Proceedings of the ASME 2021
International Mechanical Engineering Congress and Exposition
IMECE2021
November 1-5, 2021, Virtual, Online

IMECE2021-69264

STABILITY PERFORMANCE OF A STOCHASTIC TOOLPATH IN MACHINING

Tyler J. Grimm*
International Center for Automotive
Research Clemson University
Greenville, SC 29607
Email: tgrimm@clemson.edu

Nilesh Ashok Kharat
International Center for Automotive
Research Clemson University
Greenville, SC 29607
Email: nkharat@clemson.edu

Nils Potthoff
Virtual Machining
TU Dortmund University
Otto-Hahn-Straße 12
44227 Dortmund, Germany
Email: nils.potthoff@tu-dortmund.de

Laine Mears
International Center for Automotive
Research Clemson University
Greenville, SC 29607
Email: mears@clemson.edu

Petra Wiederkehr
Virtual Machining
TU Dortmund University
Otto-Hahn-Straße 12
44227 Dortmund, Germany
Email: petra.wiederkehr@tu-dortmund.de

ABSTRACT

The overall quality of a machined part relies heavily on the tool path that is used. Several methods of toolpath generation are currently employed. A more recently developed toolpath method is known as trochoidal milling, which is also known by several other terms, such as adaptive milling. This type of path benefits the machining process by attempting to reduce chip thickness on entry and exit to the workpiece. In doing so, utilization of this type of path can reduce tool wear and enables higher feed rates, thus improving machining efficiency.

Another advantage of the trochoidal approach is that it often creates paths which are relatively more smooth compared to traditionally designed paths. In order to follow the contours of the final geometry, the path can yield a significant number of direction changes which result in constantly changing forces directions on the tool. Chatter, or self-excited vibration that occurs in the tool or workpiece, can therefore be mitigated or avoided since resonance does not have time to increase the vibration's amplitude.

The trochoidal milling tool path strategy typically operates on the XY plane. The operator will assign a step-down value, which defines the Z-depth at each pass. This strategy can create issues during freeform milling: because of this step-down effect, the trochoidal path may only be able to perform clearing and not finishing. This is due to the excess material left on the workpiece when a large step-down value is used. A significant and randomized variation range of the trochoidal path is tested in this research. Using this new proposed method, stochastic behavior of the toolpath is implemented. The toolpath consists entirely of circular arcs which drive the tool in a pseudo-random fashion. As the tool nears completion of the pass, the generator will give heavier probabilistic weight to points which have not yet been machined, thereby improving the efficiency of the process. It is hypothesized that this toolpath can generate the same chip-inhibiting properties of the trochoidal path while granting the ability to perform finishing cuts. The stability of such a path is determined in this work. A key parameter of this path is the allowable radius range of the circular arcs. For example, short, tight arcs or long, relatively straight arcs can be used. The influence of these arcs is analyzed against several different met-

*Address all correspondence to this author.

rics, such as generation time, path efficiency, and chatter. The stability lobes for several radii parameters were determined. It was found that the most efficient path utilized a median parameter value, signifying a negative parabolic relationship between path efficiency and tool path radius. It was also discovered that smaller arcs result in decreased chatter. Future studies will explore the behaviors of this path when milling 3D surfaces.

1 INTRODUCTION

High precision manufacturing of components with a freeform surface such as turbine blades, engine impellers, honeycomb structures, etc., is critical in aeronautical and automobile industries to ensure functional and aesthetic requirements. Computer-Numerical Control (CNC) milling is the one of the preferred machining operations due to its capability of producing complex shapes of superior quality with high productivity in different materials. During CNC milling of freeform surface, the most crucial step is generating a coherent and optimal toolpath to ensure a desired shape and geometrical accuracy. The literature suggests various toolpath strategies and can be classified as optimization-based toolpath and constrained-based toolpath.

The optimization-based toolpath strategies aim to regulate machining output attributes such as length of toolpath, energy consumption, material removal rate, production time, and surface error. Djebali *et al.* assumed a production time proportional to the length of the toolpath and applied several optimization algorithms like Tabu search and nearest neighbor search to minimize the production time [1]. It should be noted that this approximation of production time based on toolpath length does not always apply to machining processes. The characteristics of the individual CNC machine must also be considered. The primary characteristic that would influence this assumption is the axes acceleration capabilities. For example, a toolpath which travels in many different directions with sharp transitions between paths will most likely take longer to complete than a completely linear path of similar length. However, in general, this assumption does apply.

Li *et al.* proposed a strategy to reduce the carbon footprint and energy consumption by optimizing toolpath [2]. Again, similar to the previous discussion of production time assumptions, assumptions of energy consumption based on toolpath length must also be carefully considered. In the previous example, a path with many turns would consume more energy than a linear path of comparable length due to the stopping and starting motion of the tool/workpiece. Several researchers have applied the traveling salesman problem (TSP) to optimize the toolpath. The strategy usually generates a set of cutter location points on the surface and applies TSP to generate an optimized toolpath [3, 4]. Lin *et al.* further improved the TSP-based strategy by optimizing the direction of feed and reducing the length of the toolpath [4].

