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Decentralized Time and Energy-Optimal Control of Connected and Automated Vehicles in a Roundabout

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Abstract— **We consider the problem of controlling Connected and Automated Vehicles (CAVs) traveling through a three-entry roundabout so as to jointly minimize the travel time and energy consumption while providing speed-dependent safety guarantees and satisfying velocity and acceleration constraints. We first design a systematic approach to dynamically determine all conflicting CAVs defining the safety constraints under different CAV sequencing policies. A joint optimal control and barrier function (OCBF) method is then applied to efficiently obtain a controller that optimally tracks the unconstrained optimal control solution while guaranteeing the satisfaction of all constraints. Simulation experiments performed to compare the OCBF controller to a baseline of human-driven vehicles show its effectiveness under different roundabout configurations and sequencing policies.**

I. INTRODUCTION

The performance of traffic networks critically depends on the control of conflict areas such as intersections, roundabouts and merging roadways which are the main bottlenecks in these networks [1]. Coordinating and controlling vehicles in these conflict areas is a challenging problem in terms of safety, congestion, and energy consumption [2], [3]. The emergence of Connected and Automated Vehicles (CAVs) provides a promising solution to this problem through better information utilization and more precise trajectory design. The automated control of vehicles has gained increasing attention with the development of new traffic infrastructure technologies [4] and, more recently, CAVs [1].

Both centralized and decentralized methods have been studied to deal with the control and coordination of CAVs at conflict areas. Centralized mechanisms are often used in forming platoons in merging problems [5] and determining passing sequences at intersections [6]. These approaches tend to work better when the safety constraints are independent of speed and they generally require significant computation resources, especially when traffic is heavy. They are also not easily amenable to disturbances.

Decentralized mechanisms restrict all computation to be done on board each CAV with information sharing limited to a small number of neighbor vehicles [7]–[9]. Optimal control problem formulations are often used, with Model Predictive Control (MPC) techniques employed as an alternative to account for additional constraints and to compensate for disturbances by re-evaluating optimal actions [10], [11]. The objectives in such problem formulations typically target the minimization of acceleration or the maximization of passenger comfort (measured as the acceleration derivative or jerk). An alternative to MPC has recently been proposed through the use of Control Barrier Functions (CBFs) [12], [13] which provide provable guarantees that safety constraints are always satisfied.

In this paper, we build on the use of optimal control and CBF-based methods in unsignalized intersections [14] and merging [15] to study roundabouts with all traffic consisting of CAVs. There are several similarities between merging, intersections and roundabouts. The single-lane merging problem [15] contains a single Merging Point (MP) where safety constraints must be guaranteed, while CAVs follow the same moving direction in each lane. In intersection problems, CAVs have a number of possible paths which conflict at multiple MPs restricted to a small area. In a roundabout, CAVs have the same moving direction (either clockwise or counterclockwise) but multiple possible paths which cross at multiple MPs. A roundabout problem can be dealt with as either a whole system like an intersection or it can be decomposed into several coupled merging problems.

Roundabouts are important components of a traffic network because they usually perform better than typical intersections in terms of efficiency and safety [16]. However, they can become significant bottleneck points as the traffic rate increases due to an inappropriate priority system, resulting in significant delays when the circulating flow is heavy. Previous studies mainly focus on conventional vehicles and try to solve the problem through improved road design or traffic signal control [17]–[19]. More recently, however, researchers have proposed methods for decentralized optimal control of CAVs in a roundabout. The roundabout problem is formulated as an optimal control problem with an analytical solution provided in [20]. The problem is decomposed so that first the minimum travel time is solved under the assumption that all vehicles use the same maximum speed within the roundabout. Then, fixing this time, the control input that minimizes the energy consumption is derived analytically. The general framework for decentralized optimal control of CAVs used in intersections is implemented for roundabouts in [21].

In this paper, we formulate an optimal control problem for controlling CAVs traveling through a roundabout. Unlike [20], [21], we *jointly* minimize the travel time and energy consumption and also consider speed-dependent safety constraints at a set of MPs rather than merging zones (which makes solutions less conservative by improving roadway

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utilization). In addition, to improve computational efficiency, we adopt the joint Optimal Control and Barrier Function (OCBF) approach introduced in [13]: we first derive the optimal solution when no constraints become active and subsequently optimally track this solution while also guaranteeing the satisfaction of all constraints through the use of CBFs. We first assume a First-In-First-Out (FIFO) sequencing policy over the entire system. We then divide the roundabout into separate merging problems so as to introduce different resequencing rules depending on the MP. We will show that the FIFO policy does not perform well in many "asymmetric" configurations and explore an alternative sequencing policy, termed Shortest Distance First (SDF).

