

# A Surrogate-Enhanced Scheme in Decision Making under Uncertainty in Power Systems

Yijun Xu, *Member, IEEE*, Lamine Mili, *Life Fellow, IEEE*, Mert Korkali, *Senior Member, IEEE*, Xiao Chen, Jaber Valinejad, *Student Member, IEEE*, Long Peng, *Student Member, IEEE*

**Abstract**—Facing stochastic variations of the loads due to an increasing penetration of renewable energy generation, online decision making under uncertainty in modern power systems is capturing power researchers' attention in recent years. To address this issue while achieving a good balance between system security and economic objectives, we propose a surrogate-enhanced scheme under a joint chance-constrained (JCC) optimal power-flow (OPF) framework. Starting from a stochastic-sampling procedure, we first utilize the copula theory to simulate the dependence among multivariate uncertain inputs. Then, to reduce the prohibitive computational time required in the traditional Monte-Carlo (MC) method, we propose to use a polynomial-chaos-based surrogate that allows us to efficiently evaluate the power-system model at non-Gaussian distributed sampled values with a negligible computing cost. Learning from the MC simulated samples, we further proposed a hybrid adaptive approach to overcome the conservativeness of the JCC-OPF by utilizing correlation of the system states, which is ignored in the traditional Boole's inequality. The simulations conducted on the modified Illinois test system demonstrate the excellent performance of the proposed method.

**Index Terms**—Decision making under uncertainty, surrogate model, uncertainty quantification.

## I. INTRODUCTION

DECISION making under uncertainty in modern power systems is attracting increasing attention from power researchers due to the stochastic load variations and the intermittency of the renewable generations [1]. These uncertainties can lead to dramatic fluctuations in the system states, a problem that brings challenges for a secure operation, protection, control and planning of power systems [2]–[5]. Facing these challenges, the traditional deterministic OPF approach has limited capability in providing security-ensured decisions while its robust approaches typically provide overconservative decisions at the cost of the economic benefits. Therefore, an alternative CC-OPF approach is recently advocated in the literature for its flexibility in utilizing the predefined probability to adjust the feasible region for a better balance between the security and cost of power system operation.

In general, the CC-AC-OPF is considered to be difficult to solve. Although a direct MC method is straightforward, its computational burden is prohibitively heavy for online

applications. Therefore, some researchers propose the scenario approach or adopt a predefined robust sample set to reduce the computational burden at the expense of decreased accuracy of the results [6]. Besides these sampling-based methods, many researchers rely on analytical approaches due to their capability of providing a closed-form solution that can greatly accelerate its computing speed [6], [7]. However, due to the nonlinearity of the power-system model and the non-Gaussian distribution of the uncertainty, it is extremely difficult to derive an explicit closed-form solution. This is especially true when the dependence among the uncertainties exists. To reduce its derivation complexity, a simplified DC power-flow model [6]–[8] or its improved linearized AC model [9], [10] is adopted. These also typically lead to the assumption of a stable distribution, e.g., a Gaussian distribution [7], [10].

Furthermore, the CC-OPF is typically classified as the individual chance-constrained (ICC) one and the JCC one. The former assigns the violation probabilities to each constraint individually while the latter ensures that all the constraints as a whole are satisfied simultaneously to a predefined confidence level, which gives a better characterize of the overall system performance [9]. Therefore, we choose the JCC-OPF as the focus of this paper. Typically, the JCC problem is thought to be harder to solve than the ICC one and only limited attempts have been tried [1], [8], [9]. Even the scenario approximation method is thought to be straightforward, its performance is heavily restricted by its scalability and overconservativeness [11]. Although Peña-Ordieres *et al.* [8] propose a computationally scalable method, it is not very accurate since it relies on a DC OPF model. Several papers also attempt to transform the JCCs into ICCs to solve [1], [9], [12]. However, the solutions tend to be conservative since the traditional Boole's inequality is based on the statistical independence assumption, which cannot hold in power systems.

To address the above challenges, this paper proposes a novel surrogate-enhanced scheme, for the first time, in solving the JCC-AC-OPF problem considering the dependent, non-Gaussian uncertainties. More specifically, to improve the computing efficiency of the MC framework, the response surfaces of the nonlinear AC power-flow model are simulated with a polynomial-chaos-expansion (PCE)-based surrogate [13] that enables us to evaluate the sampled values at almost no computational cost. The copula theory is further merged into the PCE to account for the dependence among high-dimensional uncertain inputs [14]. In addition, to overcome the conservativeness of the Boole's inequality, we further propose a hybrid adaptive procedure to decompose the JCC into ICCs by first properly

Y. Xu, L. Mili, J. Valinejad and L. Peng are with the Bradley Department of Electrical and Computer Engineering, Virginia Tech, Northern Virginia Center, Falls Church, VA 22043 USA (e-mail: {yijunxu,lmili,jabervlinejad}@vt.edu).

