

Data-Driven, Multi-Region Distributed State Estimation for Smart Grids

Md Jakir Hossain and Mahshid Rahnamay-Naeini

Electrical Engineering Department, University of South Florida, Tampa, Florida, USA

mdjakir@usf.edu, mahshidr@usf.edu

Abstract—Real-time wide-area monitoring of smart grids demands a low latency data processing of power system data. To enable the low latency requirements and to avoid the large overhead of communicating a large volume of time-sensitive data to central processing units, distributed and local processing of data is a promising approach that can improve system monitoring functions. Data-driven state estimation in power systems is an example of functions that can benefit from distributed processing of data and enhance the real-time monitoring of the system. In this paper, distributed state estimation is considered over multi-region, identified based on geographical distance and correlations among the state of the power system's components. Bayesian Multivariate Linear Regression (BMLR) combined with Auto-Regressive AR(p) process for distributed state estimation is considered over the multi-region power system. The performance of the distributed data-driven state estimation method and the role of regions are evaluated using the IEEE 118 test case under normal conditions as well as partially unobservable scenarios.

Index Terms—Distributed State Estimation, Smart Grid, Phasor Measurement Units, Cyber and Physical Stresses.

I. INTRODUCTION

An important function of the wide-area monitoring systems (WAMS) in power grids is monitoring the state of operational conditions of the system. The information provided by these systems to operators and other control and management functions is essential for well-informed decision making and reliable and efficient operation of power systems. The conventional state estimators, which have been widely deployed in utility control centers, have been in use to help with monitoring the state of the system. In addition to conventional model-based state estimators, data-driven state estimation (SE) methods have been proposed to use the large volume of data collected from the large deployed monitoring sensors, such as Phasor Measurement Units (PMUs).

Many of the data processing and computations related to monitoring the power systems have been traditionally performed centrally in utility-owned servers or cloud platforms. Communicating the large volume of data and their processing in central units, inevitably adds delay and inaccuracies in the SE and monitoring of the system. For certain time-sensitive functions, such delays could cost the reliability and stability of the system. To address this issue, distributed and local processing of data can provide a good solution, especially for time-sensitive functions. Data-driven SE is one of such functions that can improve the response time of the system,

particularly, to critical conditions, such as failures or cyber-stresses. The data-driven distributed SE can provide a faster and more accurate estimation of current conditions in the local regions as well as predicting future trends in state changes to identify abnormal conditions. New technologies, such as Edge or Fog computing, can provide a platform to enable these functions by local and distributed processing of data to enhance system monitoring in power grids [1], [2].

To enable local processing of data for distributed state estimation (DSE), local neighborhoods or regions need to be defined over the physical layout of the power system. This will also allow for provisioning the computational resources required for each region and their placement (for example in an edge computing platform). While the partitioning of the power system to regions can be dictated by physical, geographical, or economical constraints due to the layout of the power system and the communication and computing systems' resources, the physics of the system and relations and interactions among the components can also be considered in defining the regions as they can affect the accuracy of the estimations. In this work, we consider the geographical properties and the correlations among the PMU time series when the regions are defined over the power system for DSE purposes.

In this paper, a Bayesian Multivariate Linear Regression (BMLR) approach combined with the auto-regressive AR(p) process is considered and evaluated for DSE. The estimation method is applied to the local measurements in the defined regions to obtain the local estimates that collectively will form the SE of the whole system. The performance of the local estimates has been compared with the performance of the same method if it was to be deployed centrally using the data from the whole system. It has been shown that the local estimates can provide faster and better accuracy in the estimations compared to the case that all the information was used centrally for the estimates. We have evaluated the performance of the estimation model under normal operating conditions and partially unobservable scenarios. In this paper, partially unobservable scenarios refer to cases in which the data from a subset of PMUs become unavailable due to, for example, cyber stresses such as the denial of service (DoS) attack or physical failures of the PMUs.

The rest of the paper is organized as follows. Section II presents a brief literature review on data-driven SE. Section III presents the strategy to partition the grid into multiple regions. Section IV presents the distributed estimation model.

Finally, Sections V and VI present the numerical results and conclusion, respectively.

