# Role of dissipative effects in the quantum gravitational onset of warm Starobinsky inflation in a closed universe

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A problematic feature of low-energy-scale inflationary models, such as Starobinsky inflation, in a spatially closed universe is the occurrence of a recollapse and a big crunch singularity before inflation can even set in. In a recent work, it was shown that this problem can be successfully resolved in loop quantum cosmology for a large class of initial conditions due to a nonsingular cyclic evolution and a hysteresislike phenomenon. However, for certain highly unfavorable initial conditions, the onset of inflation was still difficult to obtain. In this work, we explore the role of dissipative particle production, which is typical in warm inflation scenario, in the above setting. We find that entropy production sourced by such dissipative effects makes hysteresislike phenomena stronger. As a result, the onset of inflation is quick in general, including for highly unfavorable initial conditions where it fails or is significantly delayed in the absence of dissipative effects. We phenomenologically consider three warm inflation scenarios with distinct forms of dissipation coefficient and from dynamical solutions and phase-space portraits find that the phase space of favorable initial conditions turns out to be much larger than in cold inflation.

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## I. INTRODUCTION

Inflation is a finite period of quasi-de Sitter accelerated expansion in the early universe, which elegantly predicts the minimal late-time curvature as well as reproduces the adiabatic, nearly Gaussian, and quasiscale-invariant spectrum of primordial density fluctuations in accordance with the observational cosmological data. An important issue in inflationary models is that of right initial conditions for the inflaton to successfully yield sufficient $e$-foldings to confirm with observations. This issue becomes more relevant in the case of low-energy inflationary models such as with Starobinsky potential [1] which are favored by observations [2,3]. Starting from Planck regime, the potential energy is suppressed in low-energy inflation models, and inflaton starts with kinetic energy domination [4,5]. If the universe is spatially closed, then such a model can undergo a recollapse before the onset of inflation and encounter the big crunch singularity. It has been expected that a quantum theory of gravity may provide some insight into this issue. Since the main problem in above scenario is the existence of a recollapse followed by a big crunch singularity, if quantum gravity effects can resolve the big crunch singularity and result in a nonsingular cyclic evolution, then one can hope that in subsequent cycles conditions on dominance of kinetic versus the potential energy alter in such a way that recollapse can be avoided and inflation can begin.

Before we investigate the above problem in this manuscript, it is important to make some remarks to set the right context of this study and discuss alternative strategies to solve the above problem. Our study is based on assuming a positive spatial curvature of the universe. It has been noted earlier that one requires a high degree of fine-tuning to start inflation in low-energy models with a positive spatial curvature [6]. Thus, in a sense, we take the most difficult case to understand the initial conditions problem because if the universe is spatially flat or spatially open the recollapse caused by intrinsic curvature is absent. In fact, the initial condition problem in such cases, especially with a compact topology, becomes much easier to address [4,7,8]. Though it has been recently claimed that a primordial spatial curvature may partially account for the observed anomaly in the temperature anisotropy spectrum at low multipoles [9], and a small amount of late-time curvature consistent with current observational data has the potential to explain the current discrepancy between dataset probing early universe and those exploring late-time universe properties [10–12], when Planck results are combined with baryon acoustic oscillations data, one finds that the current observations are consistent with a spatially flat universe [3,13]. But the almost spatial flatness of the universe in the current epoch does not imply that it was spatially flat in the preinflationary epoch. Thus, it is worthwhile to study all the cases of spatial curvature to understand the problem of initial conditions. Let us also note that if the universe has a positive spatial curvature the problem of recollapse can be avoided in low-energy inflationary models by considering an additional field

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in a quadratic potential or a similar potential which drives inflation in the beginning which is carried over by the low-energy inflation.[1] In such a model, the additional field starts from initial conditions which are potential dominated at Planck density such that the problem of recollapse is completely avoided before the low-energy inflation onsets. This strategy is expected to work for any other low-energy inflation model with an additional scalar field sourced by a potential allowing the first phase of inflation to start near the Planck density. Another possibility is to consider alternatives to low-energy inflation models, such as a chaotic inflationary model with additional cubic and quartic terms which turn out to be consistent with the Planck data [4,5].

While the above strategies exist to alleviate the problem of initial conditions in low-energy inflation models, our objective in this study is to understand whether quantum gravity effects when included can resolve this problem without any additional fields which start inflation near the Planck density. Since the big crunch singularity caused by a recollapse in the preinflationary phase is a roadblock to solve this problem, it is pertinent to incorporate quantum gravity modifications which resolve the big crunch singularity to understand the onset of inflation in low-energy inflation models. This problem was recently addressed using nonperturbative quantum gravitational effects in loop quantum cosmology (LQC) [14]. It was shown that, although a large class of unfavorable initial conditions does not result in inflation in the classical theory and lead to a big crunch singularity in a few Planck seconds, the universe successfully goes through an inflationary phase after multiple nonsingular cycles of expansion and contraction due to quantum gravity effects. The goal of the current work lies in the same direction with an aim to improve and generalize these results to demonstrate that inclusion of dissipative particle production in LQC results in a rather quick and more robust onset of inflation even for those extreme initial conditions where inflation does not occur with above quantum gravity effects.

Let us recall that the nonperturbative loop quantum gravitational effects resolve the big bang/big crunch singularities replacing them by a nonsingular bounce when the energy density reaches Planckian values [15–17]. For the spatially closed model, singularity resolution results in multiple nonsingular cycles of expansion and contraction [18–20]. It is to be noted that loop quantum gravity effects are only dominant near the classical singularities and diminish quickly at smaller energy densities, resulting in classical dynamics at the macroscopic scales. In an effective spacetime description of these quantum gravity effects, modified Friedmann equations which have been shown to capture the underlying quantum dynamics to an excellent approximation can be obtained [16,18,21,22]. From these modified Friedmann equations, one can show a generic resolution of all strong curvature singularities in isotropic and anisotropic models in LQC [23] including in the presence of spatial curvature [24]. Given that LQC robustly solves the problem of singularities, it provides an excellent stage to address the problem of resolution of onset of inflation in low-energy inflationary models in the presence of a positive spatial curvature.

An interesting feature of cosmic expansion/contraction which leads to a novel hysteresislike phenomenon in nonsingular cyclic evolution is the difference in pressure during expansion and contraction stages [25]. This phenomenon occurs even in the absence of dissipative effects for suitable scalar field potentials [26]. Hence, the universe possesses an arrow of time due to an asymmetric equation of state during the expansion-contraction phase [27] rather than entropy production due to viscous pressure as was the case in Tolman's model [25]. Of course, the most challenging issue to build such models is to overcome big bang/crunch singularities and to achieve a nonsingular evolution. This task was completed in LQC where the hysteresislike phenomenon was demonstrated for chaotic $\phi^2$ inflation [28], a result which was recently generalized for Starobinsky inflation [14]. An interesting feature of such a hysteresislike period is that, although the universe may fail to inflate at first, conditions improve in subsequent cycles for the onset of inflation because the ratio of kinetic to potential energy decreases and a subsequent equation of state $\omega$, defined as the ratio of total pressure and energy density, becomes less than $-1/3$. This causes a phase of accelerated expansion, and as a result, the recollapse is avoided. This phenomenon of occurrence of nonsingular cyclic evolution followed by inflation turns out to be a feature of a large class of initial conditions for $\phi^2$ and Starobinsky inflation models [14]. However, for the latter, the onset of inflation is found to require a much larger number of cycles in contrast to the $\phi^2$ inflation, and for certain highly unfavorable initial conditions, inflation was not found to occur even after numerous nonsingular cycles of expansion and contraction [14]. The reason for this was tied to the weak hysteresis for low-energy inflation models.

To overcome the problem of onset of inflation in such cases, we note that dissipation is an indispensable part of any physical system interacting with its environment, and there are two different dynamical realizations for inflation: cold inflation and warm inflation [29], depending on whether nonequilibrium dissipative particle production processes due to the couplings of the inflaton field with other field degrees of freedom are negligible or not during inflation. In fact, dissipative processes determine the way ultimately the vacuum energy density, stored in the inflaton field, ends up converting into radiation, thus allowing the universe to transit from the accelerating phase to the radiation-dominated epoch. In the standard inflationary or cold inflation scenarios, dissipative effects are typically ignored during the inflationary phase if any preinflationary radiation energy density is diluted. The universe then ends

[1]We thank the anonymous referee for pointing out this possibility.

up in a supercooling phase requiring a reheating mechanism [30], where the inflaton starts oscillating around the minimum of its potential and progressively dissipates its energy into other relativistic light degrees of freedom, to heat up the universe again as required by the standard big bang cosmology. On the contrary, dissipative effects may be strong enough during inflation where preinflationary radiation energy density can be sustained during inflation and also become dominant at the end of inflation whereby the universe smoothly enters into radiation dominated epoch without a need for a separate reheating period [31]. Such dissipative effects bring about much richer dynamics for inflation at both background and perturbative levels (for reviews, see, e.g., Ref. [32]) introducing warm inflation as a promising complimentary version of cold inflation by addressing some of long-lasting problems related to (post) inflationary picture in cold inflation scenarios.

