Letter

Spatially inhomogeneous magnetic superconductors

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We consider a problem of superconductivity coexistence with the spin-density-wave order in disordered multiband metals. It is assumed that random variations of the disorder potential on short length scales render the interactions between electrons to become spatially correlated. As a consequence, both superconducting and magnetic order parameters become spatially inhomogeneous and are described by the universal phenomenological quantities, whereas all the microscopic details are encoded in the correlation function of the coupling strength fluctuations. We consider a minimal model with two nested two-dimensional Fermi surfaces and disorder potentials which include both intra- and interband scattering. The model is analyzed using the quasiclassical approach to show that short-scale pairing-potential disorder leads to a broadening of the coexistence region.

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Introduction. It is a well-known fact that generally disorder is detrimental to superconductivity. Although a sufficiently small amount of potential scatterers in superconductors with an isotropic pairing wave function does not suppress the critical transition temperature and energy gap, the key result known as the Anderson theorem [1], Larkin and Ovchinnikov have shown in their seminal paper [2] that even when time-reversal symmetry is preserved the coherence peak in the density of states can be smeared by disorder-induced inhomogeneities. Although this result seems counterintuitive at first sight, one can understand it by observing that at the mean-field level their model naturally contains an effective depairing parameter. As a result, changes in the pair-potential field as well as single-particle correlation functions due to inhomogeneities are of the same form as those found earlier by Abrikosov and Gor'kov for the case of superconductors contaminated with magnetic impurities [3]. Furthermore, the hard gap in the spectrum gets also smeared due to optimal fluctuations of the order parameter, thus leading to the Lifshitz-type tail [4] in the subgap region. For refinements and extensions of the original ideas to *s*- and *d*-wave superconductors, see Refs. [5–9] as well as an extensive review [10] and references herein.

Iron-based superconductors serve as a prime example [11,12] of complex materials in which disorder seems to play a highly nontrivial role. These materials belong to a subclass of composite superconductors in which superconductivity with an isotropic $s^{\pm}$-order parameter may develop on multiple bands and it usually competes with magnetic order. There is an extensive literature on the effect of impurities on the pairing state in pnictides (see, e.g., Refs. [13–19]). Of specific interest to the present Letter, it is in the context of the physics of these materials that it was shown [20,21] that disorder may actually boost superconductivity either by changing the corresponding scattering rates or, as in the case of stoichiometric substitutions, by varying the relative anisotropy of the Fermi pockets [22,23].

Behind the physical interpretation of this effect is an idea that disorder must suppress superconductivity (SC) slower than it suppresses magnetic, in that case spin-density-wave (SDW), order. Indeed, in these materials due to the *s*-wave symmetry of the pairing amplitude, the Anderson theorem still partially applies in a sense that only interband disorder affects the SC state but SDW is affected by intraband scattering as well. This means that in the temperature-doping (T-x) phase diagram, a narrow region of concentrations of impurity atoms must be present in which superconductivity would actually be in coexistence with SDW order. In passing we note that SC-SDW coexistence in iron-based superconductors actually leads to a number of fascinating physical effects, such as an anomalous temperature and doping dependence of the heat capacity [24,25] and London penetration depth [26–28] near the point where the SDW vanishes and quantum critical fluctuations play a dominant role in determining their thermodynamic and transport response functions at low temperatures [29–32].

Almost proverbial antagonism between spin-singlet superconductivity and magnetism on one hand, and the possibility of their coexistence due to different disorder scattering rates on the other hand, brings up the question of whether allowing for spatial inhomogeneities of the order parameters, for example, would produce either the broadening of the coexistence region or, on the contrary, the narrowing of it down. In this Letter we address precisely this question and show that at least within the limits of the two-band model [20,34] that we will adopt in what follows, the spatial inhomogeneities lead, in fact, to the broadening of the coexistence region and an enhancement of SC critical temperature. Our main result is presented in Fig. 1.

