

1 Influence of Attractive Functional Groups on the Segmental 2 Dynamics and Glass Transition in Associating Polymers

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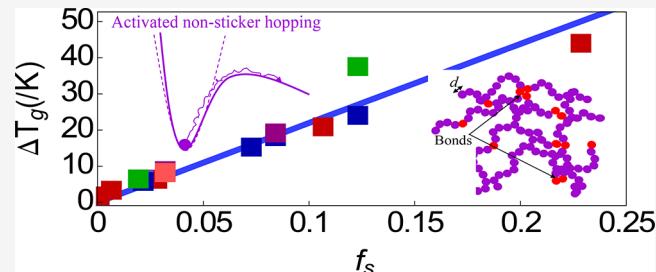
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4 **ABSTRACT:** A recently developed microscopic statistical me-
5 chanical theory for how strong short-range attractions between
6 sticky groups copolymerized in the backbone of associating
7 polymer liquids perturb the segmental dynamics and glass
8 transition temperature (T_g) is quantitatively compared with a
9 large body of experimental data on end-functionalized telechelic
10 melts of variable polymer backbone chemistry, association
11 strengths, and chain lengths. For strong associations and PDMS-
12 based telechelic melts, a universal growth in T_g is predicted and
13 experimentally observed as a function of sticker fraction or
14 telechelic chain length which does not depend on sticker chemistry. The predictions of a stronger than exponential growth of the
15 alpha relaxation time, but perturbative or negligible changes in dynamic fragility, with increasing sticker fraction are also in good
16 accord with the experimental observations. The theory is extended to the case of weaker sticker associations where the “transient
17 cross-linked” network picture based on a large time scale separation between the nonsticker alpha relaxation time and the sticker
18 bonding lifetime does not hold. The new physics is theoretically accounted for by introducing larger length scale dynamic sticker
19 fluctuations consistent with weaker physical bonding. A smaller growth in T_g with sticker fraction is predicted which depends in a
20 continuous manner on the degree of dynamical loosening of sticker bonding. The theoretical results are consistent with our
21 experimental data for PPG-based telechelic melts. The theory provides guidelines for the design of polymers with dynamic bonds
22 with controlled enhancements of T_g and segmental alpha time.



I. INTRODUCTION

23 Associating polymers with a relatively small fraction of “sticky”
24 functionalities can exhibit strongly modified dynamical and
25 rheological properties in semidilute solutions and melts. These
26 systems are drawing increasing attention for applications as
27 self-healing materials, biomaterials, rheological modifiers,
28 sensors, and actuators.^{1–12} Moreover, the polymers with
29 “sticky” (dynamic) bonds provide easier recyclability.^{13–15}
30 The “sticky” groups (stickers) attract via variable strength
31 chemistry specific interactions such as H-bonding, π – π
32 stacking, metal–ligand (M–L) interaction, ionic bond
33 formation, and even dynamic covalent bonds (e.g., vitri-
34 mers).^{16–20} Stickers can be arranged along the chain backbone
35 at regular or random intervals, or at chain ends for telechelic
36 systems, offering experimentally controllable sequence specific
37 materials. The influence of sticky groups on polymer chain
38 dynamics has been described theoretically at the coarse-
39 grained level by using the so-called sticky Rouse and sticky
40 reptation models,^{21–27} and corrections were added in the
41 refined bond lifetime renormalization model.²⁸ However, the
42 presence of stickers can strongly alter the magnitude and
43 temperature dependence of the primary alpha relaxation
44 process in polymers,^{17,28–35} which is not addressed in
45 coarse-grained theories for chain dynamics and viscoelasticity.
46 Sticker-induced modification of the alpha relaxation not only

can result in a major shift of the glass transition temperature,⁴⁷ T_g but also affects the temperature and chemistry dependences⁴⁸ of chain dynamics and rheological properties. Stickers affect⁴⁹ the local dynamics of nonsticky segments in a manner that⁵⁰ potentially depends on the melt density, sticker fraction, sticker⁵¹ monomer chemistry, and the specific backbone structure of the⁵² associating copolymer. Hence, a detailed microscopic under-⁵³ standing of how the stickers affect polymer segmental⁵⁴ dynamics is a problem of high fundamental science and⁵⁵ materials applications interest.⁵⁶

Two of us have recently formulated a microscopic, force-⁵⁷ level statistical mechanical theory of the effect of stickers on⁵⁸ the alpha relaxation time and glass transition temperature of⁵⁹ nonstickers that goes beyond all existing phenomenological⁶⁰ models.³⁶ However, to date, the new theoretical predictions⁶¹ have not been extensively compared with experimental⁶² observations. On the basis of a two-step dynamic framework⁶³

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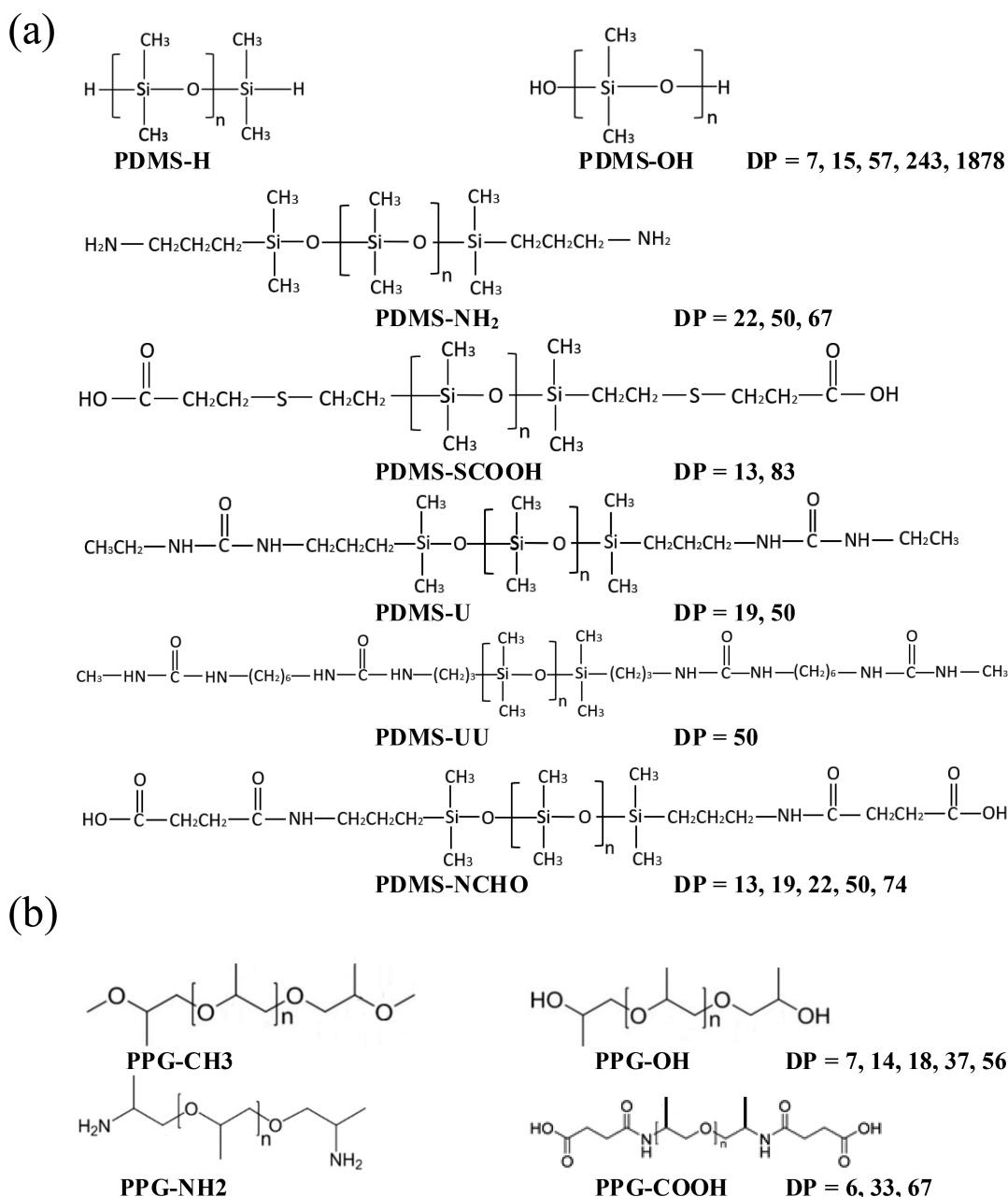


Figure 1. Telechelic (a) PDMS and (b) PPG with different end groups.

64 that involves the faster nonsticker alpha relaxation followed by
 65 a slower bond-breaking process,³⁷ the theory predicts the
 66 variation of the nonsticker alpha time, high-frequency elastic
 67 shear modulus, and change in T_g with sticker fraction.³⁶ In the
 68 strong attraction limit where the sticker lifetime significantly
 69 exceeds the nonsticker alpha time, our core hypothesis is that
 70 chemistry specific details, such as the precise nature of
 71 attraction, attraction strength (beyond a threshold), or even
 72 formation of clusters or microdomains and precise location of
 73 stickers along a polymer chain, are of secondary importance.
 74 The reason is we believe that the zeroth-order physics involves
 75 a transient cross-linked network-like picture where stickers are
 76 effectively immobilized or localized to small vibration-like
 77 motion on the time scale of the activated segmental alpha
 78 relaxation process. In essence, stickers serve as sites of
 79 immobility which enhance the kinetic constraints on the
 80 mobile nonstickers. If this simple physical picture is valid, the

precise topology of sticker clustering is expected to be of
 81 secondary importance for the nonsticker alpha relaxation
 82 process. Rather, we propose that it is the concentration of
 83 stickers that is the leading order key variable that controls
 84 nonsticker alpha relaxation dynamics and changes of T_g at
 85 least for the copolymer architectures studied here.
 86

To test the detailed predictions of the theory and the above
 87 qualitative picture, this article presents *quantitative* compar-
 88 isons with a large body of recent alpha relaxation time
 89 dielectric data for telechelic melts as a function of chain degree
 90 of polymerization (maps to the inverse degree of sticker
 91 functionalization) and sticker end group chemistry.^{28–33}
 92 Beyond testing the theory for specific systems, we aim to
 93 explore whether a general behavior exists in the data as
 94 anticipated by our theoretical work. Our study is of
 95 fundamental polymer physics interest and also provides
 96 insights relevant to the design or selection of polymer
 97