M. Korkali and X. Chen are with Lawrence Livermore National Laboratory, Livermore, CA 94550 USA (e-mail: {korkali1,chen73}@llnl.gov).

This work was supported, in part, by the U.S. NSF Grant 1917308 and by the United States Department of Energy Office of Electricity Advanced Grid Modeling Program, and performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344. Document released as LLNL-CONF-816084.

detecting the statistical active constraints and then improving the allocation of the joint chance into individual ones. Here, we use the correlation information learned from the MC samples, which also assists us to better design the tightened bounds with a quantile estimator that has no Gaussian assumption for the system states. Some simulations conducted on the modified Illinois test system demonstrate the excellent performances of the proposed method.

## II. PROBLEM FORMULATION

This section will briefly summarize the AC-OPF model associated with its extended ICC-AC-OPF and JCC-AC-OPF.

1) *AC-OPF*: Traditionally, AC-OPF is formulated as

$$\min_{\mathbf{u}_c} f(\mathbf{x}, \mathbf{u}_c) \quad (1a)$$

$$\text{s.t. } \mathbf{g}(\mathbf{x}, \mathbf{u}_c) = 0, \quad (1b)$$

$$\mathbf{h}_{\min} \leq \mathbf{h}(\mathbf{x}, \mathbf{u}_c) \leq \mathbf{h}_{\max}, \quad (1c)$$

where  $\mathbf{x} \in \mathbb{R}^N$  are vectors of state variables, e.g., the voltage magnitude and angle at a PQ bus, the voltage angle and reactive power at a PV bus, the active and reactive power at the swing bus, and the power flow along the transmission lines;  $\mathbf{u}_c \in \mathbb{R}^D$  are the vectors of the control variables, such as the generator power output and its voltage;  $f$  is a scalar objective function; and  $\mathbf{g}$  and  $\mathbf{h}$  are the vector-valued functions of equality and inequality constraints, respectively. Typically,  $f$  is modeled as a quadratic cost function expressed as  $f(\mathbf{x}, \mathbf{u}_c) = \sum_{i \in \mathcal{G}} (c_{2,i} P_{g,i}^2 + c_{1,i} P_{g,i} + c_{0,i})$ , where  $c_2$ ,  $c_1$ , and  $c_0$  denote the cost coefficients and  $P_{g,i}$  denotes the power output of the  $i$ th conventional generator, which belongs to a set of  $\mathcal{G}$ . The equality constraints are described by a set of nonlinear ac power-flow equations for an  $N_b$ -bus system. More details can be found in [10]. The inequality constraints,  $\mathbf{h}$ , need to satisfy the vector-valued lower bound,  $\mathbf{h}_{\min}$ , and the vector-valued upper bound,  $\mathbf{h}_{\max}$ . Typically, they include some hard constraints due to the physical limits such as the generator active and reactive power limits as well as some soft constraints such as the apparent power limits in the transmission lines and bus-voltage magnitudes. Till now, we have briefly summarized the AC-OPF model.

2) *CC-AC-OPF*: By incorporating the randomness of the loads and the renewable energy generation in the aforementioned equality and inequality constraints,  $\mathbf{g}$  and  $\mathbf{h}$ , we have  $\mathbf{g}(\mathbf{x}, \mathbf{u}_c, \boldsymbol{\xi})$  and  $\mathbf{h}(\mathbf{x}, \mathbf{u}_c, \boldsymbol{\xi})$ , where  $\boldsymbol{\xi} \in \mathbb{R}^S$  denotes a vector of random variables. These statistical constraints further enable us to manage the violation rate for the optimized solution under the framework of the CC-AC-OPF model.

Here, for the ICC-AC-OPF that assigns the violation probability to its individual operation set, (1c) is transformed as

$$\mathbb{P}(h_i(\mathbf{x}, \mathbf{u}_c, \boldsymbol{\xi}) \in \mathcal{H}_i) \geq 1 - \epsilon_i. \quad (2)$$

where  $\mathcal{H}_i$  denotes the prescribed  $i$ th operational set determined by  $\mathbf{h}_{\min}$  and  $\mathbf{h}_{\max}$  and  $\epsilon_i$  denotes its corresponding predefined violation rate, which is typically set to a small number, e.g., 1% – 5%, to ensure a secure operation of the power systems.