An alternative to optimization-based toolpath strategies is constraint-based toolpath strategies, which apply a pre-delineated condition such as a maximum cutting force, constant chip thickness, scallop height, and avoidance of overcuts and chatter, to generate a toolpath. Erdim *et al.* proposed a methodology that predicts chip thickness and the resulting cutting load computationally to adjust the feed rate for each segment of toolpath [5]. Ma *et al.* constrained the variation in cutting force at nodes of cutter location points to obtain a toolpath for efficient machining and improved surface quality [6]. The method of constraining the height of the scallop in ball end milling has been explored extensively in the literature [7, 8]. Liu *et al.* predicted a toolpath with constant scallop heights in 3-axis machining with a ball end mill. Recently, a strategy of constraining scallop height was applied at the boundaries of freeform surfaces while generating toolpaths [8]. Gong *et al.* developed a toolpath for freeform surface by constraining the ball end mill tangential to the component surface and avoiding overcuts [9]. The full immersion of the tool during slot or pocket milling operation results in resonance and occurrences of chatter. Otkur and Lazoglu applied a trochoidal toolpath strategy that combines the linear and circular motion of a tool uniformly to avoid high cutting loads [10]. Rauch *et al.* studied the effect of trochoidal parameters, such as trochoidal radius and step, in order to control the performance of the trochoidal toolpath [11]. In the past decade, the literature shows comprehensive evolution in the modeling of process mechanics for the trochoidal toolpath that involves modeling of chip thickness [12], cutting forces [13], stability analysis [14], and tool wear analysis [15].

Based on the literature review, it is realized that the trochoidal toolpath has excellent potential towards reducing cutting force and rapid dissipation of cutting heat. The trochoidal toolpath has been investigated thoroughly for a *2-D* surface, where it performs machining on *XY-* planes. However, the implementation of a trochoidal toolpath for freeform surfaces requires the variation in *Z-* depth at each machining layer. It thereby limits the applicability of trochoidal toolpath for roughing operation in the case of the freeform surface. In a recent study, a trochoidal toolpath was generated for machining a *3-D* slot having a freeform surface on the vertical sides [16]. Therefore, the literature suggests probable research in exploring toolpath for the freeform surface.

2 METHODS

As previously discussed, the trochoidal toolpath offers several advantages over a traditional machining process. However, it is difficult to generate trochoidal paths over a freeform surface. Therefore, a pseudo-stochastic toolpath generation method is explored herein. This method imparts random motions at the beginning of the toolpath generation when the majority of the workpiece has not been processed by the machining tool. As the tool continues to make passes over the material, only certain areas of the workpiece remain outside the desired tolerance. The toolpath will then begin to recognize these points and will become more likely to travel to those areas relative to areas which have already been machined. In this way, toolpath efficiency is gained since a completely randomized toolpath could conceivably remain within only a small portion of the workpiece. A key distinction of this path generation method relative to other methods is that self-intersections are enabled.

The primary investigation of this work is to determine if machining stability can be improved through use of a stochastic toolpath. Namely, chatter will be investigated. Trochoidal and other similar toolpathing approaches which utilize arc motions, rather than linear motions, are well known to reduce chatter due to constantly changing force vectors. It is hypothesized and tested herein that the stochastic toolpath may share this benefit, since it also can be constructed explicitly using arcs.

An obvious drawback of the stochastic toolpath is that the path length, and therefore the efficiency of the toolpath, cannot outperform conventional, linear toolpaths. Because the stochastic toolpath is self-intersecting, the tool will pass over regions of the material which have already been cut. When this occurs, there will be no contribution towards cutting the workpiece material and can be considered as wasted time/energy. Though not investigated herein, a feasible solution to this drawback is to incorporate feed rate optimization, since the tool would be able to travel much faster through areas which have previously been machined. This is currently being investigated and will be included in future publications.

The stochastic toolpath generation program has been prepared in MATLAB. The program first generates a 2D point cloud which represents a flat, uncut workpiece surface. A ball-end mill is then simulated in which the tip of the mill is positioned below the point cloud surface by the desired axial depth of cut. As the tool progresses, the points within the point cloud which intersect with the tool will be re-positioned in the axial direction to be along the surface of the tool. As previously mentioned, the toolpath first begins with nearly complete stochastic influence. After each step, the toolpath checks the entire workpiece and determines if any areas have not been machined yet or if they have not been machined as much as the rest of the workpiece. As the program identifies these points, it will tend to travel directly towards these points, rather than in a random decision. The generation method for this toolpath are outlined in greater detail in [17].

The stochastic toolpath program has been created such that either purely linear or circular motions can be used. The linear approach can be considered a more computationally efficient method; however, this type of path suffers the drawback of not having constantly changing force vectors. Therefore, this path is more likely to experience chatter than a circular path. Since the primary motivation of this work is to reduce/eliminate chatter through use of the stochastic toolpath, only the circular arc toolpath program was considered.