Model. The Hamiltonian for the model we study below is

$$\hat{\mathcal{H}} = \int \Psi^{\dagger}(\mathbf{r})\big(\hat{H}_0 + \hat{H}_{\rm mf} + \hat{H}_{\rm dis}\big)\Psi(\mathbf{r})d^2\mathbf{r}. \qquad (1)$$

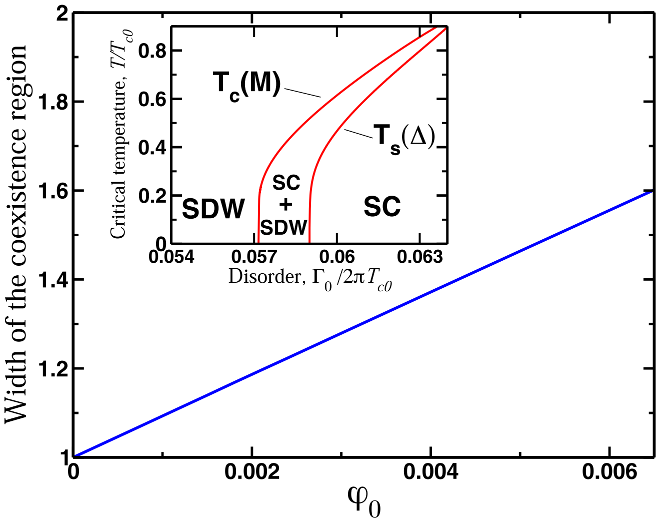

FIG. 1. Main panel: Schematic plot showing how the width of the coexistence region between the superconducting (SC) and spin-density-wave (SDW) orders varies as a function of the dimensionless parameter φ_0 describing the effects of spatial inhomogeneities in a system [Eqs. (8) and (20)]. The width is given in the units of the $\delta\Gamma_0$ which determines the one of the coexistence region in the spatially homogeneous system [34]. Inset: Phase diagram showing a region of coexistence between SC-SDW orders as found from the mean-field analysis of the Hamiltonian (1). The boundary line to the left of the coexistence region represents the superconducting critical temperature $T_c(M)$ at finite values of magnetization while the line to the right of the coexistence region represents the critical temperature $T_s(\Delta)$ of the SDW transition at finite pairing amplitude. The temperatures are given in the units of the superconducting critical temperature in a clean system.

Here, we use the eight-component spinor in the Balian-Werthammer representation [33], namely $\Psi^\dagger_{\mathbf{p}} = (\hat{c}^\dagger_{\mathbf{p}\uparrow}, \hat{c}^\dagger_{\mathbf{p}\downarrow}, -\hat{c}_{-\mathbf{p}\downarrow}, \hat{c}_{-\mathbf{p}\uparrow}, \hat{f}^\dagger_{\mathbf{p}\uparrow}, \hat{f}^\dagger_{\mathbf{p}\downarrow}, -\hat{f}_{-\mathbf{p}\downarrow}, \hat{f}_{-\mathbf{p}\uparrow})$, which contains spin-1/2 c- and f-fermionic fields with momentum $\mathbf{p}$ and describe two (one electron- and one holelike) bands, respectively [35]. $\hat{H}_0$ describes the single-particle states, and $\hat{H}_{\rm mf}$ is the interaction part taken in the mean-field approximation

$$\hat{H}_0 = -\xi_{\vec{\nabla}}\hat{\tau}_3\hat{\rho}_3\hat{\sigma}_0, \quad \hat{H}_{\rm mf} = -\Delta\hat{\tau}_3\hat{\rho}_1\hat{\sigma}_0 + \boldsymbol{M}\hat{\tau}_1\hat{\rho}_0\hat{\boldsymbol{\sigma}}. \tag{2}$$