By contrast, for all the individual operation sets,  $\{h_i\}_{i=1}^{n_{ie}}$ , that must be satisfied simultaneously for one overall violation probability,  $\epsilon$ , (1c) is transformed as

$$\mathbb{P}(\cap_{i=1}^{n_{ie}} \{h_i(\mathbf{x}, \mathbf{u}_c, \boldsymbol{\xi}) \in \mathcal{H}_i\}) \geq 1 - \epsilon, \quad (3)$$

which represents the JCC-AC-OPF [9].

Here, the JCC given by (3) can be further transformed into the ICC given by (2). The most popular way is to use the Boole's inequality that transforms one joint probability,  $\epsilon$  into  $n_{ie}$  individual chance constraints as (2) indicated by satisfying a rough approximation of  $\sum_{i=1}^{n_{ie}} \epsilon_i = \epsilon$ . Typically, due to the lack of information, every individual violation rate,  $\epsilon_i$ , is simply approximated by

$$\epsilon_i = \frac{\epsilon}{n_{ie}}. \quad (4)$$

Although simple, this strategy will yield an overconservative solution when the correlation in the system states cannot be simplified or ignored [9], [12]. This is also one of the problems addressed in this paper.

## III. RESPONSE-SURFACE METHOD IN CC-AC-OPF

In this section, we incorporate the PCE-based surrogate into the CC-AC-OPF framework.

### A. Motivation for using the Response Surface

Apparently, the most straightforward way to obtain the violation probability at its operation solution,  $\mathbf{u}_c$ , is the MC-based method, where a set of  $N_{\boldsymbol{\xi}}$  samples are drawn from the multivariate probability distribution of  $\boldsymbol{\xi}$ , yielding  $\{\boldsymbol{\xi}^{(j)}\}_{j=1}^{N_{\boldsymbol{\xi}}}$ . Then, for each  $\boldsymbol{\xi}^{(j)}$ ,  $j = 1, \dots, N_{\boldsymbol{\xi}}$ , the equality constraint,  $\mathbf{g}$ , is evaluated at the sampled values,  $\boldsymbol{\xi}^{(j)}$ , to obtain the corresponding system states  $\mathbf{x}^{(j)}$ . This further enables us to statistically assess each individual inequality constraints as

$$\mathbb{P}(h_i(\mathbf{x}, \mathbf{u}_c, \boldsymbol{\xi}) \in \mathcal{H}_i) = \sum_{j=1}^{N_{\boldsymbol{\xi}}} \frac{1}{N_{\boldsymbol{\xi}}} \chi_{\{\mathcal{H}_i\}}(h_i(\mathbf{x}^{(j)}, \mathbf{u}_c, \boldsymbol{\xi}^{(j)})), \quad (5)$$

where  $\chi$  is the characteristic function satisfying  $\chi_{\{\mathcal{H}_i\}}(h_i(\mathbf{x}, \mathbf{u}_c, \boldsymbol{\xi})) = \begin{cases} 1 & \text{if } h_i(\mathbf{x}, \mathbf{u}_c, \boldsymbol{\xi}) \in \mathcal{H}_i, \\ 0 & \text{if } h_i(\mathbf{x}, \mathbf{u}_c, \boldsymbol{\xi}) \notin \mathcal{H}_i. \end{cases}$  Similarly,

(5) can be easily extended to the joint probability distribution for the JCC-OPF problem via

$$\mathbb{P}(\cap_{i=1}^{n_{ie}} \{h_i(\mathbf{x}, \mathbf{u}_c, \boldsymbol{\xi}) \in \mathcal{H}_i\}) = \sum_{j=1}^{N_{\boldsymbol{\xi}}} \frac{1}{N_{\boldsymbol{\xi}}} \chi_{\{\mathcal{H}\}}(\mathbf{h}(\mathbf{x}^{(j)}, \mathbf{u}_c, \boldsymbol{\xi}^{(j)})), \quad (6)$$

where its characteristic function  $\chi$  is given by

$$\chi_{\{\mathcal{H}\}}(\mathbf{h}(\mathbf{x}, \mathbf{u}_c, \boldsymbol{\xi})) = \begin{cases} 1 & \text{if } \cap_{i=1}^{n_{ie}} \{h_i(\mathbf{x}, \mathbf{u}_c, \boldsymbol{\xi}) \in \mathcal{H}_i\}, \\ 0 & \text{if } \forall \{h_i(\mathbf{x}, \mathbf{u}_c, \boldsymbol{\xi}) \notin \mathcal{H}_i\}. \end{cases}$$

