Sampling of Power System Graph Signals

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***Abstract*—While sampling in classical signal processing is well-developed and studied, sampling in the Graph Signal Processing framework and its application domains are fairly new. In this paper, sampling of power grids' graph signals are discussed and analyzed in the context of two applications including the recovery of missing measurements during cyber stresses and the optimal PMU placement problem. The effects of various topological and power-dynamical factors in the system are evaluated for the selection of the sampling-set for power grid graph signals and graph signal reconstruction performance. Moreover, a novel sampling-set selection criterion based on the error introduced in the process of band-limiting the graph signal has been proposed.**

***Index Terms*—Graph signal processing, graph signal sampling, graph-frequency, smart Grid, PMU placement.**

I. Introduction

The large volume of energy data generated in modern power systems provides new opportunities for improving various functions in power systems. Energy data has been represented and processed in various forms to extract useful information for various functions in the system. One of the emerging techniques to represent and analyze such data is through a Graph Signal Processing (GSP) framework [1]. GSP is a fast-growing area, which extends the concepts of classical digital signal processing to irregular, non-Euclidean, graph domains. This special type of signal processing is becoming popular for processing of signals in various fields, such as brain connectivity network analysis [2], Electrocardiogram (ECG) signal analysis [3], image, and video processing [4], as well as power systems and similar infrastructures [5]–[7].

Due to the underlying structures and the complex dynamical inter-relations among the components in power systems, graph signal (a mapping of the graph vertices into an N–dimensional vector of real numbers) presents a suitable framework for presenting and analyzing certain types of energy data from widely deployed sensors in power systems (e.g., Phasor Measurement Units-PMUs). For instance, the voltage angles of the buses can be represented as a graph signal with voltage angle values mapped to the vertices (buses) of the power system connected over the graph of system's structure (topology).

Over the past decade, GSP has extended the concepts of classical signal processing to irregular graph domains. For instance, the interpretation of the frequency domain in the context of graph signals is one of the key foundations of GSP [1]. Another key concept in GSP is the sampling of graph signals. The concept of sampling is significant in classical signal processing for its applications in analog-to-digital conversions, data compression, and more.

Sampling of graph signals can be considered as taking values corresponding to a subset of the vertices in the graph signal. For graph signals, analogous concepts and relations to classical signal processing sampling theorems can be observed. However, the selection of the sampling-set for graph signal processing is not as straightforward as the uniform sampling in classical signal processing due to the irregularity of the graph domain. In this paper, the effects of various topological and power-dynamical factors in the system are evaluated for the selection of the sampling-set for power grid graph signals and graph signal reconstruction performance. A sampling-set selection criterion based on the error introduced in the process of band-limiting the graph signal has been proposed to improve the reconstruction performance.

Being able to down-sample the power system's graph signals and to reconstruct them using the sampled data, can enable various critical functions in power systems. Here, we will particularly discuss two applications of graph signal sampling: (1) unobservable state information recovery in case of stresses, and (2) the PMU placement problem. Specifically, the first application focuses on the state recovery when PMUs become unobservable due to, for instance, cyber attacks (e.g., Denial of Service-DoS), failure of the communication link or the physical failure of the PMU. Although the unobservability of the PMUs can be due to any of the aforementioned stresses for the discussion in this paper, we will focus on cyber attacks. This problem can be fitted to a graph signal sampling-reconstruction framework by considering the buses that are not affected by the stress in the PMU network as the sampling vertices and the unobservable buses as the non-sampling vertices. Thus the measurements from the sampling-set of buses can be used to recover the unobservable ones due to the stress. Graph signal sampling-reconstruction framework is also a good fit for solving the PMU placement problem. Specifically, the sampling-set will be the buses with PMUs and the graph signal reconstruction process can provide observability for non-sampling non-PMU buses. The main contribution of this paper can be summarized as:

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- Several criteria of sampling-set selection in the context of electrical grid including network-based and power-dynamical criteria (e.g., node degree-based, page-rank-based, and load-demand-based) and a technique based on anti-aliasing filter have been discussed and compared. It is shown that the novel sampling-set selection criterion based on the error of the anti-aliasing filter has a promising performance to minimize the reconstruction error.
- The problem of recovery of power grid's sensor measurements under stresses has been formulated as a graph signal sampling-reconstruction problem and the recovery performance has been evaluated for different scenarios including clustered and randomly scattered attacks.
- The optimal PMU placement problem has been discussed and formulated in a graph signal sampling framework. To solve the optimization, a heuristic approach based on the anti-aliasing filter error selection criterion is proposed and evaluated.