In the expressions (2) above, $\hat{\tau}_i$, $\hat{\rho}_i$, and $\hat{\sigma}_i$ are Pauli matrices operational in the band, Gor'kov-Nambu, and spin subspaces, respectively, $\xi_{\vec{\nabla}} = -\vec{\nabla}^2/(2m) - \mu$ is the single-particle dispersion, μ is a chemical potential, Δ is the superconducting order parameter, and $\boldsymbol{M}$ is the magnetization which we shall take to be along the z axis, $\boldsymbol{M} = M\boldsymbol{e}_z$. Lastly, the Hamiltonian density, which introduces disorder scattering by randomly distributed impurities in locations $\mathbf{R}_i$, is

$$\hat{H}_{\rm dis} = \sum_i [u_0(\hat{\tau}_0\hat{\rho}_3\hat{\sigma}_0) + u_\pi(\hat{\tau}_1\hat{\rho}_3\hat{\sigma}_0)]\delta(\mathbf{r} - \mathbf{R}_i). \tag{3}$$

The scattering potential u_0 accounts for disorder scattering within each band, while the second term u_π leads to the interband transitions.

Quasiclassical theory. The ground state of the Hamiltonian described by Eq. (1) can be studied using the relatively simple system of Eilenberger equations [36], that for the model under consideration can be cast into a single equation for the matrix function $\hat{\mathcal{G}}(\omega_n, \mathbf{n}, \mathbf{r})$ [37],

$$\begin{aligned}&[i\omega_n\hat{\tau}_3\hat{\rho}_3\hat{\sigma}_0; \hat{\mathcal{G}}] - [\hat{H}_{\rm mf}\hat{\tau}_3\hat{\rho}_3\hat{\sigma}_0; \hat{\mathcal{G}}] - [\hat{\Sigma}_\omega\hat{\tau}_3\hat{\rho}_3\hat{\sigma}_0; \hat{\mathcal{G}}]\\ &= iv_F(\mathbf{n}\cdot\nabla\hat{\mathcal{G}}),\end{aligned} \tag{4}$$

where ω_n is the fermionic Matsubara frequency and $[\hat{A};\hat{B}]$ represents a commutator of two matrices in each term, respectively. The self-energy part calculated to the leading accuracy within the Born approximation reads

$$\hat{\Sigma}_\omega = -i\Gamma_0\hat{\tau}_3\hat{\rho}_0\hat{\sigma}_0\hat{\mathcal{G}}\hat{\tau}_0\hat{\rho}_3\hat{\sigma}_0 + \Gamma_\pi\hat{\tau}_2\hat{\rho}_3\hat{\sigma}_0\hat{\mathcal{G}}\hat{\tau}_1\hat{\rho}_3\hat{\sigma}_0, \tag{5}$$

where $\Gamma_{0,\pi} = \pi n_{\rm imp}\nu_F u^2_{0,\pi}$ are corresponding disorder intra- and interband scattering rates with $n_{\rm imp}$ being the impurity concentration. The matrix function $\hat{\mathcal{G}}$ satisfies the normalization condition $\hat{\mathcal{G}}^2 = \hat{\tau}_0\hat{\rho}_0\hat{\sigma}_0$. Equation (4) is supplemented by the self-consistency conditions for the order parameters

$$\begin{aligned}\frac{iM}{g_{\rm m}} &= \frac{\pi T}{8}\sum_{\omega_n>0}^{\Lambda}{\rm Tr}[(\hat{\tau}_1 + i\hat{\tau}_2)(\hat{\rho}_0 + \hat{\rho}_3)\hat{\sigma}_3\hat{\mathcal{G}}],\\ \frac{i\Delta}{g_{\rm s}} &= -\frac{\pi T}{8}\sum_{\omega_n>0}^{\Lambda}{\rm Tr}[(\hat{\tau}_0 + \hat{\tau}_3)(\hat{\rho}_1 + i\hat{\rho}_2)(\hat{\sigma}_0 + \hat{\sigma}_3)\hat{\mathcal{G}}].\end{aligned} \tag{6}$$

Here, $g_{\rm m}$, $g_{\rm s}$ are the interaction constants and the trace over the matrix products also includes the integration over all directions of the Fermi velocity $\mathbf{v}_F = v_F\mathbf{n}$. As usual, the UV-cutoff Λ defines bare SC/SDW transition temperatures $(T_{c0}, T_{s0}) \sim \Lambda e^{-2/(g_{\rm s,m}\nu_F)}$.

