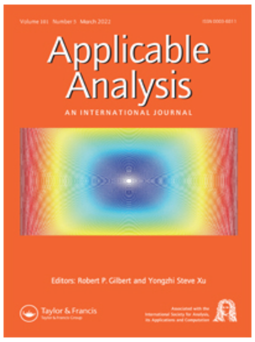

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INTRODUCTION

Introduction for special issue on modern applied analysis

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The following papers were presented at two *Special Sessions on Applicable Analysis*, at the Fall Southeastern Sectional Meeting of the AMS, October 10–11, 2020.

We firstly describe the papers which were presented at the *Special Session on Applicable Analysis of PDE Systems which Govern Fluid Flows and Flow-Structure Interactions*.

The contribution, 'Optimal Control in Poroelasticity', by L. Bociu and S. Strikwerda, deals with the analysis of optimal boundary and distributed control problems, subject to fluid flows through deformable, porous media. For such controlled elliptic-parabolic systems, the authors prove the existence and uniqueness of optimal controls, as well as derive first-order necessary optimality conditions. This control theory has important physiological implications: For example, obtained results on optimization of the pressure of the flow, and the influence of associated system parameters, could yield a greater understanding of ocular neurodegenerative pathologies, such as glaucoma.

In the work, 'Recovering Density for the Mindlin–Timoshenko System by Means of a Single Boundary Measurement', the authors J. Kurz, S. Liu, and P. Pei, consider an inverse problem for the Mindlin–Timoshenko plate system, this being a coupled PDE system, which arises in the modeling of plate vibrations, especially at high frequencies and thicker plates. In particular, the authors prove global uniqueness for the problem of recovering the plate density from a single boundary measurement, under appropriate geometrical assumptions. The line of attack here hinges in part on a diagonalization of the principal part of the system, and subsequent analysis of a system of wave-like equations.

Moreover, the contribution, 'Uniform Boundary Observability of Finite Difference Approximations of Non-compactly-coupled Piezoelectric Beam Equations', by O. Ozer and W. Horner, is a study of space-discretized finite-difference approximations of a coupled system of piezoelectric beam PDEs which account for electromagnetic effects. Although the system is known in the literature to be exactly observable by way of two boundary observations (one mechanical and one electrical), existing numerical approximations do not retain said uniform exact observability with respect to the mesh parameter, as it tends to zero. To ultimately obtain an approximation scheme that captures uniform observability with respect to the mesh parameter, a direct filtering method is adopted to eliminate artificial high-frequency eigenvalues of the approximated model. Ultimately, the work not only provides a reliable approximation scheme to design a controller or a sensor, it also gives an improved observability time for the coupled PDE system.

Furthermore, the contribution, 'Weak Solutions for a Poro-Elastic Plate System', by J. T. Webster and E. Gurvich, considers a plate model which has been obtained as a scaled limit from the

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traditional three-dimensional Biot system of poro-elasticity. The result is a '2.5' dimensional linear system, a coupling of traditional Euler–Bernoulli plate dynamics to a diffusive pressure equation in three dimensions, with the diffusion acting only in the transverse direction. The permeability coefficient is assumed to be generally time-dependent, making the problem nonautonomous and so not subject to standard abstract wellposedness theory. Utilizing the theory for weak solutions to time-dependent implicit evolutions, the authors obtain the existence of solutions. Uniqueness is obtained under additional hypotheses on the regularity properties of the permeability function.

Lastly, in the work, 'A Resolvent Criterion Approach to Strong Decay of a Multilayered Lamé-Heat System' by G. Avalos and P. Güven Geredeli, the authors establish the strong asymptotic decay of a certain fluid-structure PDE interaction which consists of respective three dimensional and two dimensional systems of elasticity coupled to a thermal equation. Each of these three dynamics evolves on its own distinct geometry. The proof of stability here is geared towards invoking a known resolvent criterion for strong decay. In particular, to ultimately apply this criterion, the authors must make appropriate use of Holmgren's type results for three-dimensional Lamé systems on nonsmooth domains.

We further describe the papers which were presented at the *Special Session on Modern Applied Analysis*.

In the contribution, 'A Kummer–Type Transformation for some $k-$hypergeometric Functions' by H. Li and Y. S. Xu, the authors apply the differential equation techniques and use integral representations. As a result, they obtain the Kummer-type transformations of $k-$hypergeometric functions, which include the known results as special cases.