For a comparison with cold inflation, it is useful to recall some of the features of warm inflation. It is interesting to note that the dissipative effect appears as a supplementary friction term in background equations allowing embedding of steeper potentials in warm inflation solving the so-called $\eta$-problem [33]. Also, it leads to several different possibilities for a graceful exit, depending on the form of potential, form of dissipation coefficient, and whether the dynamics is in the strong or weak dissipative regime [34]. Moreover, dissipative effects also modify the primordial spectrum of curvature perturbations, resulting in a smaller energy scale of inflation and reconciling steeper potentials with observational data [35]. Such appealing features of warm inflation allows it to simultaneously satisfy the so-called swampland conjectures, provided warm inflation can occur in the sufficiently strong dissipative regime [36–38]. Although it is enormously challenging to achieve a strong dissipative regime in warm inflation, two models were successfully constructed to push warm inflation into the strong dissipative regime with inspiration from particle physics [39,40]. Furthermore, the inflaton itself can be a source and responsible for cosmic magnetic field generation [41] and in combination with the intrinsic dissipative effects lead to a novel dissipative baryogenesis scenario during inflation [42]. More recently, it was shown that warm inflation enables a stable remnant of inflaton in the postinflationary epoch, which can behave either like cold dark matter accounting for all the dark matter in the universe [43] or like a quintessence at late time generating the present phase of accelerated expansion [44] (see also Refs. [45,46] for unifying all conventional ingredients of modern cosmology using dissipative effects).

The goal of this manuscript is to investigate the dissipative particle production effects[2] on preinflationary dynamics of $k = 1$ LQC and understand their role on the hysteresislike phenomena and the onset of inflation for Starobinsky potential starting from highly unfavorable initial conditions. Our goal will be to consider those cases which failed to lead to inflation in the absence of dissipative effects. In Sec. II, we give a brief review of the effective dynamics of $k = 1$ LQC and the warm inflation and discuss the way dissipative effects are implemented in $k = 1$ LQC. In Sec. III, we solve the dynamical system of equations in the presence of dissipative effects and show that even a small amount of dissipation enlarges the phase space of initial conditions for which inflation occurs. We phenomenologically investigate these solutions for three models of warm inflation: the warm little inflaton (linear temperature-dependent dissipation coefficient), variant of warm little inflaton (inverse temperature-dependent dissipation coefficient), and minimal warm inflation (cubic temperature-dependent dissipation coefficient). Moreover, we also investigate some features of the qualitative dynamics using phase-space portraits. These results show that in the presence of dissipation the hysteresislike phenomenon becomes much stronger and results in a quick onset of inflation for even those initial conditions where inflation could not start in the absence of dissipation. We conclude the manuscript with a summary of our results in Sec. IV.

## II. EFFECTIVE DYNAMICS IN $k = 1$ LQC AND WARM INFLATION

In this section, we first briefly review the effective dynamics of spatially closed LQC in the holonomy quantization [18]. This is followed by a discussion of the dynamical equations in the warm inflation scenario and the way warm inflation can be implemented in the effective spacetime description of $k = 1$ model in LQC.

### A. Effective dynamics of $k = 1$ LQC

LQC is a canonical quantization based on Ashtekar-Barbero variables. The connection $A_a^i$ and its conjugate triad $E_a^i$ which due to homogeneity and isotropy, symmetry reduce to $c$ and $p$ for the $k = 1$ Friedmann-Lemaître-Robertson-Walker model. In the improved dynamics or the $\bar{\mu}$ scheme of LQC [16], it turns out that an equivalent set of variables defined as $b = c|p|^{-1/2}$ and $v = |p|^{3/2}$ is more convenient to obtain the quantum and effective description. Here, $v$ denotes the physical volume of the unit sphere spatial manifold and is related to the scale factor of the universe as $v = 2\pi^2 a^3$. The phase-space variables $b$ and $v$ satisfy $\{b, v\} = 4\pi G\gamma$, where $\gamma$ denotes the Barbero-Immirzi parameter, whose value is generally taken to be $\gamma \approx 0.2375$ in LQC following the calculations of black hole thermodynamics in loop quantum gravity.

The effective Hamiltonian in the holonomy-based quantization of the $k = 1$ model in LQC for the lapse chosen as unity is given by

[2]For brevity, we label these effects in the following as dissipative effects. We note that the source of such dissipative effects is particle production.

$$\mathcal{H}_{\rm eff} = -\frac{3}{8\pi G\gamma^2\lambda^2} v[\sin^2(\lambda b - D) - \sin^2 D + (1+\gamma^2)D^2] + \mathcal{H}_{\rm matt} \approx 0, \tag{2.1}$$

where $D = (\lambda(2\pi^2)^{1/3}))/v^{1/3}$ and $\lambda^2 = 4(\sqrt{3}\pi\gamma)\ell_{pl}^2$. Here, we have ignored the modifications from the inverse volume effects, which turn out to be negligible in comparison to the holonomy modifications [18].[3] Before we examine the dynamics resulting from this Hamiltonian, let us note that there exists another quantization of the $k = 1$ model in LQC, which is known as the connection-based quantization [20]. Though there exist some qualitative differences in the way singularity resolution occurs in this prescription as compared to the holonomy based quantization [20,49], the main features of dynamics remain the same. Especially, the existence of hysteresis which plays an important role in the onset of inflation is robust in both the quantization prescriptions, and the difference between the two approaches turn to be small for inflationary dynamics [28]. For this reason, we consider only the effective dynamics for the holonomy quantization in this analysis.

Using Hamilton's equations, the equation of motion for volume turns out to be

$$\dot{v} = \{v, \mathcal{H}_{\rm eff}\} = \frac{3}{\gamma\lambda} v \sin(\lambda\beta - D)\cos(\lambda\beta - D), \tag{2.2}$$

which results in the following modified Friedmann equation:

$$H^2 = \frac{\dot{v}^2}{9v^2} = \frac{8\pi G}{3}(\rho - \rho_{\rm min})\left(1 - \frac{\rho - \rho_{\rm min}}{\rho_{\rm max}^{\rm flat}}\right). \tag{2.3}$$

Here, $\rho_{\rm max}^{\rm flat} = 3/(8\pi G\gamma^2\lambda^2)$ denotes the energy density at the bounce for the spatially flat model in LQC, and

$$\rho_{\rm min} = \rho_{\rm max}^{\rm flat}[(1+\gamma^2)D^2 + \sin^2(D)] \tag{2.4}$$

denotes the minimum allowed energy density in the evolution. In the classical universe, this value coincides with the value of energy density at which a classical recollapse occurs. The maximum of the energy density is given by

$$\rho_{\rm max} = \rho_{\rm min} + \rho_{\rm max}^{\rm flat}. \tag{2.5}$$

Note that in the quantum regime depending on the initial conditions a bounce as well as a recollapse can occur at $\rho_{\rm min}$ as well as at $\rho_{\rm max}$ [14].

[3]Note that a nonsingular dynamics results solely from inverse volume modifications, too, in the $k = 1$ model in LQC [47], which has been used to understand conditions for the onset of inflation [48].

The Hamilton equations for the phase-space variable conjugate to $v$ is given by

$$\dot{b} = \{b, \mathcal{H}_{\rm eff}\} = -4\pi G\gamma[\rho + P - \rho_1] \tag{2.6}$$

with

$$\rho_1 = \frac{\rho_{\rm max}^{\rm flat} D}{3}[2(1+\gamma^2)D - \sin(2\lambda\beta - D) - \sin(2D)], \tag{2.7}$$

where $P$ denotes the pressure which equals $P = -\partial\mathcal{H}_{\rm matt}/\partial v$. The dynamical equations for the scalar field matter variables with a potential $V(\phi)$ are

$$\dot{\phi} = \{\phi, \mathcal{H}_{\rm eff}\} = \frac{p_\phi}{p^{3/2}} \tag{2.8}$$

$$\dot{p}_\phi = \{p_\phi, \mathcal{H}_{\rm eff}\} = -p^{3/2}V_{,\phi}. \tag{2.9}$$

Using the above equations, it is straightforward to show that Klein-Gordon equation follows along with the standard conservation law for matter energy density.

The above dynamical equations encode nonperturbative quantum gravitational effects, which result in a nonsingular bounce of the universe in the Planck regime [14,18,28]. This results in nonsingular cycles of expansion and contraction if the matter does not violate strong energy condition, i.e., has equation of state $\omega = P/\rho$ greater than $-1/3$. For the latter type of matter content, the universe undergoes a recollapse at late times, resulting in a contraction and a big crunch singularity in the classical theory. This singularity is avoided in LQC, resulting in a bounce and another phase of expansion and a possible recollapse if the equation of state $w > -1/3$ in the expanding phase. If the recollapses occur at the macroscopic scales, the difference in the volumes of two consecutive recollapses is found to be [28]

$$\delta v_{\rm rec}^{1/3} = \frac{-\oint P dv}{(2\pi^2)^{2/3}\rho_{\rm max}^{\rm flat}\gamma^2\lambda^2}. \tag{2.10}$$

This implies that in each cycle of expansion and contraction the maximum volume $v_{\rm rec}$ changes. This occurs because of the asymmetry of the pressure during different phases of a given cycle, which results in a hysteresislike phenomenon [26,28]. This hysteresislike phenomenon has been shown to be responsible for alleviating problems with the onset of inflation for different potentials, especially low-energy scale models [14]. Before we examine this phenomenon in the presence of radiation production in warm inflationary scenarios, we summarize the latter and obtain the relevant equations in LQC.

