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Sequential Predictors for Stabilization of Bilinear Systems under Measurement Uncertainty

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***Abstract*— We build delay-compensating feedback controls for a class of nonlinear systems that include bilinear systems with arbitrarily long known constant input delays. Unlike prior sequential predictor work, we cover bilinear systems whose state measurements have uncertainty, and we prove input-to-state stability with respect to the uncertainty. We do not require constructing or estimating distributed terms in the controls. We illustrate our result in a power systems example.**

I. Introduction

This paper continues the development (which started, e.g., in [1], [3], [8], and [10]) of sequential predictors to compensate for arbitrarily long input delays. While prior sequential predictor results covered dynamics whose right sides satisfy a linear growth condition, or where this growth condition is not needed but where the uncertainty is confined to the dynamics [2], here we cover systems with uncertain measurements where this linear growth condition is not needed. Hence, the present work can be applied to significant bilinear systems that were outside the scope of previous sequential predictor results; see [4] and [5] for the importance of bilinear systems.

Our work is motivated by the ubiquity of input delays in engineering, the pitfalls arising from standard controllers that were not designed to compensate for input delays, and the computational challenges that arise from standard predictors with distributed terms. This motivated [1] and other works on sequential predictors for delay systems, whose controls use an output of a stack of ordinary differential equations. This auxiliary system of equations contains copies of the original system running on multiple time scales, and additional stabilizing terms, enabling the compensation of arbitrarily long input delays without using distributed terms in the controls. However, such results usually call for right sides of the dynamics that grow linearly in the input and state, and so would not cover bilinear systems with outputs of the form

$$\begin{cases} \dot{x}(t) = A(t)x(t) + \sum_{i=1}^{c} u_i(t-h)\big(B_i(t)x(t) + G_i(t)\big) \\ y(t) = x(t) + \Delta(t) \end{cases} \tag{1}$$

with constant delays h, unknown measurable locally essentially bounded functions Δ (representing measurement uncertainty), controls $u = (u_1, \ldots, u_c)$, and bounded continuous coefficient matrices. While such systems are often stabilizable with bounded controls u_i, knowing a bound on u is insufficient to extend previous sequential predictor papers to the bilinear system (1). This is because earlier results also need input-to-state stability (or ISS) with respect to measurement uncertainty, and because one must bound Δ.

This calls for the advances of this work, which eliminates the requirement that the right sides grow linearly in the input and state, and covers measurement uncertainty. These advances are made possible by a nontrivial variant of [2], which used a significantly different Lyapunov-Krasovskii functional construction as compared with works like [8] whose dynamics grow linearly. Using a variant of [2] with uncertainty in the measurements y (instead of in the dynamics itself, which is where it was in [2]), this overcomes a longstanding obstacle to building output feedback stabilizing sequential predictors for bilinear systems.

II. Definitions and Notation

Throughout this work, the dimensions of the Euclidean spaces are arbitrary unless we indicate otherwise, and we omit arguments of functions when no confusion would arise. The standard Euclidean norm in $\mathbb{R}^n$ and the induced matrix norm are denoted by $|\cdot|$, and $|\phi|_{\mathcal{I}}$ (resp., $|\phi|_\infty$) is the standard essential supremum of a function ϕ over an interval $\mathcal{I}$ in its domain (resp., its entire domain). Consider a dynamics

$$\dot{X}(t) = \mathcal{F}\big(t, X(t), u_{\mathcal{F}}(t-h), \Delta(t)\big), \tag{2}$$

whose state X, feedback control $u_{\mathcal{F}}$, and unknown Lebesgue measurable locally essentially bounded function Δ are valued in $\mathbb{R}^{n_1}$, $\mathbb{R}^{n_2}$, and $\mathbb{R}^{n_3}$, respectively, where $h > 0$ denotes a constant delay. Due to the delay, solutions of (2) are defined for given initial times $t_0 \geq 0$, functions Δ, and initial functions that are defined on an initial interval $\mathcal{I}^0 \subseteq (-\infty, t_0]$ such as $[t_0 - h, t_0]$. We assume that (2) is forward complete, meaning all such solutions are uniquely defined on $\mathcal{I}^0 \cup [t_0, \infty)$; our assumptions below will ensure this forward completeness condition. We use the usual standard classes $\mathcal{KL}$ and $\mathcal{K}_\infty$ of comparison functions from [7, Chapt. 4] and this standard definition of input-to-state stability (or ISS, which we also use to denote input-to-state stable):

Definition 1: The system (2) is ISS with respect to a disturbance set $\mathcal{D} \subseteq \mathbb{R}^{n_3}$ provided there are $\beta \in \mathcal{KL}$ and $\gamma \in \mathcal{K}_\infty$ so that for all initial times t_0, initial functions, and Δ's that are valued in $\mathcal{D}$, the corresponding solutions of (2) satisfy $|X(t)| \leq \beta(|X|_{\mathcal{I}^0}, t - t_0) + \gamma(|\Delta|_{[t_0,t]})$ for all $t \geq t_0$.

Let $\mathcal{B}_R$ be the closed ball of any radius $R > 0$ in Euclidean space centered at the origin, and $\mathbb{N} = \{1, 2, \ldots\}$. For subsets S_1 and S_2 of Euclidean spaces, a function $W : S_1 \times S_2 \to \mathbb{R}^n$

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is called locally Lipschitz in its second variable uniformly in its first variable provided: for each constant $R>0$, there is a constant $L_R>0$ such that $|W(s_1,s_a)-W(s_1,s_b)|\le L_R|s_a-s_b|$ for all $s_1\in S_1$ and all s_a and s_b in $\mathcal{B}_R\cap S_2$. When L_R in the preceding property can be taken to be independent of R, we use the term globally (instead of locally) Lipschitz. A function $J:[0,\infty)\times\mathbb{R}^n\to[0,\infty)$ is called uniformly proper and positive definite provided there are functions $\underline{\gamma}\in\mathcal{K}_\infty$ and $\overline{\gamma}\in\mathcal{K}_\infty$ such that $\underline{\gamma}(|x|)\le J(t,x)\le\overline{\gamma}(|x|)$ for all $x\in\mathbb{R}^n$ and $t\ge 0$. We set $\Psi_t(s)=\Psi(t+s)$ for all Ψ, $s\le 0$, and $t\ge 0$ for which $t+s$ lies in the domain of Ψ. We use $0_{\ell\times r}$ (resp., I_r) to mean the $\ell\times r$ matrix whose entries are all 0 (resp., the $r\times r$ identity matrix).

