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A Broad Doorway to the Big Tent: A Four-Strand Model for Discipline-Based Faculty Development on Inquiry-Based Learning

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ABSTRACT
Faculty professional development is an important lever for change in supporting instructors to adopt research-based instructional strategies that engage students intellectually, foster learning-supportive attitudes and habits of mind, and strengthen their persistence in mathematics. Yet the literature contains few well-rationalized models for faculty development in higher education. We describe the rationale and design for a model for discipline-based faculty development to support instructional change, and we detail our implementation of this model as applied to intensive workshops on inquiry-based learning (IBL) in college mathematics. These workshops seek to foster post-secondary mathematics instructors' adoption of IBL, to help them adapt inquiry approaches for their classrooms, and ultimately to increase student learning and persistence in science and mathematics. Based on observed faculty needs, four strands of activity help instructors develop a mental model for an IBL classroom, adapt that model to their teaching context, develop facilitation and task-design skills, and plan an IBL mathematics course. Evaluation data from surveys and observations illustrate participant responses to the workshop and its components. The model has been robust across 15 years of workshops implemented by three generations of workshop leaders and its features make it adaptive, strategic, and practical for other faculty developers.

KEYWORDS
Inquiry-based learning; workshop; teaching professional development; faculty development

1. INTRODUCTION

In the United States as in other countries, there is widespread concern about the numbers, diversity, and preparation of the workforce in science, technology, engineering, and mathematics (STEM)—all characteristics argued to be crucial for innovation and participation in the global economy, and for solving important environmental, health, and agricultural problems. Active teaching practices that engage students in meaningful work with disciplinary ideas and practices have been shown by education research to benefit undergraduates' learning, attitudes, retention of ideas, and ultimately their persistence to STEM degrees (see, e.g., reviews

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in [19,44]). Many instructors are also attracted to these approaches by the potential they see to teach critical thinking and foster social justice. Yet most students do not experience these high-impact practices, and students from underrepresented groups are least likely to do so [32]; transmission-focused, lecture-based teaching still prevails in North American classrooms [16,49].

Thus the problem for faculty developers is to persuade and prepare instructors to adopt these research-based methods [18]. Professional development focused on teaching, or teaching professional development (TPD), supports instructors' adoption of these active learning methods [33]. In turn, by influencing classroom pedagogy and curriculum, and indirectly influencing student involvement and the campus culture of teaching and learning, faculty development has a positive effect on student learning and success ([13,45]; cited in [47]). While TPD is just one of many levers for change that have shaped the movement toward research-based instruction within the complex system of STEM higher education, it is widely felt to be one of the most influential: supporting effective instructor practice is essential for any model of educational change to scale beyond a single classroom [31,34,41]. Yet many questions remain about what forms and elements of TPD are influential on college educators' practice.

To contribute to understanding of effective TPD, we present and explain a design developed for an intensive, four-day workshop on inquiry-based learning (IBL) for post-secondary mathematics instructors. We share this design in response to Giersch and McMartin's [20] analytical review, which notes the paucity of scholarly literature discussing well-rationalized models for TPD design in higher education. We recognize that experienced faculty developers know more about best practices than is captured in the literature so far. Research to demonstrate the impact of TPD on teaching is difficult: it is challenging to characterize teaching [2], teachers may need a long time to implement and become skilled with new methods, and studies depend on established, long-lived programs that invest time and resources in TPD. Given such constraints, faculty developers cannot wait for research to tell us exactly what to do; we must experiment, learn, and get on with the work of TPD. Here we contribute our learning to the TPD design literature in higher education and suggest elements that are transferable to other topics and disciplines. We describe how these elements make this TPD model adaptive to changing instructor needs and contexts, strategic in providing a general approach to teaching that accommodates instructors' diverse circumstances, and practical in offering structured entry points for new facilitators.

2. INQUIRY-BASED LEARNING AND TEACHING IN MATHEMATICS

In mathematics, IBL is a research-grounded, student-centered teaching approach that involves students in problem solving, explaining, and critiquing as they learn key disciplinary concepts and approaches. Rather than a specific method or curriculum, a set of principles guides instructor decision-making, recently codified as the "four pillars" of inquiry-based mathematics education [36,39]:

- Students engage deeply with coherent and meaningful mathematical tasks
- Students collaboratively process mathematical ideas
- Instructors inquire into student thinking
- Instructors foster equity in their design and facilitation choices.

The form of IBL shared in these workshops for mathematics educators emphasizes classroom teaching of important disciplinary concepts and habits of mind rather than small group mentorship toward open-ended discovery, as in apprentice-based undergraduate research [1,38]. It can be applied at all levels of the curriculum, and, importantly, does not rely on instructors identifying open mathematical problems on which undergraduates can make progress. In this sense, it is an "information focused" rather than "discovery focused" form of inquiry, while still inducting students into mathematical research approaches and ways of knowing [40,48].

These core IBL principles guide student and instructor activities at the same time as they accommodate a range of classroom tactics to fit diverse students and settings. Known as the "big tent" [21,22,39], this inclusive conception recognizes that higher education instructors work in wide-ranging conditions of class size, facilities, course content, student preparation, and departmental norms. Because they also have high autonomy in teaching, instructors have substantial freedom to make adaptations to suit their setting, so they are encouraged to find the mix of teaching methods that enacts the four pillars in ways suited to their course and context. Other work suggests that attention to instructors' autonomy, their diverse educational contexts, and their commitments to reaching diverse student audiences has been essential for broadening the adoption of IBL approaches [22]. The big tent may also be important in a discipline-specific setting, where the facilitators are peers with teaching expertise in that discipline but from different institutions. Here everyone is considered to be expert about what may work best in their teaching context.