### B. Warm inflationary dynamics in LQC

The dynamical realization of warm inflation is different from the cold inflation due to the presence of radiation as well as the possibility of energy exchange between inflaton and radiation energy density. Hence, the total energy density of the universe in warm inflation reads

$$\rho = \rho_\phi + \rho_r, \tag{2.11}$$

where $\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$ is the scalar field energy density with $V(\phi)$ being some potential function and $\rho_r$ is the radiation energy density. The inflaton field $\phi$ and the radiation energy density form a coupled system in warm inflationary dynamics due to dissipation of energy out of the inflaton system and into radiation. The background evolution equations are, respectively, given by [34]

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = -\Upsilon(\phi, T)\dot{\phi} \tag{2.12}$$

$$\dot{\rho}_r + 4H\rho_r = \Upsilon(\phi, T)\dot{\phi}^2. \tag{2.13}$$

Here, $\Upsilon(\phi, T)$ is the dissipation coefficient, which can be a function of both inflaton and temperature, depending on the specifics of the microscopic physics behind the construction of a warm inflation model. For a radiation or a bath of relativistic particles, the radiation energy density is given by $\rho_r = (\pi^2 g/30)T^4$, where $g$ is the effective number of light degrees of freedom ($g$ is fixed according to the dissipation regime and interactions form used in warm inflation). Such radiation production results in entropy production where the entropy density $s$ is related to radiation energy density by $Ts = (4/3)\rho_r$, i.e., is related to temperature as $s = (2\pi^2 g/45)T^3$, where we have considered a thermalized radiation bath as is typically the case in warm inflationary scenarios. Then, Eq. (2.13) can be written in terms of entropy as follows [38]:

$$T(\dot{s} + 3Hs) = \Upsilon\dot{\phi}^2. \tag{2.14}$$

As we will see in the next section, such entropy production significantly changes the hysteresislike phenomena. In fact, the term $3Hs$, which is positive in the expanding universe ($H > 0$) and negative in the contracting universe ($H < 0$), produces a larger difference in pressure during expansion/contraction stages, making the hysteresislike phase stronger in comparison with the case without dissipative effects.

Let us note that the richer dynamics of warm inflation sharpened the interest for finding explicit models aiming at overcoming two important issues found in earlier particle physics realizations of warm inflation. First is the requirement of large field multiplicities so as to be able to sustain a nearly thermal bath, and second is the difficulty in achieving achieve strong dissipative regimes ($\Upsilon \gg H$), due to the interplay between inflaton and radiation fluctuations, leading to the appearance of growing modes in the scalar curvature power spectrum, and that can render it inconsistent with the observations. The former problem was first solved with an introduction of a new class of warm inflation model building realization motivated from the ingredients used in “little Higgs” models of electroweak symmetry breaking where the inflaton is a pseudo-Nambu-Goldstone boson of a broken gauge symmetry and its potential is protected against large radiative corrections by symmetry obeyed by the model while still having enough interactions to allow thermalization of light degrees of freedom. This results in enough dissipation even if the mediators are very light with respect to ambient temperature. In such a model also known as warm little inflaton, the dissipation coefficient is given by [50]

$$\Upsilon_{\rm lin} = C_{\rm lin}T. \tag{2.15}$$

We refer to the above $\Upsilon_{\rm lin}$ as the *linear dissipation coefficient*. Although warm little inflaton was successful in producing a sustainable thermalized radiation bath utilizing just a few mediator fields, it could not obtain a strong dissipative regime, which allows steeper potentials to be embedded in warm inflation by making the energy scale of inflation smaller. To this end, a concrete model of warm inflation, the so-called minimal warm inflation [39], was recently constructed in which the inflaton has axionlike coupling to gauge fields. Since the inflaton is an axion, its shift symmetry protects it from any perturbative backreactions and thus from acquiring a large thermal mass. Hence, the thermal friction from this bath can easily be stronger than Hubble friction even for a small number of fields. The corresponding axion friction coefficient turn out to be

$$\Upsilon_{\rm cub} = C_{\rm cub}T^3. \tag{2.16}$$

Hereafter, we refer to the above $\Upsilon_{\rm cub}$ as the *cubic dissipation coefficient*. In this regard, another model was also recently proposed, inspired from an idea used in warm little inflaton where the inflaton is directly coupled to light scalar bosonic fields rather than fermionic fields, which is known as a variant of warm little inflaton [40]. Although the exact form of dissipation coefficient is complex, the leading behavior of dissipation, when the effective mass is dominated by thermal part, varies as

$$\Upsilon_{\rm inv} = C_{\rm inv}T^{-1}. \tag{2.17}$$

Hereafter, we refer to the above $\Upsilon_{\rm inv}$ as the *inverse dissipation coefficient*. We should note that $C_{\rm inv} \ll C_{\rm lin} \ll C_{\rm cub}$ since it should be fixed in such a way that the condition for a sustainable thermal bath, i.e., $T > H$, is satisfied during the inflationary phase.

Taken together, to consider the dissipative effects during both preinflationary and inflationary phases all the way from the bounce until the end of inflation, we phenomenologically implement dissipative effects into the effective equations of the spatially closed model LQC. The resulting dynamical equations are

$$\dot{v} = \frac{3}{\gamma\lambda} v \sin(\lambda b - D)\cos(\lambda b - D) \tag{2.18}$$

$$\dot{b} = -4\pi G\gamma \left[\frac{p_\phi^2}{v^2} + \frac{4}{3}\rho_r - \rho_1\right] \tag{2.19}$$

$$\dot{\phi} = -\frac{p_\phi}{v} \tag{2.20}$$

$$\dot{p}_\phi = -vV_{,\phi} - \Upsilon(\phi, T)p_\phi \tag{2.21}$$

$$\dot{\rho}_r = -\left[\frac{4}{\gamma\lambda}\sin(\lambda b - D)\cos(\lambda b - D)\right]\rho_r + \frac{\Upsilon(\phi, T)p_\phi^2}{v^2}. \tag{2.22}$$

In the next section, we will first discuss the way such dissipative effects, or equivalently entropy production, change the dynamics of preinflationary phase and also enlarge the phase space of initial conditions which result in a (warm) inflation. Then, we perform a qualitative analysis of the dynamical equations to understand the attractor behavior of the solutions and gain insights on the way dissipative effects help in the onset of inflation even starting from highly unfavorable initial conditions.

## III. DISSIPATIVE EFFECTS ON PREINFLATIONARY DYNAMICS OF $k=1$ LQC

In this section, we investigate the consequences of dissipative effects in LQC to address the problem of onset of inflation for the Starobinsky potential. We discussed in Sec. II the way nonsingular cycles of expansion and contraction result in a hysteresislike phenomenon, which arises due to differences in pressure during expansion and contraction stages of cosmic evolution. Because of this difference in pressure, the work done during one cycle can be positive or negative depending on the potential function. For sufficiently flat potentials, the work can be positive, resulting in increasing the size of the universe in the successive cycles. Because of this, even if the inflaton starts with a kinetic energy dominated condition and an equation of state close to unity, the equation of state decreases in each cycle and eventually becomes less than $-1/3$, which leads to an onset of inflation. Hence, if the universe fails to inflate after the first cycle, it can do so after subsequent cycles enlarging the phase space of initial conditions, which results in inflation [28]. Recently, it was shown that for the Starobinsky potential the universe can inflate for a large part of initial conditions; however, it should go though numerous cycles of expansion and contraction [14]. Further, for some of the initial conditions, the inflation does not commence even after a large number of nonsingular cycles. As we will see, the dissipation or entropy production leads to larger differences in pressure during expansion-contraction phase, resulting in a larger amplitude of the cycles. Therefore, we expect that entropy production due to radiation particle production makes the hysteresis phenomena stronger, leading into the universe with a bigger size in successive cycles, causing the universe to inflate after a small number of cycles. In the following, we first obtain the background solutions demonstrating the above phenomena, which is followed by discussion of phase-space portraits in qualitative dynamics of this model.

### A. Dissipative effects for Starobinsky potential

Starobinsky inflation is a prominent example of low-energy inflation models favored by current observations. In classical cosmology, this model results from adding the $R^2$ term to the action, which translates to adding the following potential in the Einstein frame

$$U(\phi) = \frac{3m^2}{32\pi}\left(1 - e^{-\sqrt{\frac{16\pi}{3}}\phi(t)}\right)^2. \tag{3.1}$$

But in LQC, the above potential is not obtained from an $R^2$ term in the action, since the covariant action in LQC does result in higher-order curvature terms but in a Palatini framework [51]. As in previous works in LQC, we consider the above potential as a phenomenological input in effective dynamics.

In the Starobinsky model, the inflation is supposed to start at energy scales far lower than the Planck scale, and as a result, the initial conditions in the Planck regime are such that kinetic energy dominates the potential energy. If one numerically solves the classical cosmological dynamics of above potential, one finds that the universe undergoes a recollapse before potential energy can dominate and encounters a big crunch singularity [4,5]. We would see that this situation changes dramatically in the effective dynamics in LQC. Below, we numerically solve dynamical equations (2.18) for various initial conditions, using the explicit Runge-Kutta algorithm and stiff-switching method in *Mathematica* with accuracy and precision goals set to 11. The initial value of $b$ (the conjugate to volume $v$) is fixed by the vanishing of the effective Hamiltonian constraint. Moreover, we also set the initial value of $p_\phi$ using the condition for the bounce, i.e., $\rho = \rho_{\max}$. Therefore, we are left with just three initial conditions on volume ($v_0$), scalar field ($\phi_0$), and initial radiation energy density ($\rho_{r0}$). We choose initial conditions such that the radiation energy density is subdominated in comparison with both the kinetic energy density and potential energy density of

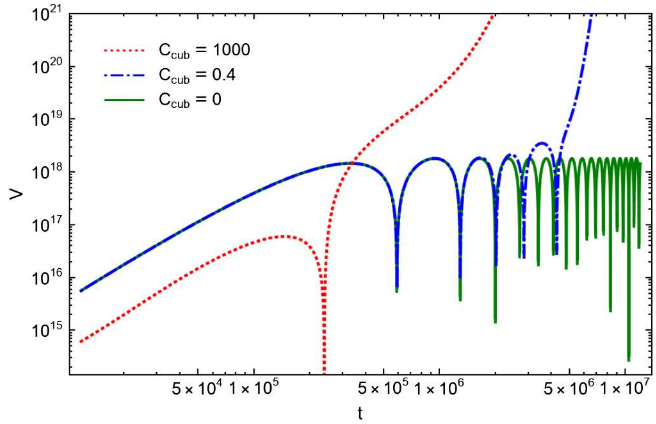


FIG. 1. The evolution of volume for different values of cubic dissipation coefficient. Initial conditions are chosen at the bounce with $v_0 = 5 \times 10^7$, $\phi_0 = -1$, $\rho_{r0} = 10^{-12}$, and $g = 17$.

the inflaton field, and the bounce happens with kinetic dominated initial conditions ($\rho_{r0} \ll U(\phi_0) \ll \dot{\phi}_0^2/2$).