III. General Result

Before presenting results for bilinear systems, we discuss our novel result for a more general class of dynamics

$$\dot{x}(t)=f(t,x(t),u(t-h)),\ \ y(t)=x(t)+\Delta(t) \tag{3}$$

whose state x, control u, and unknown Lebesgue measurable locally essentially bounded function Δ are valued in $\mathbb{R}^n$, $\mathbb{R}^c$, and $\mathbb{R}^n$ respectively, and $h>0$ is a constant delay. Assume:

Assumption 1: There are a compact neighborhood $\mathcal{U}\subseteq\mathbb{R}^c$ of $0_{c\times1}$, a continuous function $u_s:[0,\infty)\times\mathbb{R}^n\to\mathcal{U}$ that is globally Lipschitz in its second variable uniformly in its first variable, and a constant $\bar{\epsilon}>0$ such that the system

$$\dot{x}(t)=f\big(t,x(t),u_s(t,x(t)+\epsilon(t))\big) \tag{4}$$

with disturbance ϵ is ISS with respect to the disturbance set $\mathcal{B}_{\bar{\epsilon}}$. Also, $u_s(t,0_{n\times1})=0_{c\times1}$ for all $t\ge0$. □

Assumption 2: The function f is continuous, and locally Lipschitz in (x,u) uniformly in t, satisfies $f(t,0_{n\times1},0_{c\times1})=0_{n\times1}$ for all $t\ge0$, and admits a constant $k>0$ such that

$$|f(t,z_1,U)-f(t,z_2,U)|\le k|z_1-z_2| \tag{5}$$

holds for all $t\ge0$, $z_1\in\mathbb{R}^n$, $z_2\in\mathbb{R}^n$, and $U\in\mathcal{U}$ for the choice of $\mathcal{U}$ of the control set from Assumption 1. □

Throughout this work, we consider any integer $m\ge2$ and any constants $\epsilon_*>0$, $h>0$, $C_1\in(0,2m/h)$, $C_2>0$, $C_3>0$, and $\lambda_a>0$, and any constants k and $\bar{\epsilon}$ satisfying Assumptions 1-2, and then we define

$$\begin{aligned}
&p=\tfrac{m(4k+\lambda_a)}{2m-hC_1},\ \epsilon_{0,\ell}=\max\left\{1,\tfrac{\overline{C}(1+\lambda_a)h}{m}\right\},\\
&\overline{C}=\tfrac{p(1+C_3)}{C_1}\max\left\{p^2(1+C_2),k^2\left(1+\tfrac{1}{C_2}\right)\left(1+\tfrac{\lambda_a}{4}\right)\right\},\\
&\epsilon_0=\min\left\{2k\left(1-\tfrac{h\overline{C}}{km}(1+\lambda_a)\right),\tfrac{\overline{C}\lambda_a m}{2(h\overline{C}(1+\lambda_a)+m)}\right\},\\
&\hat{c}=\max\left\{\tfrac{2p^2(1+C_3)}{\epsilon_0},(1+C_3)C_2^\sharp,\tfrac{\epsilon_0}{2}\right\},\\
&\widetilde{M}=\tfrac{p^2}{2\lambda_a}+C_2^\sharp\tfrac{h}{m},\ \widetilde{N}=\left(1+\tfrac{1}{C_3}\right)\left(\tfrac{p^2}{\epsilon_0}+C_2^\sharp\tfrac{h}{m}\right),\\
&C_2^\sharp=\left(1+\tfrac{1}{C_2}\right)\left(1+\tfrac{4}{\lambda_a}\right)\tfrac{p^3}{2C_1},\ \bar{\epsilon}_*=\min\left\{0.5\epsilon_0,\tfrac{\epsilon_*}{\omega_{m-1}}\right\},\\
&\omega_0=1,\text{ and }\omega_i=\tfrac{2}{\epsilon_0}\left(\hat{c}\omega_{i-1}+\epsilon_*\right)\text{ if }1\le i\le m-1,
\end{aligned} \tag{6}$$

which will all be positive constants under the requirement (8) from our theorem. The integer m will be the number of sequential predictors, and the constants C_i's will be weighting constants in our Young's inequality applications in the appendix below. Using the constants (6) and with any constant

$$\bar{\Delta}\in\left(0,\frac{\bar{\epsilon}}{m}\sqrt{\frac{\bar{\epsilon}_*}{2\epsilon_{0,\ell}(\omega_{m-1}\widetilde{M}+\omega_{m-2}\widetilde{N})}}\right), \tag{7}$$

we prove the following output feedback stabilization result:

Theorem 1: Let $k>0$ and $\bar{\epsilon}>0$ and the function u_s be such that (3) satisfies Assumptions 1-2. Assume that

$$m>\tfrac{h\overline{C}(1+\lambda_a)}{k}. \tag{8}$$

Consider (3) in closed loop with

$$u(t)=u_s\big(t+h,z_m(t)\big), \tag{9}$$

where z_m is the last n components of the state of the system

$$\left\{\begin{array}{rcl}
\dot{z}_1(t)&=&f\left(t+\frac{h}{m},z_1(t),\Phi(t,z_m,1)\right)\\
&&-p[z_1\left(t-\frac{h}{m}\right)-y(t)]\\
\dot{z}_2(t)&=&f\left(t+\frac{2h}{m},z_2(t),\Phi(t,z_m,2)\right)\\
&&-p\left[z_2\left(t-\frac{h}{m}\right)-z_1(t)\right]\\
&\vdots&\\
\dot{z}_m(t)&=&f\left(t+h,z_m(t),\Phi(t,z_m,m)\right)\\
&&-p\left[z_m\left(t-\frac{h}{m}\right)-z_{m-1}(t)\right]
\end{array}\right. \tag{10}$$

where $\Phi(t,z_m,i)=u_s(t+h-h(m-i)/m,z_m(t-h(m-i)/m))$ for all $t\ge0$ and $i\in\{1,2,\dots,m\}$ and $z_0=x$. Then there are functions $\beta_d\in\mathcal{KL}$ and $\gamma_d\in\mathcal{K}_\infty$ such that all solutions $(x,z):[t_0-2h,\infty)\to\mathbb{R}^{(m+1)n}$ of the closed loop system given by (3) and (9)-(10), for all Lebesgue measurable essentially bounded functions $\Delta:[0,\infty)\to\mathcal{B}_{\overline{\Delta}}$ and all initial times $t_0\ge h/m$, satisfy

