



Thermal and hybridized magnons

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ABSTRACT

The investigation of the fundamental excitations of magnetically ordered systems has been at the core of the scientific career of Sergio Rezende. Here, I briefly summarize some of our own recent work that often took inspiration from his research work. In particular, I discuss how heat transported by magnons can give rise to measurable spin currents and how magnons may hybridize with other dynamic excitations. Both of these topics are currently at the core of larger research efforts within the field of magnetism.

1. Introduction

One of the main objectives of solid state physics is to identify ordered structures and understand their excitations. These excitations typically determine the physical properties of materials and provide the functionalities in technological applications. Towards this end, there has been recently an increased focus on using coherent interactions between different types of excitations for information technologies, ranging from novel highly efficient sensors, to high-bandwidth communication, and novel computational paradigms.

Of the many possible excited dynamic modes, magnetic materials and their fundamental excitations, magnons, provide unique properties that can enhance and broaden our basic understanding of magnetic phenomena. Furthermore, magnons also generate new opportunities for practical application in coherent information processing and engineering [1,2]. Magnons can couple with a wide variety of different excitations, including microwave and optical photons, phonons, and other magnons. At the same time, the frequency of magnon excitations can be conveniently tuned by changing local magnetic bias either via externally applied fields or internal fields such as magneto-crystalline anisotropies. But most importantly, there are two additional attributes of magnons, which are hard to implement in other commonly used dynamic excitations. First, magnons can host microwave-bandwidth excitations with much smaller wavelengths, down to nanometer scale, in comparison to millimeter scale wavelengths of electromagnetic microwaves. This favorable scaling provides new opportunities for manipulating microwaves with nanoscale engineering and suppressed crosstalk that is hard to eliminate in mm-wavelength traditional microwave circuits. Second, magnons can introduce unprecedented functionality, such as non-reciprocity, that is highly desirable for directional information

propagation.

In addition, magnons provide a very rich playground to explore fundamental physics. The properties of magnons can be very complex, since they are determined by competing interactions ranging from exchange coupling at an atomic length scale, to long-range dipolar interactions, which almost everybody is familiar with when they marvel at the action at a distance that we attribute to macroscopic ferromagnets. Furthermore, different types of magnetic order emphasize very different attributes of magnon dynamics. *E.g.*, while especially for nanoscale ferromagnetic systems dipolar interactions can profoundly change magnon behavior, they are less important for antiferromagnetic materials, where the net magnetizations and therefore dipolar stray magnetic fields are typically vanishing.

For the past five decades, Sergio Rezende has been a trailblazer for investigating magnon phenomena. Early on he has investigated how magnons interact with phonons and can be manipulated adiabatically through varying magnetic fields [3–5]. Furthermore, he was at the forefront of considering the role of coherency in magnons [6,7], which is nowadays experiencing a renaissance in the context of coherent information processing. Subsequently he focused on magnons in antiferromagnets [8,9], which have regained interest recently due to their relevance to spin currents in antiferromagnets [10]. Sergio Rezende also investigated extensively the strong non-linearities inherent to magnon dynamics [11,12], which can result in interesting phenomena, such as the Bose–Einstein condensation of magnons [13]. In addition, he became one of the pioneers investigating the connection between magnons and spin currents. He was among the first to show that magnetization dynamics may generate electric spin currents [14] and explored how spin polarized charge currents can excite magnons [15–17].