II. Related Works

Although GSP has become popular in various signal processing applications in the last decade, its application in power systems has been limited. Examples of the application of GSP in power system domain include graph-filter-based modeling of power system measurements [5], [6], detecting and locating false data injection attacks in the grid [8]–[10], and GSP-based analysis for the resilience of power systems [7].

The sampling technique for the graph signals has become a topic of interest for GSP researchers for the last few years. Narang and Ortega [11] showed that the spectral domain interpretation for the sampling on *k-regular bipartite graphs* is analogous to the Nyquist criterion for down-sampling of classical discrete-time signals. Although this paper considers only a special kind of simple graph structure, the results are important for the understanding of graph signal sampling in general. Anis *et al.* [12] introduced the concept of *uniqueness set* to interpret the graph signal counterpart of the Nyquist theorem for arbitrary graphs and proposed a graph-spectral domain approach for the selection of the optimal sampling-set for graph signal sampling and reconstruction. Gadde and Ortega [13] presented a probabilistic interpretation for graph signal sampling. In [14], Chen *et al.* proposed a sampling theory for band-limited finite-length graph signals, which ensures perfect reconstruction without any probability constraints. In the subsequent works [15]–[17], the authors presented extensive analyses on various aspects of graph signal sampling including comparison among various selection criteria of sampling-set, different techniques of signal recovery, and theoretical aspects of the graph signal sampling-reconstruction process. Tanaka and Eldar [18] introduced the *periodic graph spectrum (PGS)* subspace as the GSP counterpart of *shift-invariant* subspace in classical signal processing. In this work, the author proposed a generalized graph signal sampling-reconstruction technique for the smooth graph signals that lie in the PGS subspace. The application of graph signal sampling in real-world problems is still limited. Lorenzo *et al.* [19] presented sampling on a randomly generated band-limited graph signal on the IEEE 118 bus [20] topology, and on an approximately band-limited signal of a road network topology and studied the effect of different sampling-set selections. Sakiyama *et al.* [21] applied graph signal sampling for placing sensors in a network. In this paper, we have implemented the reconstruction method developed in [14]. However, for the sampling-set selection, we have studied different techniques from the topological and the power system perspective.

III. Graph signal sampling Overview

In this section, first, some of the preliminary definitions and concepts related to GSP are reviewed, which will set the stage for discussing the mathematical foundation of graph signal sampling. A more detailed review of the basic concepts of GSP can be found in our previous work [10].

A. Graph Signals

In contrast to the definition of signals by Euclidean representation of their values in classical signal processing, in GSP, a graph signal is defined by a set of values that resides on the vertices of a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{v_1, v_2, ..., v_N\}$ is the set of all vertices and $\mathcal{E} = \{e_{ij} : (i, j) \in \mathcal{V} \times \mathcal{V}\}$ is the set of all edges. The order and the size of the graph $\mathcal{G}$ can be represented by $N = |\mathcal{V}|$ and $M = |\mathcal{E}|$, where $|.|$ denotes the cardinality of a set. The graph signal is defined as a mapping of the graph vertices into an $N-$dimensional vector of real numbers, i.e., $x : \mathcal{V} \to \mathbb{R}$. The value of the graph signal at the vertex v_n is represented as $x(v_n)$; however, we will denote it as $x(n)$ for simplicity. The weight associated with the edge e_{ij} in graph $\mathcal{G}$ is defined as w_{ij} for $e_{ij} \in \mathcal{E}$, and zero, otherwise. The edge weights can be defined in different ways depending on the applications and nature of the graph signal [7]. The Laplacian matrix, $\mathbf{L}$ corresponding to a graph is defined as following (for the element at the $i-$th row and $j-$th column): $l_{ij} = \sum_{j=1}^{N} w_{ij}$ if $i = j$ and $l_{ij} = -w_{ij}$, otherwise. The graph Laplacian $\mathbf{L}$ is a real and symmetric matrix, which produces real and non-negative eigenvalues and an orthogonal set of eigenvectors. The $k-$th eigenvalue and the $k-$th eigenvector of $\mathbf{L}$ are denoted as λ_k and $\underline{\mathbf{u}}_k$.