3. PROFESSIONAL DEVELOPMENT FOR TEACHING IN HIGHER EDUCATION

The workshop we describe here is focused on implementation rather than persuasion: it is designed to encourage interested instructors to use IBL and to help them adapt and implement IBL approaches in their classrooms. The four-stranded design responds to observed instructor needs and provides active and personalized learning opportunities, using strategies such as video lesson study; discussions of educational research and course design; modeling and analysis of facilitation skills; and personal course planning time. The model accommodates diverse teaching settings and adapts readily to changing external contexts. While the model is based in a discipline, rather than in a campus teaching center, it is not math-specific; however, it is likely specific to higher education in assuming that educators have solid knowledge of the content they teach. Rather, it focuses on pedagogy, an area where most university educators have little or no formal preparation [9], and where (at least in the US) participation in professional development is generally voluntary, rather than required by employers as in K–12 schools. In sum, the wide diversity of higher education teaching environments, and instructors' high autonomy, strong disciplinary knowledge and identity [11], and dearth of education about teaching are some of the features that, we argue, distinguish this context of TPD from the needs and concerns of K–12 teachers and that thus make it important to describe TPD models in higher education [20].

This article draws on practitioner observations, records, and reflections to outline the workshop's design principles, practices as implemented, and evolution over time. Evaluation research data offer an external perspective on behaviors and outcomes for workshop participants, especially measures of how the workshop and its components supported their learning. We offer evidence from participant surveys and facilitator reflections for how the workshop functions to invite participants into the pedagogical philosophy and practice of IBL. Examples of how the workshop design was responsive to evolving participant needs and supportive of facilitators' growth bolster our claim that it is a useful model for other faculty developers—where, by model, we mean a design with a structure and rationale, and thus an example or archetype that can be emulated or imitated.

4. CONTEXT FOR THE WORK

The workshop model was first designed and implemented in summer 2006 by author SY, who tested and modified it in periodic offerings through 2010, supported by the Educational Advancement Foundation. From 2013 to 2015, SY worked with the other authors—co-facilitators MGJ and CS and evaluators CNH and SLL—to refine and formally evaluate the workshop, in a project called *SPIGOT: Supporting Pedagogical Innovation for a Generation of Transformation*, supported by the U.S. National Science Foundation (NSF). Given our focus on the design of the workshop and the thinking behind that design, this paper draws primarily on participant data from the SPIGOT project, described in detail below. We also draw on unpublished evaluation data from the 2006–2010 workshops, archived documents such as workshop agendas, and our collective experiences and reflections. In large part, this paper represents our analysis of the workshop as conceived, tested, and refined.

The use of the workshop model has continued and further enriched our understanding of how the workshop model works. As of 2021, 19 intensive IBL workshops using this model in face-to-face settings have hosted over 500 mathematics instructors from the United States and Canada, and 3 more workshops were adapted for online delivery to $\sim$65 participants in summer 2020. We thus draw on experiences and formative evaluation data from the most recent workshop series (2015–2020), also NSF-funded and known as *PRODUCT: Professional Development and Uptake through Collaborative Teams*, when such data illustrate the model's flexibility in new ways discovered as new workshop leaders applied the model. New insights about the model will no doubt emerge as we continue analyzing data from that project.

Past participants in the workshops were broadly representative of the US mathematics teaching workforce at institutions that award 2-year, 4-year, and graduate degrees in mathematics. Early-career instructors were well represented, and women's participation consistently exceeded their general representation among mathematics instructors in higher education [42]. Historically we have had little difficulty filling the workshops with people interested and ready to commit time to learning and preparing to make changes to their classroom practice. Most received travel support from their institutions, while on-site costs and some travel scholarships were covered by grants to the project, noted above.

5. APPROACH AND METHODS

The workshop design is based on observations of the needs, successes, and challenges of instructors adopting inquiry-based learning. As a newer IBL practitioner himself, with experience leading TPD for secondary school teachers, author SY attended annual national meetings of IBL instructors and noticed their interests and concerns [53]. SY's initial collaborators in the workshop included a mathematics education researcher who had studied student outcomes of IBL, two experienced IBL practitioners who had mentored newer IBL users; and an evaluator. Literature on active and inquiry-based learning informed the workshop design and messages; after workshops began, feedback from program evaluation and reflective debriefings helped to further refine it. The design thus evolved as a type of scholarship of teaching and learning (SoTL), where here the learners are new IBL instructors and the teachers are the workshop designers and leaders. As in classroom-based SoTL, reflection on one's practices, experiences, and learner responses is complemented by collegial conversation and external data. Formative evaluation data were used to refine the 2006–2010 workshops to the model that was used in the workshops beginning in 2013, when the present team joined the effort.

As evidence from workshop participants, we use evaluation data from the 2013–2015 workshop series [24], using measures developed earlier [22]. Briefly, we administer surveys before people attend the workshop, immediately after it, and again a full academic year later. The pre-survey gathers information about instructors and their teaching contexts, goals, and practices. The immediate post-survey focuses on participants' workshop experiences and gains in knowledge, skills, motivation, and beliefs, which we use to monitor workshop quality and improve the workshops. The follow-up survey again probes instructors' teaching practices, so we can document changes in practice and begin to understand what personal and contextual factors help or hinder instructors in implementing IBL. The self-report measures have been triangulated in prior studies [22] and are being validated against observations [26]. Evaluators observed the workshop to document activities and behaviors that help to explain survey comments and ratings.