Obviously, this sampling-based method will be computationally prohibitive for a large-scale power-system model. To reduce its computational burden, we propose to use the response-surface method. First, let us define a vector-valued nonlinear function  $\mathbf{y}$  that combines the functions of  $\mathbf{g}$  and  $\mathbf{h}$  for simplicity. Then, we can use  $\mathbf{y}$  to build a mapping between the uncertain random variables,  $\boldsymbol{\xi}$ , with the power system state variables,  $\mathbf{x}$ , at the fixed control variable,  $\mathbf{u}_c$ , expressed as  $\mathbf{x} = \mathbf{y}(\boldsymbol{\xi}, \mathbf{u}_c)$ . Here, if we replace this complicated function  $\mathbf{y} : \mathbb{R}^S \times \mathbb{R}^D \rightarrow \mathbb{R}^N$  with a simple functional form  $\tilde{\mathbf{y}}(\cdot)$  that captures the behavior of the complicated, high-fidelity simulation model of a power system while being computationally inexpensive to evaluate, then we call  $\tilde{\mathbf{y}}(\cdot)$  the response surface of the  $\mathbf{y}(\cdot)$  [2], [4], [13]. It holds a relationship as  $\tilde{\mathbf{y}}(\boldsymbol{\xi}, \mathbf{u}_c) \approx \mathbf{y}(\boldsymbol{\xi}, \mathbf{u}_c)$ . Using the response-surface method, the computing efficiency of the sampling-based approach can be greatly enhanced without sacrificing accuracy.

### B. Generalized Polynomial Chaos as the Surrogate

Introduced by Wiener and further developed by Xiu and Karniadakis, the generalized polynomial chaos expansion has been shown to be a cost-effective tool in modeling power-system surrogate [2], [4]. In this method, the stochastic outputs are represented as a weighted sum of a given set of orthogonal polynomial chaos basis functions constructed from the probability distribution of the input random variables. For the vector of independent, identically distributed (i.i.d.) random variables,  $\xi = [\xi_1, \xi_2, \dots, \xi_S]$ , following a standard probability distribution (e.g., the Gaussian or the beta distribution), to which a unique orthogonal polynomial, such as the Hermite or the Jacobi polynomial, is associated [13]. Let  $\Phi_i(\xi_1, \xi_2, \dots, \xi_S)$  denote this procedure's corresponding polynomial chaos basis and let  $a_i$  denote the  $i$ th polynomial chaos coefficient. For a stochastic system state,  $x$ , we have a truncated PCE as

$$x = \sum_{i=0}^{S_P} a_i \Phi_i(\xi), \quad (7)$$

where  $S_P = (S+P)!/(S!P!) - 1$ ;  $S$  is the total number of the random variables involved in the PCE; and  $P$  is the maximum order of the polynomial chaos basis functions. It is found that a relatively low maximum polynomial chaos order (typically 2) provides output results with enough accuracy [2], [4], [15]. From the polynomial chaos coefficients, the mean,  $\mu$ , and the variance,  $\sigma^2$ , of the output  $x$  can be estimated as follows:  $\mu = a_0$ ,  $\sigma^2 = \sum_{i=1}^{N_S} a_i^2 \mathbb{E}[\phi_i^2]$ . However, this property only holds true only under the i.i.d. assumption [4], [16]. Even if it has been adopted in [15] to solve an ICC-OPF, it is not applicable to our JCC approach, where the dependent inputs are considered. Here, we only use the response-surface property of the PCE as described by (7).

1) *The Orthogonal Polynomial Chaos Basis*: A set of one-dimensional polynomial chaos basis functions  $\{\Phi_i(\xi), i = 0, 1, 2, 3, \dots\}$  with respect to some real positive measure satisfy  $\int \Phi_i(\xi) \Phi_j(\xi) d\lambda \begin{cases} = 0 & \text{if } i \neq j, \\ > 0 & \text{if } i = j, \end{cases}$  where  $\lambda$  denotes a probability measure defined as the Cumulative Probability Distribution Function (CPDF) of  $\xi$ . For every CPDF, the associated orthogonal polynomials are unique. Then, a set of multi-dimensional polynomial chaos basis functions can be constructed as the tensor product of the one-dimensional polynomial chaos basis associated with each input random variable. Formally, we have  $\Phi(\xi) = \Phi(\xi_1) \otimes \Phi(\xi_2) \otimes \dots \otimes \Phi(\xi_S)$ , where  $\Phi(\xi_i)$  denotes the one-dimensional polynomial chaos basis for the  $i$ th random variable.

2) *Collocation Points*: Collocation points (CPs) are a finite sample set of  $\xi = [\xi_1, \xi_2, \dots, \xi_S]$  that are chosen to approximate the polynomial chaos coefficients. The elements of the CPs are generated by using the union of the zeros and the roots of one higher-order, one-dimensional polynomial for every random variable [2], [17]. Then, using a tensor product or sparse tensor rule, we can generate multidimensional CPs as described in [13].