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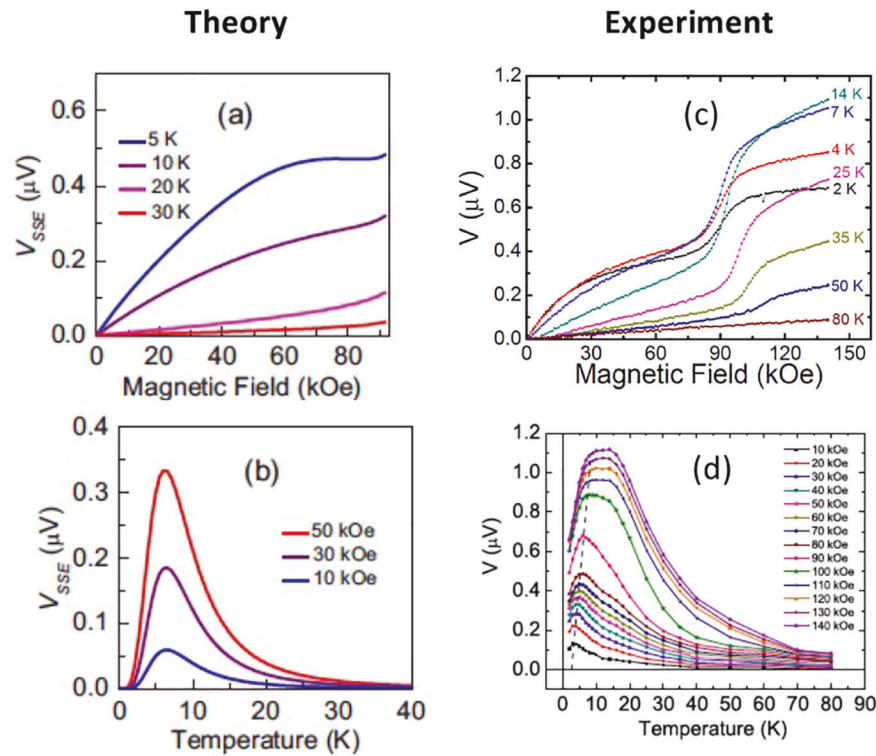


Fig. 1. Spin Seebeck effects in antiferromagnets. Theoretical (a,b) and experimental (c,d) magnetic field (a,c) and temperature (b,d) dependence of spin Seebeck signals from thermal gradients in MnF_2 . The only free parameter in the theory is for scaling of the amplitude. Adapted from Refs. [23,24].

In this paper, I will highlight two examples of my own research that were directly influenced by the extensive work of Sergio Rezende on magnon physics. First, I will discuss how magnons driven by thermal gradients can give rise to spin currents even in antiferromagnets, which was beautifully explained through a theory developed by Sergio Rezende. Next, I will discuss hybrid magnon excitations, which may provide useful new opportunities for coherent information processing.

2. Thermal magnon transport

In 2008 Uchida et al. reported experiments that were interpreted as a thermal gradient generating spin currents in ferromagnetic samples that can be injected into other adjacent non-magnetic layers [18]. While these original measurements and their interpretation remain controversial, this work nevertheless initiated interest in how thermal transport and spin transport are intertwined, a research field now known as spin caloritronics [19,20]. A more reliable example of such coupling between thermal and spin transport is given by the longitudinal spin Seebeck effect, where a thermal gradient across the interface between a ferri- or ferromagnetic insulator with a non-magnetic metal can inject an electronic spin current into the conductor [21]. This observation was nicely explained by Sergio Rezende et al., who developed a theory based on thermal magnon accumulation at the interface [22]. This model provided a quantitative explanation for the experimentally observed dependencies of the measured spin Seebeck signals on temperature and on the thickness of the ferrimagnetic insulator.

Around the same time when Sergio Rezende developed a theoretical model for the spin Seebeck effect Wu et al. explored experimentally, whether similar effects might be observable with antiferromagnetic insulators. In particular, MnF_2 was combined with Pt for thermal transport measurements, and distinct features in the thermoelectric signals were observed at the temperature/field combinations that correspond to the well established spin-flop transition in MnF_2 [see Fig. 1(c)] [24]. But surprisingly, it was observed that at low temperatures a finite spin Seebeck voltage develops already well below the spin flop transition,

which was contrary to simple intuition. It turns out that Sergio Rezende was ideally equipped to provide the theoretical explanation for this data [23]. As shown in Fig. 1, based on the magnon dispersion in MnF_2 he could adopt his theory of magnon accumulations due to thermal gradients to quantitatively explain both the magnetic field and temperature dependence of the experimental spin Seebeck signals without any additional free parameters except for an overall scaling of the signal amplitude. The explanation for the spin Seebeck signals at fields below the spin-flop transition is based on two magnon modes in the antiferromagnet with opposite net spin. At zero magnetic field these two modes are degenerate, but an applied magnetic field lifts the degeneracy, so that there is also a net imbalance of the respective magnon accumulation of these two modes due to thermal gradients. Note that Rezende et al. published the theoretical model of the data [23] about six weeks before the experimental data was actually published [24].