98 chemistry variables to control the local dynamical properties,
 99 which in turn set the time scale for the longer length and time
 100 scale chain dynamics relevant to the viscoelasticity of
 101 associating copolymer materials. To address the special case
 102 of weak association relevant to some of the experimental
 103 systems considered, the theory is extended in the current
 104 article to go beyond the assumption in ref 36 that stickers act
 105 as literally fixed cross-links on the alpha relaxation time scale of
 106 the nonstickers. The extended theory retains the idea that the
 107 precise network topology is of secondary importance.
 108 However, it considers the possibility of weaker dynamic
 109 constraints on nonsticker-activated relaxation due to small
 110 scale sticker motion on times and length scales *well below* that
 111 characteristic of the sticker alpha relaxation time (α^* process).
 112 The sticker bond breaking time scale, and the question of
 113 larger time and length scale chain dynamics, are not addressed
 114 in our present work. However, we note that a theoretical
 115 framework for the relationship between the α and α^* processes
 116 has been recently proposed and quantitatively confronted with
 117 experiments on multiblock and telechelic associating polymer
 118 melts.³⁷

119 **Section II** briefly describes the methods, synthesis, and
 120 measurement details of our experiments on the telechelic
 121 associating polymer melts, most of which has been previously
 122 discussed. Only the most important aspects of the microscopic
 123 theory developed in ref 36 are briefly summarized in **section**
 124 **III**, followed by its new extension to treat weakly associating
 125 copolymer liquids. In **section IV** we present the mapping
 126 procedure from an intermediate coarse-grained freely jointed
 127 chain (FJC) model to the polymer backbone chemistry of
 128 prime interest, poly(dimethylsiloxane) (PDMS). This calibra-
 129 tes the microscopic parameters from knowledge solely of the
 130 homopolymer reference melt behavior, thereby allowing us to
 131 make predictions for telechelics with *no* adjustable fit
 132 parameters. **Sections V** and **VI** present our predictions for
 133 the alpha relaxation time and T_g for the PDMS- and
 134 poly(propylene glycol) (PPG)-based telechelic melts, respec-
 135 tively, including quantitative comparisons with our exper-
 136 imental data.^{28–33} The article concludes with a brief discussion
 137 and summary in **section VII**.

II. EXPERIMENTAL BACKGROUND

138 **Methods and Synthesis.** Chemical structures of the investigated
 139 PDMS and PPG samples are shown in Figures 1a and 1b, respectively.
 140 DP indicates the chain degree of polymerization, n . Of all the samples
 141 investigated in this study, PDMS-UU and PDMS-NCHO show a
 142 microphase-separated structure (cf. schematic in Figure 2) as evident
 143 in the SAXS data and two glass transitions.³⁸ The synthesis protocols
 144 of all the samples are given in the Supporting Information (section I).
 145 **Broadband Dielectric Spectroscopy.** Broadband dielectric
 146 spectroscopy (BDS) in the frequency range 10^{-2} – 10^6 Hz was
 147 employed by using a Novocontrol system that includes an Alpha-A
 148 impedance analyzer and a Quatro Cryosystem temperature control
 149 unit. Most of the samples are liquid at room temperature (RT) and
 150 were placed in a parallel-plate dielectric cell made of sapphire and
 151 Invar steel with an electrode diameter of either 12 or 10 mm. For
 152 the 12 mm electrode diameter system, a capacitance of 20 pF was
 153 obtained for the empty cell with an electrode separation of 49 μm .
 154 Empty cells with 10 mm electrodes and an electrode separation of 210
 155 μm have a capacitance of 3.3 pF. PDMS-UU is a rubbery sample at
 156 room temperature (RT). Two gold-plated electrodes with a diameter
 157 of 20 mm were used to measure this sample; the separation between
 158 the electrodes was 174 μm . To avoid crystallization, most of the
 159 samples were initially quenched from RT to \sim 113 K and reheated to
 160 10 K below their respective glass transition temperature, T_g prior to

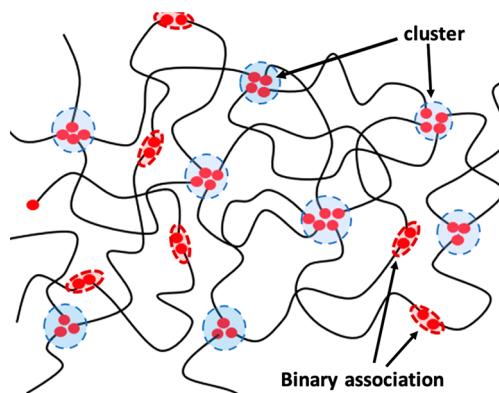


Figure 2. Cartoon of the phase-separated structures in the melt state of the associating telechelic polymers. The stickers are represented as red spheres, and polymer chains are shown as black curves. Stickers can associate with each other via binary interactions (maroon ellipses) or form phase-separated clusters (blue circles) due to their immiscibility with the polymer matrix. Out of all the systems studied here, only PDMS-UU and PDMS-NCHO showed microphase-separated domains.

161 the measurements. After each temperature increase, the samples were
 162 equilibrated for at least 10–15 min to achieve thermal stabilization
 163 within 0.2 K. More details of the experimental procedures for each of
 164 the investigated samples can be found elsewhere.^{28–33,38}

T_g and Association Activation Energy Estimation. BDS
 165 measures the segmental relaxation (alpha process) of the polymer
 166 and additionally chain relaxation in the case of PPG.²⁹ The
 167 association–dissociation process of stickers changes the overall dipole
 168 moment of the system and appears as an additional dielectric signal
 169 with a characteristic time scale given by the association/dissociation
 170 lifetime of the end groups.²⁹ Here we refer to this signal as the α^*
 171 process. Dielectric spectroscopy provides direct measurements of the
 172 time scales of these two processes and can provide rough estimates of
 173 the apparent activation energy (E_a) of the sticker dissociation defined
 174 via the Arrhenius equation:^{31,37,39–42}

$$\tau_{\alpha^*}(T) = \tau_{\alpha}(T) \exp\left(\frac{E_a}{RT}\right) \quad (1)$$

The activation energy varies strongly with molecular structure of the
 177 end groups (Figure 3). While the Arrhenius equation (eq 1) does not
 178 address the two-step nature of the α^* process involving α -relaxation
 179 followed by bond breaking,³⁷ and hence fails in some cases,³¹ it 180

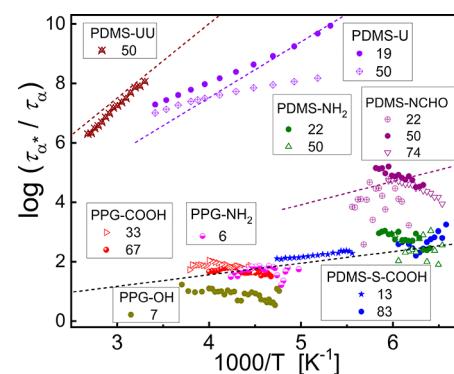


Figure 3. Logarithm of the ratio of the sticker dissociation and segmental relaxation time scales vs inverse temperature for the investigated telechelic polymers. The straight lines correspond to activation energies of 48, 36, 15, and 7.5 kJ/mol (top to bottom) per eq 1. A higher slope indicates a higher activation energy of the dissociation process.

181 provides a good zeroth-order estimate of the chemistry-specific bond-
 182 breaking energy. Extrapolation of the segmental relaxation time to τ_α
 183 ~ 100 s using the Vogel–Fulcher–Tamman (VFT) equation,
 184 $\tau_\alpha(T) = \tau_0 \exp\left(\frac{B}{T - T_0}\right)$,^{29–31} provides an estimate of T_g where the
 185 parameters of limiting relaxation time (τ_0), B , and VFT temperature
 186 T_0 for all the systems are given in refs 28 and 29. The estimated T_g
 187 and E_a for the telechelic polymers studied here are presented in
 188 Tables 1 and 2 for the PDMS and PPG systems, respectively. While

189 scale separation) and $\tau_{\alpha^*} \gtrsim \tau_\alpha$ (PPG, relatively weaker bonds \equiv 190
 191 small/modest time scale separation). Additionally, we show the 192
 193 difference in T_g defined as ΔT_g estimated as the difference between 194
 195 T_g of the polymer with associating functional end groups and T_g of the 196
 197 polymer with the same DP but with nonassociating chain ends ($-H$ or 198
 $-CH_3$). 199

III. THEORETICAL BACKGROUND

200 Our recently developed statistical mechanical theory to predict 199
 201 activated segmental relaxation and vitrification of associating 200
 202 regular AB multiblock copolymer melts (single sticker sites 201
 203 separated by a fixed number of nonstickers) has been discussed 202
 204 in detail in ref 36. Successful confrontation with alpha time 203
 205 data for randomly functionalized multiblock ionomer materials 204
 206 was also presented. Here we briefly sketch the crucial aspects 205
 207 of the prior theory and model employed. 206

Model and Large Time Scale Separation Limit.

