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Is Vector Quantization Good Enough for Access Point Placement?

Govind R. Gopal*, Gabriel Porto Villardi†, and Bhaskar D. Rao*
*Department of Electrical and Computer Engineering, University of California San Diego, La Jolla, United States
†Wireless Network Research Center, National Institute of Information and Communications Technology, Yokosuka, Japan
*ggopal@ucsd.edu, †gpvillardi@nict.go.jp, *brao@ucsd.edu

***Abstract*—We investigate the multi-faceted access point (AP) placement problems in uplink small-cell and cell-free networks in the context of throughput-optimality and to what extent the popular Lloyd algorithm from vector quantization (VQ) is suited to solve them. We develop single user throughput optimization formulations and solutions related to VQ considering the small-cell scenario, after which we expand the formulations to multiple users. For the cell-free scenario, we consider the mean throughput of all users and a max-min technique. We compare the rate performances of the AP placement solutions from both scenarios with the Lloyd algorithm. While the Lloyd algorithm is found not to strictly solve for small-cell AP locations and its approach is divergent from the cell-free perspective, we conclude from numerical experiments that it is good enough for both scenarios.**

***Index Terms*—Base station placement, Beyond 5G, cell-free, Lloyd algorithm, small-cell, throughput.**

I. Introduction

In the last two decades, massive multiple-input-multiple-output (MIMO) systems have elevated the network throughput and have gained widespread use for 5G technology [1], [2]. Distributed MIMO, as part of distributed antenna systems (DASs), offers higher average rates over its co-located counterpart [3]. Such systems can be small-cell (non-cooperative) or cell-free (cooperative) systems, the latter of which generate higher spectral efficiency [4], but suffer from limited backhaul capacity. As such, small-cell systems are still a popular choice for near future deployments of 5G technology [5].

The access point (AP) or antenna placement problem for DASs has garnered interest in the recent years, with great significance in the Beyond 5G paradigm. The main question of interest is: *How do we optimally place the APs given the distributions of users?* It can find application in large-scale events (conferences and stadiums), indoor scenarios (offices), and even in natural disaster response where existing infrastructure has been torn down. In past works, the AP placement problem for small-cell systems has optimized the per-user signal-to-noise ratio (SNR), averaged over the distribution of user positions, e.g., [6]–[10] and references therein. The authors of [6] optimize the cell averaged ergodic capacity of a DAS and use a squared distance criterion to solve for the AP positions using the standard Lloyd algorithm (with squared Euclidean distance) from vector quantization (VQ). The authors of [7] apply a VQ approach on heterogeneous wireless sensor network deployment. More recently, AP-enabled drones have gained popularity in terms of AP placement [8], [9]. A previous work by our group [10] adds throughput and delay constraints in a Lloyd-type algorithm to decrease delay without considerable loss in throughput. AP placement in the cell-free domain has been preliminarily studied in [11]. In this work, the geographical area is divided into regular grids and the throughput-optimal AP placement problem is fashioned into two compressed sensing based problems.

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The AP placement problem is diverse, in that AP locations need to be optimized for a multitude of performance indicators. For instance, throughput in both small-cell and cell-free systems addressing issues such as resource allocation, fairness, and inter-cell interference (ICI), are important. None of the above works, however, have analyzed whether the VQ framework is indeed suitable to optimize AP locations when addressing these issues. Applying the VQ approach assumes a single user model, where the distortion function considering just one user is averaged over the entire user distribution. This approach, however, is not applicable to a multi-cell system, since a user is simultaneously chosen in *each* cell to communicate with its serving AP. Hence, the VQ framework does not strictly address the multiple user aspect of small-cell AP placement. Even in the cell-free scenario in which all users are simultaneously served by all APs, the Lloyd algorithm formulation does not match the system setup.

To the best of our knowledge, the suitability of the Lloyd algorithm in VQ to both small-cell and cell-free AP placement from an optimal throughput viewpoint has not been studied in past literature. Hence, our contributions are as follows.

- We start by exploring various small-cell uplink throughput formulations in the single user case such as rate and SNR. As such, we explore the relation of VQ to problems where different distortion functions are used. Then, we expand our formulations to the multiple user scenario which addresses small-cell systems. While not ideal, the Lloyd algorithm is found to be quite successful in solving the small-cell AP placement problem.
- We then explore the average SNR maximization of all users for cell-free AP placement and compare its performance to that of the Lloyd algorithm and an existing, grid-based max-min technique. We find here also that the Lloyd algorithm, although mismatched from the placement problem, provides a reasonable solution.