In a spatially homogeneous case (4) has a solution which does not depend on coordinates. One finds that there exists the region in the values of Γ_0 where SC coexists with the SDW state. We are interested in finding out what happens to that region in the spatially inhomogeneous case. To find a solution in a general case we use the phenomenological method proposed by Larkin [38]: We assume that the coupling constants are functions of the coordinate and write them as

$$\frac{1}{\nu_F g_i(\mathbf{r})} = \left\langle\frac{1}{\nu_F g_i}\right\rangle + \lambda_i(\mathbf{r}) \tag{7}$$

($i =$ m,s). The averaging is performed over disorder distributions which we assume to be Gaussian and we also assume that $\lambda_i \ll 1$. The inhomogeneities in the coupling constants can be characterized by the following correlation function,

$$\varphi_{ij}(\mathbf{r} - \mathbf{r}') = \langle\lambda_i(\mathbf{r})\lambda_j(\mathbf{r}')\rangle, \varphi_{\mathbf{k}} = \int \varphi_{ij}(\mathbf{r})e^{-i\mathbf{kr}}d^2\mathbf{r}. \tag{8}$$

For simplicity we assume that the disorder correlators for the spin-density-wave and pairing couplings are the same. Functions $\varphi_{\mathbf{k}}$ describe the amount and size of the inhomogeneous regions.

Our plan now consists in finding the solution of (4) by perturbation theory. Since λ_i's are small, we seek for the correction to the quasiclassical function due to inhomogeneities in the form

$$\hat{\mathcal{G}}(\omega_n, \mathbf{n}, \mathbf{r}) = \langle\hat{\mathcal{G}}(\omega_n)\rangle + \delta\hat{\mathcal{G}}(\omega_n, \mathbf{n}, \mathbf{r}). \tag{9}$$

This form implies that for the order parameters we also write $\Delta(\mathbf{r}) = \langle\Delta\rangle + \delta\Delta(\mathbf{r})$ and $M(\mathbf{r}) = \langle M\rangle + \delta M(\mathbf{r})$. The first term on the right-hand side (9) is determined by the solution of the Eilenberger equation averaged over various disorder configurations, i.e., in the spatially homogeneous case, and is given by [20,32]

$$\langle\hat{\mathcal{G}}\rangle = g_\omega\hat{\tau}_3\hat{\rho}_3\hat{\sigma}_0 - f_\omega\hat{\tau}_0\hat{\rho}_2\hat{\sigma}_0 + s_\omega\hat{\tau}_2\hat{\rho}_3\hat{\sigma}_3. \tag{10}$$

Given the normalization condition for the function $\hat{\mathcal{G}}$, up to the linear order in $\hat{\mathcal{G}}_1$ it follows

$$\langle\hat{\mathcal{G}}\rangle\delta\hat{\mathcal{G}} + \delta\hat{\mathcal{G}}\langle\hat{\mathcal{G}}\rangle = 0. \tag{11}$$

This expression imposes a constraint on the matrix form for the function $\delta\hat{\mathcal{G}}$ and we choose to write it as follows,

$$\begin{aligned}\delta\hat{\mathcal{G}} =& i\alpha_x\hat{\tau}_2\hat{\rho}_1\hat{\sigma}_3 - \beta_x\hat{\tau}_3\hat{\rho}_1\hat{\sigma}_0 - \varsigma_x\hat{\tau}_1\hat{\rho}_0\hat{\sigma}_3\\ &+ a_x\hat{\tau}_3\hat{\rho}_3\hat{\sigma}_0 + ib_x\hat{\tau}_0\hat{\rho}_2\hat{\sigma}_0 + i\gamma_x\hat{\tau}_2\hat{\rho}_3\hat{\sigma}_3,\end{aligned} \tag{12}$$

with the notation $x = (\omega_n, \mathbf{n}, \mathbf{r})$. Given Eqs. (10) and (11) the functions in (12) must satisfy $g_\omega a_x - if_\omega b_x + is_\omega\gamma_x = 0$ and $g_\omega\alpha_x - is_\omega\beta_x - if_\omega\varsigma_x = 0$.