In this article, we draw on data from observation and the post-surveys to understand whether and how the workshop components contribute as part of the overall workshop design, and what other elements play a role. The evaluation measures were not designed to elicit specific feedback on each component, but many participants offered comments on what aspects of the workshop made it work (or not) for them. These were qualitatively coded for each workshop, and we report findings from these analyses along with illustrative quotations from participants. We report some indicators of the workshops' effectiveness in supporting instructors' IBL implementation, focusing on design elements, but do not detail evaluation outcomes that are reported elsewhere [24]. We include examples from the PRODUCT workshops where these serve to demonstrate how the workshop model could be used by people who learned and applied it later.

6. DESIGN OF THE WORKSHOP

The workshop design is based on four intertwined, topical strands that together support participants to learn about "big tent" IBL methods (Scheme 1). The TPD model centers on instructor learning, just as IBL instruction itself centers on student learning. Throughout all the strands, workshop leaders use inquiry *about teaching* as the way instructors learn about teaching, in parallel to the way instructors will use inquiry *about mathematics* as the means for their students to learn mathematics. Here the use of inquiry methods not only provides instructors with a rich personal experience of learning about teaching but also models tactics and strategies that instructors can themselves use in teaching mathematics and promotes a critical stance that is essential to high-quality inquiry [43]. Indeed, workshop leaders' use of collaborative, interactive, and reflective teaching methods was consistently recognized by participants as "practicing what they preach" and "teaching IBL by using IBL."

The workshop provides a structure for instructors to learn in depth about the broad IBL framework and, within it, to construct personalized teaching solutions that align with their specific needs and institutional environments. While the details and sequencing of these needs vary from person to person, we posit that several actions are necessary to implement an IBL course. Instructors must

- build or enrich their mental model of IBL instruction;
- identify course design and facilitation tactics to operationalize that mental model in their teaching context;
- examine their beliefs about instruction and students and align these beliefs with the IBL pillars;
- prepare a syllabus and materials to support their planned IBL course.

Each of the four topical strands of the workshop emphasizes one of these needed actions, although the strands work together to support instructors, as we discuss in detail below. Each strand is sequenced to generate a coherent flow of ideas within the strand, and the four strands are interwoven so as to build daily themes that address instructor needs and interests. In describing each strand, we first discuss the problem it addresses: What do instructors need that this strand offers? We then describe what leaders do in the workshop as we have implemented it, and share evidence for the role and contributions of the strand to the workshop as a whole. We then

Day 1 **Seeing IBL In Action**	Day 2 **Starting Your Course**	Day 3 **Building IBL Skills**	Day 4 **Moving Forward**
Opening Who is here? What is IBL?	*Reading 1* Addressing instructor beliefs about learning	*Reading 2* Inquiring about student thinking	*Reading 3* Fostering classroom equity
Video 1 Seeing successful models of IBL	*Video 3* Starting a course	*Video 5* Thinking about student thinking	*Video 6* Zooming out to see the big picture
Video 2 Seeing other successful models of IBL	*Video 4* Valuing students' mathematics	*Nuts and Bolts 3* Assessing student thinking	*Nuts and Bolts 4* Addressing individual questions & needs
Lunch	*Lunch*	*Lunch*	*Lunch*
Nuts and Bolts 1 Managing IBL classes	*Nuts and Bolts 2* Organizing an IBL class	*Plenary*	*Plenary*
Course Content 1 Gaining a broad view of course elements	*Course Content 2* Designing tasks	*Course Content 3* Choosing course materials & planning for student buy-in	*Course Content 4* Identifying next steps & reporting out
Wrap-up	*Wrap-up*	*Wrap-up*	*Closing*

Scheme 1. Idealized sample schedule for a four-day workshop about IBL teaching, based on the four-strand model (see Schemes 2–5 for strand details).

describe the linkages across strands and other workshop features that reinforce the workshop's broader themes. The graphical schemes that accompany these descriptions show the organization and content of an idealized workshop in a stabilized form comparable to workshops implemented in 2015–2018.

6.1. Video Strand

"I've never had an IBL class in my life, so I don't know what it looks like!" This is a common starting point for workshop participants. Written descriptions of IBL teaching, or course materials and syllabi, go only so far to communicate IBL teaching methods to those who have no working model in mind.

Session	Video 1	Video 2	Video 3	Video 4	Video 5	Video 6
Session theme	Seeing successful models of IBL	Seeing other successful models of IBL	Starting a course	Valuing students' mathematics	Thinking about student thinking	Zooming out to see the big picture
Video content	IBL classes where students present & discuss proofs – end of term, when class is going well	IBL classes using group work – end of term, when class is going well	First week of a course: Building student buy-in	Examples from early in the term: Ensuring early successes, building buy-in	Examples of student math mistakes: Using student ideas, encouraging productive failure	Observing an entire class period for flow & facilitation

Scheme 2. Sample sequence of sessions for the Video Strand.

The Video strand addresses this problem by providing concrete examples for instructors to study IBL teaching practices in the safety of the workshop setting. Practices seen in the videos, and raised for discussion, include pacing an IBL class, managing effective classroom discussions, and responding to challenging teaching situations, such as when students get stuck or are unprepared. Thus video sessions help instructors build and refine a mental model of how IBL works in a classroom and point to the skills and decisions instructors will make as they teach their own IBL course. By showing student work and students' interactions with the instructor and their peers, video provides realistic views of what is possible and allows participants to consider different scenarios for what may take place in IBL classrooms.

6.1.1. *What Happens in Video Sessions?*

Videos are selected and sequenced to provide examples of IBL and to surface common concerns. Across the set of videos, participants consider the instructor's teaching moves, students' engagement, and how the instructor's facilitation and task choices drive students' mathematical meaning-making. The first video segments show IBL courses when things are going well towards the end of the term, to provide some visions of success. Other sessions show how to manage the first class meeting, set tone and provide for early student successes, elicit and probe student ideas, and manage challenging moments when students get stuck and make mistakes. Finally, a whole class period shows how a typical class may go (Scheme 2).