In the following, we first solve the dynamical equations for the case of minimal warm inflationary model, i.e., with a cubic dissipation coefficient. In this case, the inflaton has an axionic coupling to a non-Abelian gauge theory and the sphaleron transitions between gauge vacua, existing at sufficiently high temperatures. And if the corresponding non-Abelian gauge theory has gauge group $SU(3)$, there are eight gauge bosons, each of which contribute two relativistic degrees of freedom. Including the inflaton itself, there are in total 17 relativistic degrees of freedom. So, we set the number of relativistic degree of freedom $g = 17$ [37]. In Figs. 1 and 2, we plot the evolution of volume and equation of state for three different values of $C_{\mathrm{cub}}$ in LQC. For the initial conditions, $v_0 = 5 \times 10^7$, $\phi_0 = -1$, and $\rho_{r0} = 10^{-12}$, the dynamical evolution is nonsingular for all the considered values of $C_{\mathrm{cub}}$. The initial conditions are chosen such that in the absence of dissipation, inflation does not start after various cycles of nonsingular evolution. As can be seen, in case of $C_{\mathrm{cub}} = 0$ (dissipationless universe), the universe does not enter an inflationary phase even after many cycles of nonsingular evolution. This is because the hysteresislike phenomenon is not large enough to set the scalar field at the flat part of potential function in subsequent cycles. However, dissipative effects make the hysteresis phase stronger (decreasing the number of cycles and increasing its amplitudes), whereby the universe begins the inflationary phase after a small number of cycles. This is evident in the dynamical evolution for $C_{\mathrm{cub}} = 0.4$ and $C_{\mathrm{cub}} = 1000$. We see from the former case that even a small nonzero value of $C_{\mathrm{cub}}$, resulting in small dissipative effects, has substantial effects on the hysteresislike phenomena and the onset of inflation. We find from the volume and equation of state plot that a phase of inflation starts after a few nonsingular cycles when the volume grows exponentially and the equation of state becomes less than $-1/3$. But such a small value of dissipation coefficient cannot sustain the thermal bath during inflation, and one needs a larger value of $C_{\mathrm{cub}}$. As one increases the value of $C_{\mathrm{cub}}$, the dissipative effects make the hysteresis phase very strong, and inflationary phase starts just after just one bounce. This is shown in Fig. 1 for the case of $C_{\mathrm{cub}} = 1000$. Here, we should note that the curves for $C_{\mathrm{cub}} = 0.4$ and $C_{\mathrm{cub}} = 1000$ start from the same initial volume, but because of the use of logarithmic scale in the plot, the figure does not show the same value of volume for both the curves. We note that the evolution of equation of state in Fig. 2 shows that for $C_{\mathrm{cub}} = 0$ the equation of state oscillates between 1 and 0.5 for the entire range of evolution; however, for nonvanishing dissipation coefficients, it decreases quickly below $w = -1/3$ and becomes $w \approx -1$, indicating an onset of slow-roll inflation. As one can see, the equation of state becomes $-1$ much earlier for larger dissipation coefficient.

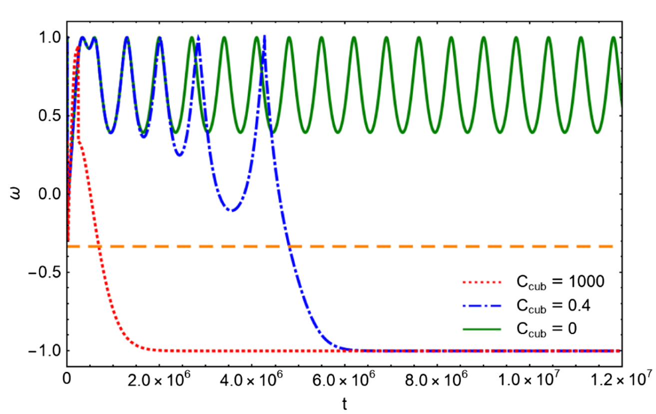


FIG. 2. The evolution of equation of state for cubic dissipation coefficient and same initial conditions as in Fig. 1.

We now discuss the case of warm little inflaton in LQC. In Figs. 3 and 4, we plot the evolution of volume and equation of state for the linear dissipation coefficient and three different values of $C_{\mathrm{lin}}$ for initial conditions $v_0 = 10^7$,

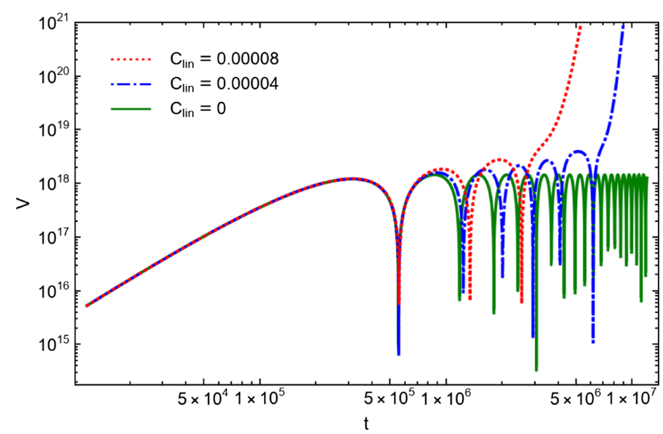


FIG. 3. The evolution of volume for different values of linear dissipation coefficient. Initial conditions are chosen at the bounce with $v_0 = 10^7$, $\phi_0 = -1.5$, $\rho_{r0} = 10^{-11}$, and $g = 12.5$.

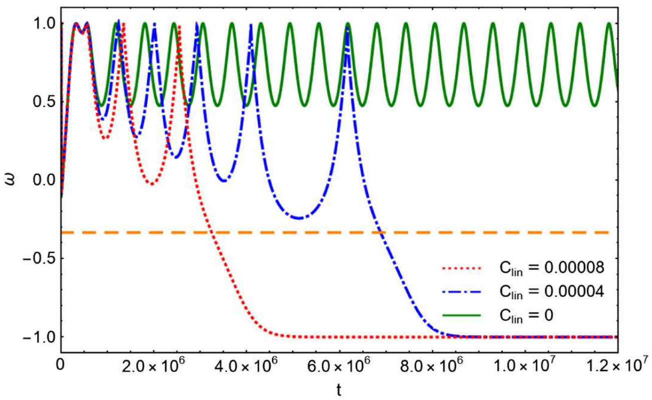


FIG. 4. The evolution of equation of state for linear dissipation coefficient and same initial conditions as in Fig. 3.

$\phi_0 = -1.5$, and $\rho_{r0} = 10^{-11}$. We also fix $g = 12.5$ using analysis in [50]. We consider nonzero values of $C_{\rm lin}$ as 0.00004 and 0.00008, which are typical values for warm inflation to happen in spatially flat spacetime. As before, the chosen initial conditions correspond to the unfavorable ones where inflation does not start in LQC even after various cycles of nonsingular evolution when dissipation is absent. This can be seen from the curve corresponding to $C_{\rm lin} = 0$, where the universe oscillates in nonsingular evolution but there is no onset of inflation since the equation of state never becomes less than $-1/3$. However, when we add dissipative effects, the hysteresis becomes stronger, and we see that the universe experiences an inflationary phase after a small number of cycles. We see that the equation of state becomes less than $-1/3$ after a few cycles for $C_{\rm lin} = 0.00004$ and $C_{\rm lin} = 0.00008$. As we increase the value of $C_{\rm lin}$, the number of cycles prior to onset of inflation decreases, and the amplitude of the cycles become larger.

Finally, we consider the variant of warm little inflaton with the inverse dissipation coefficient, which is shown in Figs. 5 and 6. As in previous cases, initial conditions are

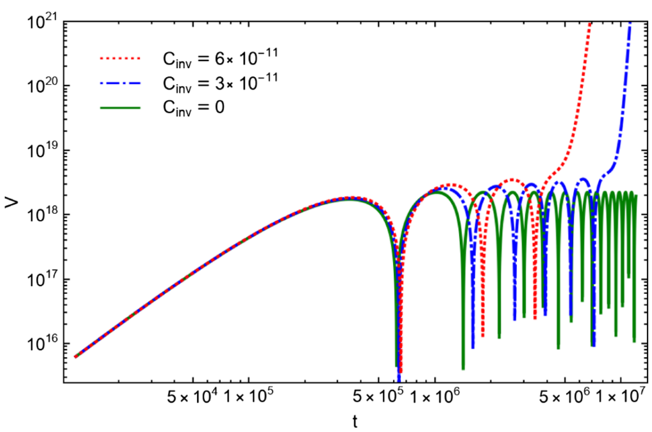


FIG. 5. The evolution of volume for different values of inverse dissipation coefficient. Initial conditions are chosen at the bounce with $v_0 = 2.5 \times 10^6$, $\phi_0 = -2$, $\rho_{r0} = 10^{-9}$, and $g = 12.5$.