$$\begin{aligned}|x(t)|\le{}&\beta_d\left(|x|_{[t_0-2h,t_0+h/m]}+|z|_{[t_0-2h,t_0+h/m]},t-t_0\right)\\&+\gamma_d\big(|\Delta|_{[t_0,t]}\big)\end{aligned} \tag{11}$$

for all $t\ge t_0$, where $z=(z_1,\dots,z_m)$. □

Remark 1: It is tempting to guess that, at least for bilinear cases, we can reduce our study of (3) to the study of systems whose right sides are globally Lipschitz in the state (which were covered in [8]), by replacing f by

$$f_{\rm new}(t,x,u)=\left\{\begin{array}{ll}f(t,x,u), & \text{if } |u|\le R\\ f\left(t,x,\frac{uR}{|u|}\right), & \text{if } |u|>R\end{array}\right. \tag{12}$$

for a bound $R>0$ on the control u_s. However, such a replacement would not solve the problems in this paper, where there is a restriction on the allowable uncertainties ϵ in Assumption 1 (making our assumption less restrictive than in [8], whose ISS assumption is required for all choices of $\epsilon(t)$) and where we must therefore compute a bound $\bar{\Delta}$ on the allowable Δ's; see (33) below. The requirement that u_s is valued in the compact set $\mathcal{U}$ is introduced to ensure that (5) is satisfied when U is a control value; see (A.3)-(A.4) below. The bound on $\mathcal{U}$ in Assumption 2 is needed for the existence of the required k when (2) is bilinear; see Remark 3 below. □

Remark 2: When $\Delta=0$, then by replacing $1+C_3$ and

$1+1/C_3$ in (6) by 1, Theorem 1 agrees with the special case of the main result in [2] for bilinear systems without uncertainty; see Remark 4 below for more comparisons with [2]. A notable distinction between works such as [9] and Theorem 1 is that we obtain a control with no distributed terms, based on variants of the sequential predictor framework that is known to be computationally cheap. □

Remark 3: The growth requirement (5) from Assumption 2 holds for our bilinear systems (1) for any bounded neighborhood $\mathcal{U} \subseteq \mathbb{R}^c$ of the origin and any bounded continuous functions A, B_i, and G_i for each i. This follows by picking

$$k = |A|_\infty + \bar{U} \sum_{i=1}^{c} |B_i|_\infty \tag{13}$$

for any bound $\bar{U}$ on the elements of $\mathcal{U}$. Then we can use [2, Lemmas 2 and 3] to check Assumption 1. □

IV. Proof of Theorem 1

Throughout the proof, our inequalities and equalities should be interpreted to hold for all $t \geq t_0$ and $t_0 \geq h/m$ along all solutions of the closed loop system from the statement of the theorem, unless otherwise noted. Recalling our definition $z_0 = x$, we introduce the error variables

$$\begin{array}{l} \mathcal{E} = (\mathcal{E}_1, \ldots, \mathcal{E}_m), \text{ where} \\ \mathcal{E}_i(t) = z_i(t) - z_{i-1}\left(t + h/m\right) \text{ for } i = 1, \ldots, m. \end{array} \tag{14}$$

The remainder of the proof has three parts.

First Part: Lyapunov-Krasovskii Functionals for $\mathcal{E}_i$. In terms of the constants from (6), we use the functionals

$$\hat{\mu}(\mathcal{E}_{i,t}) = \tfrac{1}{2}|\mathcal{E}_i(t)|^2 + \int_{t-2h/m}^{t} |\mathcal{E}_i(\ell)|^2 \mathrm{d}\ell \tag{15}$$

for $i = 1, 2, \ldots, m$ and the following analog of [2, Lemma 1] for cases where the uncertainty is in the measurements instead of being in the dynamics (and which we prove in the appendix below, and where $\mathcal{E}_{i,t}$ is the ith component of $\mathcal{E}_t$ for each i and we continue using the constants from (6)):

Lemma 1: Consider the functions $\nu(\mathcal{E}_i) = \frac{1}{2}|\mathcal{E}_i|^2$ and

$$\begin{array}{l} \mu(\mathcal{E}_{i,t}) \\ = \nu(\mathcal{E}_i(t)) + \overline{C}(1+\lambda_a) \int_{t-2h/m}^{t} \int_s^t \nu(\mathcal{E}_i(\ell))\mathrm{d}\ell \mathrm{d}s \\ \text{and } \widetilde{\mu}(\mathcal{E}_{i,t}) = \mu(\mathcal{E}_{i,t}) + \int_{t-2h/m}^{t} |\mathcal{E}_i(\ell)|^2 \mathrm{d}\ell \end{array} \tag{16}$$

for $i = 1, 2, \ldots, m$. Then, the inequalities

$$\dot{\mu}(\mathcal{E}_{1,t}) \leq -\epsilon_0 \widetilde{\mu}(\mathcal{E}_{1,t}) + \widetilde{M}|\Delta|_\infty^2 \tag{17}$$

and

$$\begin{array}{l} \dot{\mu}(\mathcal{E}_{i,t}) \leq -\frac{\epsilon_0}{2}\widetilde{\mu}(\mathcal{E}_{i,t}) + \frac{p^2(1+J_iC_3)}{\epsilon_0}|\mathcal{E}_{i-1}(t)|^2 \\ +(1+J_iC_3)C_2^\sharp \int_{t-h/m}^{t} |\mathcal{E}_{i-1}(\ell)|^2 \mathrm{d}\ell + J_i\widetilde{N}|\Delta|_\infty^2 \end{array} \tag{18}$$

hold for all $t \geq h/m$ and $i \in \{2, \ldots, m\}$, where the function J_i is defined by $J_i = 1$ if $i = 2$ and $J_i = 0$ if $i > 2$. □

The proof that ϵ_0 from (6) satisfies the conditions from Lemma 1 is based on the fact that

$$\begin{array}{rcl} \mu(\mathcal{E}_{i,t}) & \leq & \frac{1}{2}|\mathcal{E}_i(t)|^2 \\ & & + \overline{C}(1+\lambda_a)\frac{2h}{m}\frac{1}{2}\int_{t-2h/m}^{t} |\mathcal{E}_i(\ell)|^2 \mathrm{d}\ell \\ & \leq & \epsilon_{0,\ell}\hat{\mu}(\mathcal{E}_{i,t}) \end{array} \tag{19}$$

for $i = 1, \ldots, m$ and all $t \geq h/m$. We also use the bound (19) later in the proof below. From our choices (6) of our constants, we conclude from Lemma 1 that for all $i \in \{2, \ldots, m\}$ and $t \geq h/m$, we have

$$\dot{\mu}(\mathcal{E}_{i,t}) \leq -\tfrac{\epsilon_0}{2}\hat{\mu}(\mathcal{E}_{i,t}) + \hat{c}\hat{\mu}(\mathcal{E}_{i-1,t}) + J_i\widetilde{N}|\Delta|_\infty^2. \tag{20}$$