The paper is organized as follows. In Section II, the roundabout problem is formulated as an optimal control problem with safety constraints. In Section III, a decentralized framework is provided to determine the safety constraints related to a given CAV. An OCBF controller is designed in Section IV while simulation results are presented in Section V showing significant improvements in the performance of the OCBF controller compared to a baseline of human-driven vehicles. Conclusions are provided in Section VI.

II. Problem Formulation

We initiate our study of roundabouts by considering a single-lane triangle-shaped roundabout with 3 entries and 3 exits as shown in Fig. 1. We consider the case where all traffic consists of CAVs which randomly enter the roundabout from three different origins O_1, O_2 and O_3 and have assigned exit points E_1, E_2 and E_3. The gray road segments which include the triangle and three entry roads form the Control Zone (CZ) where CAVs can share information and thus be automatically controlled. We assume all CAVs move in a counterclockwise way in the CZ. The entry road segments are connected with the triangle at the three Merging Points (MPs) where CAVs from different road segments may potentially collide with each other. The MPs are labeled as M_1, M_2 and M_3. We assume that each road segment has one single lane (extensions to multiple lanes and MPs are possible following the analysis in [22]) The three entry road segments which are labeled as l_1, l_2 and l_3 have the same length L, while the road segments in the triangle which are labeled as l_4, l_5 and l_6 have the same length L_a (extensions to different lengths are straightforward). In Fig. 1, a circle, square and triangle represent entering from O_1, O_2 and O_3 respectively. The color red, green and blue represents exiting from E_1, E_2 and E_3 respectively. The full trajectory of a CAV in terms of the MPs it must go through can be determined by its entry and exit points.

A coordinator, i.e., a Road Side Unit (RSU) associated with the roundabout, maintains a passing sequence for all CAVs and records the information of each CAV. The CAVs communicate with the coordinator but are not controlled by it. All control inputs are evaluated on board each CAV in a *decentralized* way. Each CAV is assigned a unique index upon arrival at the CZ according to the passing order. The most common scheme for maintaining a passing sequence is the First-In-First-Out (FIFO) policy according to each CAV's arrival time at the CZ. The FIFO rule is one of the simplest schemes, yet works well in many situations as also shown in [23]. For simplicity, in what follows we use the FIFO queue, but we point out that any passing order policy may be used.

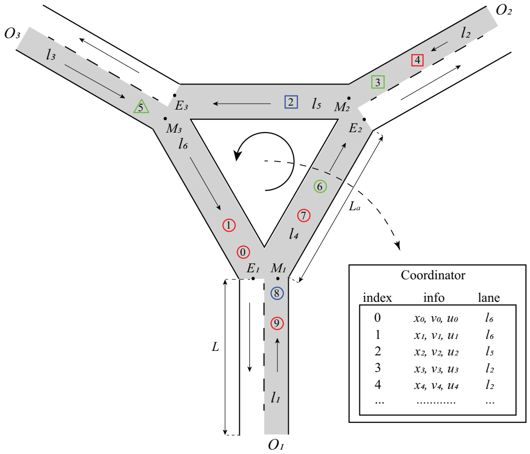

Fig. 1. A roundabout with 3 entries

Let $S(t)$ be the set of CAV indices in the coordinator queue table at time t. The cardinality of $S(t)$ is denoted as $N(t)$. When a new CAV arrives, it is allocated the index $N(t)+1$. Each time a CAV i leaves the CZ, it is dropped and all CAV indices larger than i decrease by one. When CAV $i \in S(t)$ is traveling in the roundabout, there are several important *events* whose times are used in our analysis: (i) CAV i enters the CZ at time t_i^0, (ii) CAV i arrives at MP M_k at time t_i^k, $k \in \{1,2,3\}$, (iii) CAV i leaves the CZ at time t_i^f. Based on this setting, we can formulate an optimal control problem as described next.