208 Previous work considered a liquid of regular copolymers 207
 209 composed of chains of N connected sites all of the same 208
 210 diameter d with single associating sites or “stickers” placed 209
 211 along the backbone at regular intervals with a fraction denoted 210
 212 by f_s (Figure 4A). A freely jointed-chain (FJC) model with 211 f4
 213 persistence length $l_p/d = 4/3$ was utilized, and the modest self- 212
 214 overlap of interaction sites of such an ideal chain model is 213
 215 considered⁴³ to define an effective space-filling melt packing 214
 216 fraction, ϕ_{eff} . All sites on different chains interact via the same 215
 217 hard-core repulsion. Stickers attract via an exponential pair 216
 218 potential beyond hard-core contact with a variable contact 217
 219 strength (βe in thermal energy units) of a short spatial range of 218
 $a/d = 0.10$ characteristic of a specific attraction. 219

220 Per Figure 4B, the three interchain site–site pair correlation 220
 221 functions, $g_{\alpha\beta}(r)$, their analogous partial collective static 221
 222 structure factors in Fourier space, $S_{\alpha\beta}(q)$, and the correspond- 222
 223 ing site–site direct correlation functions, $C_{\alpha\beta}(q)$, are computed 223
 224 by using the Polymer Reference Interaction Site Model 224
 225 (PRISM) integral equation theory^{44–46} with the Percus– 225
 226 Yevick (PY) closure.^{44–47} See ref 36 and references therein for 226
 227 all technical details. Our prime focus previously was the high- 227
 228 attraction regime where effectively all stickers are bonded to 228
 229 other stickers. 229

230 The formal starting point to treat activated dynamics based 230
 231 on the Elastically Collective Nonlinear Langevin Equation 231
 232 (ECNLE) theory^{48,49} is to relate the strength of kinetic 232
 233 constraints that enter calculation of the force–force time 233
 234 correlation function associated with any tagged segment to the 234
 235 equilibrium interchain site–site pair correlation functions. The 235
 236 force correlations on a nonsticker segment have multiple 236
 237 origins or dynamic pathways,^{36,50} as schematically shown in 237
 238 Figure 5. The effective force on a segment consists of an 238 f5
 239 effective intermolecular pair repulsion between all sites and an 239
 240 attractive bonding force between stickers that is explicitly 240
 241 treated based on the recently developed (and extensively 241
 242 applied to attractive colloid suspensions) Projectionless 242
 243 Dynamic Theory (PDT).^{51,52} Our focus is on the activated 243
 244 hopping of nonstickers when there is sufficient time scale 244
 245 separation that the slower stickers form physical bonds 245
 246 modeled in a localized harmonic or Gaussian manner (not 246
 247 literal pinning), the vibrational amplitude of which is a priori 247
 248 predicted from the naïve mode coupling theory (NMCT) self- 248
 249 consistent localization equations.³⁶ The ideas are schematically 249
 250 shown in Figure 4C. 250

Table 1. Total Molecular Weight and Degree of Polymerization-Dependent Dielectric T_g (BDS), Change in Glass Transition Temperature Compared to the Reference Nonassociating Polymer, and Activation Energy Determined from Eq 1 for PDMS Telechelic Associating Polymers

polymer	M_n (g/mol)	DP	T_g (K)	ΔT_g (K) ^g	E_a (kJ/mol)
PDMS-OH	400–700	7	170	38	
	700–1500	15	154	15.5	
	4200	57	146	3.5	
	18000	243	145	1.5	
	139000	1878	144	0	
PDMS-NH ₂	1738	22	151	11	9.1 ± 0.2
	3816	50	147.5	5	7.6 ± 0.3
	5592	67	146.5	3.5	8.3 ± 0.2
PDMS-SCOOH	1228	13	169.5	32	8.2 ± 0.4
	6408	83	147	4	8.1 ± 0.2
PDMS-U		19	154	14	37.3 ± 0.5
		50	148	5.5	33 ± 0.8
PDMS-NCHO	1280	13	156	18.5	19.9 ± 0.2
	1720	19	153	13	16.7 ± 0.4
	1940	22	151	11	13.5 ± 0.4
	4020	50	148	5.5	15.4 ± 0.3
	5570	74	146	3	13.7 ± 0.4
PDMS-UU		50	147.5	5	46 ± 0.2

Table 2. Molecular Weight and Degree of Polymerization-Dependent Dielectric T_g (BDS), Change in Glass Transition Temperature from Reference Nonassociating Polymer, and Activation Energy Determined from Eq 1 for PPG Telechelic Associating Polymer

polymer	M_n (g/mol)	DP	T_g (K)	ΔT_g (K)	E_a (kJ/mol)
PPG-CH ₃	468	7	172		
	874	14	183		
	1106	18	185.5		
	2208	37	194.5		
	3310	56	194.5		
PPG-OH	440	7	196	24	3.9 ± 0.2
	846	14	198	15	
	1078	18	200	14.5	
	2180	37	201	6.5	
	3282	56	201	6.5	
PPG-NH ₂	480	6	192.5	24.5	5.8 ± 0.1
	2046	33	199	6.5	
	4018	67	200	5.5	
PPG-COOH	680	6	248	80	8.4 ± 0.2
	2246	33	213.5	21	8.5 ± 0.1
	4218	67	205.5	11	7.3 ± 0.1

189 the activation energies of all PDMS telechelic systems and PPG-
 190 COOH are relatively high, that of PPG-OH is low. This indicates two
 191 distinct cases where $\tau_{\alpha^*} \gg \tau_\alpha$ (PDMS, strong bonds \equiv large time

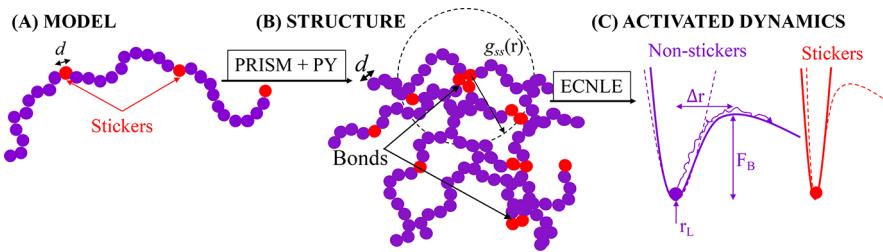


Figure 4. Schematic of the key elements of the associating copolymer theory.³⁶ (A) Regularly spaced sticker copolymer chain model. Red and purple beads represent stickers and nonstickers, respectively. (B) PRISM integral equation theory is used to obtain intermolecular site–site pair correlations functions. (C) Knowledge of (B) is input to quantify dynamic constraints and construct the dynamic free energies in the ECNLE theory of activated relaxation. Crucial local length and energy scales are the transient localization length (r_L), cage barrier height (F_B), and jump distance (Δr).

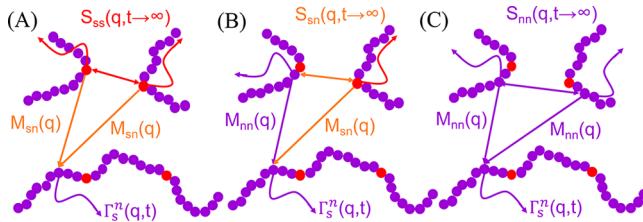


Figure 5. Schematic of interchain force–force time correlation pathways for nonsticky segment self-motion for a copolymer chain having sticky (red) and nonsticky (purple) groups. Force–time correlations decay in time in a parallel manner due to tagged nonsticker segment motion encoded in its dynamic self-correlation function, $\Gamma_s^n(q, t)$, and the collective motion of the copolymer liquid. Beyond the NMCT onset where segments first become transiently localized, the latter enters the NLE theory construction of the dynamic free energy via site-specific collective Debye–Waller factors, $S_{ij}(q, t \rightarrow \infty)$, where $(i, j) \in (s, n)$. The corresponding single particle (incoherent) dynamic structure factor becomes a Gaussian Debye–Waller (DW) factor as $\Gamma_s^n(q, t \rightarrow \infty) = \exp(-q^2 r_L^{n^2}/6)$, corresponding to the localized form of $\Gamma_s^n(q, t) = \exp(-q^2 r_n(t)^2/6)$.

251 The scalar displacement of a tagged nonsticker segment,
252 $r_n(t)$, obeys a stochastic nonlinear Langevin equation
253 (NLE),^{36,50}

$$\zeta_{s,n} \frac{d}{dt} r_n(t) - \frac{\partial F_{\text{dyn}}(r_L^s, r_n(t))}{\partial r_n(t)} + \delta f_n(t) = 0 \quad (2)$$

255 where $\zeta_{s,n}$ is the nonsticker short time friction constant, $\delta f_n(t)$ is the corresponding random fluctuating force that obeys
256 $\langle \delta f_n(0) \delta f_n(t) \rangle = 2k_B T \zeta_{s,n} \delta(t)$, and $F_{\text{dyn}}(r_L^s, r_n(t))$ is the
257 nonsticker dynamic free energy that is a function of a
258 tagged-segment instantaneous displacement and parametrically
259 depends on the predicted ensemble-averaged sticker local-
260 ization length, r_L^s . In the overdamped dynamical regime of
261 interest, a short time nonactivated process enters with a
262 characteristic time scale $\tau_s = \beta \zeta_{s,n} \sigma^2$ that is explicitly given by⁴⁸
263

$$\frac{\tau_{s,n}}{\tau_0} = 1 + \frac{1}{36\pi\eta_{\text{eff}}} \int_0^\infty dq \frac{q^2 (S_{nn}(q) - 1)^2}{S_{nn}(q) + (1 - j_0(q) + 2j_2(q))^{-1}} \quad (3)$$

264 Here, $j_n(x)$ is the spherical Bessel function of order n , and τ_0 is
265 a “bare” time scale written in a manner akin to a hard-sphere
266 fluid as $\tau_0 = \frac{g_{nn}(d)}{24\rho d^2} \sqrt{\frac{M}{\pi k_B T}}$, where $g_{nn}(d)$ is the contact value of

267 the interchain site–site pair correlation function, $g_{nn}(r)$. This
268 short time scale only enters as a prefactor in the alpha
269 relaxation hopping time and does not change with sticker
270 fraction. Although chain connectivity is considered when
271 computing structural correlations with PRISM theory which
272 enter the so-called dynamic force vertex (see below), it is
273 ignored for the dynamic hopping event.³⁶ This simplification
274 has been discussed in detail in ref 36. It is physically reasonable
275 since the jump distance is predicted to be small compared to
276 the polymer persistence length and bead diameter.
277

The nonsticker dynamic free energy is explicitly given by
278 (see ref 36 for all details)

$$\begin{aligned} F_{\text{dyn}}(r_L^s, r_n) - F_{\text{dyn}}(r_L^s, r_L^n) &= \int_{r_{L,n}}^{r_n} \frac{\partial F_{\text{dyn}}(r_L^s, r_n)}{\partial r_n} dr_n \\ &= -3 \ln\left(\frac{r_n}{r_L^n}\right) + \frac{1}{18\pi^2} \int_{r_{L,n}}^{r_n} dr_n' r_n' \\ &\quad \int_0^\infty \frac{dq}{(2\pi)^3} q^2 e^{-q^2 r_n'^2/6} [M_{nn}(q)^2 S_{nn}(q, t \rightarrow \infty) \\ &\quad + M_{sn}(q)^2 S_{ss}(q, t \rightarrow \infty) + M_{nn}(q) \\ &\quad M_{sn}(q) \{S_{sn}(q, t \rightarrow \infty) + S_{ns}(q, t \rightarrow \infty)\}] \end{aligned} \quad (4)$$

281 It considers dynamic constraints due to several types of
282 interchain forces as schematically shown in Figure 5. The
283 effective force vertices $M_{\alpha\beta}(q)$ are written in terms of the site–
284 site direct correlation function in Fourier space for hard-core
285 repulsive interactions, while attractive interactions are treated
286 explicitly based on the hybrid projectionless dynamic theory
287 (hybrid-PDT); see refs 36 and 51 for all details. The quantities
288 $S_{\alpha\beta}(q, t \rightarrow \infty)$ are the long time arrested (at the naive mode
289 coupling theory (NMCT) level) quantities which enter NLE
290 theory for the dynamic free energy as collective dynamic
291 structure factors. The latter are essentially Debye–Waller
292 factors and depend on the sticker localization length r_L^s ; e.g.,
293 for a one-component liquid $S(q, t \rightarrow \infty) = \exp\left[-\frac{q^2 r_L^s}{6S(q)}\right]$. As

294 before, we assume sticker relaxation and bond breaking is slow
295 (the α^* -process) compared to nonsticker relaxation, corre-
296 sponding to a two-step dynamical relaxation scenario.³⁷ On the
297 nonsticker relaxation time scale of interest, the stickers
298 effectively only vibrate on very small length scales around
299 their predicted localization length in the spirit of a “transient
300 network” (solid red curve in Figure 4C). Such highly restricted
301 local sticker motion is physically expected, and assumed in our
302 analysis, to be independent (to leading order) of the precise
303

303 copolymer architecture, e.g., multiblock vs telechelic. An
 304 example dynamic free energy is schematically shown in Figure
 305 4C (solid blue curve).