3) *Handling Dependence in the Response-Surface Method*: The prerequisite of an uncertainty quantification relies on a precise uncertainty modeling, in which the dependence modeling is vital. Here, we adopt the copula technique since it can be perfectly merged into the framework of the response-surface method. According to Sklar's theorem, any joint

multivariate cumulative distribution function  $F_\xi$  of an  $S$ -dimensional random vector can be expressed in terms of its marginal distributions and a copula to represent their dependence. Formally, we have

$$F_\xi(\xi) = C(F_{\xi_1}(\xi_1), F_{\xi_2}(\xi_2), \dots, F_{\xi_S}(\xi_S)), \quad (8)$$

where  $F_{\xi_i}(\xi_i)$  is the  $i$ th input marginal and  $C(\cdot)$  is a copula that describes the dependence structure between the  $S$ -dimensional input variables [14]. Accordingly, its joint multivariate density function,  $f_\xi(\xi)$ , can be obtained via  $f_\xi(\xi) = c(F_{\xi_1}(\xi_1), \dots, F_{\xi_S}(\xi_S)) \prod_{i=1}^S f_i(\xi_i)$ , where  $c$  is the  $S$ -variate copula density and  $f_i(\xi_i)$  is the marginal density for the  $i$ th variable. Here, we advocate the Gaussian copula for its simplicity and scalability for high-dimensional cases. More details are provided in [4].

With this PCE surrogate, we can efficiently evaluate the non-Gaussian dependent MC samples with almost no computational cost. Due to the limited space, the detailed step-by-step implementations of the PCE with a Gaussian copula in a power system can be found in [4].

### IV. DECISION-MAKING UNDER UNCERTAINTY

Here, we will first elaborate how the ICC-OPF is reformulated. After that, we present an adaptive procedure to transform the joint chance into individual chances. Finally, we further improve this scheme by incorporating a hybrid procedure.

#### A. Reformulation of the Individual Chance Constraints

Now, with the above stochastic testing results obtained in (6) with the PCE-surrogate, we reformulate the ICC in (2) by tightening the bounds of (1c) via

$$\mathbf{h}_{\min} + \Delta \mathbf{h}_{\min} \leq \mathbf{h}(\mathbf{x}, \mathbf{u}_c) \leq \mathbf{h}_{\max} - \Delta \mathbf{h}_{\max}. \quad (9)$$

Here,  $\Delta \mathbf{h}_{\min}$  and  $\Delta \mathbf{h}_{\max}$  denote the non-negative, adjusted lower and upper margins, respectively. Then, we can directly solve (1) with the updated bounds as a deterministic OPF problem to obtain more conservative operation solution  $\mathbf{u}_c$ .

Obviously, to achieve the designed violation probability,  $\epsilon_i$ , the way to update the margins is vital. To design the procedure, let us first detect the active constraints. As addressed by Baker and Bernstein [9], in the optimization problem, the inactive constraints are those constraints, when removed, the optimal solution will not change while the active constraints are essential in determining the optimal solution. Besides, we further propose to incorporate the violation probability in classifying the active constraints. We define these statistical active constraints as constraints whose violation probabilities exceed the predefined violation rate. Let us consider the bus voltage magnitude  $V_i$ , bounded by  $[0.9, 1.1]$  pu. Suppose an  $\epsilon = 5\%$  is predefined for its individual constraint, if the  $\mathbb{P}(V_i \leq 1.1) \leq 95\%$ , then we call it a "statistical active constraint" and vice versa. We only need to update these statistical active constraints and keep the statistical inactive constraint unchanged. For those classified as statistical active constraint, we calculate its updated margins through the quantile of its distributions via

$$\Delta h_{i\min} = Q(h_i, \mathbb{P}(h_i \in \mathcal{H}_i)) - Q(h_i, (1 - \epsilon_i)), \quad (10a)$$

$$\Delta h_{i\max} = Q(h_i, (1 - \epsilon_i)) - Q(h_i, \mathbb{P}(h_i \in \mathcal{H}_i)), \quad (10b)$$

where  $Q(h_i, \mathbb{P})$  denotes the quantile value for the distribution of  $h_i$  at a probability value of  $\mathbb{P}$ . Obviously, for the inactive constraint, we have its  $\Delta h_{i\min} = 0$  and  $\Delta h_{i\max} = 0$ . This

quantile-based updating rule is chosen since it is based on an actual stochastic testing within the MC simulation framework [10] that can avoid the Gaussian or symmetric assumption of the system responses [7], [9], [15].

### B. Transforming the JCC into an ICC

Since in this paper our focus is on joint chance constraints, we need to transform the JCC problem into an ICC problem. In this transformation, we mainly propose three major steps: active constraints detection, significant/insignificant active-constraint classification, and a fine-tuning procedure. We present them in detail in the sequel.

