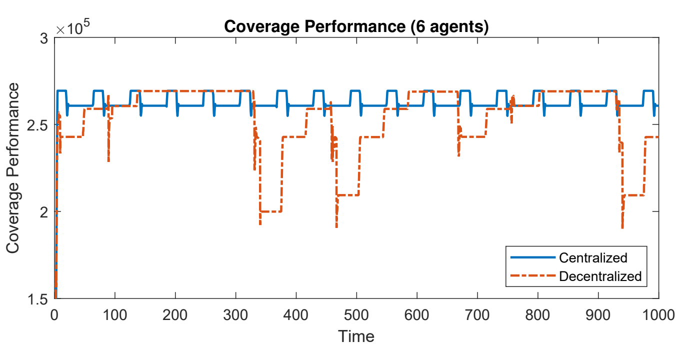

Fig. 5: Performance of Centralized and Decentralized Approaches

VII. Conclusions and Future Works

In this paper, we propose a centralized near-optimal solution to the multi-agent coverage problem with energy constrained agents. The performance between the centralized approach and decentralized approach is compared and it is shown that the centralized approach in general produces better average coverage performance than the decentralized approach. In addition, the performance gap between the proposed algorithm and the globally optimal solution becomes very small as the number of agents increases. Natural directions for future research include the consideration of collision avoidance and the effect of communication costs when agents must exchange information.

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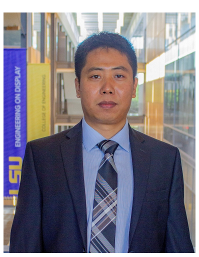

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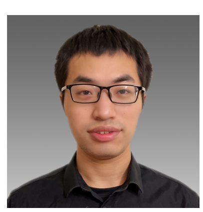

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