II. System Model

We employ the small-cell model outlined in [10], [11] and the cell-free model in [11]. Throughout this manuscript, we use bold symbols to denote vectors, $\mathbb{E}\{\cdot\}$ is the expectation operator, $||\cdot||$ represents the ℓ_2-norm of a vector, and all logarithms are to the base 2. A geographical area is considered where K single-antenna users are distributed with a probability density function $f_{\mathbf{P}}(\mathbf{p})$, with $\mathbf{p} \in \mathbb{R}^2$ as the random vector denoting the position of a user. There are M single-antenna APs that serve the users in this area, where $\mathbf{q} \in \mathbb{R}^2$ is the AP location. With $m = 1, 2, \ldots, M$ and $k = 1, 2, \ldots, K$, a narrowband fading channel is considered with

$$g_{mk} = \sqrt{\beta_{mk}} h_{mk}, \tag{1}$$

where β_{mk} and $h_{mk} \sim \mathcal{CN}(0,1)$ are the large- and small-scale fading coefficients, respectively, independent of each other and over coherent intervals. All APs are connected via error-free backhaul links to the network controller, which knows the positions of all APs and users.

A. Small-Cell Model

In the small-cell system, a cell constitutes a subset of users, with the AP in each cell serving one user at a time. For uplink transmission, the received signal at AP m with k_m denoting the index of a user associated with AP m is

$$y_m = \sum_{m'=1}^{M} \sqrt{\rho_r} g_{mk_{m'}} s_{k_{m'}} + w_m, \tag{2}$$

where ρ_r is the uplink transmit power, s_{k_m} is the data symbol with $\mathbb{E}\{|s_{k_m}|^2\} = 1$, and $w_m \sim \mathcal{CN}(0,1)$ is the additive noise. We employ a matched filter at each AP m that estimates the data symbol s_{k_m} of user k_m as $\hat{s}_{k_m} = (g^*_{mk_m}/|g_{mk_m}|)y_m$. The signal-to-interference-plus-noise ratio (SINR) achieved by user k_m at AP m is then

$$\phi_{k_m} = \frac{\rho_r \beta_{mk_m} |h_{mk_m}|^2}{1 + \rho_r \sum\limits_{\substack{m'=1 \\ m' \neq m}}^{M} \beta_{mk_{m'}} |h_{mk_{m'}}|^2}. \tag{3}$$

B. Cell-Free Model

The cell-free system differs from the small-cell system in that the users are not partitioned and each AP serves all users at the same time. The uplink received signal at AP m is

$$y_m = \sum_{k=1}^{K} \sqrt{\rho_r} g_{mk} s_k + w_m, \tag{4}$$

with the same definitions as in (2) for user k. The NC receives all received signals from the APs and constructs the ZF detector, resulting in the processed signal as $\mathbf{r} = \left(\mathbf{G}^H \mathbf{G}\right)^{-1} \mathbf{G}^H \mathbf{y}$, where $\mathbf{G} = [g_{mk}]$ is a $M \times K$ matrix consisting of the channel coefficients and $\mathbf{y} = [y_1, \ldots, y_M]^T$. The achievable per-user SNR in this case is

$$\psi_k = \frac{\rho_r}{[(\mathbf{G}^H \mathbf{G})^{-1}]_{kk}}. \tag{5}$$

III. Throughput Formulations and Solutions

In this section, we illustrate various throughput formulations for AP placement and start with the single user assumption which follows VQ and progress to the multiple user scenario, both from the small-cell viewpoint. Then, the multiple user cell-free scenario is investigated. Note that details of the small-cell formulations are provided in [12]. When applying the Lloyd algorithm for AP placement, the distortion function used is the squared Euclidean distance $d_{\text{SE}}(\mathbf{p}, \mathbf{q}_{\mathcal{E}(\mathbf{p})}) = ||\mathbf{p} - \mathbf{q}_{\mathcal{E}(\mathbf{p})}||^2$, where $\mathcal{E}$ is the encoder and $\mathcal{E}(\mathbf{p})$ denotes the index of the cell of user at $\mathbf{p}$. This is the *standard* VQ problem.