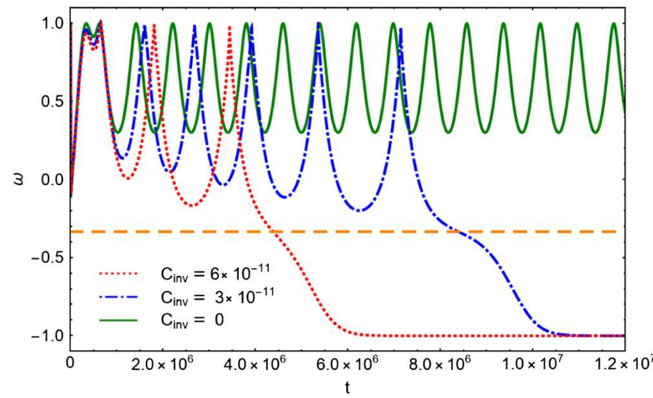


FIG. 6. The evolution of equation of state for inverse dissipation coefficient and same initial conditions as in Fig. 5.

chosen such that there is no inflationary phase even after many cycles in the absence of dissipative effects. But choosing a nonzero dissipation coefficient, even if small, leads to a striking difference in dynamics and results in a stronger phenomenon of hysteresis. In these figures, we choose $C_{\rm inv} = 3 \times 10^{-11}$ and $6 \times 10^{-11}$ and fix $g = 12.5$. The chosen values of $C_{\rm inv}$ are smaller than two other cases as we discussed previously. We find that as we increase the value of $C_{\rm inv}$ the number of cycles decreases, the amplitude of the cycles become larger, and the universe enter into the inflationary phase sooner.

To summarize the results so far, we have found the dissipative effects resulting in radiation production make the hysteresis phenomena stronger and set the condition for inflation to happen sooner for dissipation coefficients which have a cubic, linear, and inverse relationship to temperature. Though we discussed a sample of initial conditions, our results are robust to changes in initial conditions. To gain some insights on the qualitative dynamics and robustness of results we study the phase-space portraits in the following.

### B. Qualitative dynamics in phase-space portrait

It is useful to understand the phase-space portraits for qualitative dynamics by introducing the variables

$$X(t) = \chi_0\left(1 - e^{-\sqrt{\frac{16\pi G}{3}}\phi(t)}\right) \tag{3.2}$$

$$Y(t) = \frac{p_\phi(t)}{v(t)\sqrt{2\rho_{\rm max}}} Z(t) = \sqrt{\frac{\rho_r(t)}{\rho_{\rm max}}}, \tag{3.3}$$

where $\chi_0 = m\sqrt{\frac{3}{32\pi G\rho_{\rm max}}}$ and $\rho_{\rm max}$ denotes the maximum energy density (2.5) determined by the initial conditions. Our goal will be to find the inflationary attractors for different choices of dissipation coefficients and initial conditions. These have been studied earlier for cold inflation in detail in LQC [52]. The inflationary attractor

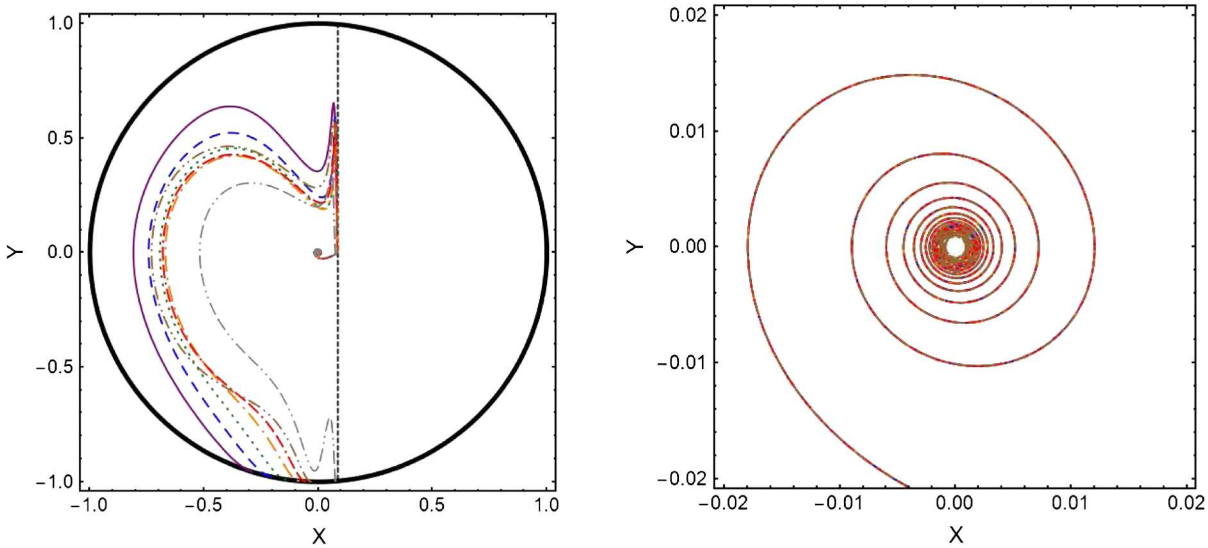


FIG. 7. Projection of three-dimensional phase-space portrait on $Z = 0$ plane for cubic dissipation coefficient with $m = 0.62$, $C_{\mathrm{cub}} = 7.5$, $v_0 = 35$, $\rho_{r0} = 10^{-3}$, and seven distinct initial conditions for $\phi_0$.

lies at $(X = 0, Y = 0)$, which corresponds to the reheating phase in cold inflation and the beginning of radiation epoch in warm inflation.

Starting with the cubic dissipation in Fig. 7, the left plot shows the projection of entire phase-space region on plane $Z = 0$, and the right plot zooms in on the attractor near the origin. The vertical dashed black line corresponds to $X = \chi_0$, and all real solutions lie to the left of this line. For a better visualization of the qualitative features it is useful to consider a large value of inflaton mass, which is chosen here to be $m = 0.62$. The solid black circular curve corresponds to the energy density of the first bounce at $t = 0$ where the initial conditions are set. The left plot in Fig. 7 shows curves corresponding to seven distinct initial conditions for $\phi_0$, while other parameters are fixed, for which the universe undergoes inflation in the presence of dissipative effects. For the same initial conditions in the absence of dissipative effects, the universe goes through many cycles without the onset of an inflationary phase. It should be noted that we chose $C_{\mathrm{cub}}$ large enough to see the end of inflation and also the postinflationary phase. So, in most of the cases, the hysteresislike phenomena go away, and the universe experiences an inflationary phase after just one bounce.

We expand on details of the above dynamics in Fig. 8, where the evolution of the potential energy density $V(\phi)$, kinetic energy density $\dot{\phi}^2/2$, and the radiation energy density $\rho_r$ are shown for the gray curve in Fig. 7. We can see from the figure that the universe starts from a bounce in the kinetic dominated regime (initial condition with equation of state $\omega \approx 1$), with $\dot{\phi}^2/2 \gg U(\phi) \gg \rho_r$, and the kinetic energy very soon dilutes away since it behaves as $a^{-6}$. In the subsequent evolution, we find that radiation energy density becomes important in comparison to kinetic and potential energies. Such a radiation dominated regime before the inflationary phase has also been

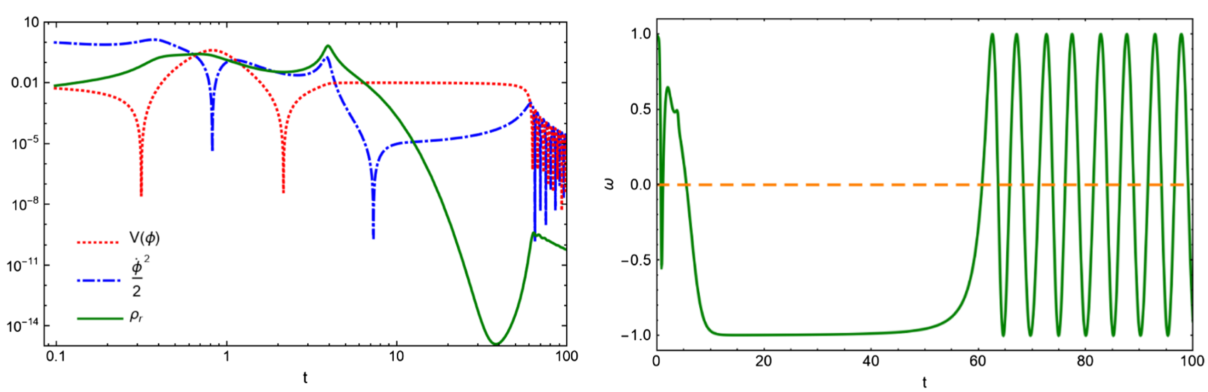


FIG. 8. Evolution of energy components as well as equation of state for cubic dissipation coefficient with $m = 0.62$, $C_{\mathrm{cub}} = 7.5$, $v_0 = 35$, $\rho_{r0} = 10^{-3}$, and $\phi_0 = 0.5$ (gray curve in Fig. 7).