306 The defining feature of ECNLE theory is the idea that the
 307 cage scale barrier hopping event controlled by the dynamic free
 308 energy generically requires the polymer segments beyond the
 309 cage scale to dynamically displace by a small amount in a long-
 310 range collective elastic manner. This contributes an additional
 311 elastic barrier which has been derived to be³⁶

$$312 \quad \beta F_e^n = 12\varphi_{\text{eff}}^n (\Delta r_{\text{eff}}^n)^2 \left(\frac{r_{\text{cage}}}{\sigma} \right)^3 K_0^n \quad (5)$$

313 where $\varphi_{\text{eff}}^n = (1 - f_s)\varphi_{\text{eff}}$. The effective jump distance associated
 314 with a segment leaving its cage sets the amplitude of the collective elastic displacement field and enters the elastic barrier quadratically. It is explicitly given by $\Delta r_{\text{eff}}^n = \frac{3}{32} \frac{(r_B^n - r_L^n)^2}{r_{\text{cage}}}$,
 316 where r_B^n is the location of nonsticker dynamic free energy barrier and K_0^n is the harmonic spring constant of the
 317 minimum (localized state) of the dynamic free energy given
 318 as $K_0^n = \frac{3k_B T}{(r_L^n)^2}$, with the cage radius $r_{\text{cage}} = 1.5d$.
 320

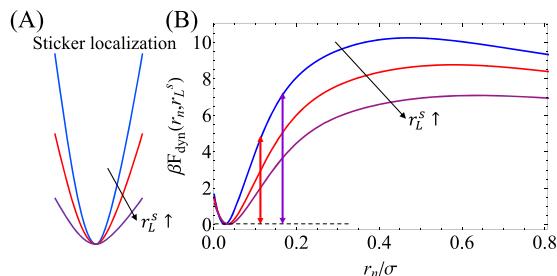
321 When the local cage barrier is larger than $\sim 1-2$ thermal energy units, the mean nonsticker alpha time is given by the
 322 simplified form of Kramers theory as

$$324 \quad \frac{\tau_\alpha}{\tau_s} = 1 + \frac{2\pi}{\sqrt{K_0^n \times K_B^n}} \exp(\beta(F_B^n + F_e^n)) \quad (6)$$

325 where K_B^n is the absolute value of the harmonic curvature of the dynamic free energy maximum at the barrier location.

326 **Beyond the Large Time Scale Separation Limit: Larger Sticker Fluctuation.** In the theory discussed so far, when constructing the nonsticker dynamic free energy the stickers are modeled as fluctuating (or vibrating) in a Gaussian manner around their predicted dynamic localization lengths (see Figure 4C). This “vibrational pinning” representation of a physical bond implies the stickers remain fully bonded on the time scale of the nonsticker alpha relaxation and mimics a transient rubber network like state. For weak enough associations this approximation must eventually fail. One can imagine stickers “dynamically moving” along their own free energy curve in the direction of larger displacements (ala a fast beta process or early stage alpha process) due to thermal fluctuations on the time scale of the nonsticker alpha relaxation.³⁶ Whether this is important or not for nonsticker dynamics is expected to be nonuniversal and polymer chemistry (e.g., association bond strength) dependent. Within our simplified framework this physics is crudely mimicked by allowing the sticker dynamic fluctuations to be larger than their strictly predicted localization length (at the minimum of the corresponding dynamic free energy), thereby weakening their dynamical constraints on nonsticker activated motion. Our basic physical picture is retained, and the larger sticker displacements (but still small compared to the nonsticker displacements characteristic of the alpha process) result in weaker dynamic constraints on nonsticker activated relaxation.

327 **Figure 6A** shows a schematic of this idea and how we implement it. Representative consequences for the nonsticker dynamic free energy at a packing fraction $\phi_{\text{eff}} = 0.58$ (where the homopolymer relaxation time is ~ 1 ns using $\tau_0 = 1$ ps) and fixed sticker fraction $f_s = 0.1$ are shown in **Figure 6B**. As the

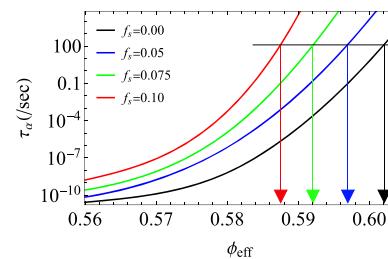


328 **Figure 6.** (A) Schematic of the weakened physical bonding constraint model where stickers are allowed to fluctuate in a Gaussian manner with a larger localization length than a priori predicted by using NMCT. (B) Representative example of the consequences of (A) for the nonsticker dynamic free energy at a fixed copolymer melt packing fraction of $\phi = 0.58$ and $f_s = 0.10$. Blue, red, and purple colors indicate sticker localization lengths of $r_L^s = r_L^s$, $r_L^s + 0.25\Delta r_n$, and $r_L^s + 0.33\Delta r_n$, respectively, all of which are well below the barrier and hence correspond to late beta or early alpha time dynamic fluctuations. The locations of $r_L^s + 0.25\Delta r_n$ and $r_L^s + 0.33\Delta r_n$ on the homopolymer dynamic free energy are shown by vertical colored arrows. At $r_L^{s'} = r_L^s + 0.33\Delta r_n$, the local cage barrier is nearly the same as if no stickers were present, per a nonassociated homopolymer melt.

329 sticker effective localization length increases ($r_L^{s'} > r_L^s$), dynamic constraints on the nonstickers decrease, resulting in a large nonsticker localization length (minimum of its dynamic free energy, $r_L^{s'}$), a lower cage barrier, F_B^n , and a smaller elastic barrier. For $r_L^{s'} = r_L^s + 0.33\Delta r_n$, where $\Delta r_n = r_B^n - r_L^n$ is the nonsticker jump distance which is a function of f_s , the cage barrier is similar to that of pure homopolymer melt for $f_s \leq 0.10$.⁵³ The collective elastic barrier and mean hopping time from Kramers theory can be calculated as before,³⁶ which then yields the nonsticker alpha relaxation time.

IV. NONSTICKER ALPHA TIME: MODEL CALCULATIONS AND PDMS HOMOPOLYMER MELTS

330 **Basic Results for Associating Copolymer Melts.** As relevant background, theoretical results for the nonsticker mean alpha time of the AB associating copolymer melt model are shown in **Figure 7** as a function of effective liquid packing fraction for various sticker fractions and $\tau_0 = 10^{-12}$ s. The alpha time grows suprarexponentially with increasing melt packing fraction and sticker concentration. For example, for $\phi_{\text{eff}} = 0.56$ a sticker fraction of $f_s = 10\%$ increases the alpha time by ~ 2 .



331 **Figure 7.** Alpha relaxation time (in seconds) as a function of effective packing fraction for various sticker fractions. The elementary time scale is taken as $\tau_0 = 1$ ps. The horizontal black line defines an experimentally relevant kinetic vitrification transition corresponding to $\tau_\alpha(\phi_g) = 100$ s. Vertical colored lines correspond to different $\phi_g(f_s)$ values.

378 decades, while for a higher $\phi_{\text{eff}} = 0.585$ the same sticker
 379 fraction increases the alpha time by ~ 7 decades. Such a large
 380 difference reflects the growing absolute and relative (to the
 381 local cage barrier) importance of collective elasticity.

382 We define an operational laboratory glass transition
 383 temperature as when $\tau_{\alpha} = 100$ s, which corresponds to ϕ_{eff}
 384 ≈ 0.60 for the homopolymer melt ($f_s = 0$). This isochronal
 385 vitrification criterion is the horizontal black line in Figure 7. A
 386 shift in the glass transition melt packing fraction in associating
 387 copolymers, $\Delta\phi_g = \phi_g(f_s) - \phi_g(0) \leq 0$, is indicated by the
 388 vertical lines in Figure 7 and implies increased dynamic caging
 389 constraints experienced by nonstickers due to the presence of
 390 more localized sticker segments.

391 **Temperature Mapping and Calibration to PDMS**
 392 **Homopolymer Melt.** To relate the alpha time results in
 393 Figure 7 to a specific chemistry and temperature under 1 atm
 394 isobaric conditions, we adopt the well-known dimensionless
 395 compressibility (S_0) mapping that employs the experimental
 396 polymer liquid equation of state (EOS),⁵⁴

$$S_{0,PY}^{\text{homopolymer}}(\phi_{\text{eff}}) \equiv S_{0,\text{expt}} = \rho k_B T \kappa_T \approx N_s^{-1} \left(-A + \frac{B}{T} \right)^{-2} \quad (7a)$$

397

$$T(\phi_{\text{eff}}) = \frac{B}{A + \frac{1}{\sqrt{N_s S_{0,PY}^{\text{homopolymer}}(\phi_{\text{eff}})}}} \quad (7b)$$

398 where the final approximate equality in eq 7a is a good analytic
 400 approximation. The parameters A and B are related to entropic
 401 and enthalpic contributions in the EOS, respectively, and N_s
 402 quantifies how many elementary functional groups a bead or
 403 segment in the FJC model represents. These parameters can be
 404 estimated from experimental data, which then provides a
 405 *polymer-specific* mapping from effective packing fraction to
 406 temperature. It has been previously shown that adopting either
 407 the numerical S_0 value computed from PRISM-PY integral
 408 equation theory of a connected polymer chain melt, or a literal
 409 mapping to a disconnected hard-sphere (HS) liquid of Kuhn
 410 segments by using $S_{0,PY}^{\text{HS}} = \frac{(1-\phi)^4}{(1+2\phi)^2}$, yields very similar results
 411 for the mapping and predicted dynamics.³⁶ We use the former
 412 approach, which results in eq 7b.

