Emergent gravity from stochastic fluctuations

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Abstract

It is possible that both the classical description of spacetime and the rules of quantum field theory emerge from a more-fundamental structure of physical law. Pregeometric frameworks transfer some of the puzzles of quantum gravity to a semiclassical arena where those puzzles pose less of a challenge. However, in order to provide a satisfactory description of quantum gravity, a semiclassical description must emerge and contain in its description a macroscopic spacetime geometry, dynamical matter, and a gravitational interaction consistent with general relativity at long distances. In this essay we argue that a framework that includes a stochastic origin for quantum field theory can provide both the emergence of classical spacetime and a quantized gravitational interaction.

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The challenges of quantum gravity

The conceptual challenges of quantum gravity motivate a rethinking of the basic premises of both quantum theory and general relativity. Puzzles arise already at the semiclassical level, for example the information paradox [1] and the firewall puzzle [2] in the context of black hole backgrounds. However, the difficulties confronted by quantum gravity are compounded when we probe beyond a semiclassical description. General relativity is a diffeomorphism-invariant theory, meaning that physical configurations are identified when spacetime points are mapped into one another in a differentiable way. Local coordinate reparametrizations provide a passive interpretation of diffeomorphisms, but in this essay we will have occasion to distinguish the passive and active viewpoints. Diffeomorphism invariance has the consequence that the Hamiltonian vanishes up to boundary terms, and in the canonical formalism rather than dynamics there are constraints on allowed states [3]. The absence of dynamics *a priori* is known as the problem of time in quantum gravity [4, 5].

Diffeomorphism invariance also leads to the challenge of identifying physical observables. If we insist that observables be invariant under active diffeomorphisms, then it appears necessary that observables be nonlocal. Observables may be integrals over spacetime, or instead may be suitably dressed [6, 7]. In a semiclassical context the identification of observables can be less problematic: In a classical spacetime background we can choose a coordinate frame, and local observables are meaningful with a fixed metric in a specified coordinate frame.

As a quantum field theory, the nonrenormalizability of general relativity perturbed about a background spacetime is well advertised. Nonrenormalizability is often taken as a hint of new physics at short distances. In the case of gravitation there is a possibility of an ultraviolet fixed point in the quantized Einstein-Hilbert theory supplemented by a finite number of additional higher-derivative corrections [8], a possibility which has earned some support from a number of recent investigations (for example, Refs. [9, 10]). Alternatively, quantum field theory may give way to a more-fundamental framework at short distances. It is the goal of this essay to argue for a quantum gravity framework of this latter type.

Emergent spacetime from short-distance physics

In the 1970s a generalization of what we would now recognize as the scalar part of the Dirac-Born-Infeld theory describing D-branes was put forward as a pregeometric theory from which gravitation was argued to emerge as an artifact of unknown short-distance physics that would also regularize ultraviolet divergences [11, 12]. The argument was based on mapping the theory to a curved-space field theory with an auxiliary spacetime metric, similar to the mapping between the Nambu-Goto and Polyakov descriptions of the bosonic string. Arguments due to Sakharov then suggest that a gravitational interaction is induced from quantum fluctuations [13].

The action for this theory of N scalar fields ϕ^a in D dimensions is given by,

$$S = \int d^D x \left(\frac{\frac{D}{2} - 1}{V(\phi^a)} \right)^{\frac{D}{2}-1} \sqrt{\left| \det \left(\sum_{a=1}^{N} \partial_\mu \phi^a \partial_\nu \phi^a \right) \right|}. \tag{1.1}$$

A number of detailed investigations have recently concluded via a certain mean-field approximation that the theory has a a curved-space vacuum, together with an emergent gravitational interaction [14–19].

In these analyses a composite metric is identified and is assumed to have a nontrivial vacuum expectation value. The vacuum metric satisfies a self-consistency condition by demanding that the expectation value of the composite metric operator agree with the presumed semiclassical vacuum metric. This is a kind of mean-field approximation. As a composite operator, the expectation value of the metric is sensitive to regulator-scale physics. For example, with dimensional regularization as a proxy for the physical regulator, self-consistency was found to demand that the expectation value of the composite metric satisfies Einstein's equations with cosmological constant plus regulator-suppressed corrections [19].

Observables, physical degrees of freedom and emergent gravity

With a semiclassical vacuum spacetime in hand, we can probe fluctuations about the vacuum. At this point we recall that this theory is coordinate-reparametrization invariant, so it is worthwhile discussing the nature of observables. The argument for nonlocality of observables in quantum gravity was based on the lack of a physical distinction of spacetimes related by active diffeomorphisms that move points on the manifold. However, in our theory

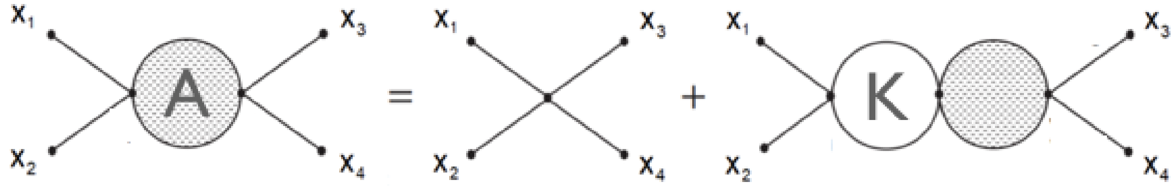

FIG. 1: Leading contributions to the four-point amplitude at large-N are summed recursively. Here, A represents the full four-point amplitude, and K represents the kernel in the recursion relation.

there is no spacetime manifold *a priori*, only a composite operator that *a posteriori* plays the role of the spacetime metric when expanded about its dynamically determined expectation value. Hence, while our theory is invariant under passive diffeomorphisms, *i.e.* coordinate reparametrizations, there is no natural notion of motion of points on a spacetime manifold except in the semiclassical description which contains the emergent spacetime geometry. In the semiclassical context local observables are well defined, so we now proceed to consider expectation values of products of fields to probe for an emergent gravitational interaction.