II. RELATED WORK

Power system state estimation has been the focus of researchers for several decades. Conventional power system SEs rely heavily on the power system models including the connectivity, attributes, and operating conditions of the components. A review of various methods for model-based power system SE can be found in [3], [4]. Many of the model-based SE techniques are based on steady-state analyses. However, steady-state analyses cannot be accurate for modern power systems due to highly dynamic and stochastic variations introduced by, for instance, Distributed Energy Generations (DEGs) and fast-changing loads. Besides, the deployment of PMUs and the availability of a large volume of measurement data, introduce new opportunities for improving and complementing the conventional model-based SE in power systems. As such, data-driven and machine learning-based SE techniques are gaining more attention in the literature [5]–[8]. Some of the benefits of data-driven approaches include robustness against frequent topology changes and missing or inaccuracies in system information, dynamic security assessments, and situational awareness against cyber/physical stresses [9]–[12]. As the focus of the current paper is on data-driven SE, next, we briefly review some of the data-driven SE methods in the literature.

In the data-driven SE context, some techniques are centralized and use the data from the entire system. Many of the centralized SE techniques are focused on addressing the challenges regarding data quality issues, such as non-Gaussian measurement noise, bad data, cyber-stresses, missing data, and also frequent topology changes. Examples of such techniques based on forecasting aided and predictive information filters are [9]–[14]. In [12], [14], the authors have proposed a centralized data-driven dynamic SE based on component's state correlations over the continuous data-streams from the PMUs. They also showed strategies to detect and locate cyber and physical stresses by observing the instantaneous changes in the cross-correlation of the PMU measurements. The works in [10], [11] address the challenge of partially unobservable systems under the influence of joint cyber and physical stresses using a minimum mean square estimator and a Bayesian linear regression framework, respectively. Among centralized techniques, Kalman Filters (KF) and its variants have also been widely adopted to address the state estimation problem [8], [15]. For instance, in [8] a variation of KF based on the Koopman Operator-Theoretic Framework has been proposed, which can capture the nonlinear dynamics of the power system on a centralized data-driven SE framework.

Communicating and centralized processing of the large volume of data generated in the system may not satisfy the response-time requirements of time-sensitive functions such as those for WAMS. To enhance the centralized data-driven SE in accuracy, robustness, and response time, various distributed data-driven SE methods have been proposed. These

techniques can be considered as fully distributed or multi-region distributed SE. An example of a fully distributed SE can be found in [16], which uses predictive information filtering. The work in [17] uses diffusion-based KF also for fully distributed SE in which a selected set of nodes in the system are allowed to share a subset of intermediate estimates to their neighbors using information propagation strategies.

Some distributed SE techniques divide the system into multiple regions/areas for SE based on various factors, such as geographical distance, operational similarity, communication resources, etc. For instance, the work in [1] discusses an early distributed stress detection and locating mechanism based on a linear predictive filter, which can be implemented over an edge computing platform for multi-area smart grids defined based on geographical distances. The authors in [18] propose a multi-area distributed SE approach with the integration of edge computing in which the local estimates have been computed using the Belief Propagation (BP) algorithm over geographically defined areas. Variants of artificial neural networks (ANNs) have also been considered for distribution system SE problems in multi-area distributed fashion [19]. Distributed cubature KF has also been studied for the SE problem in the multi-area framework for large-scale power systems in [20]. In addition to data-driven methods, distributed SE based on other techniques has also been proposed. For instance, the work in [21] presents a k-means approach that was used to partition the grid into regional subsections, and then SE was solved as a distributed convex optimization problem utilizing the Alternating Direction Method of Multipliers (ADMMs) algorithm and evaluated the performance under cyber and physical stresses.

The work in the current paper stands among the data-driven SE methods that use the local information in the defined regions over the system to provide fast and accurate state estimates. This work can enable the detection of stresses and state information recovery after stresses. Moreover, partial unobservability of the system, which can be due to cyber attacks or physical failure, has also been considered.

III. SMART GRID PARTITIONING FOR DSE

In this paper, we have considered multi-region DSE for smart grids. As such, the first step is to define and identify partitions in the system that facilitate the DSE using the local information within each region. We assume that the system consists of N buses connected using transmission lines.

As computations are supposed to be local, the geographical proximity of the PMUs is one criterion in defining the regions. To incorporate the dynamics and properties of the power system, which can affect the DSE performance, we consider the cross-correlations of PMU time series at buses as the second criterion in defining the regions. This criterion will result in more inter-related feature space for the estimation model, which can improve the estimation performance.