A. Single User Case

We consider the rate maximization problem here and define the rate using per-user SNR $\psi_{k_{\mathcal{E}(\mathbf{p})}}$ (obtained from (3) by neglecting ICI and replacing m with $\mathcal{E}(\mathbf{p})$)

$$\mathbb{E}_{\mathcal{A},\mathbf{p}} \left\{ \log\left(1 + \psi_{k_{\mathcal{E}(\mathbf{p})}}\right) \right\}. \tag{6}$$

The rate is averaged over the user position $\mathbf{p}$ and the set $\mathcal{A} = \{h_{\mathcal{E}(\mathbf{p})}, z_{\mathcal{E}(\mathbf{p})}\}$, where $h_{\mathcal{E}(\mathbf{p})}$ and $z_{\mathcal{E}(\mathbf{p})}$ are small-scale and shadow fading coefficients. We also note that the second subscript has been dropped. After simplification, we obtain

$$\underset{\mathbf{q}_1, \mathbf{q}_2, \ldots, \mathbf{q}_M}{\arg\min} \; \mathbb{E}_{\mathbf{p}} \left\{ \log\left(\left|\left|\mathbf{p} - \mathbf{q}_{\mathcal{E}(\mathbf{p})}\right|\right|^2 + \epsilon \right) \right\}, \tag{7}$$

where $\epsilon > 0$ prevents the logarithm from approaching negative infinity. This above objective function indicates that irrespective of the distortion measure (here, the logarithm of the squared distance), the VQ approach can be used. Thus, to solve (7), we design an Lloyd-type algorithm (distortion other than squared Euclidean distance) that retains the NNC step from the Lloyd algorithm, but replaces the CC step with Majorization-Minimization (MM) iterations [13] (the logarithm is concave), and is called the *MM-Lloyd algorithm* (Algorithm 1).

Algorithm 1 MM-Lloyd Algorithm

1: Initialize random AP locations $\mathbf{q}_m^{(0)}$, $\forall m$.
2: Use the NNC to determine the cells $\mathcal{C}_m^{(i+1)}$, $\forall m$

$$\mathcal{C}_m^{(i+1)} = \left\{ \mathbf{p}_k : d_{\text{SE}}\left(\mathbf{p}_k, \mathbf{q}_m^{(i)}\right) \leq d_{\text{SE}}\left(\mathbf{p}_k, \mathbf{q}_l^{(i)}\right), \forall l \neq m \right\}.$$

3: Use MM iterations to determine the AP locations $\mathbf{q}_m^{(i+1)}$, $\forall m$, with the update equations

$$\mathbf{q}_m^{(j+1)} = \frac{\sum_{\mathbf{p}_k \in \mathcal{C}_m^{(i+1)}} w_k^{(j)} \mathbf{p}_k}{\sum_{\mathbf{p}_k \in \mathcal{C}_m^{(i+1)}} w_k^{(j)}},$$

$$w_k^{(j+1)} = \frac{1}{||\mathbf{q}_m^{(j+1)} - \mathbf{p}_k||^2 + \epsilon}, \; \forall \mathbf{p}_k \in \mathcal{C}_m^{(i+1)},$$

where $\mathbf{q}_m^{(i+1)} = \mathbf{q}_m^{(j+1)}$ after convergence.
4: Repeat from step 2 until convergence.

It is interesting to note that if the average SNR is maximized instead of rate, it can be simplified to the standard VQ problem. Further, we can consider the VQ optimization problem with an exponent χ greater than 2 for the Euclidean distance (e.g., indicating millimeter wave communications), with the updated distortion function $d_\chi\left(\mathbf{p}, \mathbf{q}_{\mathcal{E}(\mathbf{p})}\right) = \left|\left|\mathbf{p} - \mathbf{q}_{\mathcal{E}(\mathbf{p})}\right|\right|^\chi$. We can solve this problem by using a Lloyd-type algorithm that uses d_χ for the NNC step and stochastic gradient descent for the CC step, and is called the *Lloyd-χ algorithm*.

B. Multiple User Small-Cell Case

The single user formulations presented above do not represent the small-cell scenario where M users (one in each cell) are served by M APs at the same time. Hence, we now consider formulations where M users are chosen together.