3. Hybridized magnons

As was discussed in the introduction, magnons can easily couple to a wide variety of excitations in solid state systems. With respect to coherent information processing they have several key advantages [1]. The coupling strength can be comparatively strong reaching values of about 10% of their excitation frequency. Given that the decoherence rate of magnons can be 0.1% or less of their excitation frequency, this means that a coherent strong coupling regime can be readily established. This opens the door to quantum transduction with many other modalities, including photons, phonons, and other magnons, while at the same time the wave properties of the magnons enable to probe phase sensitive properties. Furthermore, the relatively short wave length at microwave frequency provides opportunities for compact, on-chip devices, while at the same time the inherent chirality of magnon dynamics can be harnessed for non-reciprocal wave propagation. Below, I will discuss two specific hybrid-modes that have attracted increased recent interest: magnon-phonon and magnon-photon hybridized modes.

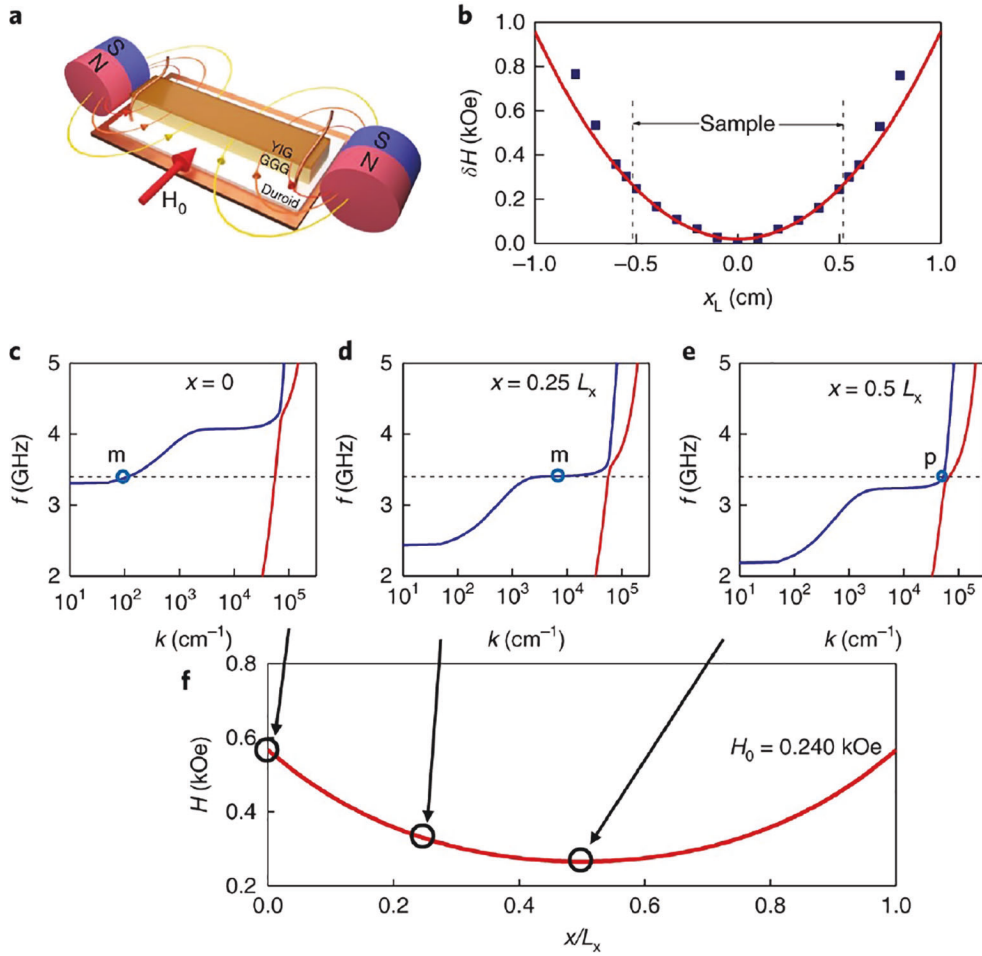


Fig. 2. Hybridized Magnon-Phonon excitations. In $\text{Y}_3\text{Fe}_5\text{O}_{12}$ (YIG) the magnon dispersion crosses the phonon dispersion, which results due to magnetoelastic coupling in hybridized magnon-phonon excitations. (a) Using inhomogeneous magnetic fields enables to directly explore the coherent conversion from magnon to phonon modes. (b) Measured magnetic field profile along the length of the YIG waveguide. (c,d,e) Position dependent magnon and phonon dispersion schematics corresponding to different positions along the YIG waveguide with varying net magnetic field as depicted in (f). Excitation with 3.4 GHz at $x/L_x = 0$ results in propagating magnon modes, which by the time they reach $x/L_x = 0.5$ have been converted into phonons. Adapted from Ref. [33].

3.1. Magnon-phonon coupling

Many magnetic materials exhibit a coupling between magnetic responses and mechanical strains, such that these materials can produce magnetic energy when stretched. In fact, such magnetoelastic phenomena were observed already in 1847 by Joule [25] and became of more widespread interest a century later, when the strong coupling between magnons and phonons, as well as resultant magnetoelastic waves were studied in great detail [4,5,26–28]. More recently, research has focused on either using phonons for coherently driving magnons, such as in acoustically driven ferromagnetic resonance [29], or the inverse effect where phonon propagation is modulated via magnons [30]. These new developments generate broad interest in the development of thin film magnetoelastic materials [31], which can be utilized in a wide variety of magnetoelectric applications, such as magnetometers, antennae, and tunable microwave inductors and filters [32].