The grid is partitioned into R regions, where the set of regions are denoted by $\mathcal{S} = \{S_1, S_2, \dots, S_R\}$. In this paper, the role of the number of regions R on the performance of

the approach has been evaluated. To solve the partitioning problem, a density-based clustering algorithm, specifically, k-means, is adopted to find the optimal non-overlapping regions given the total number of regions R . The k-means clustering partitions the PMUs while minimizing the sum of squares of distances within a region. We use the geographical coordinates of the buses as well as the cross-correlation values in the feature space for the k-means algorithm.

This method will lead to non-homogeneous partitions. However, large differences in sizes of regions may not be suitable, for example, due to limitations of the local computing and communication resources (e.g., communication bandwidth, storage, and computational power). To address this point and to evaluate the role of homogeneous and non-homogeneous regions in DSE performance, we use a size-constrained k-means algorithm that helps in generating more homogeneous regions in size. In the first step, the k-means algorithm as discussed earlier will divide the grid into R non-homogeneous regions (assuming $N \gg R$). Note that $\lfloor \frac{N}{R} \rfloor$ represents the approximate ideal size of the homogeneous partitions. In the next step, the largest partition will be identified and $\lfloor \frac{N}{R} \rfloor$ PMUs that are closest to the center of the region will be selected and assigned to this region. This region will be marked as complete and temporarily removed from the graph. This process will be repeated for the reduced graph. Specifically, in the next step k-means is applied to the reduced graph to identify $R - 1$ non-homogeneous partitions, and then the assignments for the largest partition will be finalized, and so on. This process will be repeated until R homogeneous regions are obtained. If there are PMUs left out in the last step (due to $(N \bmod R) \neq 0$), they will be assigned to their closest region.

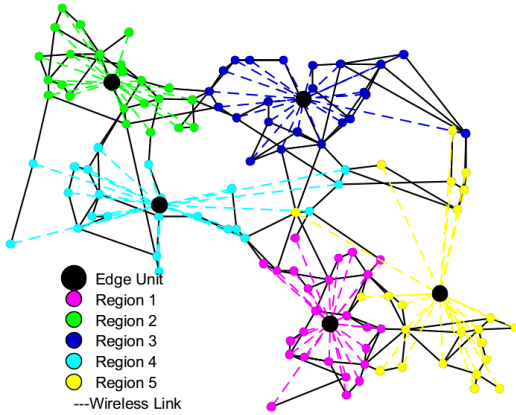


Fig. 1: Schematics of the proposed multi-region DSE framework enabled by distributed edge computing over the IEEE 118 bus system.

IV. ESTIMATION MODEL

The PMUs sample the state of the power grid's components and provide a sequence of phasor measurement observations in a form of time series. We denote the phase angles of buses in the system at time t by $\theta_t = [\theta_{1t}, \theta_{2t}, \dots, \theta_{Nt}]$ and bus voltages by $\underline{V}_t = [V_{1t}, V_{2t}, \dots, V_{Nt}]$. In this paper, we will focus on

phase angle time series, while the study can be applied to other collected attributes from the system.

Measurements from the PMUs are multivariate time series while the consecutive measurements have high auto-correlation among themselves. To capture this property of time series, we have expanded our feature space with an AR(p) process such that $\theta'_t = f(\theta_{t-1}, \theta_{t-2}, \dots, \theta_{t-p})$, where θ_{t-i} is the past observation at time $t - i$ in the time series. Then the SE problem may be described by linear regression as $\hat{\theta}_{it} = \beta^T \theta'_t + e_t$ with coefficients as β , where the goal is to find maximum likelihood (ML) estimate at node i , $\hat{\theta}_{it}$, for θ_{it} while minimizing frobenius norm $\| \beta^T \theta'_t - \hat{\theta}_{it} \|^2$.

However, the computed weight vector β in the linear regression may not be able to capture all uncertainties, especially in the case of noisy measurements, since it only gives the ML estimate. One solution to address this issue is to adopt the Bayesian approach to regression, which will learn the probability distribution of all possible β values that describe the relations between θ'_t and $\hat{\theta}_{it}$. The Bayesian linear regression with AR(p) can be realized as $\hat{\theta}_{it} \sim \mathcal{N}(\beta^T \theta'_t, \sigma^2)$. Unlike linear regression, which finds the ML estimate for coefficients β that describe the relationship between the inputs and the outputs, we are interested in computing a probability distribution for β values that describe this relationship. This can be calculated by defining prior distributions for β , and later applying the Bayes rule to calculate the posterior distribution of β . There are several choices for the prior distribution of the coefficients. In this work, we follow the Bayesian regression model described in [22]. The authors in [22] assumed conjugate normal inverse-gamma prior for β, σ^2 , and used variational inference that makes the computations faster. Under this assumption, the likelihood of the θ_{it} can be written as $\rho(\theta_{it} | \theta'_t, \beta, \sigma^2) = \mathcal{N}(\theta_{it} | \theta'_t, \beta, \sigma^2)$.