Second Part: ISS Estimate for $\mathcal{E}$ Dynamics. We next prove that with the constants ω_i defined in (6), the function

$$\mu_m^\sharp(\mathcal{E}_t) = \sum_{j=1}^{m} \omega_{m-j}\mu(\mathcal{E}_{j,t}) \tag{21}$$

is an ISS Lyapunov-Krasovskii functional for the $\mathcal{E}$ dynamics with Δ being the uncertainty in the ISS estimate. This will be done using induction and the partial sums

$$\mu_r^\sharp(\mathcal{E}_t) = \mu(\mathcal{E}_{m,t}) + \omega_1\mu(\mathcal{E}_{m-1,t}) + \ldots + \omega_r\mu(\mathcal{E}_{m-r,t}) \tag{22}$$

for $r = 1, \ldots, m-1$. Using (20) and the fact that

$$\mu_1^\sharp(\mathcal{E}_t) = \mu(\mathcal{E}_{m,t}) + \tfrac{2}{\epsilon_0}(\hat{c}+\epsilon_*)\mu(\mathcal{E}_{m-1,t}) \tag{23}$$

and the J_i notation from Lemma 1, we conclude that

$$\begin{array}{rcl} \dot{\mu}_1^\sharp & \leq & -\frac{\epsilon_0}{2}\hat{\mu}(\mathcal{E}_{m,t}) + \hat{c}\hat{\mu}(\mathcal{E}_{m-1,t}) \\ & & + \frac{2}{\epsilon_0}(\hat{c}+\epsilon_*)\left[\frac{-\epsilon_0}{2}\hat{\mu}(\mathcal{E}_{m-1,t}) + \hat{c}\hat{\mu}(\mathcal{E}_{m-2,t}) \right. \\ & & \left. + J_{m-1}\tilde{N}|\Delta|_\infty^2\right] \\ & \leq & -\frac{\epsilon_0}{2}\hat{\mu}(\mathcal{E}_{m,t}) - \epsilon_*\hat{\mu}(\mathcal{E}_{m-1,t}) \\ & & + \hat{c}\frac{2}{\epsilon_0}(\hat{c}+\epsilon_*)\hat{\mu}(\mathcal{E}_{m-2,t}) + \mathcal{G}(m)|\Delta|_\infty^2 \end{array} \tag{24}$$

if $m > 2$ and $t \geq h/m$, where $\mathcal{G}(3) = (2/\epsilon_0)(\hat{c}+\epsilon_*)\widetilde{N}$ and $\mathcal{G}(m) = 0$ if $m > 3$. On the other hand, if $m = 2$, then we can instead use (17) to check that

$$\begin{array}{rcl} \dot{\mu}_1^\sharp & \leq & -\frac{\epsilon_0}{2}\hat{\mu}(\mathcal{E}_{2,t}) - \epsilon_*\hat{\mu}(\mathcal{E}_{1,t}) \\ & & + \left[\widetilde{N} + \frac{2}{\epsilon_0}(\hat{c}+\epsilon_*)\widetilde{M}\right]|\Delta|_\infty^2 \end{array} \tag{25}$$

for all $t \geq h/m$. By induction, and by using the scaled nonpositive terms from $\dot{\mu}(\mathcal{E}_{i-1,t})$ to cancel nonnegative terms from $\dot{\mu}(\mathcal{E}_{i,t})$ for $i = 2, \ldots, m$, it follows that

$$\dot{\mu}_m^\sharp \leq -\tfrac{\epsilon_0}{2}\hat{\mu}(\mathcal{E}_{m,t}) - \epsilon_* \sum_{j=1}^{m-1} \hat{\mu}(\mathcal{E}_{m-j,t}) + \mathcal{L}|\Delta|_\infty^2 \tag{26}$$

for all $t \geq h/m$ and $m > 1$, where $\mathcal{L} = \omega_{m-1}\widetilde{M} + \omega_{m-2}\widetilde{N}$. Also, (19) gives $\omega_{m-i}\mu(\mathcal{E}_{i,t}) \leq \epsilon_{0,\ell}\omega_{m-i}\hat{\mu}(\mathcal{E}_{i,t})$ for $i = 1, \ldots, m$ and all $t \geq h/m$, and $1 \leq \omega_i \leq \omega_{i+1}$ for $i = 0, \ldots, m-2$ and $m \geq 2$, since $\hat{c} \geq \epsilon_0/2$.

It follows from (19), (26), and our choice of $\bar{\epsilon}_*$ in (6) that

$$\dot{\mu}_m^\sharp \leq -\tfrac{\bar{\epsilon}_*}{\epsilon_{0,\ell}}\mu_m^\sharp(\mathcal{E}_t) + \mathcal{L}|\Delta|_\infty^2 \tag{27}$$

for all $t \geq h/m$. By applying the method of variation of parameters to (27), we get a constant $c_a > 0$ such that

$$\begin{array}{rcl} \frac{1}{2}|\mathcal{E}(t)|^2 & \leq & \mu_m^\sharp(\mathcal{E}_t) \\ & \leq & c_a e^{\hat{\epsilon}_*(t_0-t)}|\mathcal{E}|_{[t_0-2h,t]}^2 + \frac{\mathcal{L}|\Delta|_\infty^2}{\hat{\epsilon}_*} \end{array} \tag{28}$$

for all $t \geq t_0$, where $\hat{\epsilon}_* = \bar{\epsilon}_*/\epsilon_{0,\ell}$. After multiplying (28) through by 2 and then using the subadditivity of the square root (in order to upper bound the square root of the two right side terms), we conclude that

$$|\mathcal{E}(t)| \leq e^{0.5\hat{\epsilon}_*(t_0-t)}\sqrt{2c_a}|\mathcal{E}|_{[t_0-2h,t_0]} + \sqrt{\tfrac{2\mathcal{L}}{\hat{\epsilon}_*}}|\Delta|_\infty \quad (29)$$

holds for the $\mathcal{E}$ dynamics for all $t \geq t_0$ and $t_0 \geq h/m$.