One interesting fundamental physics question had been unresolved until recently though. It is well established that magnons have an intrinsic angular momentum, which is related to the partial reversal of the spin moments that contribute to the magnon excitation. Thus, one question was how the angular momentum is conserved upon strong coupling to a phonon. Building on the early work from Sergio Rezende and Frederic Morgenthaler, where they showed that time varying magnetic fields can result in an adiabatic transformation of magnons into phonons [4,5], Sergio Rezende et al. devised an experiment to directly probe the angular momentum of these phonons [33]. As shown in Fig. 2(a), they used an $\text{Y}_3\text{Fe}_5\text{O}_{12}$ (YIG) magnon waveguide in an inhomogeneous magnetic field created by nearby permanent magnets. If the gradient of the magnetic field is sufficiently high, then it is possible

to excite propagating magnons, which convert into phonons in the low field section of the YIG waveguide. This conversion can easily be verified by the speed of the signal propagation, since the group velocity of the phonons significantly exceeds the group velocity of the magnons. Rezende et al. detected these phonons with inelastic light scattering, and by analyzing the polarization state of the light they verified that the phonons generated from magnons were circularly polarized and thus carried angular momentum as well. This remarkable demonstration suggests that phonons could possibly be used for fast and long range transmission of magnetic spin information.

3.2. Magnon-photon coupling

Since the quanta of electromagnetic waves, photons, involve time-varying magnetic fields they of course readily couple to magnetization dynamics, such as magnons. In fact, microwave based measurement techniques are key to many magnon investigations and have been central to much of the research work of Sergio Rezende over his long career. Despite this long history, this coupling has received recently renewed attention. Namely, a theoretical discussion a decade ago about how nanosized magnets may couple exceptionally strong to photonic modes of cavities [34], triggered a subsequent flurry of experimental activities, which is now known as cavity magnonics [35]. Based on early experiments with bulk YIG spheres and 3D cavities [36–39], the experimental work has progressed to the point where coherent coupling to superconducting quantum bits was established, which enabled magnon measurements down to the single quantum limit [40,41].