If left unconstrained, it would be difficult to test the likelihood of producing a chatter reducing stochastic toolpaths. This is because every toolpath that is generated would be completely different, with no effective way of correlating similarities between them. Since all toolpaths would be different, extensive testing would need to be considered to gain an understanding of what the average characteristics of the stochastic toolpath are. To further explain this, consider two different toolpaths that are randomly generated. One toolpath may produce circular arcs with an extremely large radius. This would cause the toolpath to appear linear at the relatively small scale of the workpiece surface. Conversely, another toolpath may be created with extremely small radii, which would cause the toolpath to be constantly changing directions. It is hypothesized that these two cases would produce significantly different results, and would therefore need to be tested. Furthermore, it would also be desired to test many cases that represent the middle-ground between these extremes, as well as combinations of these extremes. This would require a significant amount of testing and would not lead to any appreciable results, since even when a high performing toolpath was discovered, it would be difficult to replicate since it was generated under no constraints.

A better method for analyzing the stochastic toolpath is to impart some constraints to it. One constraint that was implemented was a radii range constraint. This required the radius that was used for the circular arcs to be within a desired range. As discussed in [17], the radius of the circular are determined such that tangency is maintained between successive arcs. Therefore, when a radius value falls outside of the desired range, a new point on the workpiece is selected. This process is repeated until a radius value is found that is within the allowable range. By adding this constraint, it is hypothesized that the variability between tests of similar parameters will also perform similarly. This would restrict the number of tests that need to be performed to only a few tests per case, since it is likely that all other tests which used the same parameters would perform similarly. Additionally, the characteristics of the toolpath can easily be generated for different geometries since the radii range can remain constant when generating these new paths.

Another constraint that was added to the simulation restricted the toolpath from traveling outside of the workpiece surface. This was termed the *Exit* constraint. Without this constraint, the toolpath was able to generate paths which extend far beyond the workpiece material, adding significant machining distance which did not contribute to cutting the workpiece. This negatively impacted the machining efficiency of the toolpath. In the event that the toolpath attempted to travel outside of the workpiece boundary by a distance greater than the tool's radius, the toolpath would select a new point. Again, this process was repeated until this condition was met.

Under certain conditions, these constraints cannot both be maintained. For example, the toolpath may be approaching the edge of the workpiece. While at this position, the only option for the toolpath is to travel forward, outside the workpiece, in order to maintain tangency with the previous path. The extent that this path must travel outside the workpiece is dependent on the minimum radius value. In this event, it was decided to relax both the constraints and allow the program to pick any point. This example is shown in Figure 1. In this figure, part a) represents the toolpath approaching the edge of the workpiece perpendicular to it. This can be considered a worst case scenario. Part b) shows that the radii range minimum and maximum values will not allow the toolpath to loop back onto the workpiece without exiting the boundary set by the *Exit* constraint. Part c) shows that both of these constraints (radii range and *Exit* constraint) are relaxed in this case and the toolpath may travel anywhere. This includes selecting a path requiring a radius value shorter than the minimum desired radius or selecting a path which extends beyond the *Exit* boundary. Note that the program was set to count the number of times case c) occurred in each simulation.

Various radii ranges were explored herein and compared against several metrics. The radii ranges that were simulated can be found in Table 1. Since no physical processes were simulated, these simulations can be considered unitless. However, these can also be thought of in terms of inches, since the experimental specimen used was 1 inch x 1 inch. In these simulations a 0.375 (or 37.5% of the specimen dimension) diameter ball-end mill tool was used. The target maximum scallop height set for all tests was 0.025 (or 2.5% of the specimen dimension).

For each radii range explored, 500 simulations were generated in order to form a distribution of results. The path length was recorded for each simulation. The time required to simulate each path was also recorded. The computer used for generating these programs had an AMD Ryzen Threadripper 1950X 16-Core processor (note that the code was not parallelized). These simulation times should be considered relative to each other, since individual computer systems' performance will vary. Another metric that was recorded was the number of times the exit constraint was relaxed and the toolpath traveled outside the workpiece dimensions.

TABLE 1: RADII RANGES SIMULATED FOR STOCHASTIC TOOLPATH

Minimum Radius	Maximum Radius
(Fraction of Workpiece Size)	
0.1	0.2
0.1	0.3
0.1	0.4
0.1	0.5
0.1	0.6
0.2	0.3
0.2	0.4
0.2	0.5
0.2	0.6
0.3	0.4
0.3	0.5
0.3	0.6
0.4	0.5
0.4	0.6
0.5	0.6

The resulting toolpath lengths were also compared to a linear toolpath that had similar maximum scallop height in one case, and similar average scallop height in another case. A conventional linear toolpath gains much of its efficiency due to a relatively consistent scallop that is formed since the path does not self-intersect. For the stochastic toolpath, there only remains a