Over time, some advantages of video as a workshop tool have become apparent, mirroring those noted in the K–12 PD literature [50]. First, it is impossible to consider teaching without considering students' thinking and behaviors. Video provides a consistent and predictable vicarious experience of how these can unfold in a real classroom, so that facilitators can plan discussions around what happens in the video. The same learning objectives may not be reliably accomplished by teaching simulations where participants role-play students, which unfold differently from group to group, and where faculty are far more expert than students will be. Second, facilitators can adapt the choice or sequence of videos to respond to participants' needs and questions. Participants' attention is focused on analyzing teaching and learning in a particular, authentic context, rather than divided between completing a mathematical task and analyzing their learning experience: they notice different things when they are watching from the balcony, so to speak, rather than dancing on the ballroom floor [27]. Finally, video is portable, convenient, and repeatable: we can watch a clip, discuss it, and watch again with new ideas in mind. For example, PRODUCT facilitators have asked participants to re-watch video of a student presenting a solution and consider how instructor or classmate responses to the presenter may be shaped by that student's identity and their own.

6.1.2. What Evidence Supports This Strand?

Video is consistently rated as "one of the best" features of the workshop because it shows "IBL in action." The Video strand is core to the workshop model because it provides relatable images and examples that serve as a shared reference point, and it highlights the consequences of instructional decisions that are examined more closely in other strands, "giving ideas of what can be done." "The video sessions helped me learn different ways to facilitate presentations and respond to students," said one respondent. Another concurred, "Seeing and analyzing videos of classrooms was the most helpful aspect—it really helped to assuage my fears of full implementation of IBL." Student behaviors seen in the videos seem to persuade participants that "regular" students like their own can participate effectively in IBL classrooms, unlike activities where seasoned mathematicians role-play IBL in working out tasks designed for students. Feedback shows that it is important for the students and instructors in the videos to be visibly diverse, and workshop leaders must point out that there are no canonical students: we all teach the particular students we get. Finally, building a video library that portrays diverse students and teachers in action with high-quality video and audio, then selecting clips of teachable moments that are well suited to workshop discussion, is not a minor effort, and collecting new classroom video is an ongoing project.

6.2. Nuts and Bolts Strand

After watching some IBL teaching in videos, a natural question arises, "How do I pull that off?" Teaching is a system, and when mental frameworks for teaching and learning change, a set of linked behavioral changes must be made to ensure that the course remains coherent and consistent. The Nuts and Bolts strand addresses these linked decisions about course logistics, as participants consider how individual course elements and their combinations enact inquiry and reflect their own philosophy of teaching and learning.

Session	Nuts & Bolts 1	Nuts & Bolts 2	Nuts & Bolts 3	Nuts & Bolts 4
Session theme	Managing IBL classes	Designing a course	Assessing student understanding	Addressing participant questions & needs
Session content	Organizing group work Choosing student presenters Think-Pair-Share as a simple tool	Analyzing syllabi for instructor choices Deciding how to organize class time	Building a syllabus Choosing what & how to assess	Rapid-fire Q&A with a panel of workshop facilitators

Scheme 3. Sample sequence of sessions for the Nuts and Bolts Strand.

Participants may be apprehensive or unaware of their course design options—assessment strategies, course syllabi, student buy-in, day-to-day course flow. Some elements—assessment, homework—have analogs in any course, but may be approached differently in an IBL course; thus the role of each course element must be thoughtfully considered [35]. Other elements are new considerations for teachers first adopting IBL. To overcome challenges such as student resistance or math anxiety, instructors must be proactive and prepared to respond if issues surface during the course. The Nuts and Bolts sessions introduce topics and help instructors make decisions about what aspects of their course design they will work on during the Course Content strand, discussed later.

6.2.1. What Happens in Nuts and Bolts Sessions?

In the Nuts and Bolts strand (Scheme 3), participants work with peers and facilitators to consider critical issues of course format and management. Using as a starting point a set of real-world examples of syllabi, assessment rubrics, and written course descriptions from a diverse set of mathematics courses, participants can ponder the pros and cons of various sets of linked choices. Time is dedicated to instructors' choices in managing small groups and whole-class discussions; selecting and managing student presentations; using simple and foundational teaching strategies such as Think-Pair-Share; deciding how to organize class time; building syllabi; assessing and grading student work; and preparing strategies to build student buy-in. In a "burning issues" panel, workshop facilitators tackle questions raised by participants in rapid-fire question-answer format.

6.2.2. What Evidence Supports This Strand?

After the Video strand, Nuts and Bolts is the workshop element participants most often mention as helpful, as in this typical comment: "The Nuts and Bolts sessions were extremely helpful. I was already sold on the IBL idea and had read a lot of the research, but the practical aspects of implementation are what I came for." In offering concrete information about the rationale for and deployment of specific teaching tactics, these sessions seem to explain participants' reports that they gain skills in the workshops [24], even though they do not practice their skills during the workshop week. With a team of several experienced facilitators sharing their IBL implementations in different courses, variations suited to different teaching contexts become visible and help to clarify how the "big tent" encompasses different tactical choices consistent with IBL principles: one size does not fit all. In this way, the Nuts and Bolts strand seeks to help instructors move from lecture to a pedagogical approach that requires changes to lesson plans and different interpersonal and facilitation skills, and it accommodates the great variety in instructors' teaching contexts by framing these as a set of decisions rather than a set of practices.

6.3. Reading Strand

Most mathematics instructors are unfamiliar with mathematics education research and base their teaching philosophies primarily on tradition or personal experience. This "math ed knowledge gap" highlights the difference between what researchers understand about how students learn mathematics, and instructors' access to and use of such knowledge to inform their teaching. Instructors may not recognize the larger failures of lecture-heavy teaching or the roots of learning challenges such as math anxiety. Thus the workshop must address fundamental instructor beliefs about how students learn and what experiences support better learning outcomes [3,12].