Vehicle Dynamics Denote the distance from the origin $O_j, j \in \{1,2,3\}$ to the current location of CAV i along its trajectory as $x_i^j(t)$. Since the CAV's unique identity i contains the information about the CAV's origin O_j, we can use $x_i(t)$ instead of $x_i^j(t)$ (without any loss of information) to describe the vehicle dynamics as

$$\begin{bmatrix} \dot{x}_i(t) \\ \dot{v}_i(t) \end{bmatrix} = \begin{bmatrix} v_i(t) \\ u_i(t) \end{bmatrix} \tag{1}$$

where v_i is the velocity CAV i along its trajectory and u_i is the acceleration (control input).

Objective 1 Minimize the travel time $J_{i,1} = t_i^f - t_i^0$ where t_i^0 and t_i^f are the times CAV i enters and exits the CZ.

Objective 2 Minimize energy consumption:

$$J_{i,2} = \int_{t_i^0}^{t_i^f} C_i(u_i(t))dt \tag{2}$$

where $C_i(\cdot)$ is a strictly increasing function of its argument.

Constraint 1 (Rear-end safety constraint) Let i_p denote the index of the CAV which immediately precedes CAV i on road segment l_k. The distance between i_p and i, $z_{i,i_p}(t) \equiv x_{i_p}(t) - x_i(t)$, should be constrained by a speed-dependent

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constraint:

$$z_{i,i_p}(t) \geq \varphi v_i(t) + \delta, \quad \forall t \in [t_i^0, t_i^f], \quad \forall i \in S(t) \tag{3}$$

where φ denotes the reaction time (as a rule, $\varphi = 1.8$ is suggested, see [24]), δ denotes the minimum safety distance (in general, we may use δ_i to make this distance CAV-dependent but will use a fixed δ for simplicity). The index of the preceding CAVs index i_p may change due to road segment changing events and is determined by the method described later in section III-B.

Constraint 2 (Safe merging constraint) Let $t_i^k,\ k \in \{1,2,3\}$ be the arrival time of CAV i at MP M_k. Let i_m denote the index of the CAV that CAV i may collide with when arriving at its next MP M_k. The distance between i_m and i, $z_{i,i_m}(t) \equiv x_{i_m}(t) - x_i(t)$, is constrained by:

$$z_{i,i_m}(t_i^k) \geq \varphi v_i(t_i^k) + \delta, \quad \forall i \in S(t), \quad k \in \{1,2,3\} \tag{4}$$

where i_m can be determined and updated by the method described in section III-B.

Constraint 3 (Vehicle limitations) The CAVs are also subject to velocity and acceleration constraints due to physical limitations or road rules:

$$\begin{aligned} v_{i,\min} \leq v_i(t) \leq v_{i,\max}, \forall t \in [t_i^0, t_i^f], \forall i \in S(t) \\ u_{i,\min} \leq u_i(t) \leq u_{i,\max}, \forall t \in [t_i^0, t_i^f], \forall i \in S(t) \end{aligned} \tag{5}$$

where $v_{i,\max} > 0$ and $v_{i,\min} \geq 0$ denote the maximum and minimum speed for CAV i, $u_{i,\max} < 0$ and $u_{i,\min} < 0$ denote the maximum and minimum acceleration for CAV i. We further assume common speed limits dictated by the traffic rules, i.e. $v_{i,\min} = v_{\min}$, $v_{i,\max} = v_{\max}$.

Similar to previous work [15], we construct a convex combination of the two objectives above:

$$J_i = \int_{t_i^f}^{t_i^0} \left[\alpha + (1-\alpha) \frac{\frac{1}{2}u_i^2(t)}{\frac{1}{2}\max\{u_{\max}^2, u_{\min}^2\}} \right] dt \tag{6}$$

where $J_{i,1}$ and $J_{i,2}$ are combined with $\alpha \in [0,1]$ after proper normalization. Here, we simply choose the quadratic function $C_i(u_i) = \frac{1}{2}u_i^2(t)$. If $\alpha = 1$, the problem degenerates into a minimum traveling time problem. If $\alpha = 0$, it degenerates into a minimum energy consumption problem.