B. Power System Measurements as Graph Signals

Although different types of graph structures have been proposed to model the topology and the dynamic interactions of components in power grids from various perspectives, in this work, the power grid is modeled as a weighted undirected graph, $\mathcal{G}$, considering the buses of the grid as the vertices and the transmission lines as the edges of the graph. Moreover, the weight of the edge e_{ij} is defined as $w_{ij} = 1/d_{ij}$ for $e_{ij} \in \mathcal{E}$, where d_{ij} is the geographical distance between the $i-$th and $j-$th buses of the grid. In power systems, measurements, such as, the voltage magnitude and phase, current magnitude and phase, and instantaneous frequency measurements corresponding to buses of the system can be modeled as graph signals. Such measurements can be obtained, for instance, from the PMUs mounted on the buses. The analyses presented in this paper are for the voltage angle measurements of buses, which are modeled as graph signals, i.e., $x(n)$ denotes the voltage angle measurement (in degrees) of the $n-$th bus of the grid.

C. The Graph Spectral Domain

In this work, the spectral domain of a graph is defined by the graph Fourier transform. The graph Fourier transform (GFT) and the the inverse graph Fourier transform (IGFT) of a graph signal $x(n)$ is defined by the following analysis and synthesis equations, respectively:

$$X(\lambda_k) = \langle x(n), u_k(n) \rangle = \sum_{n=1}^{N} x(n) {u_k}^*(n), \quad (1)$$

$$x(n) = \sum_{k=1}^{N} X(\lambda_k) u_k(n), \quad (2)$$

where $u_k(n)$ is the $n-$th element of $\underline{\mathbf{u}}_k$, the $k-$th eigenvector of the Laplacian $\mathbf{L}$. In other words, $u_k(n)$ is a graph signal that acts a basis signal for GFT, analogous to the complex exponential in the case of classical Fourier transform. Analogous to the concept of bandwidth for the signals defined in the Euclidean domain, the bandwidth, λ_B of a graph signal can be defined as: If $X(\lambda_k) = 0$, for $k > B$, then λ_B is called the *bandwidth* of the graph signal $x(n)$. In this case, the graph signal is said to be band-limited to the graph frequency, λ_B. The set $\{\lambda_k : \lambda_k \leq \lambda_B\}$ contains B number of significant graph-frequency components in the graph signal.

D. Sampling of Band-limited Graph Signals

Sampling a graph signal can be defined as considering graph signal values corresponding to a subset $\mathcal{S}$ of the set of all vertices $\mathcal{V}$. According to the Nyquist criterion in classical signal processing, while down-sampling a signal by a factor d, the signal needs to be band-limited within $\frac{\pi}{d} radian/sample$ for being able to be perfectly reconstructed from its down-sampled version [22]. If the signal is not band-limited to $\frac{\pi}{d} radian/sample$, overlapping would occur in the spectral domain during the down-sampling process causing aliasing. To avoid aliasing signals can be made to be band-limited by discarding insignificant high-frequency contents over $\frac{\pi}{d} radian/sample$. If the frequency component beyond $\frac{\pi}{d} radian/sample$ is not insignificant, the signal should not be down-sampled at a rate of d. Similarly, down-sampling of graph signal creates aliasing in the graph-spectral domain unless the signal is band-limited to a certain frequency. Narang and Ortega [11] showed that for $k-$*regular bipartite* graphs the phenomenon is the same as Nyquist criteria when every d vertices are sampled, However, for the arbitrary graphs, the scenario is not directly analogous to the $\frac{\pi}{d}$ limit. For the method implemented in [14], if the graph signal is band-limited to B graph-frequency components, then the number of sampling points N_s should not be less than B (i.e. $N_s \geq B$).