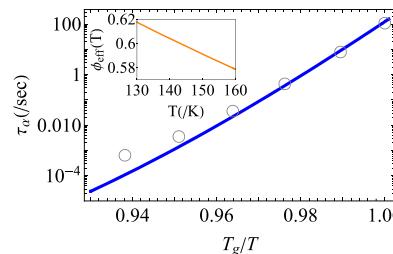
413 In practice, the purely end-functionalized telechelic melt
 414 systems of present interest exhibit a degree of polymerization
 415 (DP)-dependent glass transition temperature.^{28–33} Because the
 416 DP is inversely proportional to the fraction of end groups, we
 417 write $f = a/\text{DP}$, where a is a proportionality constant. In
 418 principle, its precise value is related to the level of coarse
 419 graining employed in the theory model (i.e., what a model
 420 bead corresponds to in terms of real chemical monomers) and
 421 hence the detailed polymer chemistry. One can view a as a
 422 polymer-specific adjustable parameter in our theoretical
 423 analysis. However, we have attempted to realistically estimate
 424 it using homopolymer PDMS parameters⁵⁴ and find

$$a \approx \frac{C_N}{N_s} \sim 1.6$$
 using a PDMS characteristic ratio (C_N) of
 425 ~ 6.3 , and N_s is the number of elementary functional groups in
 426 a lightly coarse-grained bead which we find to be close to ~ 4 .

427 To focus on the effects of physical bonding of associating
 428 end groups on the change of the glass transition temperature,
 429 we calibrate $N_s(f)$ (here f is inversely related to the
 430 homopolymer degree of polymerization, N) to ensure the
 431 theory and model reproduce the experimental reference f –

432 dependent PDMS *homopolymer* glass transition temperature,
 $T_g(f)$, and fragility, $m(f)$, in the *absence* of associating end
 434 groups. The experimental PDMS homopolymer melt glass
 435 transition temperature and fragility values⁵⁴ employed for this
 436 calibration are given in the *Supporting Information* (section
 437 II). From PDMS EOS data⁵⁴ it is known that $A = 0.675$ and B
 438 = 1057 K in eqs 7a and 7b which are employed for all end
 439 group fractions f . We then fix $\tau_0 = 10^{-12}$ s and vary N_s to
 440 reproduce T_g and m for each variable DP homopolymer melt
 441 of interest with an end group fraction, f . As discussed in the
 442 SM, a modest variation of $N_s \sim 4.3$ to ~ 3.5 changes T_g from
 443 ~ 144 K at $f \sim 0$ (long chain limiting behavior) to ~ 130 K at f
 444 ~ 0.20 .
 445

446 Figure 8 shows a typical result for the temperature-
 447 dependent homopolymer alpha relaxation time in the deeply
 448



449 **Figure 8.** Theoretical prediction for the alpha relaxation time
 450 compared to experiment for the long chain limit of the PDMS
 451 homopolymer melt studied in ref 54. The theory based on the coarse-
 452 grained FJC model deviates from experimental results at high
 453 temperature (as expected), and hence in this work we focus on the
 454 deeply supercooled regime relatively close to T_g . The inset shows the
 455 mapping from effective packing fraction to temperature for PDMS.

456 supercooled regime and the limiting long chain behavior where
 457 T_g and fragility become independent of DP. Also shown in the
 458 inset is the corresponding $\phi_{\text{eff}} - T$ mapping obtained by using
 459 eq 7b. Because we only calibrate the model parameters with
 460 respect to the $\tau_{\alpha} = 100$ s state (for both T_g and m), the
 461 theoretical results in Figure 8 represent *predictions* for all
 462 temperatures above T_g . At high enough temperatures
 463 deviations are found, as expected based on using a coarse-
 464 grained model, but overall, the theory agrees well with
 465 experiment in the *deeply* supercooled regime close to kinetic
 466 vitrification which is of prime interest in this work.
 467

468 Once we have calibrated the model to account for the f -
 469 dependent homopolymer melt T_g and fragility behavior, the
 470 theory can be used to make *no* adjustable parameter
 471 predictions for associating copolymer melts based on assuming
 472 that the telechelic EOS is the same as for pure PDMS. The
 473 latter approach is adopted both for simplicity and to allow
 474 predictability given the lack of associating copolymer
 475 experimental EOS data to “precisely” carry out the mapping
 476 for each telechelic individually.
 477

V. APPLICATION TO PDMS-BASED TELECHELIC MELTS

478 **Shift in T_g .** The DP and sticker fraction dependent change
 479 in glass transition temperature is
 480

$$\Delta T_g = T_g(f_s = f) - T_g(f) = \{T_g(f \rightarrow \infty) - T_g(f)\} \\ + \{T_g(f_s = f) - T_g(f \rightarrow \infty)\} \quad (8)$$

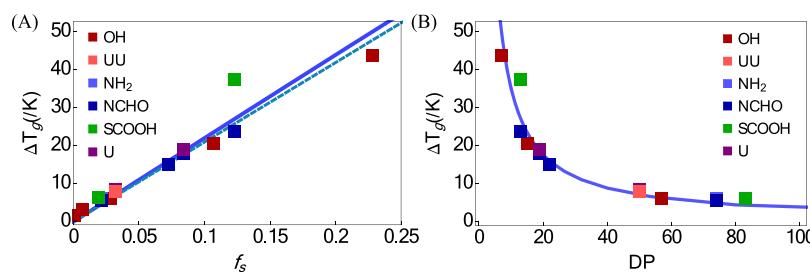


Figure 9. (A) Change of T_g compared to the DP-dependent homopolymer melt as a function of sticker fraction for various sticker chemical moieties attached to a PDMS backbone. Different end groups are color-coded by different squares as indicated. The a priori theoretical prediction (not a fit, no adjustable parameters) is the solid blue curve. The dashed blue curve is an empirical linear fit through all the experimental points and follows the predicted theory curve very closely. (B) The plot shows the same change in T_g as a function of DP where $DP = 1.60/f$. The blue curve is the theoretical prediction.

where the reference $T_g(f)$ in eq 8 corresponds to the DP-dependent homopolymer glass transition temperature. The contribution $T_g(f \rightarrow \infty) - T_g(f)$ reflects changes in dynamical constraints due to nonassociating homopolymer chain length and always contributes positively; i.e., T_g grows and then saturates with degree of polymerization. The term $T_g(f_s = f) - T_g(f \rightarrow \infty)$ captures the changes due to the enhanced dynamical constraints in an associating polymer due to physical bonding of stickers relative to the $DP \rightarrow \infty$ limit (highest T_g of a homopolymer). It can in principle be positive (strongly associating systems), close to zero (moderately associating systems), or even negative (repelling or weakly associating end groups). For PDMS, both contributions are positive and increase the glass transition temperature. However, we will also discuss a PPG telechelic in section VI where the second contribution in eq 8 can be positive or negative depending on the precise nature of the associating end group. Of course, dynamic caging constraints experienced by nonstickers will always increase with sticker concentration, resulting in an increase of the copolymer T_g .

Theoretical predictions for PDMS telechelic melts as a function of sticker fraction are shown in Figure 9A. Numerically, for $f_s = 0.1$ one sees T_g changes by ~ 20 K, while at $f_s = 0.20$, T_g is larger by ~ 40 K. Remarkably, we find the growth in T_g follows an almost perfect linear trend with sticker fraction. This is not a trivial “chain end” approach as in phenomenological free volume models (e.g., Flory–Fox^{56,57}); indeed, those models attempt to mimic the decrease of T_g as chain length decreases, the opposite direction of T_g changes in the associating systems.

To compare the theoretical results with experiment, we convert telechelic degree of polymerization (DP or n) to sticker fraction, f_s . Because, as discussed above, DP is inversely related to sticker fraction, one has $f_s = a/DP$ where $a \approx 1.6$. All our analysis below uses this conversion between theoretical sticker fraction and experimental DP.

Figure 9A plots the experimental change in T_g as a function of sticker fraction for six choices of sticky end group (see data in Table 1) which have variable strengths of attraction between stickers as well as some degree of microphase separation for PDMS-NCHO and PDMS-UU. Overall, we find remarkably good agreement with theory, especially given this is *not* a fit, and no adjustable parameters have been introduced. This agreement for the telechelic melts considered is consistent with the assumed sufficiently large time scale separation between the α and α^* processes adopted in the theoretical analysis where the precise nature of attractive functionality does not matter to zeroth order. It is also consistent with our physical

idea that higher scale structuring (clusters, microphase separation) is not important to leading order for the questions of interest. A linear fit of all the experimental data points is also shown, which agrees almost perfectly with theory. The same results are plotted as of T_g vs DP in Figure 9B showing the same degree of agreement, where the theoretical f_s is converted to DP by using $DP = 1.60/f_s$.

Alpha Relaxation Time: Temperature and Sticker Fraction Dependences. We quantitatively confront theory and PDMS-NCHO-COOH melt experiments for the alpha time as a function of inverse temperature (normalized by T_g) in Figure 10. VFT fits of the experimental data points are

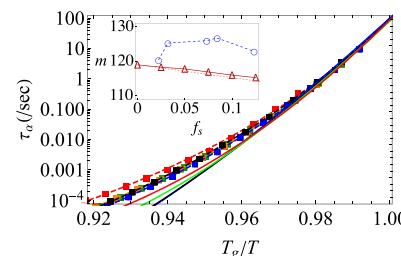


Figure 10. Angell plot of the experimental alpha time data and theory predictions for sticker fractions of 0, 0.05, 0.075, and 0.10 (solid curves are theory, from blue to red, bottom to top) and 0.022, 0.032, 0.073, 0.084, 0.123 (experiment, points). Dashed curves are VFT fits through the experimental points. Inset shows the fragility of the experimental PDMS-NCHO systems (blue circles) obtained from the VFT fits and the corresponding theoretical predictions (red).

shown as dashed curves, while theory predictions are the solid curves. The level of theory–experiment agreement for the full temperature dependence of the alpha time is reasonable for all sticker fractions studied experimentally which range from $f_s = 0.028$ –0.124.