To do this one expands the theory about the expectation value of the composite metric operator, in which case the theory has an effective description as a field-theory in curved spacetime with scalar potential $V(\phi^a)$, supplemented by higher-dimension interactions. One can then use curved-space quantum field theory techniques to calculate perturbative contributions to correlation functions and scattering amplitudes. This was done [19] in a generalization of a flat-space analysis of Ref. [14], and it was discovered that a sum over leading contributions at large-N (see Fig. 1) contains the propagator of a massless spin-two field coupled to matter energy-momentum tensors as in general relativity, plus corrections that are suppressed by a regulator-dependent scale. This demonstrates the emergent gravitational interaction in this framework. The Planck mass which determines the strength of the gravitational interaction follows from the scattering amplitude and is sensitive to the ultraviolet regulator:

$$(M_{\mathrm{P}})^{D-2} = \frac{1}{(4\pi)^{D/2}} \frac{N}{D-4} \left(\frac{m^{D-2}}{3} \right) = \frac{V_0 D}{6m^2}, \tag{1.2}$$

where D is the number of spacetime dimensions, V_0 is a parameter in the potential, and m is the mass of the N scalar fields, taken equal for simplicity (but not required). In the case that the effective cosmological constant is tuned to zero nonlinear self-gravitational couplings were also found to agree with general relativity at long distances [17].

Prequantum pregeometry

The analysis sketched above suggests that if ultraviolet divergences are physically regularized then both a semiclassical description described by quantum field theory in a curved-space background and a gravitational interaction about that background are emergent features of this theory. At distances comparable to the regulator scale the semiclassical description breaks down. While this is suggestive, we still do not have a quantum theory of gravity without a physical regulator. In the analysis summarized above dimensional regularization was used as a proxy for a physical regulator, but at short distances the $(D-4)$-dependent corrections become more significant, and there is no reason to expect them to satisfy properties like unitarity that are important for the interpretation of the quantum field theory. Other covariant regulators, for example Pauli-Villars fields, have similar problems unless the regulator masses are taken to infinity.

It is possible that quantum field theory is itself only an approximate description of physical law. We might imagine that at short distances quantum field theory gives way to a more fundamental description that also serves to regularize field-theory divergences, in which case in a pregeometric framework like the one described above the theory would contain an emergent gravitational interaction. Edward Nelson observed [20–22] that a probabilistic description consistent with the Schrödinger equation and the Born rule follows from a time-reversible stochastic process in which the drift velocity obeys a stochastic version of Newton's second law $\mathbf{F} = m\mathbf{a}$. Guerra and Ruggiero subsequently did the same for relativistic quantum field theory [23, 24]. Some recent work extending Nelson's ideas to a curved-spacetime setting seems promising [25], but this program is not yet complete.

Suppose that stochastic processes defining states of the pregeometric theory exist. If the stochastic process were not continuous as in stochastic mechanics but discrete, causing fluctuations of the fields at random events, then the discreteness could provide a regulator for the equivalent quantum field theory description that would emerge from the stochastic process. And, with such a regulator incorporated into the pregeometric theory described earlier we would have a complete framework for quantum gravity coupled to matter.

But if the process is discrete, how is it to be consistent with coordinate-reparametrization invariance? Here we borrow from the causal set approach to quantum gravity. In order to reconcile the discreteness of the causal set with Lorentz invariance in the neighborhood of each point, the points are distributed as a "Poisson sprinkling" in the spacetime [26]. Whether or not one of these points exists in a given small Lorentz-invariant volume follows Poisson statistics. Because the probability distribution is defined per Lorentz-invariant volume, there is no preferred Lorentz frame. We can utilize the same observation in the context of a stochastic formulation of quantum field theory. Some attempts along these lines have already been made [27], but a complete description is still lacking.

Final thoughts

The possibility of a stochastic origin for quantum field theory is compelling for a number of reasons. As we have argued, discreteness of the stochastic process corresponding to a state of the quantum field theory can serve to regularize ultraviolet divergences of the theory. Such a physical regulator can then be responsible for the emergence of a gravitational interaction consistent with general relativity at long distances, as we have seen in a specific example here. The ontology of a stochastic field theory is quite straightforward: the basic objects are fields that are defined everywhere in a continuum. Every state corresponds to a certain stochastic process describing the evolution of the field configuration, with drift satisfying a stochastic formulation of the classical equations of motion. The stochastic process is generally nonlocal due to entanglement, but then again so is quantum mechanics so nonlocality of this sort should not faze us.

The action given by Eq. (1.1) has vanishing energy-momentum tensor $T_{\mu\nu} = 0$, as expected by reparametrization invariance. Beginning with an arbitrary field theory coupled to an auxiliary spacetime, the condition $T_{\mu\nu} = 0$ determines the composite metric (or vielbein up to local Lorentz transformations) in terms of the fields. Hence, the pregeometric framework can be generalized, although a stochastic regulator for reparametrization–invariant theories with fermions and gauge fields is still lacking.

If gravitation is an emergent phenomenon as advocated here, then physics at ultra-short distances is nongravitational. In the context of the early universe, we might expect a phase transition after which gravitation emerges and gives rise to standard cosmology. The absence of gravitation at early times might shed light on the observation, emphasized by Penrose for many years, that the early universe appears to have been in a remarkably fine-tuned state [28]. If the early universe was in a state in which gravitation was absent, then the maximum-entropy state of the early universe could have been uniform rather than clumped, allowing for a more-natural start to standard cosmology.

Acknowledgments

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