The Reading strand exposes participants to education research literature showing how IBL and other active learning strategies support more students to succeed than in traditional lecture courses. These findings help instructors to respond if students and colleagues question their instructional design choices, and equip them with resources beyond their personal experiences to continue their professional growth after the four-day workshop. For this reason, PRODUCT facilitators came to call this strand "Literature to Practice," emphasizing the goal of reading rather than the activity itself. While both the name and the content of this strand have evolved more than other strands over time, its core goal of addressing what instructors believe about how their students learn has remained intact. This function thus mirrors the emphasis in K–12 PD on attention to student thinking [50].

6.3.1. What Happens in Reading Sessions?

The Reading strand uses education literature to address participants' assumptions about how students learn and why they fail. Three articles, assigned as advance reading, are complemented by brief presentations of related research, and then participants discuss the findings. Using literature shows how teaching can be informed by a scientific approach to learning that goes beyond anecdotal experience. Because it's easier to argue with an author who is not present than with a colleague who is, discussing ideas from the literature helps to contain the rise of emotions when deeply held beliefs are challenged. These sessions are placed early on workshop days 2–4 (Schemes 1 and 4) to ground participants in research before moving toward practice, through clinical observation in the Video strand and the practical applications in the Nuts and Bolts and Course Content strands.

In early years, both the evidence for active learning and mathematics instructors' awareness of this research were less developed than they are today. As this evidence has accumulated and awareness has grown through disciplinary endorsements (e.g., [14]) and high-profile review studies (e.g., [19]), we shifted readings somewhat from directly confronting participants' own beliefs about learning to preparing them for conversations with students or colleagues about learning, and to helping them understand what scholars know about some nuances of effective practice. Whereas early workshops made a case for using group work and discussion in mathematics courses as a way to hear what students think, now workshops can leverage participants' prior knowledge of such approaches to deepen understanding of how to probe student thinking. Most recently, PRODUCT facilitators introduced a reading focused on equity in mathematics classrooms [10]. This prompts participants to consider how to implement group work and discussions in ways that promote equitable student outcomes and combat structures of privilege that students face.

6.3.2. What Evidence Supports This Strand?

Observations and post-survey comments show that these sessions help participants articulate the beliefs that underpin their teaching decisions, and steer them to the education literature as a source of guidance for those decisions. While some express impatience with reading the research, because they are already persuaded of the value of IBL, it is not uncommon for participants to report on follow-up surveys that they used the literature to address students' or colleagues' resistance or skepticism. In this way, the Reading strand thus addresses needs that often feel less urgent to instructors during the workshop yet provides important support needed for their implementation.

6.4. Course Content Strand

After building a mental model of IBL and its practices, instructors must get off the balcony and back onto the dance floor, to plan their course and prepare student-ready IBL course materials. They must learn how to choose, adapt or create math tasks with the right level of challenge for their courses. Adapting the IBL framework into a practical, ready-to-implement form can require large changes to existing courses.

The Course Content strand provides guidance, time and opportunity to design a target course using IBL methods and materials—crucial planning so that a first attempt to use IBL is more likely to succeed. Participants clarify the particulars of their course and make choices based on its audience and purpose. By the workshop's end, they have made some key decisions and progressed far enough to be confident in completing course preparation on their own. This decision-making raises additional issues that facilitators can address in other strands or in follow-up

Session	Reading 1	Reading 2	Reading 3
Session theme	Reflecting on instructor beliefs	Focusing on student learning	Fostering classroom equity
Session content	Overview of research results about student outcomes of IBL & how students learn Typical anchor reading: *The coverage issue*, Yoshinobu & Jones (2012)	Inquiring about student thinking Typical anchor reading: *On developing a rich conception of variable*, Trigueros & Jacob (2008)	Seven teaching practices to foster classroom equity Typical anchor reading: *How a detracked mathematics approach promoted respect, responsibility, and high achievement*, Boaler (2006)

Scheme 4. Sample sequence of sessions for the Reading Strand [10,52,54].

conversations. The Course Content strand recognizes instructors' autonomy—they are in charge of their courses and will make most decisions about them—but also supports their needs for thinking time and collegial conversation as they align these decisions with an IBL approach.

6.4.1. *What Happens in Course Content Sessions?*

Course Content sessions tend to be practical and concrete, dealing with everyday course elements: the syllabus, sequence of math tasks, assessments (Scheme 5). After an initial orientation to a library of IBL course materials, participants consider how they will structure class time and then can begin to select or construct suitable course materials. Because most IBL courses are not taught from textbooks, participants must sequence and adjust the difficulty of problems to meet their own students' preparation. In early workshops, we did not have sample materials for all courses, so participants had to write their problem sequences; now they can draw on our growing library of course resources as well as published materials in the *Journal for Inquiry-Based Learning in Mathematics*. Some time and guidance are given to building "starter problems"—smaller problems or exercises that lead students toward a larger problem or goal theorem in a course—or to adapting standard textbook problems for inquiry. A planning outline helps instructors track their course planning decisions and select those that feel most challenging as a focus for their work time in the supported workshop environment.

Open and semi-structured work time within this strand builds rapport among facilitators and participants, as facilitators circulate and check in with participants working in small groups on similar target courses. As they learn about participants' teaching beliefs and tricky situations, they can offer personalized advice and brainstorming. Allowing time for work and planning serves to apply and solidify participants' understanding, cement their commitment to the change, and develop realistic expectations of the preparation they will need to do.