Let $x(n)$ be a graph signal approximately band-limited to λ_B, i.e. $X(\lambda_{k_u}) \ll X(\lambda_{k_l})$, for $k_u > B$ and $k_l \leq B$. Since the signal does not have significant frequency contents beyond λ_B, discarding those frequency components would not distort the signal notably; however, similarly to the case of sampling in classical signal processing, these insignificant frequency components cause aliasing during the sampling process, which makes reconstruction impossible. To avoid this situation and to be able to reconstruct the original signal from its samples, we discard the high-frequency components of the original signal using an anti-aliasing graph filter. The frequency response of the proposed anti-aliasing graph filter is $H(\lambda_k) = 1$, for $\lambda_k \leq \lambda_B$ and zero otherwise. The band-limited graph signal $x_{BL}(n)$, which is obtained by filtering the original graph signal $x(n)$ can be described in the GFT domain by $X_{BL}(\lambda_k) = H(\lambda_k) X(\lambda_k)$. The set of vertices to be sampled, $\mathcal{S}$, is an indexed set with the $i-$th member of the set denoted as s_i. As such, the sampled graph signal can be expressed as $x_s(n) = x_{BL}(n)$ if $n \in \mathcal{S}$ and $x_s(n) = 0$, otherwise.

The selection of vertices to be sampled, $\mathcal{S}$, can be based on various criteria considering the topology and physics of the system. In this work, we have compared different types of criteria for the selection of $\mathcal{S}$ (discussed in Section IV). The reconstruction process estimates the original band-limited signal values from the sampled signal $x_s(n)$. The reconstructed signal can be defined as $x_{re}(n) = \mathscr{R}(x_s(n))$, where $\mathscr{R}$ is the reconstruction operator that acts on the sampled signal. Note that the aforementioned descriptions of $x_s(n)$ and $x_{re}(n)$ provide conceptual definition of the graph signal sampling and reconstruction process. In this work, we have implemented both of the operations following the approach suggested in [14] based on matrix multiplications as discussed next.

The sampling process corresponds to the matrix multiplication $\underline{\mathbf{s}} = \mathbf{\Psi} \underline{\mathbf{x}}_{BL}$, where $\underline{\mathbf{x}}_{BL}$, a $N \times 1$ vector, is the vector form of the graph signal $x_{BL}(n)$ and $\mathbf{\Psi}$ is a $N_s \times N$ sparse matrix. The entry at the $i-$th row and $j-$th column of $\mathbf{\Psi}$ is defined as: $\psi_{ij} = 1$ if, $j = s_i$, and $\psi_{ij} = 0$, otherwise. The $N_s \times 1$ vector $\underline{\mathbf{s}}$ contains the non-zero values of the sampled signal in the order of the indexed set, $\mathcal{S}$. The reconstruction process is implemented by $\underline{\mathbf{r}} = \mathbf{U_{N_s}} (\mathbf{\Psi} \mathbf{U_{N_s}})^{-\mathbf{1}} \underline{\mathbf{s}}$, where $\mathbf{U_{N_s}}$ is a $N \times N_s$ matrix containing the first N_s eigenvectors, $\{\underline{\mathbf{u}}_k : k \leq N_s\}$ of the Laplacian matrix $\mathbf{L}$ in its N_s columns. For the application of graph signal sampling in this power grid we define $x_{re}(n) = x(n)$ if $if \;\; n \in \mathcal{S}$ and $x_{re}(n) = r_n$, otherwise, where r_n is the $n-$th element of the vector $\underline{\mathbf{r}}$.

IV. Implementation of Graph Signal Sampling

In this work, we have implemented the technique described in the previous section for power system graph signals. The voltage angle measurement of each bus for the IEEE 118 bus system has been considered as the graph signal, $x(n)$. Simulations have been performed in MATPOWER 6.0 [23].