1) Random user selection: Let $\underline{\mathbf{p}} \triangleq \{\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_M\}$ be the set of M user locations chosen independently from $f_{\mathbf{P}}(\mathbf{p})$. Assuming no interaction among the users, then we can minimize the sum of distortions incurred by each user with its closest AP, as follows

$$\begin{aligned}\underset{\mathbf{q}_1,\mathbf{q}_2,\ldots,\mathbf{q}_M}{\arg\min}\ & \mathbb{E}_{\underline{\mathbf{p}}}\left\{\sum_{m=1}^{M} d(\mathbf{p}_m, \mathbf{q}_{\mathcal{E}(\mathbf{p}_m)})\right\} \\ &= \underset{\mathbf{q}_1,\mathbf{q}_2,\ldots,\mathbf{q}_M}{\arg\min}\ M\cdot\mathbb{E}_{\mathbf{p}}\left\{d(\mathbf{p}, \mathbf{q}_{\mathcal{E}(\mathbf{p})})\right\}, \quad (8)\end{aligned}$$

where the simplification uses the fact that each user is i.i.d. The final objective function, considering d_{SE} as the distortion function, thus is the same as the standard VQ problem.

2) Random selection of one user per cell without ICI: If we now consider that one user is randomly picked from each cell with its serving AP, then again assuming no interactions among the users (no ICI) and the joint distribution of users $f_{\underline{\mathbf{P}}}(\underline{\mathbf{p}}) = \prod_{m=1}^{M} f_{\mathbf{P}_m}(\mathbf{p}_m|\mathbf{p}_m \in \mathcal{C}_m)$, the optimization is

$$\underset{\mathbf{q}_1,\mathbf{q}_2,\ldots,\mathbf{q}_M}{\arg\min}\ \mathbb{E}_{\underline{\mathbf{p}}}\left\{\sum_{m=1}^{M} d(\mathbf{p}_m, \mathbf{q}_m)\right\}. \quad (9)$$

This optimization problem using d_{SE} can be simplified to the standard VQ optimization problem, except that d_{SE} between a user at $\mathbf{p}$ and an AP at $\mathbf{q}_m$ is pre-multiplied with a weight derived to be $w_m = 1/\Pr(\mathbf{p} \in \mathcal{C}_m) = K/N_m$, where N_m is the number of users in $\mathcal{C}_m$. A Lloyd-type algorithm, called the *weighted MSE (WMSE) Lloyd algorithm* (Algorithm 2), is thus designed which uses the above weighted distortion function for the NNC step while the CC step is unaltered from the Lloyd algorithm. *Note that in this algorithm, the weights do not remain constant and are learnt in every iteration.*

3) Random selection of one user per cell with ICI: Representative of our small-cell system model, we can now consider that due to ICI, the users do interact among each other, leading to the objective function to minimize as follows

$$\sum_{m=1}^{M}\mathbb{E}_{\underline{\mathbf{p}}}\left\{d(\mathbf{p}_m, \mathbf{q}_m, \underline{\mathbf{p}}'_m)\right\} = \sum_{m=1}^{M}\int\cdots\int d(\mathbf{p}_m, \mathbf{q}_m, \underline{\mathbf{p}}'_m) f_{\underline{\mathbf{P}}}(\underline{\mathbf{p}})\mathrm{d}\underline{\mathbf{p}}, \quad (10)$$

where the (general) distortion function uses the term $\underline{\mathbf{p}}'_m$ which denotes the set of user positions other than the user at $\mathbf{p}_m$ and $f_{\underline{\mathbf{P}}}(\underline{\mathbf{p}})$ is as above. Since the distortion function between a user and its serving AP depends on the positions of the other (interfering) users, the problem is not readily solvable. In [12], we provide a solution to the above problem by designing a Lloyd-type algorithm, where an approximation of the ICI term is added to the SNR term in the distortion function.

Algorithm 2 WMSE Lloyd Algorithm

1: Initialize random AP locations $\mathbf{q}_m^{(0)}$, $\forall m$.
2: Use the NNC to determine the cells $\mathcal{C}_m^{(i+1)}$, $\forall m$

$$\mathcal{C}_m^{(i+1)} = \left\{\mathbf{p}_k : w_m d_{\text{SE}}\left(\mathbf{p}_k, \mathbf{q}_m^{(i)}\right) \le w_j d_{\text{SE}}\left(\mathbf{p}_k, \mathbf{q}_l^{(i)}\right), \forall l \ne m\right\}.$$

3: Use the CC to determine the AP locations $\mathbf{q}_m^{(i+1)}$, $\forall m$

$$\mathbf{q}_m^{(i+1)} = \frac{1}{\left|\mathcal{C}_m^{(i+1)}\right|}\sum_{\mathbf{p}_k \in \mathcal{C}_m^{(i+1)}} \mathbf{p}_k.$$

4: Repeat from step 2 until convergence.