Results for the dynamic fragility, $m = \frac{d \log \tau_a}{d(T_g/T)}$ at $T = T_g$, are shown in the inset. Globally, and nontrivially, it is nearly constant to leading order. The theory predicts a perturbative but systematic decrease in the fragility with increasing sticker fraction. As expected, the overall agreement between theory and experiment for fragility is good but not perfect. More generally, given all the simplifications made in the modeling, and the fact that fragility is a subtle feature since it is the ratio of an inverse temperature derivative quantity to T_g , the level of agreement seems reasonable. The prediction of a small 4% decrease in fragility with sticker fraction versus roughly constant fragility observed in experiment could be due to

many hard to disentangle factors such as (i) the theory employs an associating multiblock polymer model while experiments are performed on telechelics, some of which show microphase separation, (ii) a coarse-grained FJC model is employed that ignores the chemistry of the real end groups and PDMS monomers, (iii) the experimental equation of state (EOS) is presumably weakly sticker fraction dependent factor which is ignored, (iv) in reality there is not an infinite separation of time scales between the α and α^* process but we assume there is in our adopted strict two-step dynamic scenario, and (v) other chemical effects such as possible backbone stretching, changes of C_N with telechelic DP, and so on. Remarkably, all these effects apparently have very small consequences, indicating a high degree of "self-averaging" of subnanometer scale chemical structure for the dynamical properties we study.

We note that although deviations of the experimental results from the theory predictions are found at higher temperatures (smaller T_g/T values), they are qualitatively identical with those present for the reference pure homopolymer PDMS melt discussed in section IV. Thus, we believe that these deviations relate primarily to the coarse-grained model adopted, although the other second order factors (i–v) mentioned above could also contribute. More crucially, as one transitions from a low to high sticker fraction, the deviations are similar in magnitude as seen in Figure 10 (cf. blue vs red plots at fixed T_g/T values). Hence, to zeroth order, the observed deviations do not depend on sticker fraction or the physics of bond formation.

Theory and experiment are further confronted in Figure 11 for the alpha time as a function of sticker fraction at several

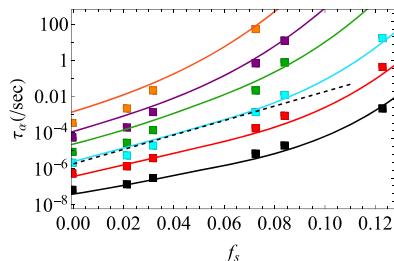


Figure 11. Alpha relaxation time (seconds) as a function of sticker fraction at various T/T_g ratios of 1.20, 1.16, 1.14, 1.12, 1.10, 1.08 (theory, black to orange curves) and 1.20, 1.16, 1.14, 1.12, 1.1, 1.08 (experiment, black to orange points) for PDMS-NCHO-COOH. The data points with $\tau_\alpha > 10$ s from experiments^{28–33} are obtained from extrapolated VFT fits of the discrete experimental data. A linear fit through the first four cyan experimental points (dashed black line) is shown as an example to visually illustrate the supraexponential growth of the alpha time with respect to sticker fraction observed experimentally. The latter agrees well with theoretical predictions (coincident with the initial growth of cyan curve).

fixed reduced temperatures, $\frac{T}{T_g}$, where T_g is the DP-dependent glass transition temperature of the homopolymer melt. Experimental points are shown as filled squares, with the $\tau_\alpha > 10$ s points obtained from extrapolated VFT fits to the experimental data at temperatures close to the glass transition for the PDMS-NCHO samples.⁵⁸ Overall, there is very good accord between theory (no fitting parameter) and experiment for all trends, and the agreement is (remarkably) almost quantitative. In detail, at the lowest sticker fraction studied in the experiments ($f_s = 0.022$), as $\frac{T}{T_g}$ changes from 1.20 to 1.08,

the alpha time increases by ~4 decades, while at $f_s = 0.073$ the alpha time grows by ~7 decades, and the difference increases even stronger at higher stickers fraction (Figure 11). These trends agree well with the theoretical predictions. The experimental data also show a stronger than exponential growth of the alpha time with sticker fraction as shown by an illustrative black dashed line through the cyan points in Figure 11. This is as predicted by the theory, a trend that has been experimentally observed for multiblock random copolymer melts and previously shown to be well captured by our same theory.³⁶

VI. APPLICATION TO PPG-BASED TELECHELIC MELTS: Shift in T_g

In this section we first present model calculations of how our theoretical predictions for the change of the glass transition temperature depend on introducing enhanced dynamic fluctuations of the stickers based on the new extension of the theory reported in section III. The theory is then applied to understand the behavior of the PPG-based weakly associating telechelic melts.

Change in T_g for Larger Sticker Fluctuation. Figure 12 plots the predicted change in nonsticker glass transition

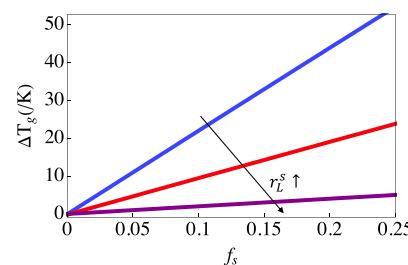


Figure 12. Theoretical predictions for the change of the nonsticker T_g due to the presence of stickers associated with enhanced sticker fluctuations as a function of sticker fraction for different values of $r_L^s = r_L^s$ (blue), $r_L^s + 0.25\Delta r_n$ (red), and $r_L^s + 0.33\Delta r_n$ (purple) using the PDMS homopolymer mapping described in the main text (cf. Figure 5, blue line).

temperature for PDMS as a function of sticker fraction for three values of sticker fluctuation amplitude. The blue curve is the same as the blue solid curves in Figure 9 with the a priori predicted value of $r_L^s = r_L^s$. The other two curves illustrate how larger sticker fluctuations ("looser and weaker bonds") change the theoretical results. If $r_L^s = r_L^s + 0.33\Delta r_n$ (a displacement well below the nonsticker dynamic free energy barrier), then essentially all the constraints due to the presence of bonded stickers are predicted to disappear. This indicates a dynamic crossover to a regime of weak sticker associations which have little or no effect on the nonsticker alpha time. Importantly, such behavior can emerge without the need for the sticker alpha time becoming equal to its nonsticker analogue; i.e., the α and α^* relaxation events do not need to have an identical time scale. Qualitatively, we physically rationalize the predicted behavior by noting that $0.33\Delta r_n > a = 0.1d$ where a is the range of attractive interaction between stickers. Hence, this size of a sticker dynamic fluctuation is larger than the attraction range that defines a physical bond, and hence the predicted destruction of kinetic constraints on nonstickers due to sticker bonding seems physically intuitive. How fragility decreases as a

631 function of sticker fraction for increasing degrees of sticker
632 bond fluctuations is discussed in the SM.

633 **Comparison of Theory and Experiment for PPG**
634 **Systems.** To predict the glass transition temperature changes
635 for the weakly associating PPG telechelic melts, we adopt the
636 same packing fraction to temperature mapping discussed in
637 **section III.** However, the EOS state data appear to be
638 unavailable for the PPG homopolymer. We thus use $A = 0.627$
639 and $B = 1353$ K parameters for PIB,⁵⁴ $\tau_0 = 10^{-12}$ s, and
640 calibrate N_s to best reproduce the DP-dependent glass
641 transition temperature of homopolymer PPG with non-
642 associating chain ends. For the mapping used, we again
643 adopt $f = a/DP$ where $a \sim 1.3$ based on $C_N \sim 6.8$ and $N_s \sim 5.2$
644 appropriate for PPG.⁵⁴ We note that using the specific
645 mapping parameters for any other homopolymer does not
646 change any of our results significantly since the adopted
647 calibration step ensures we capture the correct DP-dependent
648 homopolymer melt T_g .

649 The theoretical results for the shift of T_g are reported in
650 **Figure 13** for various end groups of the PPG telechelic melt.

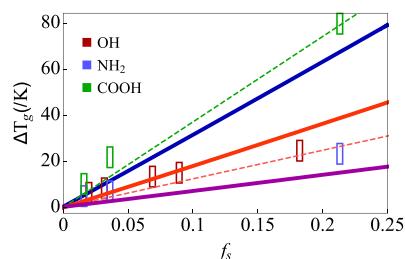


Figure 13. Change of T_g compared to the DP-dependent homopolymer melt as a function of sticker fraction for various sticker moieties attached to a PPG backbone. Theoretical predictions are given by the blue curve when stickers are localized at their respective a priori predicted localization lengths (r_L^s) corresponding to tight physical bonds. Red and purple curves correspond to theoretical results for larger sticker dynamic fluctuations of $r_L^{s'} = r_L^s + 0.25\Delta r_L^s$ and $r_L^{s'} = r_L^s + 0.33\Delta r_L^s$, respectively. Experimental data are shown as the solid squares. A fit through the experimental COOH system points is indicated by the green dashed line, while those for NH₂ and OH are given in red (treating them as single set of data).

651 The blue solid line corresponds to when stickers fluctuate at
652 the tight localization lengths that are a priori predicted. This
653 result agrees well with our COOH-based experiments on the
654 PPG telechelic melt. We believe that strong H-bonding
655 between COOH groups is responsible for creating stronger
656 bonding constraints and, hence, the larger growth in T_g .

657 The other two curves in **Figure 13** are results when sticker
658 fluctuations are larger than their respective localization lengths
659 corresponding to weakened bonding constraints on the time
660 scale of the activated relaxation of nonsticker segments.
661 Specifically, we deduce that if stickers displace distances
662 close to 1/4 to 1/3 of nonsticker hopping jump distance (a
663 relatively small fractional distance), the theory captures well
664 the experimentally observed smaller growth in T_g for PPG
665 systems with NH₂ and OH end groups (theoretical red and
666 purple curves, respectively). We note that when stickers are
667 allowed to fluctuate with localization lengths close to a
668 displacement of maximum force on the dynamic free energy of
669 pure homopolymers (its inflection point in **Figure 6B**), i.e.,
670 $r_L^{s'} \sim r^*(f_s \rightarrow 0)$, dynamic caging constraints due to bonding
671 are essentially entirely lost and the local barrier is nearly the

672 same as that of the homopolymer (not plotted). In this case we 673
673 see no change of T_g due to such dynamically weak associations. 674
674 This limit corresponds to even weaker bonding constraints 675
675 than appear to be appropriate for PPG with NH₂ or OH end 676
676 groups in **Figure 13**.