The first step towards obtaining our main result is to insert expressions (9), (10), and (12) into (4) and average both parts of the equation over the disorder distribution function keeping the leading nonvanishing terms which contain nontrivial corrections. There will be three resulting equations with one of them being redundant due to the normalization condition. The remaining two equations can be written compactly using the components of the vector $\vec{\Pi}$,

$$\Pi_z f_\omega - \Pi_x g_\omega = \langle a_x\delta\Delta\rangle, \quad \Pi_z s_\omega - \Pi_y g_\omega = \langle a_x\delta M\rangle, \tag{13}$$

where $\Pi_x = \langle\Delta\rangle + \Gamma_m f_\omega$, $\Pi_y = \langle M\rangle - \Gamma_t s_\omega$, $\Pi_z = \omega_n + \Gamma_t g_\omega$, and $\Gamma_{t,m} = \Gamma_0 \pm \Gamma_\pi$. The fact that only the interband scattering rate Γ_π enters into the first equation is a manifestation of the Anderson theorem, i.e., if we set $\Gamma_\pi \to 0$, then we recover the corresponding equation for the BCS model [2].

In order to compute the local (disorder-induced) correlation functions featured in Eqs. (13), we go back to the Eilenberger equation (4) and keep the terms up to the first order in the components of $\delta\hat{\mathcal{G}}$. The solution of the Eilenberger equation can be conveniently found by going into the momentum representation

$$\begin{aligned}a_k &= -\frac{\Pi_z[f_\omega\delta\Delta(\mathbf{k}) + s_\omega\delta M(\mathbf{k})]}{(v_F/2)^2(\mathbf{nk})^2 + \vec{\Pi}^2},\\ ib_k &= \frac{\Pi_y f_\omega\delta M(\mathbf{k}) - \delta\Delta(\mathbf{k})(\Pi_z g_\omega + \Pi_y s_\omega)}{(v_F/2)^2(\mathbf{nk})^2 + \vec{\Pi}^2},\\ i\gamma_k &= -\frac{\Pi_y f_\omega\delta\Delta(\mathbf{k}) - \delta M(\mathbf{k})(\Pi_z g_\omega + \Pi_x f_\omega)}{(v_F/2)^2(\mathbf{nk})^2 + \vec{\Pi}^2},\end{aligned} \tag{14}$$

where now $k = (\omega_n; \mathbf{n}, \mathbf{k})$. It is easy to check that these relations satisfy the corresponding constraint condition. In order to find the expressions which are valid for an arbitrary values of $k_F l$ (k_F is a Fermi momentum and l is the mean free path), in (14) one needs to replace $\delta\Delta(\mathbf{k}) \to \delta\Delta(\mathbf{k}) - i\Gamma_m\langle b_k\rangle_\mathbf{n} - (f_\omega/g_\omega)\Gamma_t\langle a_k\rangle_\mathbf{n}$, $\delta M(\mathbf{k}) \to \delta M(\mathbf{k}) - i\Gamma_t\langle\gamma_k\rangle_\mathbf{n} - (s_\omega/g_\omega)\Gamma_t\langle a_k\rangle_\mathbf{n}$ (here, $\langle\cdots\rangle_\mathbf{n}$ denotes the averaging over all directions of $\mathbf{n}$), and solve (14) for $\langle a_k\rangle_\mathbf{n}$, $\langle b_k\rangle_\mathbf{n}$, and $\langle\gamma_k\rangle_\mathbf{n}$ after averaging them over $\mathbf{n}$. All these expressions can be found in the closed form. Lastly, we note that the expressions for the remaining three functions α_k, β_k, and ς_k are of no importance to us since they do not contribute to the self-consistency equations for their averages over the directions of the Fermi velocity vanish identically.