Session	Course Content 1	Course Content 2	Course Content 3	Course Content 4
Session theme	Considering course design	Reverse engineering a course	Course materials & student buy-in plan	Addressing individual questions & needs
Session content	Identifying key elements of a course Reviewing IBL course case studies Work time	Goal problems & starter problems Designing tasks or prompts Work time	Investigating course materials & design choices Choosing Day 1 activities Building student buy-in Work time	Preparing a list of next steps Reporting to the workshop group Work time

Scheme 5. Sample sequence of sessions for the Course Content Strand.

6.4.2. What Evidence Supports This Strand?

Post-workshop surveys show that planning time is highly valued, an opportunity to internalize and begin applying new ideas to their courses and reflect with peers. "I am leaving with a strong plan for my course," noted one participant. Said another, "I like how we were pushed to have much of the course planning completed before we left." Participants also value their interactions with the workshop team, who are commonly described as "helpful" and "approachable," "exceptional as facilitator and coach, without playing expert." In follow-up surveys, most respondents say they implemented the materials and plans in a course they later taught.

6.5. Other Workshop Features

As Scheme 1 shows, the four strands are interwoven across the workshop days in such a way as to highlight daily themes. Day 1 offers a broad view, seeking to help instructors build a mental model of IBL teaching and begin to identify the decisions they will need to make. Day 2 emphasizes learning objectives and how to design or choose student tasks that will achieve these objectives—decisions that must be made in advance—while Day 3 emphasizes decision-making in the moment with its focus on probing student thinking. Day 4 highlights next steps in fostering an inclusive classroom atmosphere and planning ahead for implementation. Other workshop design features enhance these themes and complement the four main strands.

- An opening session introduces the big tent philosophy of IBL and acquaints everyone with each other.
- Two plenary sessions add local flavor to the workshop. A guest speaker may describe a particular course; a panel of past IBL students may share their

experiences and answer questions; or facilitators may lead an activity emphasizing mathematical thinking to model the IBL learning experience.

- Daily reflection time provides closure, invites feedback, and collects burning questions. The facilitators then meet together to review the day, discuss participant feedback, and plan adjustments based on this formative assessment.
- Facilitators make purposeful use of specific active learning structures to assign groups, foster participation, and manage discussion, both to enhance participant learning and to model structures that instructors can use with their students. To make this modeling visible, these structures are called out and recorded on a poster in the workshop room; anyone can add to the list at any time. Importantly, the workshop uses inquiry approaches to teach about teaching—the content of the workshop—not to teach mathematics.
- The workshop arrangements build community through shared housing and meals. Substantial breaks allow time for a personal conversation, an errand, or a walk. At the closing ceremony, the team awards completion certificates and congratulatory high fives. These elements foster trust and collegiality and acknowledge the hard intellectual work of redesigning instruction.

Participant comments also reflect synergies among these design elements, as in this typical example, "The workshop was run with an inquiry-based learning approach and this gave me the opportunity to know how it feels like to be a student in that environment. Having that experience is so valuable to be able to understand how to structure your own class." Another commented, "Building a community of faculty, all working on a common goal at schools all over the country—it is a great feeling to be a part of this and to know that you have tons of support for implementing inquiry-based learning."

6.6. Follow-Up Support for IBL Implementation

In the year after the workshop, the workshop leaders support participants through a cohort-specific group email list, periodically prompting participants to share their teaching experiences. The list's main purpose is to make implementing IBL doable. On average, 83% of participants contributed to the list over the year. A social network analysis of the list messages [25] shows that these discussions offer intellectual and emotional support in a safe space for brainstorming and trouble-shooting with peers and the facilitator team. As instructors share their struggles and successes, the list functions to normalize the ups and downs of trying a new teaching approach, so that people do not get discouraged and give up. Making visible many variations of IBL renders the big tent philosophy concrete and supports instructors trying more or less profound implementations. The list also connects workshop participants to other opportunities for learning and sharing about IBL, such as short courses, conference sessions, and IBL community events.

6.7. Data-Driven Decision-Making

Formative feedback from workshop participants has been essential to improving the implementation of the workshop over time. Such data-driven decision-making has led to simple changes, such as adjusting schedules to better accommodate informal conversation and rest, and longer-term efforts to build the libraries of video and course materials.

During the 2013–2015 workshops, evaluators CNH and SL reported participant feedback to workshop leaders SY, MGJ, and CS, and documented improvements as these leaders refined the workshop design and implementation. For example, participant data identified common concerns about IBL, so the leaders chose discussions and examples to directly address those concerns. To address student resistance, the most common and least easily dispelled concern, leaders more directly addressed ways to shape students' affective responses to new kinds of instruction and developed a checklist for participants to create their student buy-in plan. To better support instructors working in diverse teaching settings, leaders incorporated wider-ranging examples of IBL in varied contexts, helping participants to more readily see their options within the IBL big tent. They gathered more materials from a growing range of IBL courses. The impact of these changes could be seen in subsequent workshops [23]. In turn, the facilitators' use of feedback helped the evaluators improve their strategies for reporting and visualizing key indicators.

For summative evaluation, overall quality ratings for the workshops were consistently high [24]. While drops in such ratings can signal trouble, the open-ended comments are more informative, showing where improvements were needed or had made a difference. Indeed, negative comments about particular sessions diminished each year, so that by 2015, the only remaining issue was that participants wanted more: more examples, more videos, more time. Instructors' self-reported IBL knowledge, skills, and beliefs rose during the workshop; their motivation to use IBL began and remained high. Skills continued to rise in the follow-up year as participants implemented IBL [24].