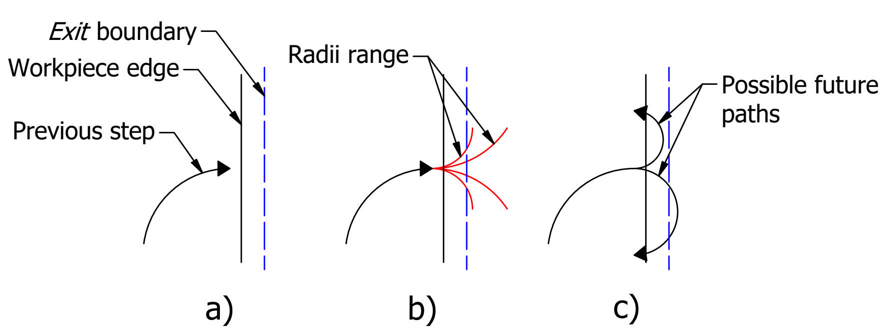
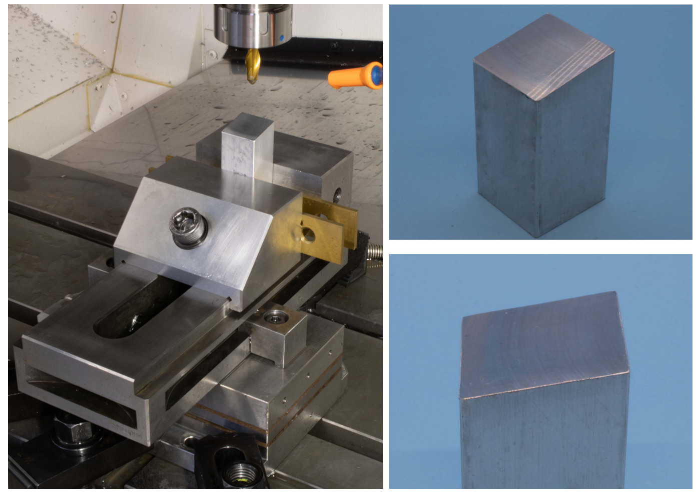


FIGURE 1: EXAMPLE CASE OF RELAXED CONSTRAINT

few peaks that are near the maximum scallop height, with much more of the material machined closer to the desired surface. In general, a maximum scallop height is usually called out as a desired parameter in engineering specifications. In this case, the linear toolpath will significantly outperform the stochastic toolpath since the linear toolpath does not waste time machining material closer to the desired workpiece after it is within the maximum scallop height tolerance limits. However, when viewed from the perspective of average scallop height, the stochastic toolpath length becomes much more similar to the linear toolpath length. Though often not required in engineering specifications, the average scallop height may be of concern in certain cases, such as lens manufacturing. Therefore, both cases were considered. These results will be represented as a proportion stochastic path length to conventional path length (*i.e.* a value of one represents a stochastic toolpath that has the same path length as a comparable conventional toolpath; a value of two represents a stochastic toolpath that has twice the path length as a comparable conventional toolpath). The process used to determine comparable linear path lengths based on stochastic paths can be found in [17].



(a) (b)

FIGURE 2: (A) EXPERIMENTAL SETUP AND (B) WORKPIECE WITH ANGLED SURFACE

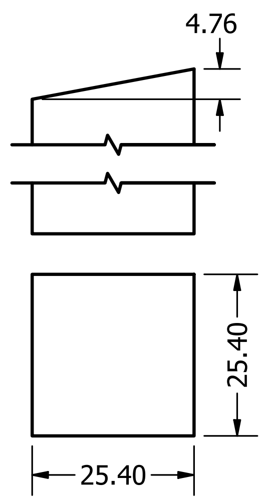


FIGURE 3: SCHEMATIC OF WORKPIECE WITH ANGLED SURFACE; DIMENSIONS IN MM

2.1 Experimental Methodology

The influence of a stochastic toolpath on chatter was investigated in the machining of 6061-T6 aluminum. The workpiece was a rectangular bar of dimensions 25.4 mm x 25.4 mm. The tool used was a 2-flute, carbide ball-end mill with a titanium nitride (TiN) coating. Other tool specifications can be found in Table 2. The workpiece was secured to a Kistler 9257B force dynamometer, which was used to measure all machining forces discussed herein. The workpiece and test setup and can be seen in Figure 2.

The workpiece that was used for experiments had an angled surface, as shown in Figure 3. The angled surface was used to simulate machining a workpiece with varying depths of cut. Furthermore, while the creation of stability lobes were not performed herein, future work will investigate this. In order to construct stability lobes, a workpiece similar to this angled block is typically used. In anticipation of this future testing, a similar block was used herein. For this study, machining this angled block does offer additional insight to the performance of the stochastic toolpath since this represents a more realistic case (*i.e.* the toolpath is not likely to be used to machine a constant depth of cut). In reality, if it is desired to maintain a constant depth of cut and machine a flat surface, it would be much more advantageous to use facing mills or flat-end mills, instead of ball-end mills. It is more likely that the workpiece may start as a flat surface, but the desired surface would be a complex surface. While machining this type of surface, the depth of cut would constantly change while conforming to the contours of the final geometry. A similar effect is true even when a roughing profile is first machined.