In our work, we have designed the anti-aliasing filter to obtain $x_{BL}(n)$ to be band-limited within $B = N_S$ graph-

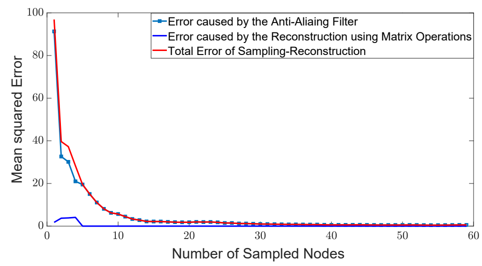

Fig. 1. Decomposition of reconstruction error for graph-signal sampling as a function of number of sampled nodes, N_s.

frequency components. In this sampling reconstruction process, the total error in the reconstructed signal consists of two errors: 1) error caused by the anti-aliasing filter for band-limiting the graph signal, 2) error for reconstructing of the non-sampled vertices in the matrix multiplication process. We observe that, as N_s increases, the first type of error decreases since more high-frequency components are being allowed by the anti-aliasing filter. However, the second error depends mainly on the selection of the sampling-set, $\mathcal{S}$, and is relatively negligible compared to the other error. The comparison of the two types of error is shown in Fig. 1.

We have implemented several criteria for the selection of the sampling-set, $\mathcal{S}$, and proposed a novel sampling-set selection criterion based on the average error introduced by the anti-aliasing filter calculated from the historical data in different buses. To present the selection criteria, let us define an operator, $\mathscr{F}$ that operates on a finite-length real-valued vector to obtain the indices of the values sorted in descending order. For example, consider a vector, $\underline{\mathbf{q}} = [77 \quad 92 \quad 28 \quad 55]^T$, then $\mathscr{F}(\underline{\mathbf{q}}) = [2 \quad 1 \quad 4 \quad 3]^T$. Next, we will discuss various selection criteria. Note that except the random selection criterion, the rest of the sampling-set selection strategies are new techniques proposed and evaluated for sampling of power grid graph signals in this paper.

1) Random Selection of $\mathcal{S}$: Among the N vertices, N_s vertices are selected randomly (based on the uniform distribution over the vertices) to be sampled as discussed in [15], [24].

2) Degree-based Selection of $\mathcal{S}$: The vertices with higher node-degree are selected to be sampled first. Let, $\underline{\mathbf{d}}$ be the vector form of the graph signal $d(n)$, where $d(n)$ indicates the node-degree of the $n-$th vertex. Hence, if $\underline{\mathbf{d}'} = \mathscr{F}(\underline{\mathbf{d}})$ then the sampling-set can be defined as:

$$\mathcal{S} = \{v_n \in \mathcal{V} : n \in \{\text{First} \quad N_s \quad \text{elements of} \quad \underline{\mathbf{d}'}\}\}. \qquad (3)$$

3) Page-rank-based Selection of $\mathcal{S}$: The vertices with higher page-rank centrality measure values are selected to be sampled first. Let $\underline{\mathbf{p}}$ be the vector form of the graph signal $p(n)$, where $p(n)$ indicates the page-rank value of the $n-$th vertex. If $\underline{\mathbf{p}'} = \mathscr{F}(\underline{\mathbf{p}})$, the sampling-set can be defined as:

$$\mathcal{S} = \{v_n \in \mathcal{V} : n \in \{\text{First} \quad N_s \quad \text{elements of} \quad \underline{\mathbf{p}'}\}\}. \qquad (4)$$

4) Load Demand-based Selection of $\mathcal{S}$: Let $\underline{\ell}$ be the vector form of the graph signal $l(n)$, where $l(n)$ indicates the load demand of the $n-$th bus. If $\underline{\ell'} = \mathscr{F}(\underline{\ell})$ then the sampling-set can be defined as:

$$\mathcal{S} = \{v_n \in \mathcal{V} : n \in \{\text{First} \quad N_s \quad \text{elements of} \quad \underline{\ell'}\}\}. \qquad (5)$$