A key measure of impact was the proportion of participants who implemented IBL. Among 2013–2015 participants who answered the follow-up survey, 95% had implemented at least some IBL methods, and 62% had taught at least one course that they considered "full IBL." These self-report figures are broadly corroborated by analysis of participants' email comments, which show that at least 72% of all participants implemented IBL in the first year—far exceeding early predictions of 15–20% implementation, and with remarkably little variation by characteristics of the instructor or their institution [24]. Email follow-up support was particularly strong and seemed to increase implementation [22,25]. Finally, participants reported the numbers of courses and students reached by their IBL work. With a mean per instructor of 2.5 courses and over 60 students experiencing IBL in the first year alone, the numbers quickly add to hundreds of courses and thousands of students benefiting each year, nationwide, and continue to mount over time. More detailed discussion of these outcomes is available from the sources cited. Additional longitudinal analyses across multiple generations of the workshop incorporate data from the PRODUCT workshop series also based on the model described here, and this expanded data set is the subject of ongoing work that applies a psychological model to analyze changes in individuals' instructional behavior [6,7].

7. DISCUSSION

Drawing on the descriptions and evidence provided above, we discuss how the model is adaptive, strategic, and practical. It is adaptive because new workshop activities and content can be implemented in response to changes in participants' needs and contexts over time; strategic because it promotes a "big tent" approach to teaching and learning that adapts to instructors' diverse settings and circumstances, and practical because it offers a way to prepare new workshop leaders.

7.1. An Adaptive Approach to Workshop Implementation

The four-strand workshop design is both robust and flexible. It has been effective across two dozen distinct instantiations over 15 years of work with instructors, while some specifics of implementation have evolved with changes in the external context. These go beyond improvements in facilitators' abilities to implement the workshop, to encompass adaptations made within the four strands that respond to changes over time in participants' needs and contexts—reflecting shifts in both the workshop audience and their external context. Such changes arise from both participant data and facilitators' insights, especially as facilitators gain experience and begin to recognize for themselves when it may be helpful, for example, to rework an activity, shift emphases in guiding the discussion of a video, or replace a reading, as PRODUCT facilitators did in a variety of situations. Their making these adaptations within the general and conceptual four-strand design demonstrates that this workshop design is not idiosyncratic; it offers a more general model that other faculty developers can use.

As an example of changing external context, as active learning strategies became more mainstream and visible in US mathematics education, participants expressed less concern about resistance to IBL from their colleagues or department heads. Yet on surveys most participants still report only passive tolerance from colleagues, not active departmental support for IBL teaching—so we continue to equip them with some awareness of the research base supporting IBL, to help them meet concerns in their home department, while lessening our emphasis on the research base as a tool to persuade them. The Reading strand has been the main vehicle for making this adaptation over time. As individual readings are shifted to respond to instructors' changing needs and contexts, the core goals remain—to draw out instructor beliefs and to engage with the educational literature as a resource for learning beyond instructors' own personal experience.

We also saw on surveys that, over time, participants came to the workshop with more general knowledge of active learning, but less personal experience with IBL approaches—an example of a shift in the audience. The workshop builds on their initial knowledge and interest to emphasize a set of principles for inquiry learning, the four pillars of inquiry-based mathematics education, as described above: student engagement and sense-making, instructor inquiry into student thinking and design for equity [36,39]. The big tent approach to IBL is likewise crucial for meeting instructors' teaching needs and opportunities. We cannot offer instructors specific steps on how to teach with IBL; rather, we help them understand and enact the four pillars in ways that are comfortable for them as teachers and suited to their students. Participants learn how to engage their students in rich mathematical tasks primarily in the Course Content strand. In Nuts and Bolts, they learn to organize class time to support student collaboration and sense-making. They study student thinking and classroom culture in the Video and Reading strands. Across all the strands, participants consider how to manage class discussions and activities in ways that value all students' contributions and elevate diverse voices. Together, these elements help instructors to overcome several of the constraints to the use of IBL identified in other research [48], especially in generating philosophical buy-in, supporting instructors' transition to inquiry, and helping them develop approaches to support students' transition. The workshop also strengthens teachers' beliefs and practices, which Spronken-Smith and colleagues identified as enabling factors [48]. In drawing out and making use of what instructors may already know about active learning, the workshop helps them move forward in their practice.

The incorporation of classroom equity topics into the workshop offers another example of how the workshop model is adaptive. In recent years, instructors' awareness and concern about inequitable educational outcomes have risen, as has their desire to teach in more inclusive ways. At the same time, research has highlighted ways that active learning environments can be more equitable than lecture-based courses yet simultaneously foster new experiences of inequity, in ways we don't currently understand [17,29,39,51]. During the PRODUCT workshop series, individual facilitators began incorporating inclusive teaching practice as an explicit topic in some sessions, especially the Reading strand, and evaluation data made clear that this topic needed to be more fully integrated into all the strands. To broaden all the facilitators' capacity around inclusive teaching, two experienced facilitators developed support materials and led a mini-workshop for the other facilitators, with project support. The goal was to help everyone identify natural opportunities within the workshop structure where equity concerns could be noticed and called out (e.g., in analyzing classroom video) or more inclusive approaches could be modeled or highlighted (e.g., in forming groups or calling on participants). In subsequent years, evaluation data showed steady improvements to participants' comments about and appreciation of this aspect of the workshop, and an increasing number of facilitators reported greater comfort in raising or responding to equity concerns, even if they did not feel they were experts.

7.2. A Strategic Approach to Teaching-related Professional Development

We argue that workshop participants' high rate of implementing IBL is a direct consequence of this principled and flexible approach to big-tent inquiry teaching. Show-and-tell dissemination of instructional innovations is not effective, because instructors do not simply adopt a teaching method wholesale, but rather adapt it to their teaching context and identity, sometimes introducing "lethal mutations" that make the method less effective [28]. Here, IBL is presented as a set of principles that can be implemented in different settings, and workshop participants are helped to find the particular version of IBL that works for them—opening a "broad door" to the "big tent." While fashioning a personalized adaptation is a harder lift for instructors than adopting a curriculum wholesale, it results in a more reflective teacher whose implementation is more personalized and more likely to be sustained.