All experimental testing was performed in an Okuma GenosM560-V 3-axis vertical machining center. The parameters used throughout testing can be found in Table 2. Note that the axial depth of cut varied along the Y-axis direction, such that there was a maximum depth of 4.76 mm, which is equal to the radius of the cutting tool, and a minimum depth of zero. Flood coolant (Blaser Synergy 735) was used.

The experimental test set was limited to only three sets of radii ranges. These ranges can also be found in Table 1. Example

TABLE 2: TOOL ATTRIBUTES AND MACHINING PARAMETERS

Tool Attributes (Carbide Ball End Mill)	
Cutter Diameter	9.5 mm
Helix Angle	30°
No. of Flutes	2
Flute Length	25.4 mm
Tool Stick-out	33 mm
Coating	Titanium Nitride (TiN)
Machining Parameters	
Feed	1600 mm/min
Spindle Speed	10000 RPM
Feed per Tooth	0.08 mm/tooth
Surface Speed	300 m/min
Axial Depth of Cut (DOC)	0 - 4.76 mm

paths can be seen in Figure 4. Note that some of these paths travel far outside the workpiece, signifying the relax in constraints that was previously discussed. A desired maximum scallop height of 0.64 mm was set in the toolpath generation program, which was similar to the simulation cases. After performing the simulations described in the previous section, the three shortest toolpaths for each parameter set were experimentally tested. This represents a brute force optimization method, wherein many simulations are performed and the shortest one produced is used.

During experimental testing, the force dynamometer was set at a sample rate of 100 kHz. This is a much greater sample frequency than is typically used in machining experiments; however, it needed to be ensured that all chatter was captured. Therefore, the sample rate was set at approximately the maximum capabilities of the data acquisition device. Subsequent to testing, the force data was analyzed. FFT analysis was also performed on the force data to determine any effects on chatter/instability during machining.

3 RESULTS AND DISCUSSION
3.1 Simulation Results

As previously discussed, several radii ranges were simulated many times in order to form a distribution of results. The distribution of resulting path lengths, the stochastic path length com-

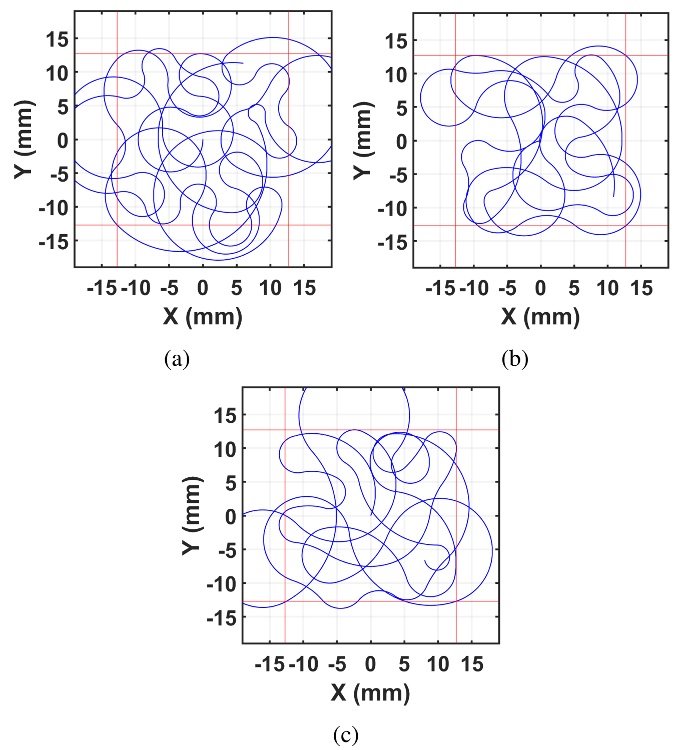


FIGURE 4: EXAMPLE TOOLPATHS WITH DIFFERENT RADII RANGE: (A) 2.54 TO 10.16 MM; (B) 5.08 TO 12.70 MM; (C) 7.62 TO 15.24 MM

parisons to conventional path lengths, and the number of times the exit constraint was relaxed, were all analyzed. As stated in the introduction, path length cannot be directly related to machining time. The resulting machining time will depend on the acceleration capabilities of the CNC machine used for milling. However, the path length does provide a simple approximation.

From these results, it was found that the path lengths for all parameters tested were generally similar. No discernible trend could be determined from these results. This result provides good indication that the radii range selected will not significantly alter the path length. When considering the way the toolpath generation program was created, this result does make sense. Regardless of the radii range, the toolpath will always target high points towards the end of the code. Because of this, the toolpath will always finish around the same time.