By defining $\beta \equiv \frac{\alpha}{2(1-\alpha)} \max\{u_{\max}^2, u_{\min}^2\}, \alpha \in [0,1)$ and proper scaling, we can rewrite this minimization problem as

$$J_i(u_i) = \beta(t_i^f - t_i^0) + \int_{t_i^0}^{t_i^f} \frac{1}{2}u_i^2(t)dt \tag{7}$$

where β is the weight factor derived from α. Then, we can formulate the optimal control problem as follows:

Problem 1: For each CAV i following the dynamics (1), find the optimal control input $u_i(t)$ that minimizes (7) subject to constraints (1), (3), (4), (5), the initial condition $x_i(t_i^0) = 0$, and given t_i^0, v_i^0 and $x_i(t_i^f)$.

III. Decentralized Control Framework

Compared to the single-lane merging or intersection control problems where the constraints are determined and fixed immediately when CAV i enters the CZ, the main difficulty in a roundabout is that the constraints generally change after every event (defined earlier). In particular, for each CAV i at time t only the merging constraint related to the next MP ahead is considered. In other words, we need to determine at most one i_p to enforce (3) and one i_m to enforce (4).

In order to solve **Problem 1** for each CAV i, we need to first determine the corresponding i_p and i_m (when they exist) required in the safety constraints (3) and (4). Once this task is complete and (3) and (4) are fully specified, then **Problem 1** can be solved. In what follows, this first task is accomplished through a method designed to determine the constraints in an event-driven manner which can be used in either of the two approaches above and for any desired sequencing policy. An extended queue table, an example of which is shown in Table I corresponding to Fig. 1, is used to record the essential state information and identify all conflicting CAVs. We specify the state-updating mechanism for this queue table so as to determine for each CAV i the corresponding i_p and i_m. Then, we develop a general algorithm for solving **Problem 1** based on the OCBF method in Section IV.

A. The Extended Coordinator Queue Table

Starting with the coordinator queue table shown in Fig. 1, we extend it to include additional columns for each CAV i including the current road segment, the original road segment, the MPs on the CAV trajectory, as well as i_p and i_m. The precise definitions of i_p and i_m are given below:

TABLE I

THE EXTENDED COORDINATOR QUEUE TABLE $S(t)$

$S(t)$								
idx	state	curr.	ori.	1st MP	2nd MP	3rd MP	i_p	i_m
0	$\boldsymbol{x}_0$	l_6	l_1	M_1, M	M_2, M	M_3, M		
1	$\boldsymbol{x}_1$	l_6	l_1	M_1, M	M_2, M	M_3, M	0	
2	$\boldsymbol{x}_2$	l_5	l_2	M_2, M				
3	$\boldsymbol{x}_3$	l_2	l_2	M_2	M_3	M_1		2
4	$\boldsymbol{x}_4$	l_2	l_2	M_2	M_3		3	
5	$\boldsymbol{x}_5$	l_3	l_3	M_3	M_1			1
6	$\boldsymbol{x}_6$	l_4	l_1	M_1, M				
7	$\boldsymbol{x}_7$	l_4	l_1	M_1, M	M_2	M_3	6	4
8	$\boldsymbol{x}_8$	l_1	l_1	M_1	M_2			7
9	$\boldsymbol{x}_9$	l_1	l_1	M_1	M_2	M_3	8	

- i_p: Index of the CAV that immediately precedes CAV i in the same road segment (if such a CAV exists).
- i_m: Index of the CAV that may conflict with CAV i at the next MP. CAV i_m is the last CAV that passes the MP ahead of CAV i. Note that if i_m and i are in the same road segment, then i_m $(= i_p)$ is the immediately preceding CAV. In this case, the safe merging constraint is redundant and need not be included.

Event-driven Update Process for $S(t)$: The extended coordinator queue table $S(t)$ is updated whenever an event (as defined earlier) occurs. Thus, there are three different update processes corresponding to each triggering event:

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- A new CAV enters the CZ: The CAV is indexed and added to the bottom of the queue table.
- CAV i exits the CZ: All information of CAV i is removed. All rows with index larger than i decrease their index values by 1.
- CAV i passes an MP: Mark the MP with M and update the current road segment value *curr* of CAV i with the one it is entering.