C. Multiple User Cell-Free Case

Here, we study the throughput formulations in the cell-free scenario where we recall that as opposed to the small-cell regime, all K users are simultaneously served by all M APs. For simplicity, we will assume that the users are approximately orthogonal such that the SNR equation (5) simplifies and removes any inter-user dependence.

1) Average SNR maximization: We begin by maximizing the average per-user SNR, with the following optimization

$$\underset{\mathbf{q}_1,\mathbf{q}_2,\ldots,\mathbf{q}_M}{\arg\max}\ \mathbb{E}_{\mathcal{B},\mathbf{p}}\left\{\psi_k\right\}, \quad (11)$$

where ψ_k is the per-user SNR from (5) and $\mathcal{B} = \{z_{mk}, \forall m\}$. Since the users do not interact with one another, the solution of the above optimization can be derived to be a colocated placement where all M APs appear at the same location assuming coherent combining at the APs. This is similar to the near-colocated solution found in the grid-based max-sum algorithm in [11] that yields high sum rate. The result here is independent of the area in which the served users are present. Thus, although (11) maximizes the sum SNR, the users that are far away from the AP location are severely affected by low throughput. Hence, the max-SNR solution for AP placement presented above is unsuitable for cell-free AP placement when fairness is taken into consideration.

2) VQ based AP placement: We can now undertake the VQ approach for AP placement by applying the Lloyd algorithm. The placement is the same as in the small-cell scenario and this single user strategy minimizes the distance of a user to its closest AP on average for the entire user distribution. By this design, the Lloyd algorithm ensures that there is one AP that is close to each user, thus providing a distributed AP solution. We also point out that the fact that neighboring APs can cooperate is not explicitly accounted for in the optimization problem, which iterates that the VQ approach is mismatched from the cell-free placement problem. Fortuitously, high resolution theory from VQ dictates that the AP density is proportional to a power of the user density and so, the many APs in areas of high user densities can cooperate with each other. However, this also means that heavy-tailed user distributions that scatter users over a large area cause lower minimum rates.

3) Minimum throughput maximization: With the VQ approach, though one obtains a distributed AP placement and can expect one AP to be close to any user, it is not clear from the formulation the trade-off made between sum (or average) rate and minimum rate. One can thus maximize the minimum rate of all users. Consequently, as outlined in [11], the max-min problem for cell-free AP placement can be designed by assuming a grid where APs can be located and using a compressed sensing based algorithm to solve for the AP locations. The grid structure leads to an approximation and grid refinement is needed to solve the max-min problem.

IV. Simulation Results

A. Small-Cell Systems

The simulation framework uses $M = 8$ APs and $K = 2000$ users, which are distributed in a geographical area of size 2×2 km^2, according to a Gaussian mixture model (GMM) $f_{\mathbf{P}}(\mathbf{p}) = \sum_{l=1}^{L} p_l \mathcal{N}\left(\mathbf{p}|\boldsymbol{\mu}_l, \sigma_l^2\mathbf{I}\right)$, where $\mathbf{I}$ is the identity matrix and L is the number of mixture components. Here, p_l is the weight, $\boldsymbol{\mu}_l$ is the mean, and σ_l is the standard deviation of mixture component l. We use $L = 3$, $\boldsymbol{\mu}_1 = [0.5, -0.5]^T$, $\boldsymbol{\mu}_2 = [0, 0.5]^T$, $\boldsymbol{\mu}_3 = [-0.5, 0]^T$, $\sigma_1 = \sigma_2 = \sigma_3 = 100$, $p_1 = 0.6$, and $p_2 = p_3 = 0.2$. The path loss model and paramaters used are from [12]. The uplink transmit power is $\rho_r = 200$ mW and pursuant to the small-cell model, one randomly selected user from each cell is served by its associated AP. For the Lloyd and Lloyd-type algorithms presented, we use the same initializations [14] for unbiased comparison and set the maximum number of iterations to 50. To evaluate the performances of the algorithms, we utilize the achievable per-user rate, given by $R_{k_m} = \mathbb{E}\left\{\log_2\left(1 + \phi_{k_m}\right)\right\}$, with ϕ_{k_m} from (3). Achievable rate is calculated for each algorithm through Monte Carlo simulations with 1000 iterations. Cumulative distribution function (CDF) plots are generated for the rate measure, though normalized by the largest value so as to focus on the relative performance of the considered algorithms.