The result of similar path lengths between all tests raises the question of whether or not the exit constraint was relaxed a different number of times for the different parameters. For example, a test with a relatively high radii range would seem to not be capable of selecting points without traveling outside the workpiece, whereas a test with a low radii would be able to quickly turn when approaching the edges of the workpiece. However, this hypothesis was found to be incorrect. Again, no significant

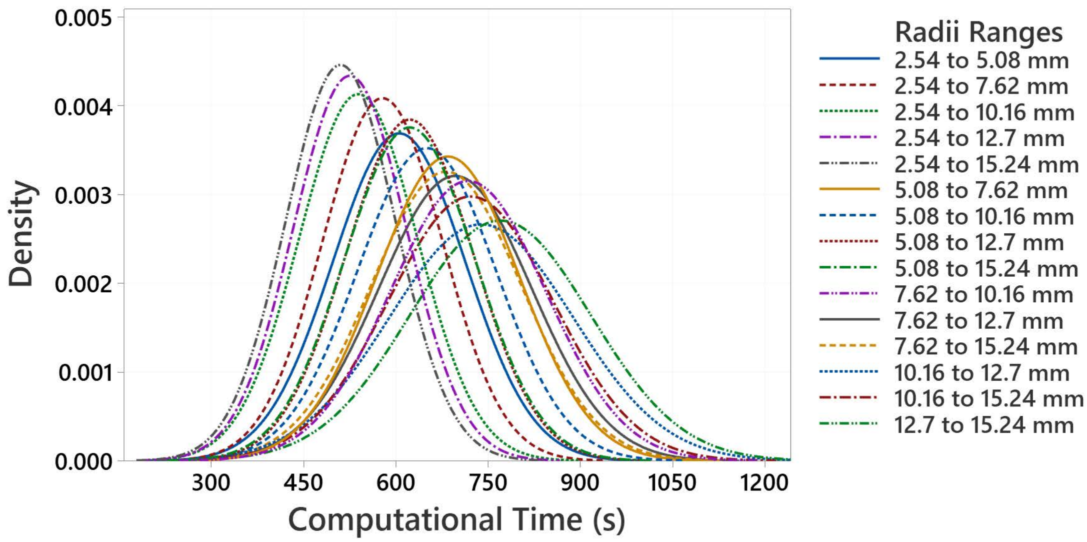


FIGURE 5: COMPUTATION TIME DISTRIBUTION

difference could be deduced from these results. Though, this hypothesis may prove to be correct for other, more extreme ranges. The average scallop value for all tests were also found to be similar between all parameters. The average scallop height and the path lengths are closely related.

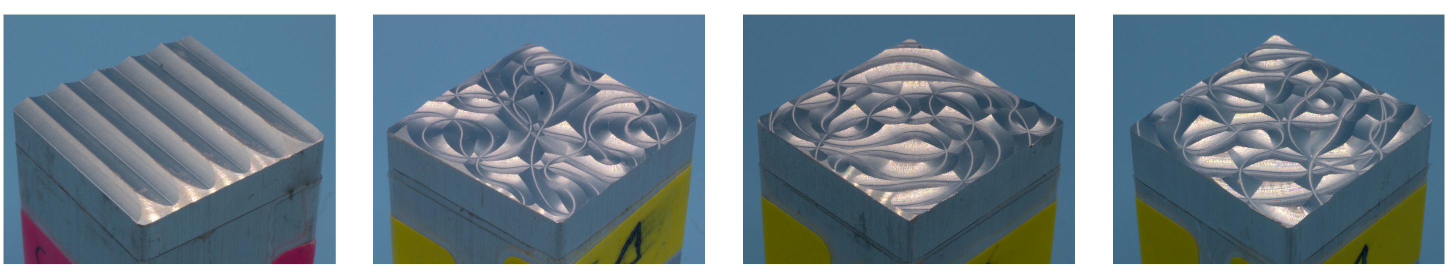
The resulting computation time distribution can be seen in Figure 5. Again, this data should only be considered relative to each other since individual computer system will vary. This distribution did result in a significant difference. It can be seen from the figure that as the difference between the maximum and minimum radii range values increases, the computational time decreases. In other words, as the radii range constraint is relaxed, enabling a larger selection of radius values to be chosen, the computation time is decreased.

In conclusion, the selection of radii range only influences the computational time. These parameters do not generally influence the path length, nor the average scallop height. Without considering the resulting effects on chatter, a radii range with a large difference between the maximum and minimum values should be, as this will offer the fastest computation time.

3.2 Experimental Results

A limited number of paths that were simulated were experimentally tested, as described in the previous section. Example workpiece surfaces of this testing can be seen in Figure 6. It can be seen that the surface finish of these samples is relatively smooth, indicating that there was not any major chatter that occurred during testing.

Separate from surface finish, the surface variability of these different tests are significantly different. It is obvious from these images that the conventional linear toolpath produces long scallops, whereas the stochastic toolpath produces small peaks throughout the surface. Similar surface results were also produced in [17].