VII. SUMMARY AND CONCLUSIONS

We have quantitatively tested predictions of our recently proposed³⁶ microscopic theory of how attractive forces and physical bonding in associating copolymer liquids affect segmental relaxation and the glass transition temperature against experimental data for telechelic PDMS- and PPG-based associating polymers. Theoretical results for the alpha relaxation time as a function of packing fraction are mapped to temperature space by using the polymer EOS that allows us to predict how the glass transition temperature changes for real associating materials. Remarkably, the large time scale separation between the nonsticker alpha relaxation and slower bond-breaking process for PDMS telechelic melts renders the systems oblivious to the specific type of attraction or even microphase separation for the question of the change of glass transition temperature as a function of sticker fraction. The data thus support the proposed physical idea that such systems behave as cross-linked networks on the time scale of the nonsticker alpha relaxation. More quantitatively, our microscopic *parameter free* theoretical predictions for the associating melts agree remarkably well with experiment, not only for how T_g changes with telechelic chain length but also for other important questions such as the sticker-induced exponential growth of the alpha relaxation with sticker fraction that becomes stronger upon approaching T_g , and the perturbative or almost negligible changes of dynamic fragility with sticker fraction. These findings also support our hypothesis that changes of the nonsticker structural relaxation time are not affected (to leading order) by the precise transient network topology, at least for the copolymer architectures we have studied to date.

The theory has been extended in this work to lift the assumption of strong time scale separation between the nonsticker alpha time and the sticker bond breaking lifetime by allowing stickers to fluctuate within their predicted dynamic free energy to modestly larger length scales characteristic of the late beta or early alpha relaxation process of stickers while still retaining the same basic transient network like picture. This results in *dynamical* weakening of sticker bonding, which in turns reduces their effect on the activated relaxation of nonsticker activated relaxation. This advance provides an understanding of experimental observations of weakly associating PPG telechelic melts for -OH and -NH₂ end groups which exhibit much reduced T_g enhancements. On the other hand, for the strongly associating case of PPG-COOH, the theoretical prediction for the enhancement of T_g due to physical bond formation again agrees remarkably well with experiment.

Future joint theoretical-experimental work can explore the consequences of the presented level of understanding for the problem of material design and synthesis of associating polymers and polymers with dynamic covalent bonds. The present work also sets the stage to theoretically address other questions for melts of polymers with dynamic bonds, such as elastic reinforcement and change of flow viscosity where longer length/time-scale chain dynamics is strongly impacted by the nonperturbative changes of local segmental dynamics.

733 ■ ASSOCIATED CONTENT

734 ■ Supporting Information

735 The Supporting Information is available free of charge at
736 <https://pubs.acs.org/doi/10.1021/acs.macromol.2c00080>.

737 (i) A brief summary of the synthesis protocols of
738 telechelic systems and (ii) details of the theory modeling
739 of the reference homopolymer melt DP-dependent T_g
740 and fragility (PDF)

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769 Notes

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777 ■ REFERENCES

778 (1) Wang, C.; Wu, H.; Chen, Z.; McDowell, M. T.; Cui, Y.; Bao, Z.
779 Self-Healing Chemistry Enables the Stable Operation of Silicon
780 Microparticle Anodes for High-Energy Lithium-Ion Batteries. *Nat. Chem.* **2013**, *5* (12), 1042–1048.
781 (2) Olsen, B. D.; Kornfield, J. A.; Tirrell, D. A. Yielding Behavior in
783 Injectable Hydrogels from Telechelic Proteins. *Macromolecules* **2010**,
784 *43* (21), 9094–9099.
785 (3) Cao, P.-F.; Li, B.; Hong, T.; Townsend, J.; Qiang, Z.; Xing, K.;
786 Vogiatzis, K. D.; Wang, Y.; Mays, J. W.; Sokolov, A. P.; Saito, T.
787 Superstretchable, Self-Healing Polymeric Elastomers with Tunable
788 Properties. *Adv. Funct. Mater.* **2018**, *28* (22), 1800741.

789 (4) Cordier, P.; Tournilhac, F.; Soulié-Ziakovic, C.; Leibler, L. Self- 789
790 Healing and Thermoreversible Rubber from Supramolecular 790
791 Assembly. *Nature* **2008**, *451* (7181), 977–980. 791
792 (5) Campanella, A.; Döhler, D.; Binder, W. H. Self-Healing in 792
793 Supramolecular Polymers. *Macromol. Rapid Commun.* **2018**, *39* (17), 793
794 1700739. 794
795 (6) Huang, Y.-F.; Xu, J.-Z.; Xu, J.-Y.; Zhang, Z.-C.; Hsiao, B. S.; Xu, 795
796 L.; Li, Z.-M. Self-Reinforced Polyethylene Blend for Artificial Joint 796
797 Application. *J. Mater. Chem. B* **2014**, *2* (8), 971. 797
798 (7) Kim, M.; Chen, W. G.; Kang, J. W.; Glassman, M. J.; Ribbeck, 798
799 K.; Olsen, B. D. Artificially Engineered Protein Hydrogels Adapted 799
800 from the Nucleoporin Nsp1 for Selective Biomolecular Transport. 800
801 *Adv. Mater.* **2015**, *27* (28), 4207–4212. 801
802 (8) Ma, M.; Guo, L.; Anderson, D. G.; Langer, R. Bio-Inspired 802
803 Polymer Composite Actuator and Generator Driven by Water 803
804 Gradients. *Science* **2013**, *339* (6116), 186–189. 804
805 (9) Wei, M.-H.; Li, B.; David, R. L. A.; Jones, S. C.; Sarohia, V.; 805
806 Schmitigal, J. A.; Kornfield, J. A. Megasupramolecules for Safer, 806
807 Cleaner Fuel by End Association of Long Telechelic Polymers. *Science* 807
808 **2015**, *350* (6256), 72–75. 808
809 (10) Rosales, A. M.; Anseth, K. S. The Design of Reversible 809
810 Hydrogels to Capture Extracellular Matrix Dynamics. *Nat. Rev. Mater.* 810
811 **2016**, DOI: [10.1038/natrevmats.2015.12](https://doi.org/10.1038/natrevmats.2015.12). 811
812 (11) Li, C.-H.; Wang, C.; Keplinger, C.; Zuo, J.-L.; Jin, L.; Sun, Y.; 812
813 Zheng, P.; Cao, Y.; Lissel, F.; Linder, C.; You, X.-Z.; Bao, Z. A Highly 813
814 Stretchable Autonomous Self-Healing Elastomer. *Nat. Chem.* **2016**, *8* 814
815 (6), 618–624. 815
816 (12) Hagenau, A.; Suhre, M. H.; Scheibel, T. R. Nature as a 816
817 Blueprint for Polymer Material Concepts: Protein Fiber-Reinforced 817
818 Composites as Holdfasts of Mussels. *Prog. Polym. Sci.* **2014**, *39* (8), 818
819 1564–1583. 819
820 (13) Zhang, Z. P.; Rong, M. Z.; Zhang, M. Q. Polymer Engineering 820
821 Based on Reversible Covalent Chemistry: A Promising Innovative 821
822 Pathway towards New Materials and New Functionalities. *Prog. 822
823 Polym. Sci.* **2018**, *80*, 39–93. 823
824 (14) Folmer, B. J. B.; Sijbesma, R. P.; Versteegen, R. M.; van der 824
825 Rijt, J. A. J.; Meijer, E. W. Supramolecular Polymer Materials: Chain 825
826 Extension of Telechelic Polymers Using a Reactive Hydrogen- 826
827 Bonding Synthon. *Adv. Mater.* **2000**, *12* (12), 874–878. 827
828 (15) Filippidi, E.; Cristiani, T. R.; Eisenbach, C. D.; Waite, J. H.; 828
829 Israelachvili, J. N.; Ahn, B. K.; Valentine, M. T. Toughening 829
830 Elastomers Using Mussel-Inspired Iron-Catechol Complexes. *Science* 830
831 **2017**, *358* (6362), 502–505. 831
832 (16) Albertazzi, L.; van der Zwaag, D.; Leenders, C. M. A.; Fitzner, 832
833 R.; van der Hofstad, R. W.; Meijer, E. W. Probing Exchange Pathways 833
834 in One-Dimensional Aggregates with Super-Resolution Microscopy. 834
835 *Science* **2014**, *344* (6183), 491–495. 835
836 (17) Chen, Q.; Tudry, G. J.; Colby, R. H. Ionomer Dynamics and 836
837 the Sticky Rouse Model. *J. Rheol.* **2013**, *57* (5), 1441–1462. 837
838 (18) Holten-Andersen, N.; Harrington, M. J.; Birkedal, H.; Lee, B. 838
839 P.; Messersmith, P. B.; Lee, K. Y. C.; Waite, J. H. PH-Induced Metal- 839
840 Ligand Cross-Links Inspired by Mussel Yield Self-Healing Polymer 840
841 Networks with Near-Covalent Elastic Moduli. *Proc. Natl. Acad. Sci. U.* 841
842 S. A. **2011**, *108* (7), 2651–2655. 842
843 (19) Tang, S.; Wang, M.; Olsen, B. D. Anomalous Self-Diffusion and 843
844 Sticky Rouse Dynamics in Associative Protein Hydrogels. *J. Am. 844
845 Chem. Soc.* **2015**, *137* (11), 3946–3957. 845
846 (20) Zhang, Z.; Huang, C.; Weiss, R. A.; Chen, Q. Association 846
847 Energy in Strongly Associative Polymers. *J. Rheol.* **2017**, *61* (6), 847
848 1199–1207. 848
849 (21) Leibler, L.; Rubinstein, M.; Colby, R. H. Dynamics of 849
850 Reversible Networks. *Macromolecules* **1991**, *24* (16), 4701–4707. 850
851 (22) Semenov, A. N.; Rubinstein, M. Thermoreversible Gelation in 851
852 Solutions of Associative Polymers. 1. Statics. *Macromolecules* **1998**, *31* 852
853 (4), 1373–1385. 853
854 (23) Rubinstein, M.; Semenov, A. N. Thermoreversible Gelation in 854
855 Solutions of Associating Polymers. 2. Linear Dynamics. *Macromolecules* **1998**, *31* (4), 1386–1397. 855

857 (24) Rubinstein, M.; Semenov, A. N. Dynamics of Entangled
858 Solutions of Associating Polymers. *Macromolecules* **2001**, *34* (4),
859 1058–1068.