Experiment 1. We now compare the performances of the MM-Lloyd, Lloyd-χ with $\chi = 4$, and WMSE Lloyd algorithms with the Lloyd algorithm. The final AP locations for all algorithms in Fig. 1 show that the AP locations of both the MM-Lloyd and WMSE Lloyd algorithms are placed closer to the GMM centers compared to the Lloyd algorithm and those of the Lloyd-χ algorithm are placed farther away. These effects are due to the logarithm in the MM-Lloyd algorithm which suppresses the contribution of users at larger distances (cell edge users) and the higher exponent in the Lloyd-χ algorithm that amplifies the contribution. For the WMSE Lloyd algorithm, the weights cause the objective function in (9) to be independent of the cell probabilities as opposed to that in the Lloyd algorithm. Thus, each cell can have a larger number of users leading to the APs situated closer to the GMM centers where the density of users is higher. These effects are quantified in Fig. 2 where we observe that in spite of different

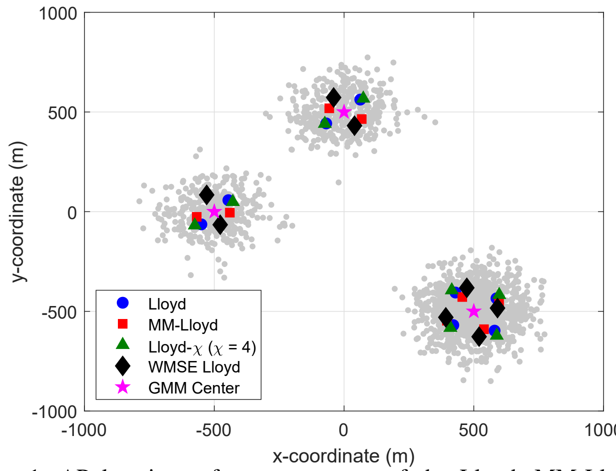

Fig. 1: AP locations after convergence of the Lloyd, MM-Lloyd, Lloyd-χ ($\chi = 4$), and WMSE Lloyd algorithms with $M = 8$.

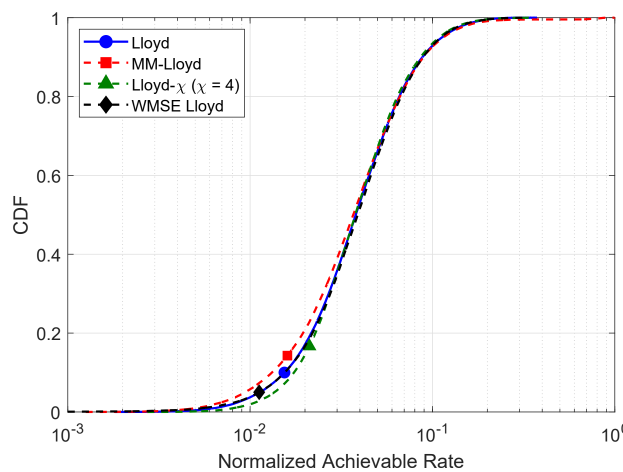

Fig. 2: CDF plots of per-user achievable rate for the Lloyd, MM-Lloyd, Lloyd-χ ($\chi = 4$), and WMSE Lloyd algorithms with $M = 8$.

TABLE I: Improvements in Average Achievable Rates for the Lloyd-Type Algorithms over the Lloyd Algorithm in the Small-Cell Case

Algorithm	MM-Lloyd	Lloyd-χ ($\chi = 4$)	WMSE Lloyd
Average Rate	4.26%	-0.64%	1.14%

AP placements, the rate performances of all algorithms are not overly dissimilar and that there is only a small difference in the lower rate values. We also calculate the percentage change in average achievable rate values for the Lloyd-type algorithms over the Lloyd algorithm, in Table I. It shows that the average rate values are not significantly different, with a maximum difference of 4% by the MM-Lloyd algorithm.

B. Cell-Free Systems

We use a simulation setup similar to the small-cell model, but with $M = 32$ APs, the GMM parameters $\sigma_1 = \sigma_2 = \sigma_3 = 200$ and $p_1 = p_2 = p_3 = 1/3$, and the pathloss model from [15, (4.34)]. For performance, we utilize the per-user achievable rate $R_k = \mathbb{E}\left\{\log_2\left(1 + \psi_k\right)\right\}$, with ψ_k from (5). The rate values are plotted for varying user transmit powers.