Third Part: ISS-Like Estimate for Closed Loop x Dynamics. We use the new variable

$$\mathcal{E}^\sharp(t) = \sum_{\ell=0}^{m-1} \mathcal{E}_{m-\ell}\big(t + \ell\tfrac{h}{m} - h\big) \quad (30)$$

and we choose a constant $\lambda_* \in (0,1)$ that satisfies

$$\bar{\Delta} = \lambda_* \tfrac{\bar{\epsilon}}{m}\sqrt{\tfrac{\hat{\epsilon}_*}{2\mathcal{L}}}, \quad (31)$$

which exists because of our condition on the bound $\bar{\Delta}$ on Δ from (7) . Since $1/\lambda_* > 1$, the exponential ISS condition in (29) then produces a constant $\mathcal{T} > 0$ such that

$$|\mathcal{E}(t)| \leq \tfrac{1}{\lambda_*}\sqrt{\tfrac{2\mathcal{L}}{\hat{\epsilon}_*}}\bar{\Delta} \quad (32)$$

for all $t \geq t_0 + \mathcal{G}_\Delta$ and such that we also get

$$|\mathcal{E}^\sharp(t)| \leq m|\mathcal{E}|_{[t-h,t-h/m]} \leq \tfrac{m}{\lambda_*}\sqrt{\tfrac{2\mathcal{L}}{\hat{\epsilon}_*}}\bar{\Delta} = \bar{\epsilon} \quad (33)$$

for all $t \geq t_0 + h + \mathcal{G}_\Delta$, where $\mathcal{G}_\Delta = \mathcal{T}(|x|_{[t_0-2h,t_0+h/m]} + |z|_{[t_0-2h,t_0+h/m]})$, by (31) and our condition $|\Delta|_\infty \leq \bar{\Delta}$. To find a formula for the constant $\mathcal{T}$, notice that if we set

$$\bar{e} = \frac{\overline{\Delta}}{\sqrt{2c_a}}\left(\frac{1}{\lambda_*} - 1\right)\sqrt{\frac{2\mathcal{L}}{\hat{\epsilon}_*}}, \quad (34)$$

then it follows from (29) that (32) holds if $|\mathcal{E}|_{[t_0-2h,t_0]} \leq \bar{e}$; while if $|\mathcal{E}|_{[t_0-2h,t_0]} > \bar{e}$, then (32) is satisfied if $t - t_0 \geq (2/\hat{\epsilon}_*)\ln(|\mathcal{E}|_{[t_0-2h,t_0]}/\bar{e})$ (again by (29)), which holds if $t - t_0 \geq (2/\hat{\epsilon}_*)|\mathcal{E}|_{[t_0-2h,t_0]}/\bar{e}$ (using the fact that $\ln(r) \leq r$ for all $r \geq 1$), which holds if $t - t_0 \geq (2m/(\bar{e}\hat{\epsilon}_*))(|x|_{[t_0-2h,t_0+h/m]} + |z|_{[t_0-2h,t_0+h/m]})$. Therefore, we can choose $\mathcal{T} = 2m/(\bar{e}\hat{\epsilon}_*)$. The rest of the proof of the theorem closely follows the last part of the proof of [2, Theorem 1], and so is omitted here.

Remark 4: The work [2] covers systems $\dot{x}(t) = f(t, x(t), u(t-\tau), \delta(t))$ with uncertainty δ. Its sequential predictors are defined as in (10) except with f replaced by $f_0(t,x,u) = f(t,x,u,0)$. This produces a perturbation in the dynamics for only the first error variable $\mathcal{E}_1$. By contrast, here Δ enters the dynamics of the first predictor z_1 (through the measurement $y(t) = x(t) + \Delta(t)$), leading to perturbations in the dynamics for $\mathcal{E}_1$ and $\mathcal{E}_2$, hence two denominator terms in (7). This contrasts with [2], which has only one term in the denominator of the upper bound on the uncertainty, coming from the perturbation only being in $\mathcal{E}_1$. □

V. Dc/dc Converter Application

We study a dc/dc converter dynamic, of the type that are widely used in electrified transportation and renewable energy systems [6]. The converter we consider is a buck/boost bidirectional dc/dc converter which is capable of stepping the input voltage up and down and providing bidirectional power flow between the input source and the output load. When the converter operates in boost mode, the current flows from the source to the load, and in buck mode, the current reverses direction and flows from the load to the source.

The control input is the switch's duty ratio, which is the ratio of the switch's conduction time to the switching period. The duty ratio is determined by processing the sampled current and voltage measurements according to the control strategy on a digital signal processor (or DSP). Sampling rates range from 50 to 100 micro seconds for acceptable control performance. There is a delay between the time sampled measurements are received and the time control decisions are made. These delays in the control input, if not compensated, can degrade controller's performance and in some cases push the closed-loop control system to instability; see, e.g., Fig. 3 below. It is common to consider a control input delay of one sampling period, i.e., 50 to 100 microseconds.

Using Kirchoff's laws, the dynamics of the dc/dc buck/boost converter can be expressed as

$$L\tfrac{di}{dt} = -dV_{\rm dc} - Ri + V_{\rm in}, \;\; C_{\rm dc}\tfrac{d}{dt}V_{\rm dc} = -i_0 + di \quad (35)$$

with control d (which is also known as the switching function) where $V_{\rm in}$ is the input source voltage, i_0 is the load current, i is the input current, $C_{\rm dc}$ is the dc-link capacitance, $V_{\rm dc}$ is the dc-link voltage, and R and L are the input source internal resistance and inductance, respectively; see Fig. 1. Hence, R, L, $C_{\rm dc}$, i_0, and $V_{\rm in}$ are given constants. Consider

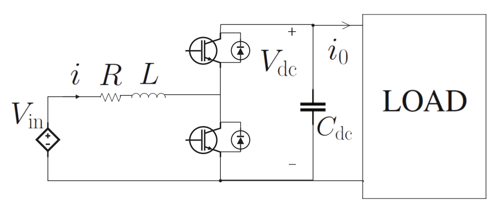

Fig. 1. Dc/dc Converter.

constant reference values I and $V_{\rm dcref}$ of i and $V_{\rm dc}$, respectively, which therefore satisfy $0 = -DV_{\rm dcref} - RI + V_{\rm in}$ and $0 = -i_0 + DI$ for some reference value D of the control input d. This produces the bilinear dynamics

$$\begin{cases} \dot{x}_1(t) \;=\; \frac{1}{L}\big[-Dx_2(t) \\ \qquad\qquad -u(t-h)(x_2(t) + V_{\rm dcref}) - Rx_1(t)\big] \\ \dot{x}_2(t) \;=\; \frac{1}{C_{\rm dc}}\big[Dx_1(t) + u(t-h)(x_1(t) + I)\big] \end{cases} \quad (36)$$

for the error states $x_1 = i - I$ and $x_2 = V_{\rm dc} - V_{\rm dcref}$ and control $u = d - D$. The control goal is to design the control u to compensate for a given constant delay $h > 0$, which will be realized if we render (36) in closed loop with the control from our theorem ISS with respect to the perturbation Δ in the measurement $y = x + \Delta$, where $x = (x_1, x_2)$ and where D, $V_{\rm dcref}$, R, and I are suitable positive constants.