B. Determination of Safety Constraints

Recall that for each CAV i in the CZ, we need to consider two different safety constraints (3) and (4). First, by looking at each row $j < i$ and the corresponding current road segment value *curr*, CAV i can determine its immediately preceding CAV i_p if one exists. This fully specifies the rear-end safety constraint (3). Next, we determine the CAV which possibly conflicts with CAV i at the next MP it will pass so as to specify the safe merging constraint (4). To do so, we find in the extended queue table the last CAV $j < i$ that will pass or has passed the same MP as CAV i. In addition, if the CAV is in the same road segment as CAV i, it coincides with the preceding CAV i_p. Otherwise, we find i_m, if it exists. As an example, in Table I (a snapshot of Fig. 1), CAV 8 has no immediate preceding CAV in l_1, but it needs to yield to CAV 7 (although CAV 7 has already passed M_3, when CAV 8 arrives at M_3 there needs to be adequate space between CAV 7 and 8 for CAV 8 to enter l_4). CAV 9 however, only needs to satisfy its rear-end safety constraint with CAV 8.

It is now clear that we can use the information in $S(t)$ in a systematic way to determine both i_p in (3) and i_m in (4). Thus, there are two functions $i_p(e)$ and $i_m(e)$ which need to be updated after event e if this event affects CAV i. The index i_p can be easily determined by looking at rows $j < i$ in the extended queue table until the first one is found with the same value *curr* as CAV i. For example, CAV 9 searches for its i_p from CAV 8 to the top and sets $i_p = 8$ as CAV 8 has the *curr* value l_1. Next, the index i_m is determined. To do this, CAV i compares its MP information to that of each CAV in rows $j < i$. The process terminates the first time that any one of the following two conditions is satisfied:

- The MP information of CAV i_m *matches* CAV i. We define i_m to "match" i if and only if the last marked MP or the first unmarked MP of CAV i_m is the same as the first unmarked MP of CAV i.
- All prior rows $j < i$ have been looked up and none of them matches the MP information of CAV i.

Combining the two updating processes for i_p and i_m together, there are four different cases as follows:

1. Both i_p and i_m exist. In this case, there are two possibilities: (i) $i_p \neq i_m$. CAV i has to satisfy the safe merging constraint (4) with $i_p < i$ and also satisfy the rear-end safety constraint (3) with $i_m < i$. For example, for $i = 7$, we have $i_p = 6$ and $i_m = 4$ (M_2 is the first unmarked MP for CAV 7 and that matches the first unmarked MP for CAV 4). (ii) $i_p = i_m$. CAV i only has to follow i_p and satisfy the rear-end safety constraint (3) with respect to i_p. Thus, there is no safe merging constraint for CAV i to satisfy. For example, $i = 4$ and $i_p = i_m = 3$.

2. Only i_p exists. In this case, there is no safe merging constraint for CAV i to satisfy. CAV i only needs to follow the preceding CAV i_p and satisfy the rear-end safety constraint (3) with respect to i_p. For example, $i = 1$ and $i_p = 0$.

3. Only i_m exists. In this case, CAV i has to satisfy the safe merging constraint (4) with the CAV i_m in $S(t)$. There is no preceding CAV i_p, thus there is no rear-end safety constraint. For example, $i = 5$, $i_m = 1$ (M_3 is the first unmarked MP for CAV 5 and that matches the last marked MP for CAV 1 with no other match for $j = 4, 3, 2$).

4. Neither i_p nor i_m exists. In this case, CAV i does not have to consider any safety constraints. For example, $i = 2$.

C. Sequencing Policies with Sub-coordinator Queue Tables

In order to allow possible resequencing when a CAV passes an MP, we introduce next a sub-coordinator queue table $S_k(t)$ associated with each M_k, $k = 1, 2, 3$. $S_k(t)$ coordinates all the CAVs for which M_k is the next MP to pass or it is the last MP that they have passed. We define CZ_k as the CZ corresponding to M_k that consists of the three road segments directly connected to M_k. A sub-coordinator queue table can be viewed as a subset of the extended coordinator queue table except that the CAVs are in different order in the two tables. As an example, Table II (a snapshot of Fig. 1) is the sub-coordinator queue table corresponding to M_1 (in this case, still based on the FIFO policy).