Note that in the Bayesian approach a general assumption is that the individual data streams from the PMUs have Gaussian distribution, which may not be accurate and valid in all cases. Modeling individual time series with their true distribution may improve the estimation performance. However, in this case, the mathematical model may become intractable and will have a higher computational cost.

V. DISTRIBUTED MULTI-REGION STATE ESTIMATION

For the local processing of data in distributed computing units, we consider edge computing as an example supporting platform of multi-region distributed SE. First, the smart grid is divided into regions and the center of each region μ_r (based on its geographical coordinates) is adjusted to its nearest PMU for optimal placement of edge computing unit. Each edge computing unit is connected via wireless links to PMUs in the region. It collects the associated regional PMU measurements (such as V, θ) and processes the data for one step ahead SE using the model discussed in section IV. Since data is being processed by local edge nodes instead of being transferred to the central cloud servers, the overall communication latency will be reduced and faster decision making can be achieved from the local estimations [1]. Finally, local estimates can be

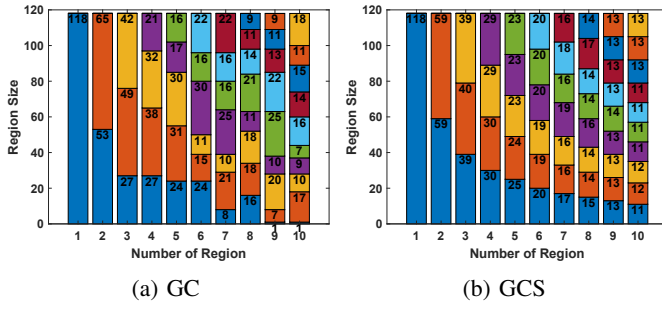


Fig. 2: Stacked representation of different region sizes for, a) non-homogeneous partitioning, and b) homogeneous partitioning. Each color represents a different region. For example when the number of regions is ten then different colors represent the ten regions.

used locally for responding to a situation identified or can be communicated to the central systems and also be combined to achieve the overall system estimate. Figure 1 shows the general schematics of the multi-region SE on the IEEE 118 bus system. In this figure, each colored node of the graph represents a bus in the system and the black solid lines represent a transmission line. Different colors assigned to the nodes specify the regions defined over the system. Note that in this paper, we do not consider the coordinated and collaborative SE among the edge servers of different regions. We assume that the central system is also not helping with the distributed estimations. However, such methods and the communication among the edge servers of different regions and the central system in a distributed manner can also be considered in DSE estimation [19]–[21]. These aspects are some of our current ongoing research.

VI. RESULTS

In this paper, the IEEE 118 bus system has been used to demonstrate the performance of the proposed technique. We have simulated a large dataset of PMU time series in both normal and also under partially unobservable scenarios using MATPOWER [23] simulation toolbox. The considered unobservable scenarios (due to physical failures or DoS stresses) are described in detail later in this section. We have used real load profiles from the New York Independent System Operator (NYISO) and sampled at 30Hz to generate quasi-static PMU time series by solving power flow at each sample. From the simulation, the bus phase angle time series have been recorded as the state variable.

Using the simulated data the spatial cross-correlations of state variables have been calculated. Using the partitioning strategy discussed in Section III, the grid topology is divided into non-overlapping regions. We have solved the partitioning in both non-homogeneous and homogeneous partition sizes using 1) geographical distance (G), 2) geographical distance and cross-correlation (GC), 3) geographical distance in homogeneous partitioning (GS), and 4) geographical distance and cross-correlation in homogeneous partitioning (GCS). We have compared the performance of estimation for these partitioning