We next use Theorem 1 to build the required control u. To check that its Assumption 1 is satisfied, we apply [2, Lemma 3] with $n = 2$, $c = 1$, the constant coefficient matrices

$$A = \begin{bmatrix} -R/L & -D/L \\ D/C_{\rm dc} & 0 \end{bmatrix}, \; B_1 = \begin{bmatrix} 0 & -1/L \\ 1/C_{\rm dc} & 0 \end{bmatrix}, \quad (37)$$
$$\text{and } G_1 = \begin{bmatrix} -V_{\rm dcref}/L \\ I/C_{\rm dc} \end{bmatrix},$$

the diagonal matrix $P = 0.5\text{diag}\{L, C_{\text{dc}}\}$, $c_1 = R$, $c_2 = 0$, $\delta = 0$, $\mathcal{H}_* = |(V_{\text{dcref}}, I)|$, and $M_1(t, x) = Ix_2 - V_{\text{dcref}}x_1$. It then follows from [2, Lemma 3] that the requirements of Assumption 1-2 are met, with $k = |A| + (\pi/2)\bar{\omega}|B_1|$ and $u_s(t, x) = -\bar{\omega}\arctan(M_1(t, x))$ for any constant $\bar{\omega} > 0$.

In Fig. 2, we plot Mathematica simulations for the state x of (36) using the control from Theorem 1 with 0 initial states for each z_i, and the preceding choices. We chose $h = 0.0001$, $V_{\text{dcref}} = 400$, $D = 0.597$, $I = 20$, $L = 0.001$, $\bar{\omega} = 0.01$, $R = 0.08$, and $C_{\text{cdc}} = 0.0015$, which satisfy the requirements from Theorem 1 with $C_1 = 1.38/h$, $C_2 = 0.4$, and $m = 2$ sequential predictors, where for simplicity, we took the measurement uncertainty $\Delta = 0$ as in Remark 2.

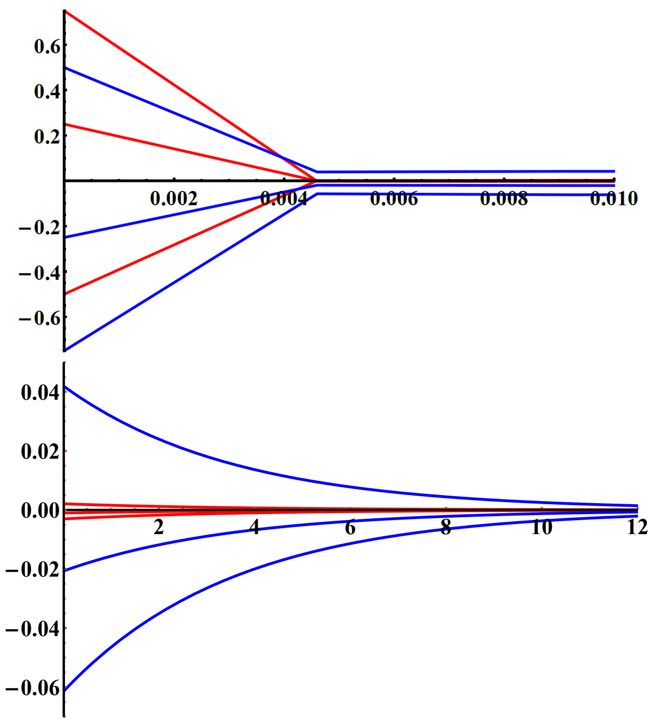

Fig. 2. First (Red) and Second (Blue) Components of (36), with Initial States $(0.75, -0.75)$, $(-0.5, 0.5)$, and $(0.25, -0.25)$, On Time Intervals $[0, 0.01]$ (Top Panel) and $[0.01, 12]$ (Bottom Panel).

We expressed the convergence in two phases in Fig. 2, to show the different performance on the interval $[0, 0.01]$ (during which only one error state converges closely to 0) and the second phase during $[0.01, 12]$ (when both states convergence to the equilibrium). These simulations show the effectiveness of our method. While the components of our initial states at time $t_0 = 0$ range from -0.75 to 0.75, the corresponding $x_i(t)$ components are valued in $[-0.06, 0.042]$ by time 0.01, and this explains why the starting values of the $x_i(t)$'s are contained in $[-0.06, 0.042]$ in the bottom panel of Fig. 2 which starts at time 0.01. While h is small, it is significant relative to the system dynamics, and we can compensate for any constant $h > 0$. For example, if we increase h to $h = 0.001$, then our assumptions are satisfied with $m = 4$ and with all other constants kept the same.

It is tempting to surmise that since h is small, the system would exhibit good performance without predictors. However, this would not be correct, because our approach improves on the control performance that we would have obtained without delay compensation. This is illustrated in our Mathematica simulation in Fig. 3 below, where we replaced the last sequential predictor z_2 in u by x (by simulating (36) with the control $u(t-h) = \bar{\omega}\arctan(V_{\text{dcref}}x_1(t-h) - Ix_2(t-h))$ and the parameter values stated above), which corresponds to not compensating for the delay, and kept everything else the same as the first simulation. Since Fig. 3 illustrates the lack of convergence in the absence of delay compensation, it also helps motivate our methods.

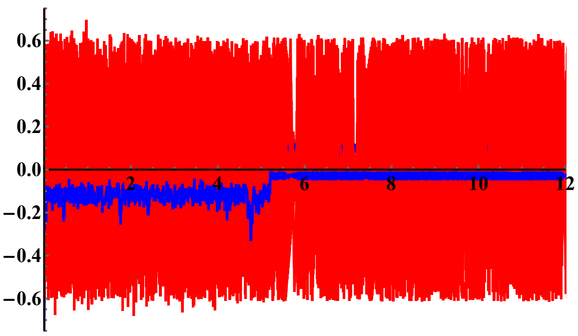

Fig. 3. First (Red) and Second (Blue) Components of (36), with Initial States $(0.75, -0.75)$, $(-0.5, 0.5)$, and $(0.25, -0.25)$, On Time Interval $[0, 12]$ without Delay Compensation.