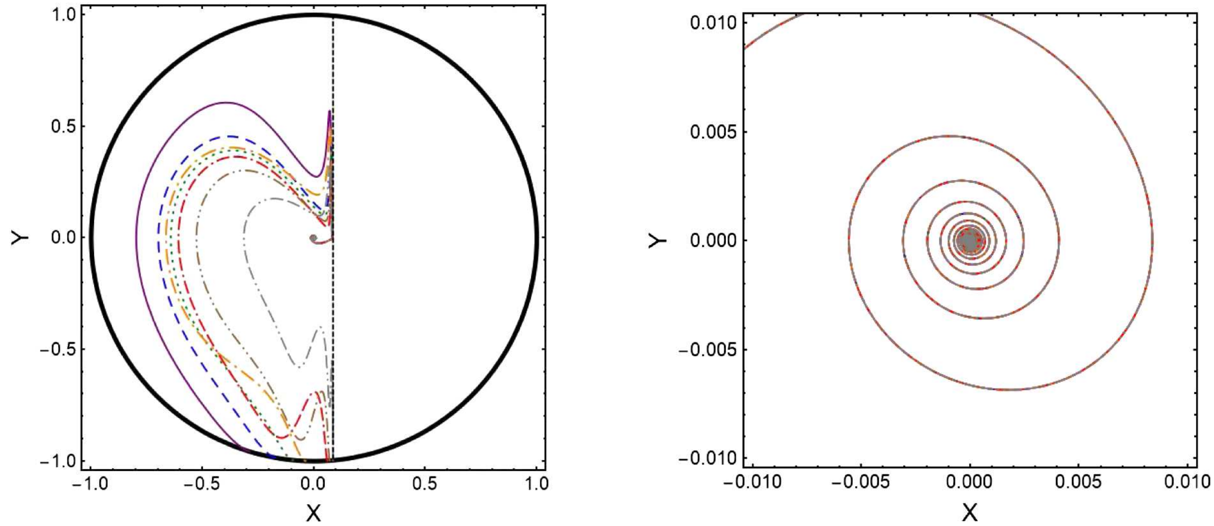


FIG. 9. Projection of three-dimensional phase-space portrait on the $Z = 0$ plane for the linear dissipation coefficient with $m = 0.62$, $C_{\mathrm{lin}} = 0.8$, $v_0 = 35$, $\rho_{r0} = 10^{-3}$, and seven distinct initial conditions for $\phi_0$.

reported for warm inflation in spatially flat LQC [53] (see also Ref. [54] for a review on warm inflation in spatially flat LQC). After some cycles, when the hysteresislike phenomenon causes an onset of inflation, the radiation energy density becomes subdominant in evolution. Note that, contrary to cold inflation, radiation is concurrently produced during the inflationary phase, and it may reach an equality with potential energy density at the end of inflation if dissipative effects are strong enough to sustain the thermal bath, and the universe smoothly enters into a radiation-dominated epoch without subsequent (p)reheating phase. However, as is clear in Fig. 8, the universe does not enter into a radiation-dominated epoch at the end of inflation. This is because the considered value of $C_{\mathrm{cub}}$, chosen due to computational constraints, is not large enough, and hence the inflationary phase is a cold type, and a reheating mechanism is a must. Hence, although a small dissipation coefficient could not sustain thermal bath during inflation leading to warm-type inflation, it has substantial effect on preinflationary dynamics and the onset of inflation, which is evident from the existence of a $\omega \approx -1$ phase from the plot of the equation of state. We further note that if the dissipation coefficient is chosen large enough the warm inflation starts quickly and the radiation energy density reaches values similar to the potential energy density at the end of inflation.

Figure 9 illustrates the projection of the entire phase-space portrait on $Z = 0$ plane for the linear dissipation coefficient and $m = 0.62$. The left plot shows that seven distinct initial conditions (different values of $\phi_0$ while other parameters are fixed) starting from the first bounce all exhibit an attractor behavior and come to the center of circle (fixed point). In Fig. 10, the evolution of energy density components shows that inflation ends in a radiation-dominated epoch due to dissipative effects. Although such dissipative effects are large enough to sustain thermal bath during inflation, they are not large enough to suppress kinetic energy during inflation and terminate the universe in

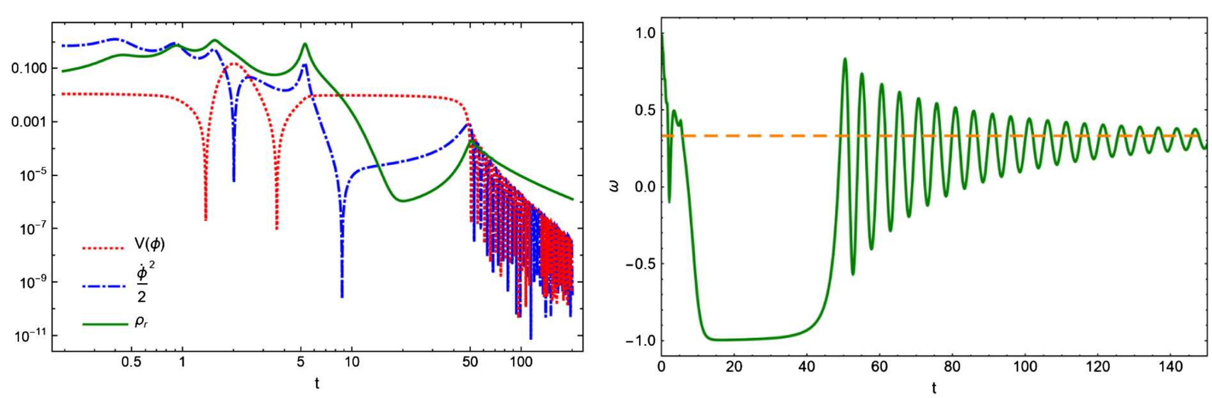


FIG. 10. Evolution of energy components as well as equation of state for linear dissipation coefficient with $m = 0.62$, $C_{\mathrm{lin}} = 0.8$, $v_0 = 35$, $\rho_{r0} = 10^{-3}$, and $\phi_0 = 1.55$ (gray curve in Fig. 9).

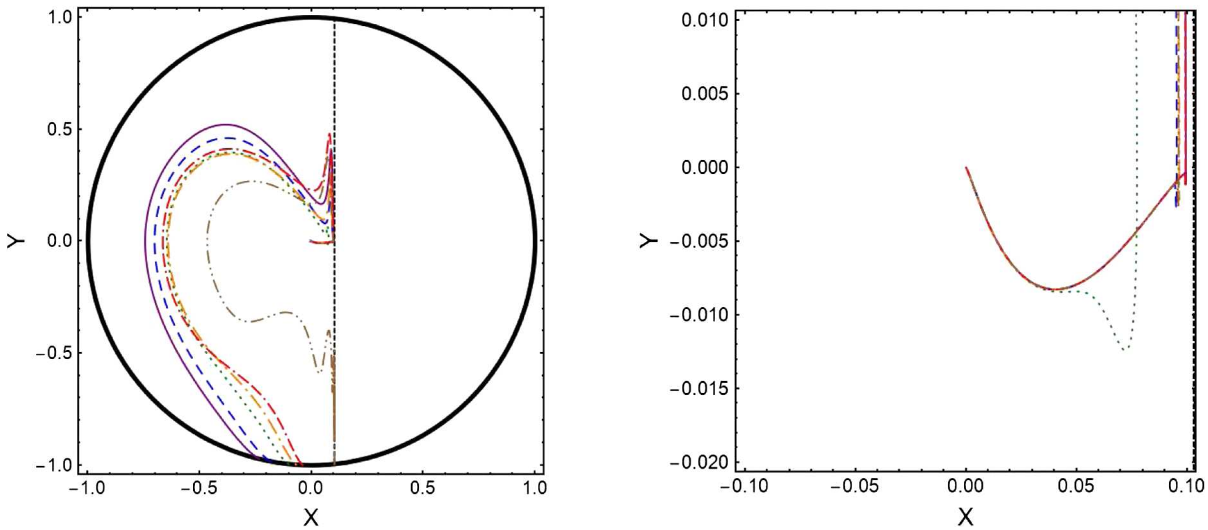


FIG. 11. Projection of three-dimensional phase-space portrait on $Z = 0$ plane for inverse dissipation coefficient with $m = 0.79$, $C_{\rm inv} = 0.2$, $v_0 = 30$, $\rho_{r0} = 10^{-3}$, and six distinct initial conditions for $\phi_0$.

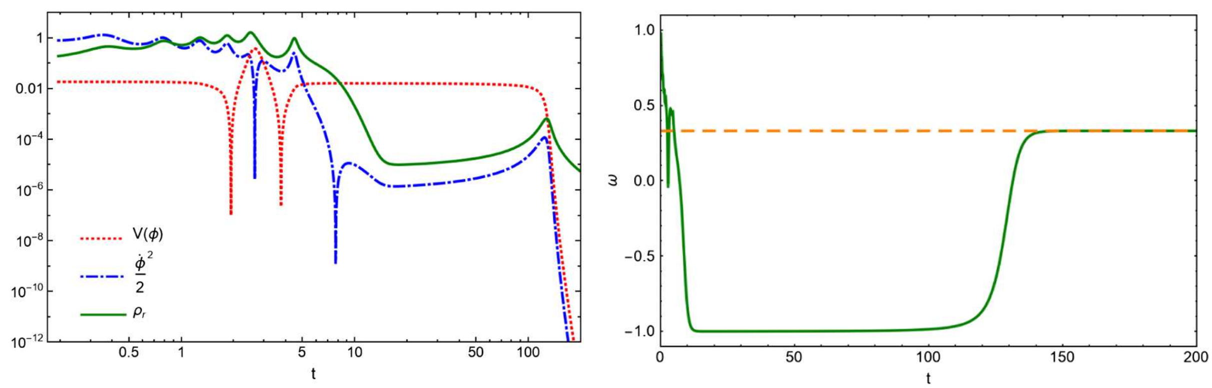


FIG. 12. Evolution of energy components as well as equation of state for inverse dissipation coefficient with $m = 0.79$, $C_{\rm inv} = 0.2$, $v_0 = 30$, $\rho_{r0} = 10^{-3}$, and $\phi_0 = 2.2$ (brown curve in Fig. 11).

a radiation-dominated epoch. Hence, there is a very short kinetic-dominated regime before the universe transits into a radiation epoch, which is typical in the warm inflation scenario when the dissipation effect is small (such a kinetic-dominated regime after warm inflation has also been seen in Ref. [45]). However, such a kinetic-dominated regime does not have any adverse implications since it is very short. Moreover, since in this model such a kinetic-dominated regime occurs around the minimum of potential, we see spiral behavior, which is typical in cold inflation due to the reheating phase. However, this oscillatory phase plays no role in making the universe hot, and the universe enters into a radiation-dominated regime due to radiation production during inflation and not a reheating phase. We also find that there is a radiation-dominated regime before the inflation phase, as was seen in the case of the cubic dissipation coefficient.