860 (25) van Ruymbeke, E.; Vlassopoulos, D.; Mierzwa, M.; Pakula, T.;
861 Charalabidis, D.; Pitsikalis, M.; Hadjichristidis, N. Rheology and
862 Structure of Entangled Telechelic Linear and Star Polyisoprene Melts.
863 *Macromolecules* **2010**, *43* (9), 4401–4411.

864 (26) Ahmadi, M.; Hawke, L. G. D.; Goldansaz, H.; van Ruymbeke,
865 E. Dynamics of Entangled Linear Supramolecular Chains with Sticky
866 Side Groups: Influence of Hindered Fluctuations. *Macromolecules*
867 **2015**, *48* (19), 7300–7310.

868 (27) Cates, M. E. Reptation of Living Polymers: Dynamics of
869 Entangled Polymers in the Presence of Reversible Chain-Sission
870 Reactions. *Macromolecules* **1987**, *20* (9), 2289–2296.

871 (28) Ge, S.; Tress, M.; Xing, K.; Cao, P.-F.; Saito, T.; Sokolov, A. P.
872 Viscoelasticity in Associating Oligomers and Polymers: Experimental
873 Test of the Bond Lifetime Renormalization Model. *Soft Matter* **2020**,
874 *16* (2), 390–401.

875 (29) Xing, K.; Tress, M.; Cao, P.-F.; Fan, F.; Cheng, S.; Saito, T.;
876 Sokolov, A. P. The Role of Chain-End Association Lifetime in
877 Segmental and Chain Dynamics of Telechelic Polymers. *Macromolecules*
878 **2018**, *51* (21), 8561–8573.

879 (30) Xing, K.; Chatterjee, S.; Saito, T.; Gainaru, C.; Sokolov, A. P.
880 Impact of Hydrogen Bonding on Dynamics of Hydroxyl-Terminated
881 Polydimethylsiloxane. *Macromolecules* **2016**, *49* (8), 3138–3147.

882 (31) Tress, M.; Xing, K.; Ge, S.; Cao, P.; Saito, T.; Sokolov, A. What
883 Dielectric Spectroscopy Can Tell Us about Supramolecular Networks.
884 *Eur. Phys. J. E* **2019**, DOI: 10.1140/epje/i2019-11897-4.

885 (32) Xing, K.; Tress, M.; Cao, P.; Cheng, S.; Saito, T.; Novikov, V.
886 N.; Sokolov, A. P. Hydrogen-Bond Strength Changes Network
887 Dynamics in Associating Telechelic PDMS. *Soft Matter* **2018**, *14* (7),
888 1235–1246.

889 (33) Tress, M.; Ge, S.; Xing, K.; Cao, P.-F.; Saito, T.; Genix, A.-C.;
890 Sokolov, A. P. Turning Rubber into a Glass: Mechanical Reinforce-
891 ment by Microphase Separation. *ACS Macro Lett.* **2021**, *10* (2), 197–
892 202.

893 (34) Mordvinkin, A.; Döhler, D.; Binder, W. H.; Colby, R. H.;
894 Saalwächter, K. Rheology, Sticky Chain, and Sticker Dynamics of
895 Supramolecular Elastomers Based on Cluster-Forming Telechelic
896 Linear and Star Polymers. *Macromolecules* **2021**, *54* (11), 5065–5076.

897 (35) Samanta, S.; Kim, S.; Saito, T.; Sokolov, A. P. Polymers with
898 Dynamic Bonds: Adaptive Functional Materials for a Sustainable
899 Future. *J. Phys. Chem. B* **2021**, *125* (33), 9389–9401.

900 (36) Ghosh, A.; Schweizer, K. S. Microscopic Theory of the Effect of
901 Caging and Physical Bonding on Segmental Relaxation in Associating
902 Copolymer Liquids. *Macromolecules* **2020**, *53* (11), 4366–4380.

903 (37) Ghosh, A.; Schweizer, K. S. Physical Bond Breaking in
904 Associating Copolymer Liquids. *ACS Macro Lett.* **2021**, *10* (1), 122–
905 128.

906 (38) Ge, S.; Samanta, S.; Tress, M.; Li, B.; Xing, K.; Dieudonné-
907 George, P.; Genix, A.-C.; Cao, P.-F.; Dadmun, M.; Sokolov, A. P.
908 Critical Role of the Interfacial Layer in Associating Polymers with
909 Microphase Separation. *Macromolecules* **2021**, *54* (9), 4246–4256.

910 (39) Gold, B. J.; Hövelmann, C. H.; Lühmann, N.; Székely, N. K.;
911 Pyckhout-Hintzen, W.; Wischnewski, A.; Richter, D. Importance of
912 Compact Random Walks for the Rheology of Transient Networks.
913 *ACS Macro Lett.* **2017**, *6* (2), 73–77.

914 (40) Shabbir, A.; Javakhishvili, I.; Cerveny, S.; Hvilsted, S.; Skov, A.
915 L.; Hassager, O.; Alvarez, N. J. Linear Viscoelastic and Dielectric
916 Relaxation Response of Unentangled UPy-Based Supramolecular
917 Networks. *Macromolecules* **2016**, *49* (10), 3899–3910.

918 (41) Wu, S.; Liu, S.; Zhang, Z.; Chen, Q. Dynamics of Telechelic
919 Ionomers with Distribution of Number of Ionic Stickers at Chain
920 Ends. *Macromolecules* **2019**, *52* (6), 2265–2276.

921 (42) Mordvinkin, A.; Suckow, M.; Böhme, F.; Colby, R. H.; Creton,
922 C.; Saalwächter, K. Hierarchical Sticker and Sticky Chain Dynamics in
923 Self-Healing Butyl Rubber Ionomers. *Macromolecules* **2019**, *52* (11),
924 4169–4184.

925 (43) Schweizer, K. S.; Curro, J. G. Integral Equation Theory of 926
926 Polymer Melts: Intramolecular Structure, Local Order, and the 926
927 Correlation Hole. *Macromolecules* **1988**, *21* (10), 3070–3081. 927

928 (44) Schweizer, K. S.; Curro, J. G. Integral Equation Theories of the 928
929 Structure, Thermodynamics, and Phase Transitions of Polymer 929
930 Fluids. *Adv. Chem. Phys.* **2007**, *98*, 1–142. 930

931 (45) Kolbet, K. A.; Schweizer, K. S. Real Space Structure of 931
932 Associating Polymer Melts. *Macromolecules* **2000**, *33* (4), 1443–1458. 932

933 (46) Sung, B. J.; Yethiraj, A. Integral Equation Theory of Random 933
934 Copolymer Melts: Self-Consistent Treatment of Intramolecular and 934
935 Intermolecular Correlations. *J. Chem. Phys.* **2005**, *122* (23), 234904. 935

936 (47) Hansen, J.-P.; McDonald, I. R. *Theory of Simple Liquids*; Jordan 936
937 Hill Elsevier Science: 2014. 937

938 (48) Mirigian, S.; Schweizer, K. S. Elastically Cooperative Activated 938
939 Barrier Hopping Theory of Relaxation in Viscous Fluids. I. General 939
940 Formulation and Application to Hard Sphere Fluids. *J. Chem. Phys.* **2014**, *140* (19), 194506. 941

942 (49) Mirigian, S.; Schweizer, K. S. Elastically Cooperative Activated 942
943 Barrier Hopping Theory of Relaxation in Viscous Fluids. II. Thermal 943
944 Liquids. *J. Chem. Phys.* **2014**, *140* (19), 194507. 944

945 (50) Viehman, D. C.; Schweizer, K. S. Theory of Gelation, 945
946 Vitrification, and Activated Barrier Hopping in Mixtures of Hard 946
947 and Sticky Spheres. *J. Chem. Phys.* **2008**, *128* (8), 084509. 947

948 (51) Dell, Z. E.; Schweizer, K. S. Microscopic Theory for the Role of 948
949 Attractive Forces in the Dynamics of Supercooled Liquids. *Phys. Rev. 949
950 Lett.* **2015**, DOI: 10.1103/PhysRevLett.115.205702. 950

951 (52) Ghosh, A.; Schweizer, K. S. Microscopic Theory of the 951
952 Influence of Strong Attractive Forces on the Activated Dynamics of 952
953 Dense Glass and Gel Forming Fluids. *J. Chem. Phys.* **2019**, *151* (24), 953
954 244502. 954

955 (53) Here the logic is Δr is a function of ϕ_{eff} and stickers are 955
956 allowed to fluctuate to a displacement equivalent to 1/4 or 1/3 of the 956
957 distance required for the full hop (the α^* relaxation event). So, our 957
958 choices maintain a fixed ratio of the jump length scale and adjusts 958
959 (jump length)/ d per the value of ϕ (maps to temperature). If one 959
960 chooses to implement this idea assuming, e.g., $r_L^{s'} = r_L^s + 0.10d$, or 960
961 $r_L^s + 0.20d$, this does not change the results qualitatively. 961

962 (54) Mirigian, S.; Schweizer, K. S. Dynamical Theory of Segmental 962
963 Relaxation and Emergent Elasticity in Supercooled Polymer Melts. 963
964 *Macromolecules* **2015**, *48* (6), 1901–1913. 964

965 (55) Hintermeyer, J.; Herrmann, A.; Kahlau, R.; Goiceanu, C.; 965
966 Rössler, E. A. Molecular Weight Dependence of Glassy Dynamics in 966
967 Linear Polymers Revisited. *Macromolecules* **2008**, *41* (23), 9335– 967
968 9344. 968

969 (56) Fox, T. G.; Loshaek, S. Influence of Molecular Weight and 969
970 Degree of Crosslinking on the Specific Volume and Glass Temper- 970
971 ature of Polymers. *J. Polym. Sci.* **1955**, *15* (80), 371–390. 971

972 (57) Fox, T. G.; Flory, P. J. Second-Order Transition Temperatures 972
973 and Related Properties of Polystyrene. I. Influence of Molecular 973
974 Weight. *J. Appl. Phys.* **1950**, *21* (6), 581–591. 974

975 (58) To be more accurate, because the temperatures calculated as 975
976 xT_g can be any T between the experimentally measured points, the 976
977 data were interpolated between the nearest points using the VFT 977
978 expression. In this case the error bar is not very large. The VFT fit 978
979 parameters can be found in references mentioned in the main text. 979