TABLE II
THE SUB-COORDINATOR QUEUE TABLE $S_1(t)$

$S_1(t)$								
idx	info	curr.	ori.	1st MP	2nd MP	3rd MP	i_p	i_m
6	$\boldsymbol{x}_6$	l_4	l_1	M_1, M				
7	$\boldsymbol{x}_7$	l_4	l_1	M_1, M	M_2	M_3	6	
8	$\boldsymbol{x}_8$	l_1	l_1	M_1	M_2			7
0	$\boldsymbol{x}_0$	l_6	l_1	M_1, M	M_2, M	M_3, M		
1	$\boldsymbol{x}_1$	l_6	l_1	M_1, M	M_2, M	M_3, M	0	
9	$\boldsymbol{x}_9$	l_1	l_1	M_1	M_2	M_3	8	

The event-driven update process for $S(t)$ is given in detail in [25]. Note that CAV j may appear in multiple sub-coordinator queue tables with different i_p and i_m values. However, only the one in $S_k(t)$ where M_k is the next MP CAV j will pass is used to update the extended coordinator queue table $S(t)$. The information of CAV j in other sub-coordinator queue tables is necessary for determining the safety constraints as CAV j may become CAV i_p or i_m of other CAVs.

Resequencing rule: The sub-coordinator queue table allows resequencing when a CAV passes a MP. A resequencing rule generally designs and calculates a criterion for each CAV and sorts the CAVs in the queue table when a new event happens. For example, FIFO takes the arrival time in the CZ as the criterion while the Dynamic Resequencing (DR) policy [26] uses the overall objective value in (7) as the criterion.

We propose here a straightforward yet effective (see Section V) resequencing rule for the roundabout as follows. Let

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$\tilde{x}_i^k \equiv x_i - d_j^k$ be the position of CAV i relative to M_k, where d_j^k denotes the fixed distance from the entry point (origin) O_j to merging point M_k along the trajectory of CAV i. Then, consider

$$y_i(t) = -\tilde{x}_i^k(t) - \varphi v_i(t) \tag{8}$$

This resequencing criterion reflects the distance between the CAV and the next MP. The CAV which has the smallest $y_i(t)$ value is allocated first, thus referring to this as the Shortest Distance First (SDF) policy. Note that $\varphi v_i(t)$ introduces a speed-dependent term corresponding to the speed-dependent safety constraints. Other resequencing policies can also be easily implemented with the help of the sub-coordinator queue tables.

IV. Joint optimal control and control barrier function controller (OCBF)

We now return to the solution of **Problem 1**, i.e., the minimization of (7) subject to constraints (1), (3), (4), (5), the initial condition $x_i(t_i^0) = 0$, and given t_i^0, v_i^0 and $x_i(t_i^f)$. The problem formulation is complete since we have used the extended coordinator table to determine i_p and i_m (needed for the safety constraints) associated with the closest MP to CAV i given the sequence of CAVs in the system. After introducing the sub-coordinator queue tables, we also allow some resequencing for CAVs passing each MP and focus on the CZ associated with that MP. Thus, each such problem resembles the merging control problem in [15] which can be analytically solved. However, as pointed out in [15], when one or more constraints become active, this solution becomes computationally intensive. The problem here is exacerbated by the fact that the values of i_p and i_m change due to different events in the roundabout system. Therefore, to ensure that a solution can be obtained in real time while also guaranteeing that all safety constraints are always satisfied, we adopt the OCBF approach which is obtained as follows: (i) an optimal control solution is first obtained for the *unconstrained* roundabout problem (as reported in [15] such solutions are computationally efficient to obtain, typically requiring $\ll$ 1sec using MATLAB). (ii) This solution is used as a reference control which is optimally tracked subject to a set of CBFs, one for each of the constraints (3), (4), (5). Using the forward invariance property of CBFs, this ensures that these constraints are always satisfied. This whole process is carried out in a decentralized way. The detailed process and derivation of the OCBF method is included in [25].

V. Simulation Results

In this section, we use Vissim, a multi-modal traffic flow simulation platform, as a baseline to compare a roundabout performance with human-driven vehicles to our OCBF controller. We build the scenario shown in Fig. 1 in Vissim and use the same vehicle arrival patterns in the OCBF controller for consistent comparison purposes.