With the help of the first equation (14) we can now express the disorder correlation functions (13) in terms of the order parameter correlators. For brevity, we represent it in terms of the two-component field $\hat{\Phi}(\mathbf{r}) = [\delta\Delta(\mathbf{r}), \delta M(\mathbf{r})]$:

$$\langle\hat{\Phi}(\mathbf{r})\hat{\Phi}(\mathbf{r}')\rangle = \int\frac{d^2\mathbf{k}}{(2\pi)^2}\begin{bmatrix}\mathcal{D}_\mathbf{k} & \mathcal{C}_\mathbf{k}\\ \mathcal{C}_\mathbf{k} & \mathcal{M}_\mathbf{k}\end{bmatrix}e^{i\mathbf{k}(\mathbf{r}-\mathbf{r}')}. \tag{15}$$

In their turn, the correlators (15) can be expressed in terms of the correlators of the interaction constants (8) by solving the following system of linear equations derived from the self-consistency conditions:

$$\begin{aligned}&\pi T\sum_{\omega_n>0}^{\infty}\left(\frac{g_\omega}{\omega_n + 2\Gamma_t g_\omega} + \chi_y s_\omega - \vec{p}_\omega\vec{\chi}\right)\delta M(\mathbf{k})\\ &+\pi T\sum_{\omega_n>0}^{\infty}\chi_y f_\omega\delta\Delta(\mathbf{k}) = -\langle M\rangle\lambda_\mathrm{m}(\mathbf{k}),\\ &\pi T\sum_{\omega_n>0}^{\infty}\left(\frac{g_\omega}{\omega_n + 2\Gamma_\pi g_\omega} + \chi_x f_\omega - \vec{p}_\omega\vec{\chi}\right)\delta\Delta(\mathbf{k})\\ &+\pi T\sum_{\omega_n>0}^{\infty}\chi_y f_\omega\delta M(\mathbf{k}) = -\langle\Delta\rangle\lambda_\mathrm{s}(\mathbf{k}).\end{aligned} \tag{16}$$

Here, functions χ_α are the components of the vector $\vec{\chi} = (\chi_x, \chi_y, \chi_z)$ with $\chi_j = (\Pi_j/|\vec{\Pi}|)[(v_F k/2)^2 + \vec{\Pi}^2]^{-1/2}$ and $\vec{p}_\omega = (f_\omega, s_\omega, g_\omega)$.

In what follows, we are primarily interested in finding how inhomogeneity-induced correlations influence the coexistence region. For this purpose, we only need to analyze the critical temperatures $T_c(M)$, which determines the onset of the SC emerging from the preexisting SDW state, and $T_s(\Delta)$, which sets the boundary between the coexistence region and purely SC state in the temperature-doping phase diagram. Therefore, we only need to analyze the expressions for the disorder correlators when one of the order parameters is zero.

Results for $T_c(M)$ boundary. In this case $\langle M\rangle \neq 0$, $\langle\Delta\rangle = 0$, and we also set $\delta\Delta = 0$, which means that the first equation (13) is satisfied identically, while the second equation can be parametrized as follows,

$$\Pi_z s_\omega - \Pi_y g_\omega = -\eta_\mathrm{m}\langle M\rangle g_\omega s_\omega, \tag{17}$$

where we parametrized the correlator as $\langle a_x\delta M\rangle = -\eta_\mathrm{m}\langle M\rangle g_\omega s_\omega$. The expression for the parameter η_m which is applicable for arbitrary values of $k_F l$ is

$$\eta_\mathrm{m} = \langle M\rangle\int_0^\infty\frac{d^2\boldsymbol{k}}{(2\pi)^2}\frac{\vec{p}_\omega\vec{\chi}\mathcal{M}_\mathbf{k}}{(1 + 2\Gamma_t\chi_z - \Gamma_t\vec{p}_\omega\vec{\chi})}, \tag{18}$$

where we used the identity $\vec{p}_\omega\vec{\chi} = \chi_z/g_\omega$, introduced

$$\mathcal{M}_\mathbf{k} = \varphi_\mathbf{k}\left[\pi T_c\sum_{\omega_n>0}^{\infty}\left(\frac{g_\omega}{\omega_n + 2\Gamma_t g_\omega} + \chi_y s_\omega - \vec{p}_\omega\vec{\chi}\right)\right]^{-2}, \tag{19}$$