Based on these developments, an obvious question to ask is whether it is possible to integrate magnetic components into co-planar micro-

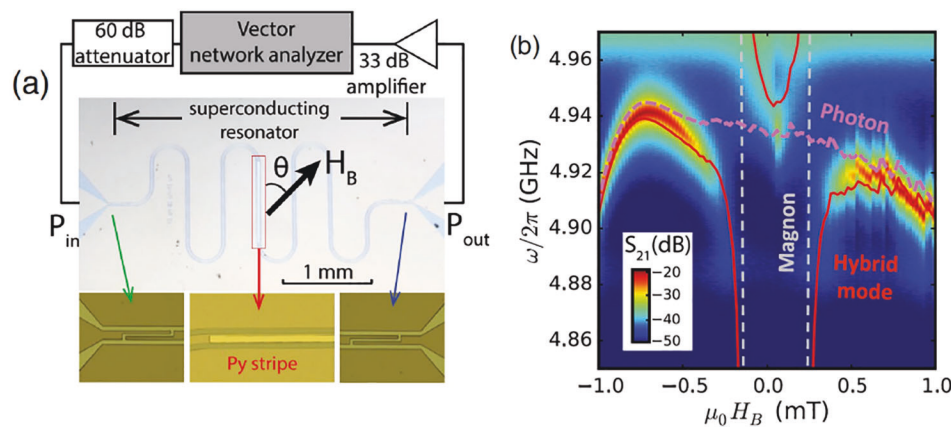


Fig. 3. Hybridized Magnon-Photon excitations. (a) Schematic of the measurements of magnon-photon coupling in planar hybrid devices. The optical images show an overview and details of the hybrid device consisting of a NbN superconducting microwave resonator with a permalloy ($\text{Ni}_{80}\text{Fe}_{20}$) magnetic structure integrated on top of the waveguide. (b) Microwave absorption spectrum of the hybrid device shown in (a). A clear hybridization gap is observed where the magnon dispersion crosses the weakly field dependent photon resonance mode of the microwave resonator. Adapted from Ref. [42].

wave circuitry for direct coherent microwave interactions. In order to address this question, Li et al. developed superconducting microwave resonators based on NbN that were integrated with permalloy ($\text{Ni}_{80}\text{Fe}_{20}$) magnetic structures [42] as shown in Fig. 3(a). The corresponding microwave absorption measurements are shown in Fig. 3(b) and show a clear hybridization gap between the photon mode of the cavity and the magnon mode of the magnetic element. A detailed quantitative analysis shows a high cooperativity of 68, which indicates the strong coherence of the magnon-phonon interaction. More importantly, when the coupling constant of 152 MHz is normalized by the spin density of the permalloy structure, this results in a coupling constant per spin of $g_0/2\pi = 26.7$ Hz. This value is several orders of magnitude larger compared to prior measurements on bulk YIG spheres, which resulted in $g_0/2\pi = 0.038$ Hz [37]. Furthermore, further improvements in the superconducting cavity using a lumped element design, enables to increase this coupling even further to $g_0/2\pi = 263$ Hz [43]. Thus these early results show that magnetic systems can be readily integrated into compact on-chip microwave devices and possibly provide new functionalities for coherent information processing systems, including quantum computation and communication. Since non-reciprocal phenomena are a key aspect that distinguishes magnons from other dynamic excitations, the practical application of magnons in these systems will benefit significantly from the development of small scale non-reciprocal coupling mechanisms [44], which will allow to incorporate noise reducing schemes already at the chip level.

4. Conclusion

When we do scientific research, we literally always stand on the shoulders of giants, who explored nature before us. Sergio Rezende and his scientific accomplishments are a prime example of this. His work on magnon physics continues to have relevance even for some of the most contemporary aspects of research in magnetism. I illustrated this point in this article with two selected examples. Thus we are truly lucky to have such a talented scientist in our midst, who has taught us so much about the Fundamentals of Magnonics [45]. At the same time, after all these years, magnons still can surprise us and there are many current research areas, such as antiferromagnets, quantum coherent hybrid systems, two-dimensional materials, and many others, that will keep us busy for many decades to come!

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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