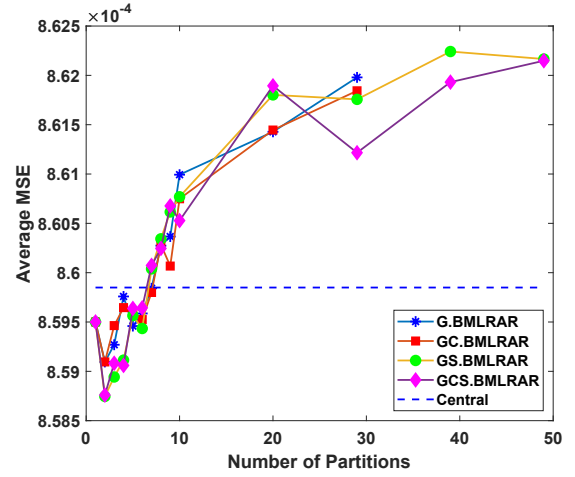


Fig. 3: Average MSE over all the buses compared for different partitioning strategies and for different number of regions.

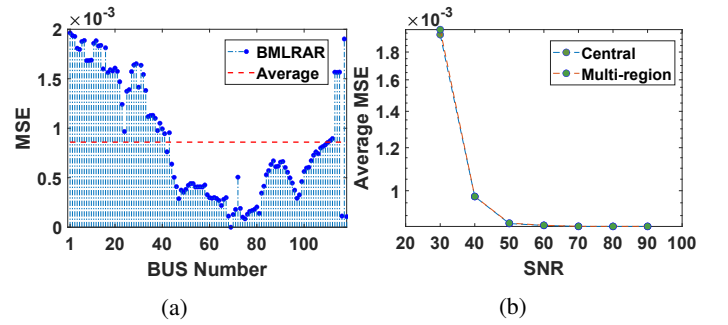


Fig. 4: a) MSE at each bus for five regions, $R = 5$, and b) average MSE as a function of added noise (SNR) for central estimation and multi-region estimation for R in the range of 2 to 10 and GCS partitioning technique.

scenarios. Figure 2 represents the region sizes (i.e., number of buses inside each region) as a function of the number of regions. It can be seen from figure 2(a) that in non-homogeneous case the size of regions are not consistent and some regions have almost twice the number of components of others. In figure 2(b) homogeneous partitioning results in more size consistency.

The performance of DSE in terms of average mean square error (MSE) is depicted in figure 3 as a function of the number of regions for different partitioning strategies in both homogeneous and non-homogeneous partition sizes. The average MSE is taken over all the buses of the system. In this figure, the dashed line is the estimation error if the model was to be applied centrally using the data from the whole system. From the figure, we see that partitioning the grid can improve the overall estimation performance depending on the number/size of partitions. As the defined regions consider the cross-correlation among the PMUs, in the smaller region sizes the feature space for the model is more densely correlated thus improving the overall average estimation accuracy. However,

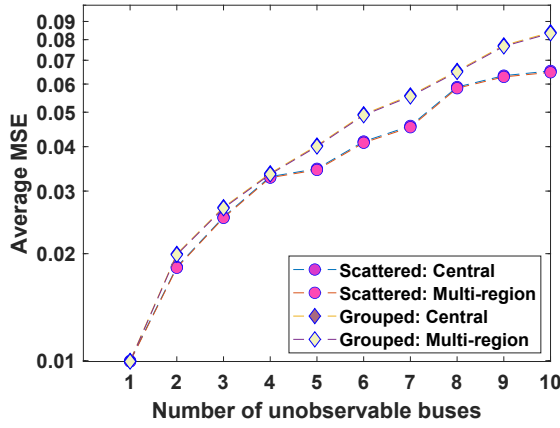


Fig. 5: Average MSE for different stress sizes (number of unobservable buses) in two different stress scenarios (scattered and grouped). The results are shown for the central estimation and averaged multi-region estimation for R from 2 to 10 and GCS partitioning technique.

as the number of regions increases the model will have access to less information compared to the larger partition sizes thus the overall estimation error increases again. The variation of estimation error among different partitioning is small but the results show that incorporating correlation into the partitioning process slightly improves the overall estimation accuracy. The best estimation accuracy occurs when the number of partitions is homogeneous and small (less than ten). As such, we will focus on a small number of GCS-based partitions for the rest of the analyses.

Note that the estimation accuracy values are different at various buses of the system as shown in figure 4(a). Specifically, the state of some buses is difficult to estimate, which can be due to their complex interaction dynamics with other components. The dotted line shows the average performance for all buses. Figure 4(b) shows the average estimation performance as a function of added Gaussian white noise to the PMU time series data. This added noise represents measurement noise or communication channel noise. As expected the performance increases with the increase in Signal to Noise Ratio (SNR). The results also show that the estimator maintains good performance for SNR values larger than 40db. Since our results show small variations of estimation error under noise for different partition sizes, figure 4(b) only depicts the MSE for the central case and for averaged over the number of regions in the range 2 to 10.