Simulation 1: The first simulation focuses on the performance of the OCBF controller. The basic parameter settings are as follows: $L_a = 60m$, $L = 60m$, $\delta = 10m$, $\varphi = 1.8s$, $v_{\max} = 17m/s$, $v_{\min} = 0$, $u_{\max} = 5m/s^2$, $u_{\min} = -5m/s^2$. This scenario considers a *symmetric* configuration in the sense that $L_a = L$. The traffic in the three incoming roads is generated through Poisson processes with all rates set to 360 CAVs/h. Under these traffic rates, vehicles will sometimes line up waiting for other vehicles in the roundabout to pass. A total number of approximately 200 CAVs are simulated. The simulation results of the performance of OCBF compared to that in Vissim are listed in Table III.

TABLE III
Objective function comparison for a symmetric roundabout

Items	OCBF		Vissim	
Weight	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.1$	$\alpha = 0.2$
Ave. time (s)	13.7067	13.3816	20.6772	
Ave. energy	16.0698	24.6336	33.2687	
Ave. obj.[1]	35.1084	66.4511	61.9893	97.8850

[1] Ave. obj = $\beta\times$ Ave. time + Ave. energy, $\beta = \frac{\alpha \max\{u_{\max}^2, u_{\min}^2\}}{2(1-\alpha)}$

In this simulation, FIFO is chosen as the sequencing policy in the OCBF method. As seen in Table III, the travel time of CAVs in the roundabout improves about 34% using the OCBF method compared with that of Vissim when $\alpha = 0.1$ (with some additional improvement when $\alpha = 0.2$). The CAVs using the OCBF method consume 52% and 26% less energy than that in Vissim with α set to 0.1 and 0.2 respectively. A larger α means more emphasis on the travel time than the energy consumption, which explains the shorter travel time and the larger energy consumption. When it comes to the total objective, the OCBF controller shows 44% and 32% improvement over the human-driven performance in Vissim when α equals to 0.1 and 0.2 respectively. This improvement in both the travel time and the energy consumption is to be expected as the CAVs using the OCBF method never stop and wait for CAVs in another road segment to pass as in Vissim.

Simulation 2: The second simulation compares the performance of OCBF under different sequencing rules in an *asymmetric* configuration. The parameter settings are the same as the first case except that $L = 100m$. The weight is set to $\alpha = 0.2$. The simulation results of the performance of OCBF with FIFO and OCBF with the SDF sequencing policy, as well that in Vissim, are shown in Table. IV.

TABLE IV
Objective function comparison of different resequencing rule for an asymmetric roundabout

Items	OCBF+FIFO	OCBF+SDF	Vissim
Ave. time (s)	16.4254	14.7927	24.6429
Ave. energy	56.9643	23.1131	30.8947
Ave. obj.	108.2937	69.3403	107.9038

Table IV shows that a CAV using OCBF with FIFO spends around 33% less travel time but 84% more energy than that in Vissim. The average objective values of the two cases are almost the same, indicating that OCBF with FIFO works

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poorly in an asymmetric roundabout. For example, when a CAV enters segment l_4, it has to wait for another CAV that has entered l_2 just before it to run 40 more meters for safe merging. This is unreasonable and may also result in some extreme cases when the OCBF problem becomes infeasible. This problem can be resolved by choosing a better sequencing policy such as SDF. As shown in Table IV, OCBF+SDF outperforms OCBF+FIFO, achieving an improvement of 40% in travel time, 26% in energy consumption and 36% in the objective value compared to that in Vissim.

Simulation 3: The purpose of this experiment is to study the effect of traffic volume with details included in [25]. Simulation results show that the imbalanced traffic causes longer travel times and more energy consumption. However, when OCBF+SDF is applied to the system, the imbalanced traffic brings almost no performance loss and becomes more balanced after passing the roundabout.

VI. Conclusion

We have presented a decentralized optimal control framework for controlling CAVs traveling through a roundabout to jointly minimize both the travel time and the energy consumption while satisfying speed-dependent safety constraints, as well as velocity and acceleration constraints. An OCBF controller, combining an unconstrained optimal control solution with CBFs, is designed and implemented to track the desired (unconstrained) trajectory while guaranteeing that all safety constraints and vehicle limitations are satisfied. Significant improvements are shown in the simulation experiments which compare the performance of the OCBF controller to a baseline of human-driven vehicles. Future research is directed at studying different sequencing policies, as well as considering the centrifugal discomfort caused when road segments are curved and extending the model to more complex roundabouts as well as to a multi-lane version which allows lane changing and overtaking.

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