(a) (b) (c) (d)

FIGURE 6: MACHINED WORKPIECE: (A) CONVENTIONAL TOOLPATH; (B) STOCHASTIC TOOLPATH (RADII 2.54 TO 10.16 MM); (C) STOCHASTIC TOOLPATH (RADII 5.08 TO 12.70 MM); (D) STOCHASTIC TOOLPATH (RADII 7.62 TO 15.24 MM)

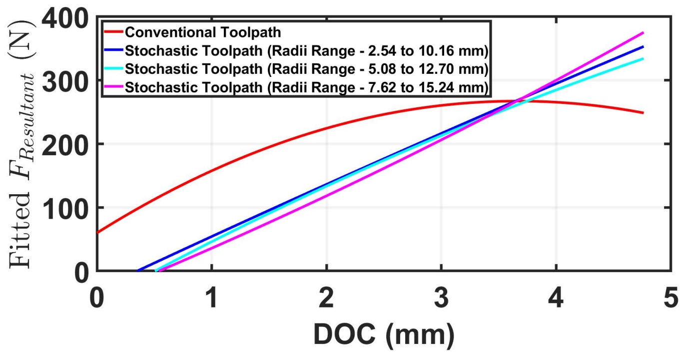


FIGURE 7: Average Force

Plot of forces experienced during the stochastic machining can be seen in Figure 7. This figure was produced by associating the position of the tool in the Y axis with the instantaneous axial depth of cut (DOC). The lines in these plots represent trend lines generated from the magnitude of all of the data points that were collected in all axes. As can be seen, as the axial DOC is increased, the machining forces also increase. This trend is similar between all test cases. However, the amplitude of these forces differed though.

An FFT analyses of the force data collected from each test was performed. This revealed that the stochastic toolpath is significantly effective at reducing chatter. The conventional toolpath's FFT results compared to the stochastic toolpath that resulted in the most improvement can be seen in Figure 8. This test that produced the greatest chatter reduction had a radii range of 7.62 – 15.24 mm. This test resulted in a 84.27% reduction in force for the first harmonic. All other tests produced similar reductions in force. On average, all of the stochastic toolpaths tested reduced the first harmonic force by $81.56 \pm 0.02\%$.

From these FFT results, it can be concluded that the radii range does not have any significant impact on the resulting chatter reduction benefits. No significant difference could be deduced between tests with different radii ranges. Therefore, since the radii range selection does not affect chatter, the radii range may be selected based on computational time considerations. As previously discussed, the largest radii range should be selected. In the case explored herein, the highest performing radii range was 2.54 – 15.24 mm, which had the largest span of all tests considered. The extreme of this case can also be considered, which would be an infinite span representative of completely eliminating the radii range constraint. This study has therefore shown that the radii range constraint should not be used with this type of toolpath generation.

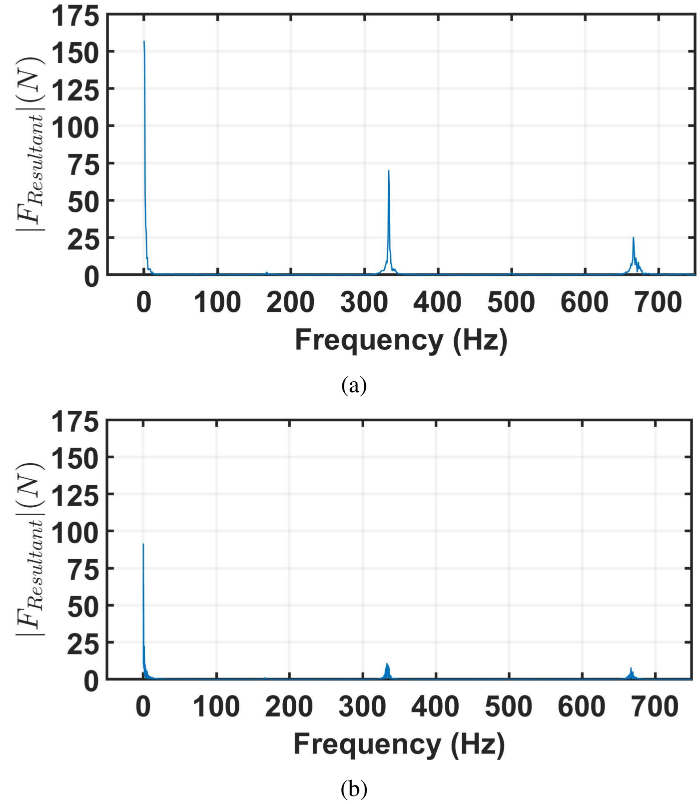


FIGURE 8: FFT - SINGLE SIDED AMPLITUDE SPECTRUM: (A) CONVENTIONAL TOOLPATH; (B) STOCHASTIC TOOLPATH

4 CONCLUSIONS

The stochastic toolpath first introduced in [17] was explored for its ability to reduce chatter in a milling operation. New constraints were added to the stochastic toolpath generation method. It was found that:

- The path length produced by the stochastic toolpath generator does not depend on the radii range that is used.
- The computational time to generate a path is reduced when the radii range set spans a large range.
- Chatter in a milling operation can be significantly reduced by utilizing a stochastic toolpath approach.
- Chatter reduction does not depend on the radii range selected.
- A radii range of 2.54 – 15.24 mm produced the best results in this study.