1) *Detection of the Active Constraints:* Given the predefined joint chance,  $\epsilon$ , as we briefly mentioned above, the Boole's inequality is widely used to transform  $\epsilon$  into  $n_{ie}$  individual chances via  $\epsilon_i = \frac{\epsilon}{n_{ie}}$ . Obviously, in a large-scale optimization problem such as the OPF problem, the number of  $n_{ie}$  can be extremely large if we do not have any prior information since the number of the inequality in a power system model can go very high. For example, in a 200-bus power system, by only considering the constraints for the maximum voltages with the  $\epsilon$  set as 2%, we have  $\epsilon_i = \frac{2}{200} = 0.01\%$ . Statistically speaking, a probability as small as 0.01% cannot be easily quantified in practice.

Fortunately, since we can efficiently evaluate MC samples using the PCE surrogate, we propose to conduct an online MC testing first. By using it, we can not only detect the joint and individual probabilities as indicated in (6) and (5), but also detect the number of active constraints, which is denoted by  $n_{ac}$ . Then, an improved version of Boole's inequality using active constraints is obtained as

$$\epsilon_i = \frac{\epsilon}{n_{ac}}. \quad (11)$$

This step will greatly relieve the conservativeness of the Boole's inequality.

2) *Significant/Insignificant Active Constraints Classification:* Here, using (11), another issue will appear when we distribute the joint probability to each individual probability. Suppose for every active constraint, the detected violation probability is calculated as set of  $\{\epsilon_i^{ac}\}_{i=1}^{n_{ac}}$ . If  $\epsilon_i^{ac} \geq \epsilon_i$ , then we can use (10) to tighten its corresponding bound. However, if  $\epsilon_i^{ac} \leq \epsilon_i$ , then this active constraint does not have enough capability to adjust the assigned probability,  $\epsilon_i$ . Therefore, we further propose to adopt another step to screen out the insignificant active constraints that do not have enough capability for this adjustment. Motivated by Holm who used a sequential multiple testing in hypothesis rejection [18], we propose a simple sequential testing for detecting significant and insignificant active constraints and assign joint probability as illustrated in Algorithm 1. Here,  $\epsilon_{ad}$  represents a joint probability distribution that does not include the probability for the insignificant active constraints. Till now, after the sequential testing and adjustment of the joint probability, we can directly transform this joint probability into an individual probability for a significant active constraint via

$$\epsilon_i = \frac{\epsilon_{ad}}{n_{sac}}, \quad i = 1, \dots, n_{sac}. \quad (12)$$

3) *Fine-tuning Procedure:* Now, let us further improve (12) by using the correlation information of all the significant active constraints,  $n_{sac}$ . It is proved by induction in [12] that, based on Fréchet's inequality, a less conservative way to distribute the joint probability to individual ones can be obtained via

### Algorithm 1 Sequential Testing Algorithm

---

```

1: All  $\{\epsilon_i^{ac}\}_{i=1}^{n_{ac}}$  are sorted in order of smallest to largest;
2: Define the number of significant active constraints as  $n_{sac}$ 
   and the number of insignificant active constraints as  $n_{isac}$ ;
3: Initiate  $n_{sac} = n_{ac}$ ,  $n_{isac} = 0$  and  $i = 1$ ;
4: while  $i \leq n_{ac}$  do
5:   if  $\epsilon_i^{ac} < \frac{\epsilon}{n_{sac}}$  then
6:     Mark as insignificant constraint;
7:      $\epsilon_{ad} = \epsilon - \epsilon_i^{ac}$ ;
8:      $n_{sac} = n_{sac} - 1$  and  $n_{isac} = n_{isac} + 1$ ;
9:   else
10:    Mark as significant constraint;
11:   end if
12: end while

```

---

considering the correlation for the intersection. Following this rule, (12) can be further improved via

$$\epsilon_i = \frac{\epsilon_{ad} + (n-1)\mathbb{P}(\cap_{i=1}^{n_{sac}} \{h_i \notin \mathcal{H}_i\})}{n_{sac}}, \quad i = 1, \dots, n_{sac}. \quad (13)$$

Here,  $(n-1)\mathbb{P}(\cap_{i=1}^{n_{sac}} \{h_i \notin \mathcal{H}_i\})$  can be easily approximated in the MC samples obtained via the surrogate method. Till now, the proposed method will yield a less conservative decision than the one obtained via the traditional Boole's inequality.