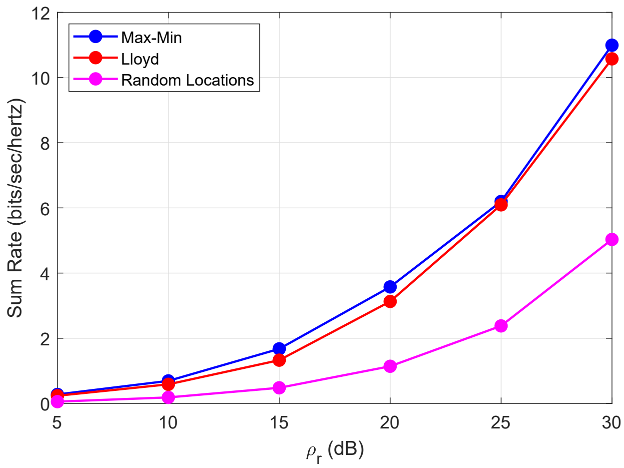

Fig. 3: Sum rates versus transmit power ρ_r for the max-min and Lloyd algorithms with $M = 32$

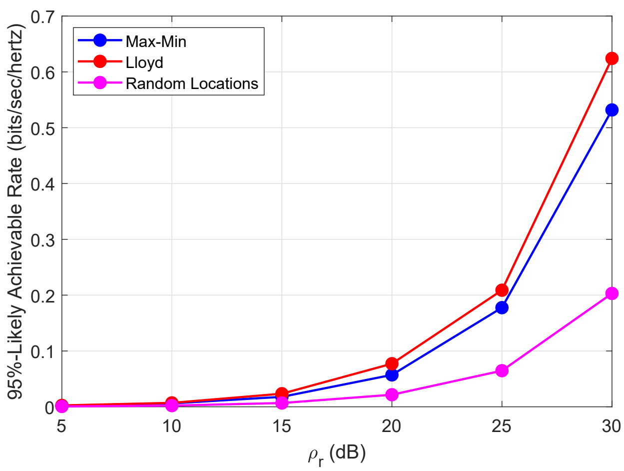

Fig. 4: 95%-likely rates versus transmit power ρ_r for the max-min and Lloyd algorithms with $M = 32$

Experiment 2. Here, we quantify the relative rate performances of the max-min [11] (with 2500 grid points) and Lloyd algorithms, and in Fig. 3, we plot the sum rates. It is observed here that both the max-min and Lloyd algorithms provide comparable performances. Fig. 4, on the other hand, plots the 95%-likely rate, which is the 5^{th} percentile of the CDF of the rate values, as a function of the transmit power, where the Lloyd algorithm shows only a slight improvement over the max-min algorithm.

We study briefly this minimum rate performance of the two algorithms. Although Fig. 4 shows that the Lloyd algorithm fares better in terms of the 95%-likely rate than the max-min algorithm, this occurrence is due to the selection of the GMM configuration as well as the grid resolution (number of grid points). Accordingly, the max-min algorithm has superior performance to the Lloyd algorithm, when the grid resolution is high and the GMM is heavy-tailed, i.e., where there are some users further away from where the majority are (figure omitted due to space constraints). This occurs because the Lloyd algorithm places APs while minimizing both the Euclidean distances as well as the probability of cells. In contrast, the max-min algorithm always places APs such that the minimum rate of each user is satisfied. However, the Lloyd algorithm still provides a reasonable solution keeping in mind the implementation complexities related to the increased grid resolution in the max-min algorithm.

V. Conclusion

In this paper, we have considered both the small-cell and cell-free AP placement problems for throughput optimality and have addressed the appropriateness of the standard Lloyd algorithm from vector quantization (VQ) to these problems. We showed in the small-cell scenario that the VQ-related single user and multiple user cases without inter-cell interference (ICI), could be solved by the VQ approach and the Lloyd algorithm performed similarly (only up to a 4% difference) from the other solutions. In the cell-free scenario, we compared the Lloyd algorithm with a max-min solution and found that their rate performances were comparable. In conclusion, the VQ approach, while not ideal, is a good solution to the small-cell and cell-free AP placement problems.

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