5 FUTURE WORK

The reduction in toolpath efficiency is one of the greatest drawbacks of the stochastic toolpath process. In order to overcome this limitation, feed rate optimization should be pursued. Since the stochastic toolpath has a chip thickness that is constantly varied due to its self-intersecting nature, the feed rate may be increased when passing over regions of the workpiece which have already been machined. This will enable significant improvements relative to constant feed rate stochastic toolpaths. Utilizing this method could also enable the stochastic toolpath to outperform linear toolpaths, an achievement which is presently impossible without feed rate optimization.

ACKNOWLEDGEMENTS

The investigations are based on the research project "Stochastic Modeling of the Interaction of Tool Wear and the Machining Affected Zone in Nickel-Based Superalloys and Application in Dynamic Stability", which is kindly funded by the German Research Foundation (DFG – 400845424) and the National Science Foundation under Grant No. 1760809. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

REFERENCES

[1] Djebali S, Segonds S, Redonnet JM, and Rubio W. Using the global optimisation methods to minimise the machining path length of the free-form surfaces in three-axis milling. *International Journal of Production Research*, 53(17):5296–5309, 2015.

[2] Li L, Deng X, Zhao J, Zhao F, and Sutherland JW. Multi-objective optimization of tool path considering efficiency, energy-saving and carbon-emission for free-form surface milling. *Journal of Cleaner Production*, 172:3311–3322, 2018.

[3] Castelino K, D'Souza R, and Wright PK. Toolpath optimization for minimizing airtime during machining. *Journal of manufacturing systems*, 22(3):173–180, 2003.

[4] Lin Z, Fu J, Shen H, Gan W, and Yue S. Tool path generation for multi-axis freeform surface finishing with the LKH TSP solver. *Computer-Aided Design*, 69:51–61, 2015.

[5] Erdim H, Lazoglu I, and Ozturk B. Feedrate scheduling strategies for free-form surfaces. *International Journal of Machine Tools and Manufacture*, 46(7-8):747–757, 2006.

[6] Ma JW, Song D, Jia ZY, Hu GQ, Su WW, and Si LK. Toolpath planning with constraint of cutting force fluctuation for curved surface machining. *Precision Engineering*, 51:614–624, 2018.

[7] Liu W, Zhou LS, and An LL. Constant scallop-height tool path generation for three-axis discrete data points machining. *The International Journal of Advanced Manufacturing Technology*, 63(1-4):137–146, 2012.

[8] Jia ZY, Zhao XX, Ma JW, Chen SY, Qin FZ, and Liu Z. Toolpath generation in sub-regional processing with constraint of constant scallop-height at boundary for complex curved surface. *Precision Engineering*, 55:217–230, 2019.

[9] Gong H and Wang N. 5-axis flank milling free-form surfaces considering constraints. *Computer-Aided Design*, 43(6):563–572, 2011.

[10] Otkur M and Lazoglu I. Trochoidal milling. *International Journal of Machine Tools and Manufacture*, 47(9):1324–1332, 2007.

[11] Rauch M, Duc E, and Hascoet JY. Improving trochoidal tool paths generation and implementation using process constraints modelling. *International Journal of Machine Tools and Manufacture*, 49(5):375–383, 2009.

[12] Niaki FA, Pleta A, and Mears L. Trochoidal milling: investigation of a new approach on uncut chip thickness modeling and cutting force simulation in an alternative path planning strategy. *The International Journal of Advanced Manufacturing Technology*, 97(1):641–656, 2018.

[13] Pleta A, Niaki FA, and Mears L. A comparative study on the cutting force coefficient identification between trochoidal and slot milling. *Procedia Manufacturing*, 26:570–579, 2018.

[14] Niaki FA, Pleta A, Mears L, Potthoff N, Bergmann JA, and Wiederkehr P. Trochoidal milling: investigation of dynamic stability and time domain simulation in an alternative path planning strategy. *The International Journal of Advanced Manufacturing Technology*, 102(5):1405–1419, 2019.

[15] Pleta A, Nithyanand G, Niaki FA, and Mears L. Identification of optimal machining parameters in trochoidal milling of inconel 718 for minimal force and tool wear and investigation of corresponding effects on machining affected zone depth. *Journal of Manufacturing Processes*, 43:54–62, 2019.

[16] Zhaoyu LI, Pengcheng HU, Fubao XIE, and Kai T. A variable-depth multi-layer five-axis trochoidal milling method for machining deep freeform 3D slots. *Robotics and Computer-Integrated Manufacturing*, 68:102093, 2021.

[17] Grimm TJ and Mears L. Simulation of a stochastic toolpathing strategy. *Computer-Aided Design*, 2021.