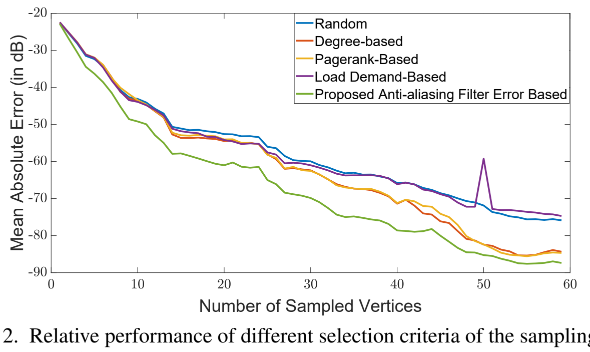

Fig. 2. Relative performance of different selection criteria of the sampling-set, $\mathcal{S}$ in terms of the mean absolute reconstruction error (in dB).

5) Anti-Aliasing Filter-based Selection of $\mathcal{S}$: According to this criterion for selecting $\mathcal{S}$, data on the output of anti-aliasing filtering applied to instances of system's graph signals have been collected. The vertices are then sorted based on the average amount of error introduced by the filter. From the sorted set, the vertices with the largest average errors are selected as the sampling-set, $\mathcal{S}$. The rationale behind this criterion is that as the anti-aliasing filter discards the high graph-frequency components from a graph signal, the vertices with a larger amount of errors are corresponding to the regions where signal values are rapidly changing with respect to the neighboring vertices. As such, retaining the values on those vertices keeps the overall sampling-reconstruction error lower. Let $\underline{\mathbf{a}}$ be the vector form of the graph signal, where $a(n)$ indicates the average error caused by the anti-aliasing filter at the $n-$th bus. If $\underline{\mathbf{a}'} = \mathscr{F}(\underline{\mathbf{a}})$ then the sampling-set is:

$$\mathcal{S} = \{v_n \in \mathcal{V} : n \in \{\text{First} \quad N_s \quad \text{elements of} \quad \underline{\mathbf{a}'}\}\}. \qquad (6)$$

Fig. 2 illustrates the performance of the sampling-reconstruction process for these criteria in terms of the mean absolute sampling-reconstruction error expressed in (dB) for a different number of sampling vertices, N_s. It can be observed that the performance of the bus load demand-based criterion is quite similar to the uniform random selection of sampling nodes [15], [24]. However, the topology-based criteria (node degree and page-rank based) performs better than the random selection and load-demand-based criterion. The proposed criterion based on anti-aliasing filter error outperforms both the load demand-based and the topology-based criteria.

V. Graph Signal Sampling Applications

A. Missing Data Recovery due to Cyber Stresses

The graph signal sampling and reconstruction process implemented in the previous sections can be applied to recover or estimate the system measurements during stresses, for instance, cyber-attack on the PMUs. In our previous work [25], we showed that correlations among the states of buses (i.e., PMU time series) can be used to estimate the missing PMU measurements in the smart grid. Here, we demonstrated an alternative approach using graph signal sampling.

Let $\mathcal{A}$ be the set of all buses under cyber-attack in the grid at time t_A. Our goal is to estimate the bus voltage angle of any bus $i \in \mathcal{A}$ at any time instant, t, for $t > t_A$ using the graph signal reconstruction technique discussed in the previous section. In the graph signal sampling-reconstruction framework, we consider the buses under cyber-attack as non-sampling vertices and the buses, which are not under cyber-attack, as the sampling vertices, i.e., $\mathcal{A} = \mathcal{V} \backslash \mathcal{S}$ and $\mathcal{V} \backslash \mathcal{A} = \mathcal{S}$. Fig. 3 shows an example of the missing voltage angle recovery using graph signal sampling. Fig. 4 illustrates how the location of the attack affects the recovery performance. Using this approach, it can be observed that clustered cyber attacks can cause more recovery error than random attacks of the same size. Moreover, the vulnerable locations in the grid can be identified from the buses with higher reconstruction error.