VI. Conclusions

We provided new sequential predictor based delay compensating methods for a class of dynamics that include bilinear systems with perturbed measurements. This solves a longstanding open problem for delay compensation for bilinear systems with measurement uncertainty. Our method avoids the use of distributed terms in controls, while still compensating for arbitrarily long constant input delays. Our power systems example illustrated the good performance of our method under realistic operating conditions. We hope to extend this analysis to cover bilinear systems with perturbed asynchronously sampled measurements.

Appendix: Proof of Lemma 1

Using the definition of the $\mathcal{E}_i$'s from (14) gives

$$\begin{aligned}&\dot{\mathcal{E}}_1(t) = -p\mathcal{E}_1\left(t - \tfrac{h}{m}\right) + p\Delta(t)\\ &+ f\left(t + \tfrac{h}{m}, z_1(t), u\left(t - \tfrac{h(m-1)}{m}\right)\right)\\ &- f\left(t + \tfrac{h}{m}, x\left(t + \tfrac{h}{m}\right), u\left(t - \tfrac{h(m-1)}{m}\right)\right)\end{aligned} \tag{A.1}$$

and

$$\begin{aligned}\dot{\mathcal{E}}_i(t) \;=\;& -p\mathcal{E}_i\left(t - \tfrac{h}{m}\right) + p\mathcal{E}_{i-1}(t) - pJ_i\Delta\left(t + \tfrac{h}{m}\right)\\ &+ f\left(t + i\tfrac{h}{m}, z_i(t), u\left(t - \tfrac{h(m-i)}{m}\right)\right)\\ &- f\left(t + i\tfrac{h}{m}, z_{i-1}\left(t + \tfrac{h}{m}\right), u\left(t - \tfrac{h(m-i)}{m}\right)\right)\end{aligned} \tag{A.2}$$

when $i > 1$. We first study the $\mathcal{E}_1$-subsystem (A.1).

The Fundamental Theorem of Calculus yields

$$\begin{aligned}&\dot{\mathcal{E}}_1(t) = -p\mathcal{E}_1(t) + p\int_{t-\frac{h}{m}}^{t}\dot{\mathcal{E}}_1(\ell)\mathrm{d}\ell + p\Delta(t)\\ &+ f\left(t + \tfrac{h}{m}, z_1(t), u\left(t - \tfrac{h(m-1)}{m}\right)\right)\\ &- f\left(t + \tfrac{h}{m}, x\left(t + \tfrac{h}{m}\right), u\left(t - \tfrac{h(m-1)}{m}\right)\right).\end{aligned} \tag{A.3}$$

Then Assumption 2 and Young's inequality yield

$$\begin{array}{rcl}\dot{\nu}(t) & \leq & (k-p)|\mathcal{E}_1(t)|^2 + p\int_{t-\frac{h}{m}}^{t} \mathcal{E}_1(t)^\top \dot{\mathcal{E}}_1(\ell)\mathrm{d}\ell \\ & & +p|\mathcal{E}_1(t)|\,|\Delta(t)| \\ & \leq & (k-p)|\mathcal{E}_1(t)|^2 + p|\mathcal{E}_1(t)|\,|\Delta(t)| \\ & & +p\int_{t-\frac{h}{m}}^{t}\left[\frac{C_1}{2}|\mathcal{E}_1(t)|^2 + \frac{1}{2C_1}|\dot{\mathcal{E}}_1(\ell)|^2\right]\mathrm{d}\ell \\ & = & \left(k-p+\frac{phC_1}{2m}\right)|\mathcal{E}_1(t)|^2 \\ & & +\frac{p}{2C_1}\int_{t-\frac{h}{m}}^{t}|\dot{\mathcal{E}}_1(\ell)|^2\mathrm{d}\ell + p|\mathcal{E}_1(t)|\,|\Delta(t)|.\end{array} \quad \text{(A.4)}$$

Next note that (A.1) and Assumption 2 give $|\dot{\mathcal{E}}_1(\ell)| \leq p\left|\mathcal{E}_1\left(\ell - \frac{h}{m}\right)\right| + k|\mathcal{E}_1(\ell)| + p|\Delta(\ell)|$ for all $\ell \geq 0$. Hence,

$$\begin{array}{rcl}|\dot{\mathcal{E}}_1(\ell)|^2 & \leq & (1+C_2)p^2|\mathcal{E}_1(\ell-h/m)|^2 \\ & & +\left(1+\frac{1}{C_2}\right)\Big(k^2|\mathcal{E}_1(\ell)|^2 \\ & & +p^2|\Delta(\ell)|^2 + 2kp|\mathcal{E}_1(\ell)||\Delta(\ell)|\Big) \\ & \leq & (1+C_2)p^2|\mathcal{E}_1(\ell-h/m)|^2 \\ & & +\left(1+\frac{1}{C_2}\right)\Big(k^2\left(1+\frac{\lambda_a}{4}\right)|\mathcal{E}_1(\ell)|^2 \\ & & +p^2(1+4/\lambda_a)|\Delta(\ell)|^2\Big)\end{array} \quad \text{(A.5)}$$

for all $\ell \geq 0$, by Young's inequality. Therefore, (A.4) gives

$$\begin{array}{l}\dot{\nu}(t) \leq \left(k-p+\frac{phC_1}{2m}\right)|\mathcal{E}_1(t)|^2 + p|\mathcal{E}_1(t)||\Delta(t)| \\ +\frac{p^3(1+C_2)}{2C_1}\int_{t-2h/m}^{t-h/m}|\mathcal{E}_1(\ell)|^2\mathrm{d}\ell + C_2^\sharp\int_{t-h/m}^{t}|\Delta(\ell)|^2\mathrm{d}\ell \\ +\frac{pk^2}{2C_1}\left(1+\frac{1}{C_2}\right)\left(1+\frac{\lambda_a}{4}\right)\int_{t-h/m}^{t}|\mathcal{E}_1(\ell)|^2\mathrm{d}\ell\end{array} \quad \text{(A.6)}$$

for all $t \geq \frac{h}{m}$. Young's Inequality also gives

$$p|\mathcal{E}_1(t)||\Delta(t)| \leq \frac{\lambda_a}{2}|\mathcal{E}_1(t)|^2 + \frac{p^2}{2\lambda_a}|\Delta(t)|^2, \quad \text{(A.7)}$$

which we can use to upper bound the second right side term in (A.6) with our choices of p, $\bar{C}$, and $\tilde{M}$ from (6) to get

$$\begin{array}{l}\dot{\nu}(t) \leq -k|\mathcal{E}_1(t)|^2 + \frac{p^3(1+C_2)}{2C_1}\int_{t-2h/m}^{t-h/m}|\mathcal{E}_1(\ell)|^2\mathrm{d}\ell \\ \quad +\frac{pk^2}{2C_1}\left(1+\frac{1}{C_2}\right)\left(1+\frac{\lambda_a}{4}\right)\int_{t-h/m}^{t}|\mathcal{E}_1(\ell)|^2\mathrm{d}\ell + \widetilde{M}|\Delta|_\infty^2 \\ \quad \leq -2k\nu(\mathcal{E}_1(t)) + \overline{C}\int_{t-2h/m}^{t}\nu(\mathcal{E}_1(\ell))\mathrm{d}\ell + \widetilde{M}|\Delta|_\infty^2\ .\end{array}$$