In Fig. 11, we illustrate the projection of the entire phase-space portrait on the $Z = 0$ plane for the inverse dissipation coefficient and $m = 0.79$. We find that in this case, although there is an attractor behavior and all initial conditions starting from the first bounce come to the center of the phase-space plot, there is no spiral behavior. This is because the universe enters into a radiation-dominated regime due to strong dissipative effects. There is no oscillatory behavior, which is typical of cold inflation or what is found for previous cases of the dissipation coefficient. In particular, the kinetic energy always remains subdominated during inflation due to strong dissipative effects, and inflation ends when radiation becomes equal to potential energy (see Fig. 12). Furthermore, we find that there is also a radiation-dominated regime before the inflationary phase, as was seen for cubic and linear dissipation coefficients.

## IV. CONCLUSIONS

The onset of inflation in low-energy inflationary models in classical theory is challenging in spatially closed models because of the recollapse of the universe before inflation can set in. Indeed, one requies a high degree of fine-tuning of initial conditions for the onset of inflation in such a case [6]. On the other hand, if the spatial curvature is zero or negative, this problem is nonexistent, and the onset of inflation becomes highly probable [4,7,8]. The recollapse in a spatially closed model causes a big crunch singularity, and a closed universe ends in a big crunch singularity in a few Planck seconds before the beginning of an inflationary phase. This is a longstanding problem whose resolution becomes important given that current observational data favor low-energy inflation models and a slight positive spatial curvature of the universe. There are various ways to overcome this issue in classical cosmology. Apart from the case of considering models with a spatially flat and spatially open universe, one can consider a low-energy inflation model coupled with an additional scalar field which drives an early phase of inflation near the Planck density, which is taken over later by low-energy inflation. Further, one can introduce additional higher-order terms in the quadratic potential to fit with Planck data [4,5]. But if one aims to understand this issue for a single field set up in low-energy inflation models, two issues need to be addressed simultaneously. The first is a successful and a generic resolution of singularities, and the second is a mechanism to create favorable conditions for inflation to begin. The challenges underlying the first problem are well known and require insights from nonperturbative quantum gravity. The latter problem is also nontrivial, given that inflation in low-energy inflation models begins at very small energy scales compared to Planck scale due to which initial conditions in the Planck regime are kinetic energy dominated, which leads to a recollapse of the universe. The above problem was recently analyzed in LQC [14] where nonperturbative quantum gravity effects are known to result in a generic resolution of all strong curvature singularities [23]. In particular, the big bang/big crunch singularities are resolved and replaced by a nonsingular bounce [15,16,18]. It was found that for the Starobinsky inflation potential the universe in LQC cycles through various periods of expansion and contraction, resulting in an onset of inflation even when inflation starts from kinetic-energy-dominated initial conditions which result in a big crunch in a few Planck seconds. At the heart of this resolution likes a hysteresislike phenomenon, which changes the ratio of kinetic and potential energy in subsequent cycles in such a way that the equation of state even when starting from $\omega \approx 1$ becomes less than $-1/3$. The universe then enters a phase of accelerated expansion, a recollapse is avoided, and inflation sets in.

Although hysteresislike phenomena can enlarge the phase space of initial conditions for a plateaulike potential, with the Starobinsky potential as the most known one, the inflationary phase either occurs after many cycles or does not happen due to flatness of the potential for some unfavorable initial conditions [14]. A pertinent question is whether there exists a mechanism which can result in the onset of inflation in LQC even for such unfavorable initial conditions. The goal of the paper was to successfully answer this question. Motivated by Tolman's model in which the hysteresislike phase happens due to entropy production sourced by viscous pressure and also warm inflationary dynamics, we considered spatially closed LQC in which the scalar field concurrently dissipates its kinetic energy into a radiation field starting from the first bounce. Hence, there are two contributions for work done in each cycle: one from an asymmetric equation of state of scalar field during the expansion-contraction phase and the other from entropy production due to dissipative effects. Because of dissipative effects, one expects the phenomena of hysteresis to become much stronger and inflation to set in far more easily.

We worked in the setting of effective spacetime description in LQC and obtained Hamilton's equations with nonperturbative quantum gravity corrections in the presence of dissipative effects for the $k = 1$ model using holonomy quantization. Hamilton's equations were numerically solved for the Starobinsky potential and three different dissipation coefficients inspired from quantum field theory all the way from the first bounce until the end of inflation. These were with cubic, linear, and inverse temperature dependence. We found that even a small value of dissipation makes the hysteresislike phenomena strong. The effect is such that inflation sets in not only a few cycles but also for those initial conditions which are extremely unfavorable for inflation to begin even in LQC without radiation production. Moreover, we find that as we make the dissipative effects large enough the hysteresislike phenomenon goes away and the universe inevitably enters into the inflationary phase after just one bounce. To gain insights on the qualitative dynamics of the universe from the bounce until the end of inflation, we studied the phase-space portraits for different initial conditions and all three dissipation coefficients. In all three cases, we found that all initial conditions experience an attractor dynamics showing that the universe goes through the inflationary phase. In other words, the universe starting from the first bounce with stifflike initial conditions dilutes its kinetic energy due to both Hubble friction and dissipative effects and transfers it to radiation field, as opposed to a dissipationless universe, whereby after some cycles the radiation energy density becomes the dominant energy component. However, such a radiation-dominated epoch continues only for a very short period since radiation energy density decays as $a^{-4}$. Then, the potential energy becomes dominant, resulting in an inflationary phase. Moreover, if the dissipative effects are large enough, the inflationary phase will be of warm type, whereby the universe smoothly enters into a radiation-dominated epoch without the need for a separate reheating epoch. Our analysis shows that with the presence of dissipative effects nonsingular quantum gravitational dynamics results in an onset of inflation for low-energy inflationary models even from highly unfavorable initial conditions. In comparison to the cases where dissipation is absent, we find hat inflation sets in much quicker due to stronger hysteresislike phenomena. Since our results establish phenomenological viability of low-energy inflationary models in spatially closed universes at the level of background dynamics, it will be interesting to investigate the model at perturbative level to confront its predictions with observational data.

## ACKNOWLEDGMENTS

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[1] A. A. Starobinsky, New type of isotropic cosmological models without singularity, Phys. Lett. B **91**, 99 (1980).

[2] P. A. R. Ade *et al.* (Planck Collaboration), Planck 2015 results. XX. Constraints on inflation, Astron. Astrophys. **594**, A20 (2016).

[3] P. A. R. Ade *et al.* (Planck Collaboration), Planck 2018 results. X. Constraints on inflation, Astron. Astrophys. **641**, A10 (2020).

[4] A. D. Linde, Inflationary cosmology after Planck 2013, arXiv:1402.0526.

[5] A. D. Linde, On the problem of initial conditions for inflation, Found. Phys. **48**, 1246 (2018).

[6] A. D. Linde, Can we have inflation with $\Omega > 1$?, J. Cosmol. Astropart. Phys. 05 (2003) 002.

[7] A. D. Linde, Creation of a compact topologically nontrivial inflationary Universe, J. Cosmol. Astropart. Phys. 10 (2004) 004.

[8] W. E. East, M. Kleban, A. Linde, and L. Senatore, Beginning inflation in an inhomogeneous Universe, J. Cosmol. Astropart. Phys. 09 (2016) 010.

[9] B. Bonga, B. Gupt, and N. Yokomizo, Inflation in the closed FLRW model and the CMB, J. Cosmol. Astropart. Phys. 10 (2016) 031; W. Handley, Primordial power spectra for curved inflating Universes, Phys. Rev. D **100**, 123517 (2019).

[10] J. Ooba, B. Ratra, and N. Sugiyama, Planck 2015 constraints on the non-flat ΛCDM inflation model, Astrophys. J. **864**, 80 (2018).

[11] E. Di Valentino, A. Melchiorri, and J. Silk, Planck evidence for a closed Universe and a possible crisis for cosmology, Nat. Astron. **4**, 196 (2020).

[12] W. Handley, Curvature tension: Evidence for a closed Universe, Phys. Rev. D **103**, L041301 (2021).

[13] G. Efstathiou and S. Gratton, The evidence for a spatially flat Universe, Mon. Not. R. Astron. Soc. **496**, L91 (2020).

[14] L. Gordon, B. F. Li, and P. Singh, Quantum gravitational onset of Starobinsky inflation in a closed Universe, Phys. Rev. D **103**, 046016 (2021).

[15] A. Ashtekar, T. Pawlowski, and P. Singh, Quantum Nature of the Big Bang, Phys. Rev. Lett. **96**, 141301 (2006).

[16] A. Ashtekar, T. Pawlowski, and P. Singh, Quantum nature of the big bang: Improved dynamics, Phys. Rev. D **74**, 084003 (2006).

[17] A. Ashtekar, A. Corichi, and P. Singh, Robustness of key features of loop quantum cosmology, Phys. Rev. D **77**, 024046 (2008).

[18] A. Ashtekar, T. Pawlowski, P. Singh, and K. Vandersloot, Loop quantum cosmology of $k = 1$ FRW models, Phys. Rev. D **75**, 024035 (2007).

[19] L. Szulc, W. Kaminski, and J. Lewandowski, Closed FRW model in loop quantum cosmology, Classical Quantum Gravity **24**, 10 (2007).

[20] A. Corichi and A. Karami, Loop quantum cosmology of $k = 1$ FRW: A tale of two bounces, Phys. Rev. D **84**, 044003 (2011).

[21] P. Diener, B. Gupt, and P. Singh, Numerical simulations of a loop quantum cosmos: Robustness of the quantum bounce and the validity of effective dynamics, Classical Quantum Gravity **31**, 105015 (2014).

[22] P. Diener, A. Joe, M. Megevand, and P. Singh, Numerical simulations of loop quantum Bianchi-I spacetimes, Classical Quantum Gravity **34**, 094004 (2017).