To evaluate the performance of the DSE under partial unobservability, we have considered scenarios that can be resulted from cyber stresses (such as DoS stresses) or the physical failure of PMU devices or disconnections in their communication links. In such cases, the local and the central servers will not receive any data or only receive the channel noise instead of the actual measurements from the PMU. We have considered two different scenarios of partial unobservability:

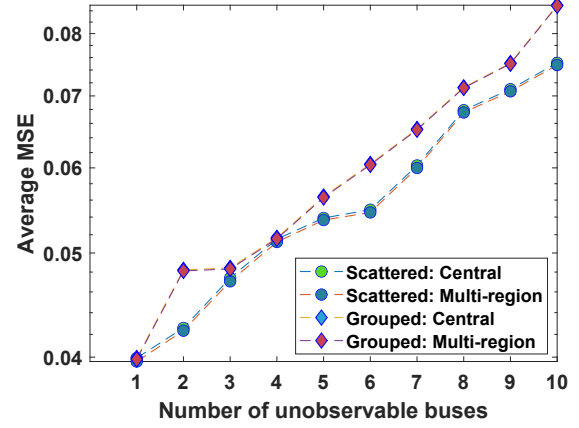


Fig. 6: Average MSE of unobservable buses for different stress sizes (number of unobservable buses) in two different stress scenarios (scattered and grouped). The results are shown for the central estimation and averaged multi-region estimation for R from 2 to 10 and GCS partitioning technique.

a) when the unobservable PMUs are scattered throughout the grid randomly, and b) when the unobservable PMUs are localized geographically (for instance due to localized events such as earthquakes or attacks). We have simulated 100 stress scenarios for each case and each stress size (i.e., number of unobservable PMUs). The average MSE over all the buses under different stress scenarios for different stress sizes is represented in figure 5. As the variations of estimation error are very small for different numbers of partitions, in this figure, we have presented MSE for the central estimator and for the multi-area estimator averaged over the number of regions in the range 2 to 10. It can be observed that the estimation error rises as the number of unobservable buses increases in both stress scenarios. However, the grouped stress cases impose more strain on the estimator as the number of correlated features becoming unavailable increases in a region. In the simulations, we have considered less than ten percent of the buses may become unobservable as a result of the stress; however, these results can be extended to any number of stressed buses. Overall the estimator retains promising performance under a small number of unobservable buses in both scenarios. Figure 6 shows similar results for the MSE of recovering the state of unobservable buses from other PMU data streams in both stress scenarios.

The estimation error of our proposed approach, which is $\cong 8.6 \times 10^{-4}$ (averaged over all buses for the number of partition < 10), is comparable to the state of the art physics-driven estimation techniques. For instance, the performance (the average estimation error over all the buses) of the adaptive multi-area distributed Quasi-newton (A-DQN) algorithm presented in [24] is $\cong 5.8 \times 10^{-4}$, which is slightly better than the proposed technique in the current paper due to collaboration consideration between the regions. However, the proposed technique in the current paper outperforms

the physics-driven hybrid linear multi-area state estimation techniques [25], which is based on the traditional weighted least square approach, with a performance of $\cong 1.38 \times 10^{-2}$ for average estimation error over all the buses.

VII. CONCLUSION

In this work, we discussed a data-driven, multi-region distributed SE framework for the smart grid. We applied a multivariate Bayesian linear regression method combined with AR(p) for one step ahead state prediction. We discussed that for multi-region distributed SE the grid needs to be partitioned into regions with geographical and power system considerations (such as correlation among the PMU time series). We considered both homogeneous and non-homogeneous region sizes in our study. We showed that the distributed SE can achieve better estimates compared to its central counterpart when the grid is partitioned into few regions. We also discussed that the proposed method can be implemented over a distributed computing platform, such as edge computing. The proposed framework can lead to low latency and faster data processing, which results in improved wide-area monitoring for smart grids. We also considered partially unobservable scenarios that can result from cyber or physical stresses on PMUs and showed that the distributed estimation approach can handle the estimations under such scenarios well. The future goal is to provide a distributed SE framework with communication and coordination among the regions to improve situational awareness and robustness against different cyber and physical stresses.

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