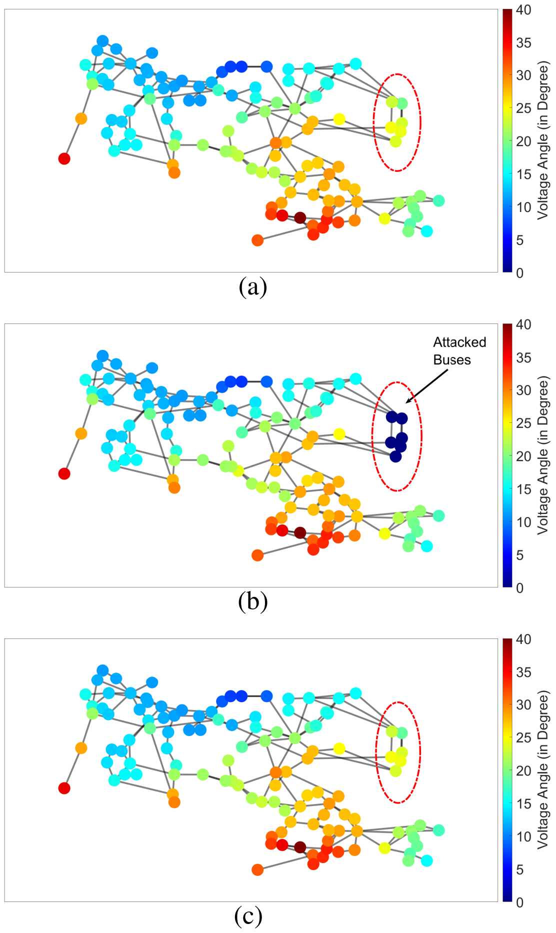

Fig. 3. An example of missing PMU measurements recovery by graph signal reconstruction: (a) the band-limited actual voltage angles measurements, (b) the measurements under cyber-attack at bus no. 59 to 64 (shown in dark blue), (c) the recovered measurements.

B. Optimal PMU Placement Problem

The optimal PMU placement problem involves selecting a subset of the buses for mounting PMUs to collect component measurements. In practice, a PMU can measure the complex voltage of the bus on which it is mounted, the currents entering or leaving through all the branches connected to the bus, and the instantaneous frequency of operation. In this paper, only the bus voltage angles are considered for reconstruction of the signal and the evaluation of PMU placement strategies. The key objective is to collect the maximum amount of data reflecting the grid dynamics to provide observability of the state of the system with the minimum number of PMUs. Based on various requirements of monitoring functions in the power system, various PMU placement techniques have been proposed in the literature [26]. In this work, the PMU placement problem within the graph signal sampling-reconstruction framework has been studied to find the GSP-based optimal placement of PMUs for reconstruction of the state of the whole system. The theoretical minimum number of required PMUs depends on the smoothness of the graph signal associated with the PMU measurement values (e.g., voltage magnitude, angle, frequency, etc.). If the graph signal at any time instant is band-limited to B graph frequency components, the theoretical minimum number of PMUs to be placed for the perfect recovery of the graph signal at each time instant is B. However, since the graph signals in power grids are not ideally band-limited, we design and use the anti-aliasing filters to analyze the reconstruction performance as a function of B. The value of B can be selected depending on the required precision of estimation and the details of the high-frequency components of the graph signal.

Let us consider $\mathcal{P} \subset \mathcal{V}$ to be the set of all the buses with PMUs mounted on them and representing the sampling-set, $\mathcal{S}$. The reconstruction process is equivalent to estimating the measurements of the buses with no PMU from the measurements of the PMU buses. Mathematically, this process can be captured in the form of $x_{re}(p') = \mathscr{R}(x(p)), \quad \forall p \in \mathcal{P}, \quad \forall p' \in \mathcal{V} \backslash \mathcal{P}$, where $\mathscr{R}$ represents the estimation function described in Section III that estimates the measurements of the buses with no PMUs from the measurement of the PMU buses. In this framework, the PMU placement problem can be formulated as an optimization problem of minimizing the graph signal reconstruction error with the minimum number of PMUs as follows:

$$\min_{\mathcal{P}} \sum_{p' \in \mathcal{V} \backslash \mathcal{P}} [x_{re}(p') - x(p')]^2 + \lambda |\mathcal{P}|, \tag{7}$$