[23] P. Singh, Are loop quantum cosmos never singular?, Classical Quantum Gravity **26**, 125005 (2009); Curvature invariants, geodesics and the strength of singularities in Bianchi-I loop quantum cosmology, Phys. Rev. D **85**, 104011 (2012); S. Saini and P. Singh, Resolution of strong singularities and geodesic completeness in loop quantum Bianchi-II spacetimes, Classical Quantum Gravity **34**, 235006 (2017); Generic absence of strong singularities in loop quantum Bianchi-IX spacetimes, Classical Quantum Gravity **35**, 065014 (2018).

[24] P. Singh and F. Vidotto, Exotic singularities and spatially curved loop quantum cosmology, Phys. Rev. D **83**, 064027 (2011).

[25] R. C. Tolman, Static solutions of Einstein's field equations for spheres of fluid, Phys. Rev. **55**, 364 (1939).

[26] V. Sahni and A. Toporensky, Cosmological hysteresis and the cyclic Universe, Phys. Rev. D **85**, 123542 (2012).

[27] V. Sahni, Y. Shtanov, and A. Toporensky, Arrow of time in dissipationless cosmology, Classical Quantum Gravity **32**, 182001 (2015).

[28] J. L. Dupuy and P. Singh, Hysteresis and beating phenomena in loop quantum cosmology, Phys. Rev. D **101**, 086016 (2020).

[29] A. Berera and L. Z. Fang, Thermally Induced Density Perturbations in the Inflation Era, Phys. Rev. Lett. **74**,

1912 (1995); A. Berera, Warm Inflation, Phys. Rev. Lett. **75**, 3218 (1995); A. Berera, M. Gleiser, and R. O. Ramos, A First Principles Warm Inflation Model that Solves the Cosmological Horizon/Flatness Problems, Phys. Rev. Lett. **83**, 264 (1999).

[30] L. Kofman, A. D. Linde, and A. A. Starobinsky, Reheating After Inflation, Phys. Rev. Lett. **73**, 3195 (1994); L. Kofman, A. D. Linde, and A. A. Starobinsky, Towards the theory of reheating after inflation, Phys. Rev. D **56**, 3258 (1997).

[31] A. Berera, Interpolating the stage of exponential expansion in the early Universe: A possible alternative with no reheating, Phys. Rev. D **55**, 3346 (1997).

[32] A. Berera, I. G. Moss, and R. O. Ramos, Warm inflation and its microphysical basis, Rep. Prog. Phys. **72**, 026901 (2009); S. Bartrum, M. Bastero-Gil, A. Berera, R. Cerezo, R. O. Ramos, and J. G. Rosa, The importance of being warm (during inflation), Phys. Lett. B **732**, 116 (2014); I. G. Moss and C. Xiong, On the consistency of warm inflation, J. Cosmol. Astropart. Phys. 11 (2008) 023; L. M. H. Hall, I. G. Moss, and A. Berera, Scalar perturbation spectra from warm inflation, Phys. Rev. D **69**, 083525 (2004); M. Bastero-Gil, A. Berera, I. G. Moss, and R. O. Ramos, Cosmological fluctuations of a random field and radiation fluid, J. Cosmol. Astropart. Phys. 05 (2014) 004; R. O. Ramos and L. A. da Silva, Power spectrum for inflation models with quantum and thermal noises, J. Cosmol. Astropart. Phys. 03 (2013) 032; M. Bastero-Gil, A. Berera, and R. O. Ramos, Shear viscous effects on the primordial power spectrum from warm inflation, J. Cosmol. Astropart. Phys. 07 (2011) 030; M. Motaharfar, E. Massaeli, and H. R. Sepangi, Power spectra in warm G-inflation and its consistency: Stochastic approach, Phys. Rev. D **96**, 103541 (2017); I. G. Moss and C. Xiong, Non-Gaussianity in fluctuations from warm inflation, J. Cosmol. Astropart. Phys. 04 (2007) 007.

[33] A. Berera, Warm inflation at arbitrary adiabaticity: A Model, an existence proof for inflationary dynamics in quantum field theory, Nucl. Phys. **B585**, 666 (2000).

[34] S. Das and R. O. Ramos, On the graceful exit problem in warm inflation, Phys. Rev. D **103**, 123520 (2021).

[35] M. Bastero-Gil and A. Berera, Warm inflation model building, Int. J. Mod. Phys. A **24**, 2207 (2009); M. Benetti and R. O. Ramos, Warm inflation dissipative effects: Predictions and constraints from the Planck data, Phys. Rev. D **95**, 023517 (2017); M. Bastero-Gil, A. Berera, R. Hernndez-Jimnez, and J. G. Rosa, Dynamical and observational constraints on the warm little inflaton scenario, Phys. Rev. D **98**, 083502 (2018); M. Bastero-Gil, S. Bhattacharya, K. Dutta, and M. R. Gangopadhyay, Constraining warm inflation with CMB data, J. Cosmol. Astropart. Phys. 02 (2018) 054.

[36] M. Motaharfar, V. Kamali, and R. O. Ramos, Warm inflation as a way out of the swampland, Phys. Rev. D **99**, 063513 (2019); V. Kamali, M. Motaharfar, and R. O. Ramos, Warm brane inflation with an exponential potential: A consistent realization away from the swampland, Phys. Rev. D **101**, 023535 (2020); S. Das and R. O. Ramos, Runaway potentials in warm inflation satisfying the swampland conjectures, Phys. Rev. D **102**, 103522 (2020).

[37] S. Das, G. Goswami, and C. Krishnan, Swampland, axions, and minimal warm inflation, Phys. Rev. D **101**, 103529 (2020).

[38] R. Brandenberger, V. Kamali, and R. O. Ramos, Strengthening the de Sitter swampland conjecture in warm inflation, J. High Energy Phys. 08 (2020) 127.

[39] K. V. Berghaus, P. W. Graham, and D. E. Kaplan, Minimal warm inflation, J. Cosmol. Astropart. Phys. 03 (2020) 034.

[40] M. Bastero-Gil, A. Berera, R. O. Ramos, and J. G. Rosa, Towards a reliable effective field theory of inflation, Phys. Lett. B **813**, 136055 (2021).

[41] A. Berera, T. W. Kephart, and S. D. Wick, GUT cosmic magnetic fields in a warm inflationary Universe, Phys. Rev. D **59**, 043510 (1999).

[42] R. H. Brandenberger and M. Yamaguchi, Spontaneous baryogenesis in warm inflation, Phys. Rev. D **68**, 023505 (2003); M. Bastero-Gil, A. Berera, R. O. Ramos, and J. G. Rosa, Warm baryogenesis, Phys. Lett. B **712**, 425 (2012); Observational implications of mattergenesis during inflation, J. Cosmol. Astropart. Phys. 10 (2014) 053.

[43] J. G. Rosa and L. B. Ventura, Warm Little Inflaton becomes Cold Dark Matter, Phys. Rev. Lett. **122**, 161301 (2019); M. Levy, J. G. Rosa, and L. B. Ventura, Warm inflation, neutrinos and dark matter: A minimal extension of the Standard Model, arXiv:2012.03988.

[44] K. Dimopoulos and L. Donaldson-Wood, Warm quintessential inflation, Phys. Lett. B **796**, 26 (2019); J. G. Rosa and L. B. Ventura, warm little inflaton becomes dark energy, Phys. Lett. B **798**, 134984 (2019).

[45] G. B. F. Lima and R. O. Ramos, Unified early and late Universe cosmology through dissipative effects in steep quintessential inflation potential models, Phys. Rev. D **100**, 123529 (2019).

[46] P. M. Sá, Triple unification of inflation, dark energy, and dark matter in two-scalar-field cosmology, Phys. Rev. D **102**, 103519 (2020).

[47] P. Singh and A. Toporensky, Big crunch avoidance in $k = 1$ semi-classical loop quantum cosmology, Phys. Rev. D **69**, 104008 (2004).

[48] J. E. Lidsey, D. J. Mulryne, N. J. Nunes, and R. Tavakol, Oscillatory Universes in loop quantum cosmology and initial conditions for inflation, Phys. Rev. D **70**, 063521 (2004); D. J. Mulryne, N. J. Nunes, R. Tavakol, and J. E. Lidsey, Inflationary cosmology and oscillating Universes in loop quantum cosmology, Int. J. Mod. Phys. A **20**, 2347 (2005).

[49] J. L. Dupuy and P. Singh, Implications of quantum ambiguities in $k = 1$ loop quantum cosmology: Distinct quantum turnarounds and the super-Planckian regime, Phys. Rev. D **95**, 023510 (2017).

[50] M. Bastero-Gil, A. Berera, R. O. Ramos, and J. G. Rosa, Warm Little Inflaton, Phys. Rev. Lett. **117**, 151301 (2016).

[51] G. J. Olmo and P. Singh, Effective action for loop quantum cosmology a la Palatini, J. Cosmol. Astropart. Phys. 01 (2009) 030.

[52] B. F. Li, P. Singh, and A. Wang, Qualitative dynamics and inflationary attractors in loop cosmology, Phys. Rev. D **98**, 066016 (2018).

[53] L. L. Graef and R. O. Ramos, Probability of warm inflation in loop quantum cosmology, Phys. Rev. D **98**, 023531 (2018).
[54] R. Herrera, Warm inflationary model in loop quantum cosmology, Phys. Rev. D **81**, 123511 (2010); L. N. Barboza, L. L. Graef, and R. O. Ramos, Warm bounce in loop quantum cosmology and the prediction for the duration of inflation, Phys. Rev. D **102**, 103521 (2020); M. Benetti, L. Graef, and R. O. Ramos, Observational constraints on warm inflation in loop quantum cosmology, J. Cosmol. Astropart. Phys. 10 (2019) 066; V. Kamali, S. Basilakos, A. Mehrabi, M. Motaharfar, and E. Massaeli, Tachyon warm inflation with the effects of loop quantum cosmology in the light of Planck 2015, Int. J. Mod. Phys. D **27**, 1850056 (2018).