and rescaled $\mathcal{M}_{\mathbf{k}} \to \langle M\rangle^2 \mathcal{M}_{\mathbf{k}}$. Without loss of generality, we consider

$$\varphi_{\mathbf{k}} = \varphi_0 e^{-(kr_c/2)^2}, \tag{20}$$

where φ_0 and r_c are the phenomenological parameters characterizing the magnitude and scale of the inhomogeneities. Although both $\langle M\rangle$ and η_m must be solved for self-consistently, from Eq. (17) we see that inhomogeneities produce the shift in the scattering rate $2\Gamma_t \to 2\Gamma_t + \eta_{\rm m}\langle M\rangle$. This means that at least at very small values of $\eta_{\rm m}$, $T_c(M)$ must increase compared to its value in the spatially homogeneous case for suppression of $\langle M\rangle$. Qualitatively this implies a boost for superconductivity. The actual magnitude of the parameter $\eta_{\rm m}$ crucially depends on the correlation radius r_c: When the correlation radius $k_F r_c \sim 1$ and $r_c \ll v_F/\langle\Delta\rangle$, we expect $\eta_{\rm m} \ll 1$.

Results for $T_s(\Delta)$ boundary. In this case $\langle\Delta\rangle \neq 0$, $\langle M\rangle = 0$, and thus we have

$$\Pi_z f_\omega - \Pi_x g_\omega = -\eta_{\rm s}\langle\Delta\rangle g_\omega f_\omega, \tag{21}$$

where the dimensionless parameter $\eta_{\rm s}$ is given by

$$\eta_{\rm s} \approx \langle\Delta\rangle \int_0^\infty \frac{d^2\boldsymbol{k}}{(2\pi)^2}\frac{\vec{p}_\omega \vec{\chi}\mathcal{D}_{\mathbf{k}}}{(1-\Gamma\vec{p}_\omega\vec{\chi})}, \tag{22}$$

and the rescaled $\mathcal{D}_{\mathbf{k}} \to \langle\Delta\rangle^2\mathcal{D}_{\mathbf{k}}$ correlator is

$$\mathcal{D}_{\mathbf{k}} = \varphi_{\mathbf{k}}\left[\pi T_s \sum_{\omega_n>0}^{\infty}\left(\frac{g_\omega}{\omega_n + 2\Gamma_\pi g_\omega} + \chi_x f_\omega - \vec{p}_\omega\vec{\chi}\right)\right]^{-2}. \tag{23}$$

We note that expression (22) acquires such a simple form only when we assume that $\Gamma_\pi \ll \Gamma_0 = \Gamma$. This approximation is not restrictive as Γ_π is primarily responsible for bending the SC dome of $T_c(\Gamma_\pi)$ at larger dopings, and has a weaker influence of the physics near the coexistence region. In addition, we observe again that inhomogeneities lead to an increase in the interband scattering rate $2\Gamma_\pi \to 2\Gamma_\pi + \eta_{\rm s}\langle\Delta\rangle$, so it is not *a priori* clear whether it will yield the suppression or boost of $T_s(\Delta)$. To resolve this question we need to employ a self-consistent approach.

Self-consistent method. Parameters $\eta_{\rm m,s}$ are functions of the Matsubara frequency and therefore Eqs. (18) and (22) must be solved self-consistently with (17) and (21) along with the equation for $\langle\Delta\rangle$ and $\langle M\rangle$. However, in the case of strong inhomogeneities the main contribution to the integral comes from the region of momenta $k \sim r_c^{-1}$ (r_c is the disorder correlation radius) and the frequency dependence of these parameters can be neglected. In Fig. 2 we show the results of the self-consistent solution of the equations above for the critical temperature $T_c(M)$ and $T_s(\Delta)$ correspondingly as functions of parameter φ_0. As we have expected, $T_c(M)$ increases with an increase in the magnitude of inhomogeneities, while $T_s(\Delta)$ decreases with an increase in φ_0. This means that within the linear approximation we have adopted, spatial inhomogeneities have a much more profound effect on the magnetic transition than on superconductivity.