Recalling our choice of $\mu(\mathcal{E}_{1,t})$ from (16), it follows that for all $t \geq h/m$, we have

$$\begin{array}{l}\frac{d}{dt}\mu(\mathcal{E}_{1,t}) \leq -2k\nu(\mathcal{E}_1(t)) + \widetilde{M}|\Delta|_\infty^2 \\ +\overline{C}\left(\int_{t-2h/m}^{t}\nu(\mathcal{E}_1(\ell))\mathrm{d}\ell + \frac{2h(1+\lambda_a)}{m}\nu(\mathcal{E}_1(t))\right) \\ -\overline{C}(1+\lambda_a)\int_{t-2h/m}^{t}\nu(\mathcal{E}_1(\ell))\mathrm{d}\ell\ .\end{array} \quad \text{(A.8)}$$

This gives

$$\begin{array}{l}\frac{d}{dt}\mu(\mathcal{E}_{1,t}) \leq 2k\left[-1+\frac{h\overline{C}}{km}(1+\lambda_a)\right]\nu(\mathcal{E}_1(t)) \\ -\lambda_a\overline{C}\int_{t-2h/m}^{t}\nu(\mathcal{E}_1(\ell))\mathrm{d}\ell + \widetilde{M}|\Delta|_\infty^2.\end{array} \quad \text{(A.9)}$$

Therefore, our lower bound (8) on m from our theorem and our choice of ϵ_0 in (6), combined with the bound

$$\begin{array}{l}\widetilde{\mu}(\mathcal{E}_{1,t}) \leq \\ \nu(\mathcal{E}_1(t)) + 2\left(1+\frac{h\overline{C}(1+\lambda_a)}{m}\right)\int_{t-2h/m}^{t}\nu(\mathcal{E}_1(\ell))\mathrm{d}\ell,\end{array} \quad \text{(A.10)}$$

give (17) along all trajectories of the $\mathcal{E}_1$ dynamics.

Similarly, (A.2) and the relation $2rs \leq \lambda_a r^2/4 + 4s^2/\lambda_a$ for all $r \geq 0$ and $s \geq 0$ give

$$\begin{array}{rcl}|\dot{\mathcal{E}}_i(\ell)|^2 & \leq & (1+C_2)p^2|\mathcal{E}_i(\ell-h/m)|^2 \\ & & +\left(1+\frac{1}{C_2}\right)\left(k|\mathcal{E}_i(\ell)| + p\Delta_i^\sharp(\ell)\right)^2 \\ & \leq & (1+C_2)p^2|\mathcal{E}_i(\ell-h/m)|^2 \\ & & +\left(1+\frac{1}{C_2}\right)k^2(1+\lambda_a/4)|\mathcal{E}_i(\ell)|^2 \\ & & +\left(1+\frac{1}{C_2}\right)p^2(1+4/\lambda_a)|\Delta_i^\sharp(\ell)|^2\end{array} \quad \text{(A.11)}$$

for all $\ell \geq 0$ and $i \in \{2,3,\ldots,m\}$, where $\Delta_i^\sharp(\ell) = |\mathcal{E}_{i-1}(\ell)| + J_i|\Delta(\ell+h/m)|$. Therefore, by replacing the index 1 by i and also replacing Δ by $\Delta_i^\sharp$ in the analysis that led to (A.6)-(A.7), it follows that the function $\mu(\mathcal{E}_{i,t})$ satisfies

$$\begin{array}{rcl}\frac{d}{dt}\mu(\mathcal{E}_{i,t}) & \leq & -\epsilon_0\widetilde{\mu}(\mathcal{E}_{i,t}) + p|\mathcal{E}_i(t)||\Delta_i^\sharp(t)| \\ & & +C_2^\sharp\int_{t-h/m}^{t}|\Delta_i^\sharp(\ell)|^2\mathrm{d}\ell \\ & \leq & -\frac{\epsilon_0}{2}\widetilde{\mu}(\mathcal{E}_{i,t}) + \frac{p^2}{\epsilon_0}|\Delta_i^\sharp(t)|^2 \\ & & +C_2^\sharp\int_{t-h/m}^{t}|\Delta_i^\sharp(\ell)|^2\mathrm{d}\ell\end{array} \quad \text{(A.12)}$$

for all $t \geq h/m$, where the second inequality in (A.12) used Young's inequality to get $p|\mathcal{E}_i(t)||\Delta_i^\sharp(t)| \leq \epsilon_0|\mathcal{E}_i(t)|^2/4 + p^2|\Delta_i^\sharp(t)|^2/\epsilon_0$ and $C_2^\sharp$ is as defined in (6). Hence, the relation

$$|\Delta_i^\sharp|^2 \leq (1+J_iC_3)|\mathcal{E}_{i-1}(\ell)|^2 + \left(1+\frac{J_i}{C_3}\right)|\Delta|_\infty^2 \quad \text{(A.13)}$$

gives

$$\begin{array}{rcl}\frac{d}{dt}\mu(\mathcal{E}_{i,t}) & \leq & -\frac{\epsilon_0}{2}\widetilde{\mu}(\mathcal{E}_{i,t}) + \frac{p^2}{\epsilon_0}(1+J_iC_3)|\mathcal{E}_{i-1}(t)|^2 \\ & & +C_2^\sharp(1+J_iC_3)\int_{t-h/m}^{t}|\mathcal{E}_{i-1}(\ell)|^2\mathrm{d}\ell \\ & & +J_i\left(1+\frac{J_i}{C_3}\right)\left[\frac{p^2}{\epsilon_0} + C_2^\sharp\frac{h}{m}\right]|\Delta|_\infty^2,\end{array} \quad \text{(A.14)}$$

which proves the lemma.

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