4) *Further Discussion on a Hybrid Procedure:* It can be seen from the above section that the proposed method can not only be applied to the ICC-OPF but also to the JCC-OPF. We also need to mention that if we set each individual probability,  $\epsilon_i$ , to the joint probability,  $\epsilon$ , instead of using the aforementioned strategy. The solution obtained by the ICC will be less conservative than that obtained by the JCC method, but more conservative than the traditional deterministic method [10]. This means that, using the control decision obtained in the ICC method, we can reduce the number of the detected active constraints compared with that obtained from the deterministic method. Therefore, we would like to suggest that, before we transform the JCC problem into an ICC problem, we solve an ICC problem with each  $\epsilon_i$  being equal to  $\epsilon$  first. This procedure brings a better initial decision for the JCC-AC-OPF while reducing the number of active constraints. As for the initial decision in the ICC-OPF, we can follow the tradition by solving a deterministic OPF that directly takes the means of the uncertainties,  $\bar{\xi}$ , in the solver [10].

### V. CASE STUDIES

Using the proposed method, various case studies are conducted on a modified 200-bus Illinois power system, geographically situated in the central part of the U.S. state of Illinois. Its detailed data is provided in MATPOWER 7.1. Here, it is assumed that the loads follow a Gaussian distribution with mean equal to the original bus loads and standard deviation equal to 5% of their mean. 4 wind farms, each with a rated power of 50 MW, are added at Buses 5, 15, 100, and 140, respectively, to introduce the randomness in the OPF model. They are modeled as Weibull distributions with their shape and scale parameters are set to  $\{7.41, 2.06\}$  [2]. The Gaussian copula is used to model the dependent samples. The capacity of transmission line between Buses 29 and 30 is increased

from 100 MVA to 102 MVA, and unit commitment is not considered. Besides, we further utilize parallel computing with the proposed method.

First, we test the performance of the proposed method under different acceptable violation probabilities,  $\epsilon$ , using 10,000 as the sample size. The simulation results of the proposed method are validated with the MC method with 10,000 samples of power-flow cases to measure its joint violation probability. The simulation results are provided in Table I. It can be seen that the proposed method can provide a quite accurate solution, which are also less conservative compared with that obtained from the traditional Boole's inequality.

Therefore, as shown in Table II, the proposed method can also obtain a less costly solution compared with that obtained through the Boole's method. By increasing  $\epsilon$ , the cost can be further reduced even though the proposed method is more costly than the traditional deterministic approach. Figure 1 demonstrates the boxplots for the voltage magnitude at Buses 11 through 20 with an upper limit of 1.1 pu as an example (note that all these buses are PQ buses). We can see that the JCC-AC-OPF can provide a more secure operation solution that leads to a few violations while the traditional deterministic method will place the system under an insecure operating condition.

Finally, we would like to mention that the surrogate technique brings significant improvement in the computing time since the proposed method can finish the simulation in approximately 12 s while the traditional MC method will spend around 1 h. The two-order-of-magnitude speedup factor shows its capability for online applications.

TABLE I  
VALIDATION ON THE MODIFIED 200-BUS SYSTEM

|                  | Boole's | Proposed method |
|------------------|---------|-----------------|
| $\epsilon = 2\%$ | 1.44%   | 1.80%           |
| $\epsilon = 3\%$ | 1.64%   | 2.57%           |

TABLE II  
COST COMPARISON FOR THE PROPOSED METHOD

|                     | Deterministic       | Boole's              | Proposed method      |
|---------------------|---------------------|----------------------|----------------------|
| $f(\epsilon = 2\%)$ | $3.574 \times 10^4$ | $3.864 \times 10^4$  | $3.8335 \times 10^4$ |
| $f(\epsilon = 3\%)$ | $3.574 \times 10^4$ | $3.8535 \times 10^4$ | $3.8259 \times 10^4$ |

## VI. CONCLUSIONS

In this paper, we propose a surrogate-enhanced approach in solving the JCC-AC-OPF problem. This approach can efficiently achieve the operation decision that balances the security and economy in the grid operations while being computational cheap to evaluate. The simulation results reveal the excellent performance of the proposed method.

## REFERENCES

- [1] X. Geng and L. Xie, "Data-driven decision making in power systems with probabilistic guarantees: Theory and applications of chance-constrained optimization," *Annu. Rev. Control*, vol. 47, pp. 341–363, 2019.
- [2] Z. Ren, W. Li, R. Billinton, and W. Yan, "Probabilistic power flow analysis based on the stochastic response surface method," *IEEE Trans. Power Syst.*, vol. 31, no. 3, pp. 2307–2315, May 2016.