Summary and discussion. In conclusion, we have considered the impact of spatial pairing-potential correlations induced by short-scale disorder fluctuations on the interplay of the SDW-SC competition in multiband metals. We found that quantitative effects stemming from the physics of short scales are the enhancement of superconducting T_c in the optimally doped region and a widening of the coexistence phase. These conclusions are rather robust and fairly universal as a microscopic form of the disorder correlation function is not essential. It is only the correlation radius and strength of correlations that determine the relevant parameters of the model.

The extent of the results presented in this Letter is limited by two major factors. First, we considered only a minimal two-band model. A more elaborate treatment will bring additional features, most notably a possible disorder-induced topological change of the superconducting gap structure [39,40], the appearance of a narrow dome of $s + is'$ time-reversal broken superconductivity separating the gapped and nodal regions [41], as well as the effects of nematic correlations [42]. All these phenomena have profound observed experimental signatures. However, these complications do not change the main conclusion of this work concerning the effect of short-range disorder fluctuations on the width of the coexistence region. Indeed, the multiband character simply brings additional renormalizations of Γ_π, and thus a steeper

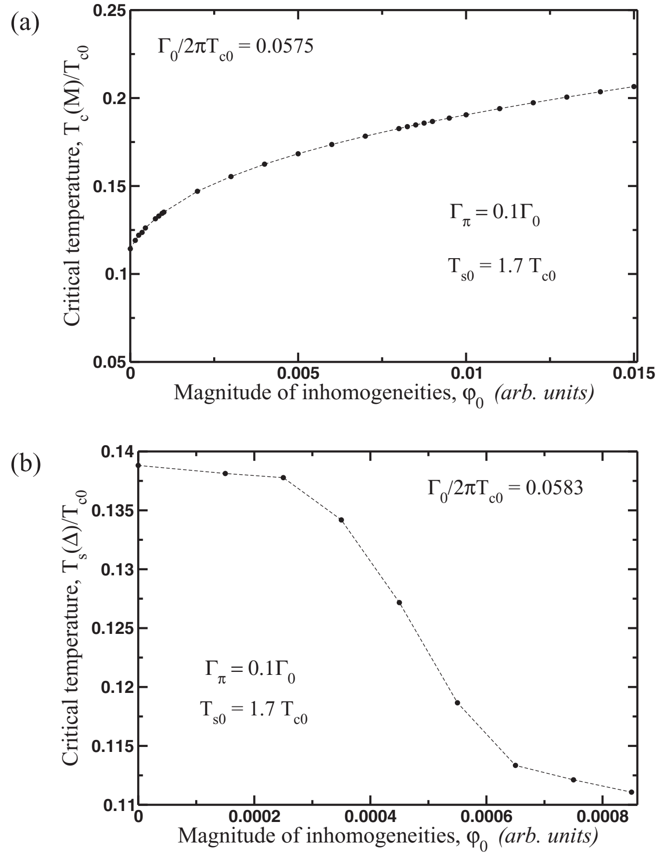

FIG. 2. (a) Results of the self-consistent solution of Eqs. (17) and (18) for the superconducting critical temperature inside the SDW phase. (b) Results of the self-consistent solution of Eqs. (21) and (22) for the SDW transition temperature inside the superconducting phase. These results have been obtained by neglecting the dependence of the parameters $\eta_{\rm m,s}$ on the Matsubara frequency.

suppression of T_c in the overdoped region, but has qualitatively the same weaker effect in the domain of optimal doping, as supported by our numerical self-consistent analysis. These details can be further tackled quantitatively based on the quasiclassical theory of the three-band modeling of magnetic order in iron pnictides [43] extended to superconducting scenarios. Second, we considered only weak impurities treated at the level of the Born approximation, thus missing the physics of the induced Yu-Shiba-Rusinov localized bound or miniband subgap states [44–46] that can be captured by a full $\hat{T}$-matrix analysis. This is still an open problem to address in the context of SDW-SC coexistence and the density of states subgap structure that we leave for further investigation.

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