where λ is the Lagrange multiplier and $|\mathcal{P}|$ denotes the cardinality of the set $\mathcal{P}$. In the practical setting, in addition to minimizing the error of estimating the measurement values at the buses with no PMUs, several aspects are to be considered regarding the observability and implementations issues. These aspects can be considered as the constraints of the optimization problem in (7). In this paper, we are considering two of these aspects as examples: 1) since placing a PMU at one bus ensures full observability of the voltage angle of its $1-$hop neighbors, if a PMU is placed at bus n, the $1-$hop neighbors of n are not considered as PMU bus, 2) a radial bus (i.e., vertex with degree 1) is not considered as a PMU bus [27]. The optimization in (7) can be expanded based on the reconstruction process discussed in Section III-D and shown to be an NP-hard problem [19]. Here, we propose a heuristic for sampling-set selection based on the anti-aliasing filter error criterion described in Section IV-5. According to this technique, we consider buses one by one for placing PMUs in the sequence of the vector, $\mathbf{a}'$. If a bus is a radial one or is at $1-$hop distance of an already placed PMU, the bus will be skipped and the next bus is considered for placing a PMU.

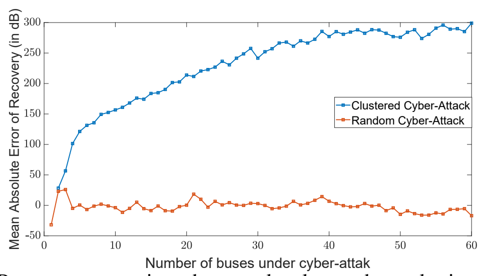

Fig. 4. Recovery errors in cyber-attack: clustered attacks introduce larger errors than random attacks of the same size.

Fig. 5 illustrates the mean absolute reconstruction errors as a function of the number of PMU placed in the grid for directly applying the anti-aliasing filter error based criterion and for the modification of the anti-aliasing filter error based criterion considering the two previously stated aspects of PMU placement. According to the modified criterion, when some of the buses are equipped with PMUs, the voltage angle of their $1-$ hop neighbors can be directly calculated using Kirchhoff's law, and the voltage angles of the rest of the buses are estimated using the graph signal reconstruction method. In Fig. 5, the mean absolute reconstruction errors for the modified case are calculated for the unobservable buses only. From Fig. 5 the number of PMUs can be chosen depending on the desired application and the budget. In our case, we suggest placing 36 PMUs in IEEE 118 bus system according to the modified anti-aliasing filter error based criterion with an average error of 0.5^0 for the non-PMU bus voltage estimation, which is acceptable for many applications (e.g. real-time performance monitoring and trending, small-signal stability monitoring, voltage stability monitoring/assessment, etc [28]).

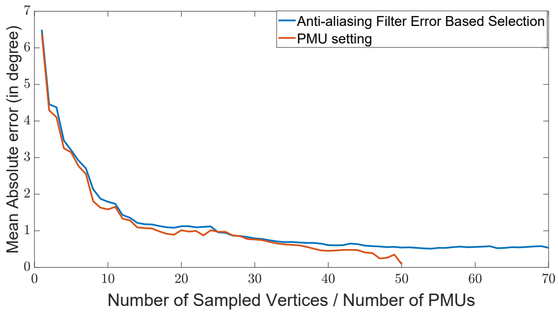

Fig. 5. Error of reconstruction in PMU placement setting.

VI. Conclusion

In this paper, a graph signal sampling technique has been discussed to apply to the graph signals associated with the voltage angle measurements in power grids. Several criteria based on the topology and power-dynamical properties of the electric grid for selecting the sampling-set have been studied to evaluate their effects on the graph signal reconstruction performance. Specifically, a criterion based on the anti-aliasing filter error has been proposed that minimizes the reconstruction error of sampling. The application of the proposed sampling framework in recovering missing measurements during cyber stresses has been proposed and studied. As the second application, the PMU placement problem has been captured in a graph signal sampling framework. An anti-aliasing filter error-based criterion has been proposed for PMU placement to minimize the measurement reconstruction error.

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[28] PARTF: PMU Data Quality: A Framework for the Attributes of PMU Data Quality and a Methodology for Examining Data Quality Impacts to Synchrophasor Applications.