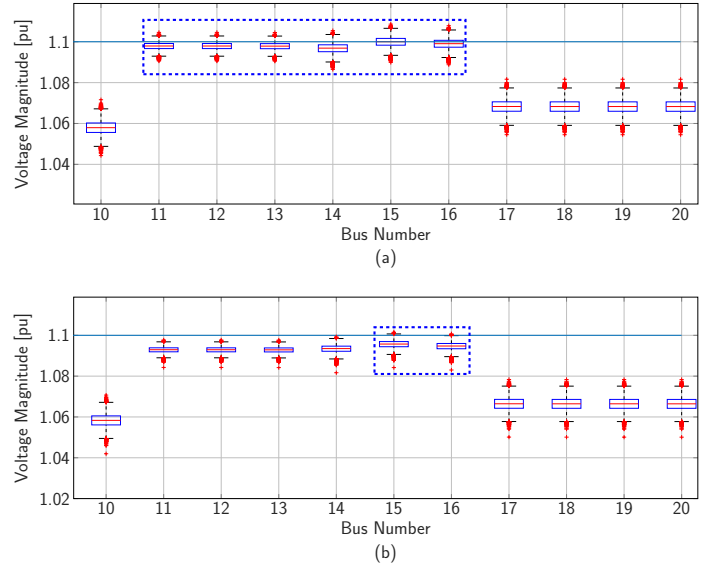


Fig. 1. Boxplots of bus-voltage magnitude at Buses 11 through 20 with (a) the deterministic method and (b) the proposed method.

- [3] Z. Hu *et al.*, "Uncertainty quantification in stochastic economic dispatch using Gaussian process emulation," in *2020 IEEE Power & Energy Soc. Innovative Smart Grid Tech. Conf. (ISGT)*, 2020.
- [4] F. Ni, P. H. Nguyen, and J. F. G. Cobben, "Basis-adaptive sparse polynomial chaos expansion for probabilistic power flow," *IEEE Trans. Power Syst.*, vol. 32, no. 1, pp. 694–704, Jan. 2017.
- [5] Y. Xu, Z. Hu, L. Mili, M. Korkali, and X. Chen, "Probabilistic power flow based on a Gaussian process emulator," *IEEE Trans. Power Syst.*, vol. 31, no. 4, pp. 3278–3281, Jul. 2020.
- [6] Y. Zhang, S. Shen, and J. L. Mathieu, "Distributionally robust chance-constrained optimal power flow with uncertain renewables and uncertain reserves provided by loads," *IEEE Trans. Power Syst.*, vol. 32, no. 2, pp. 1378–1388, Mar. 2017.
- [7] B. Li, M. Vrakopoulou, and J. L. Mathieu, "Chance constrained reserve scheduling using uncertain controllable loads part II: Analytical reformulation," *IEEE Trans. Smart Grid*, vol. 10, no. 2, Mar. 2019.
- [8] A. Peña-Ordieres, D. K. Molzahn, L. Roald, and A. Wächter, "DC optimal power flow with joint chance constraints," *IEEE Trans. Power Syst.*, 2020.
- [9] K. Baker and A. Bernstein, "Joint chance constraints in AC optimal power flow: Improving bounds through learning," *IEEE Trans. Smart Grid*, vol. 10, no. 6, pp. 6376–6385, Nov. 2019.
- [10] L. Roald and G. Andersson, "Chance-constrained AC optimal power flow: Reformulations and efficient algorithms," *IEEE Trans. Power Syst.*, vol. 33, no. 3, pp. 2906–2918, May 2018.
- [11] G. C. Calafiore and M. C. Campi, "The scenario approach to robust control design," *IEEE Trans. Autom. Control*, vol. 51, no. 5, pp. 742–753, Aug. 2006.
- [12] K. Baker and B. Toomey, "Efficient relaxations for joint chance constrained AC optimal power flow," *Electr. Power Syst. Res.*, vol. 148, pp. 230–236, 2017.
- [13] D. Xiu, *Numerical Methods for Stochastic Computations: A Spectral Method Approach*. Princeton University Press, 2010.
- [14] M. Jan-Frederik and M. Scherer, *Simulating Copulas: Stochastic Models, Sampling Algorithms, and Applications*, 2nd ed. Singapore: World Scientific, 2017.
- [15] T. Mühlpfordt, L. Roald, V. Hagenmeyer, T. Faulwasser, and S. Misra, "Chance-constrained AC optimal power flow: A polynomial chaos approach," *IEEE Trans. Power Syst.*, vol. 34, no. 6, pp. 4806–4816, Nov. 2019.
- [16] C. Cui, K. Liu, and Z. Zhang, "Chance-constrained and yield-aware optimization of photonic ICs with non-Gaussian correlated process variations," *IEEE Trans. Comput. Aided Des. Integr. Circuits Syst.*, 2020.
- [17] D. Xiu, *Numerical Methods for Stochastic Computations: A Spectral Method Approach*. Princeton, NJ: Princeton University Press, 2010.
- [18] S. Holm, "A simple sequentially rejective multiple test procedure," *Scand. J. Stat.*, pp. 65–70, 1979.