This approach is consistent with Kennedy's [30] description of "strategic" TPD programs for K–12 teachers that convey a goal and teach illustrative practices to achieve that goal. She finds that strategic programs are more effective than prescriptive TPD programs, perhaps because the former teach a rationale to help teachers decide for themselves when and why to implement the practices. This strategic approach to TPD is possible because we conceive of IBL as a comprehensive approach and philosophy of teaching rather than a recipe, a set of procedures, or a list of tips, tricks, and tactics. Like most of the K–12 studies Kennedy examined, our TPD provided > 30 h of contact time, plus a year of follow-up support. Also like those studies, we measured teaching practices some time after exposure to the PD, attempting to account for the delayed and developmental influences of TPD on teaching. But the core of Kennedy's argument is that the theory of action behind a TPD program may be more important than program design features that are "necessary … but far from sufficient" [50, p. 794] and may in fact be "unreliable predictors of program success" [30, p. 971]. The four strands of this model focus on building instructor knowledge, strengthening their beliefs, and elevating their attention to the planning, practices and skills needed to implement IBL in a strategic, context-appropriate manner. This focus on strategic decision-making, rather than specific materials or procedures, is one element of this model that is transferable to other faculty development activities. The four strands themselves are also transferable core workshop elements:

- an experiential backbone to build mental models and establish common vocabulary, here provided by the Video strand;
- conceptual and theoretical grounding for classroom choices, here offered in the Reading strand;
- attention to classroom logistics and tactics and their implications for planning, here represented by Nuts and Bolts; and
- personal reflection and planning time, as in the Course Content strand.

Shorter workshop designs might adapt this model through steps such as reducing work time, providing conceptual grounding more concisely through summaries and pre-reading or take-home materials, and streamlining the "nuts and bolts," e.g., by selecting one type of course on which to focus. The same components could also be offered within a department or region, using an extended workshop design or learning community that explores each strand over a year or longer. With obvious modifications to the content, the design is applicable to faculty development on other teaching strategies or in other STEM fields.

7.3. A Practical Approach to Involving New Leaders

A final feature that makes the model robust is a modular design that can be readily taken on by new leaders—now extending to a third generation of workshop leaders, numbering over 25, via the PRODUCT project. Initially, new leaders take well-defined roles leading one strand; this distributes the work and simplifies onboarding of new facilitators. When they are not leading a session, team members are observing, learning about participants' needs, and circulating to help during work time. Indeed, watching each other lead different sessions offers "balcony time" of their own that helps them to notice workshop dynamics, bootstrap their skills, and appreciate team members' contributions. As they come to understand how the strands interconnect and reinforce central principles of IBL teaching, leaders develop their style; they become comfortable leading other strands, working with different teammates, and making cross-strand connections; they find new language and activities for communicating key ideas and become confident to take on other TPD roles. Dividing workshop leadership roles into conceptually coherent strands is a tactic that other workshop designers can also use.

Some of our current work focuses on this leadership capacity-building and will be the subject of future articles. Initial indicators are positive, as evaluation data from later instances of the workshop show that they continue to be well received and one-year implementation rates remain high [5,37]. This evidence offers support to our claim that the workshop design is portable to other projects: new people can quickly learn the model and lead it with good success. Indeed, in summer 2020, team members quickly and skillfully converted the intensive, face-to-face workshop for online delivery when gatherings and travel were halted by the COVID-19 pandemic. They drew upon the four pillars of IBL to hone their learning objectives and restructured the four workshop strands to devise two remote versions of the workshop—an intensive four-day workshop and an extended mini-course—that served differing needs of participants working from home with a mix of asynchronous and synchronous activities. Evaluation data indicated that the workshops guided by this framework and delivered online were as well received as the face-to-face format has been for many years [4]. This experience served as an inadvertent test of how well the PRODUCT workshop leaders had internalized and personalized the model and could adapt it for new and challenging circumstances. Their adaptations to workshop delivery are described in a practical handbook by Daly et al. [15].

8. CONCLUSION

Multiple elements of the workshop design make it a model that can be considered and adapted by other professional developers as they design workshops to support instructors in implementing a research-based approach to teaching. Workshop leaders can

- conceptualize workshop goals and select activities to meet instructor needs for knowledge, skills, and supportive beliefs as identified from experience and the literature;
- focus on guiding principles and the variety of ways those principles can be enacted to foster instructors' strategic decision-making within diverse teaching contexts;
- place workshop activities in a four-stranded model that aligns with instructor needs, structures participants' experience, and divides facilitator responsibilities into manageable units;
- teach the workshop in ways that model the teaching that instructors are encouraged to do, and be explicit about doing so;
- provide online, cohort-based support to participants in their first academic year of implementation.

The workshop model focuses on individual teachers and does not directly address the changes in higher education systems that are needed to support them [8], such as changes to faculty rewards systems that would better value evidence-based teaching, or changes to classroom facilities that would make it easier for students to interact. TPD must necessarily interact with and complement larger efforts to change higher education, both in individual institutions and in a discipline as a whole [34,46]. Distinct from K–12 school districts or state systems, providing professional development for teaching in higher education is not widely recognized as an institutional responsibility. Many efforts to date have, like this one, come from grant-funded, discipline-based efforts that are often short-lived. Thus, even as we offer the design and rationale of this workshop as a contribution to the literature, we recognize that both literature and reality are lacking sustainable models that integrate TPD with other levers of change to generate widespread uptake of research-based active instructional strategies.

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