The contribution, 'The SMGT Equation from the Boundary: Regularity and Stabilization', by M. Bongarti, I. Lasiecka, and R. Triggiani, deals with a mixed (initial-boundary value) problem is considered for SMGT (for G. G. Stokes (1851), F. K. Moore and W. E. Gibson (1960), P. A. Thompson (1972)) equation, which is a third-order linear equation of the form

$$\psi_{ttt} + \alpha\psi_{tt} - c^2\Delta\psi - b\Delta\psi_t = 0 \quad \text{in } (0;T] \times \Omega,$$

where Ω is a bounded domain in $\mathbb{R}^n$.

It is assumed that $\gamma = \alpha - \frac{c^2}{b} \in L^\infty(\Omega)$. The physically most interesting case is given by $\alpha \in L^\infty(\Omega)$ with b, c^2 that are positive constants.

Part A of the paper provides optimal regularity of solutions of the mixed problem with the Dirichlet or Neumann boundary conditions from $L^2(0,T;L^2(\partial\Omega))$ and zero initial data. Stabilization is studied in Part B for the problem with Neumann dissipative boundary conditions and critical parameter $\gamma(x) \geq 0$ a.e. in Ω. Uniform stabilization is established under minimal checkable geometric conditions and strong stabilization is established without imposing geometric conditions.

The work, 'The Krein–von Neumann Extension Revisited', by G. Fucci, F. Gesztesy, K. Kirsten, L. Littlejohn, R. Nichols, and J. Stanfill represents the continuation of the seminal work by M. G. Krein on nonnegative self-adjoint extensions of the operator. He proved that, among all nonnegative self-adjoint extensions of a nonnegative symmetric operator, there exist two which are the largest and smallest in the sense of order between nonnegative self-adjoint operators. The largest extension is the celebrated Friedrichs extension, and the smallest extension is the Krein–von Neumann extension. Due largely in part to their extremal nature, considerable work has been done by many authors to provide explicit characterizations of the Friedrichs and Krein–von Neumann extensions, especially in the concrete setting of Sturm–Liouville operators. The paper is devoted to an explicit characterization of the Krein–von Neumann extension of a generally singular, strictly positive minimal Sturm–Liouville operator. The characterization is stated in terms of generalized boundary values at the interval endpoints. To illustrate the characterization, three explicit examples corresponding to a generalized Bessel operator, a Jacobi operator, and a singular operator relevant to acoustic black hole phenomena are discussed.

The paper, 'Sign-Changing Points of Solutions of Homogeneous Sturm–Liouville Equations with Measure-Valued Coefficients', by A. Ghatasheh and R. Weikard, contains two generalizations of the classical results in Sturm–Liouville theory, i.e. the Sturm separation theorem and the Sturm comparison theorem. These theorems specify the location of zeros of the solutions. The authors consider the homogeneous Sturm–Liouville equations with measure-valued coefficients. It appears that, under some conditions, the corresponding Cauchy problem has a unique solution, the sign-changing points are isolated, and some versions of the Sturm separation theorem and Sturm comparison theorem take place. Differential equations with measure-valued coefficients represent a unification for both the discrete and continuous cases. Also, these equations appear in modern Physics since they describe the so-called delta and delta' interaction.

Moreover, the contribution, 'Malaria Modeling and Optimal Control Using Sterile Insect Technique and Insecticide-Treated Net', by L. Cai, L. Bao, L. Rose, J. Summers, and W. Ding, presents a system of ordinary differential equations to represent malaria transmission with human and mosquito compartments. A compartment for released sterile mosquitoes is introduced, and then a careful calculation of the basic reproductive number is given. Two control interventions, one affecting the release rate of sterile mosquitoes and the other for bed net coverage, are introduced. Using a goal to minimize a combination of the infected human population and the cost of implementing the controls, the optimal strategies are characterized and illustrated numerically. Such results can suggest effective strategies for managing malaria outbreaks.

Furthermore, the contribution, 'On Non-homogeneous Robin Reflection for Harmonic Functions', by M. Aldawsari and T. Savina, is devoted to the reflection principle for 2D harmonic functions defined in the neighborhood of the real-analytic curves subject to the non-homogeneous Robin (impedance) boundary conditions on that curve. Robin-to-Neumann mapping and Robin reflection about an arc of a real-analytic curve are constructed as the integral operators, whose kernel contains the Schwartz function of the curve. The authors conduct a comparison of the results with the known reflection formula for the homogeneous Robin condition. Finally, they consider examples of reflection he examples of reflection about some specific arcs.

Lastly, in the work, 'On the Sturm-Liouville Problem Describing an Ocean Waveguide Covered by Pack Ice' by B. Belinskiy, D. Hinton, L. Weerasena, and M. Khan, the authors consider a Sturm–Liouville problem that appears from the study of acoustic wave propagation in a layered ocean waveguide covered by pack ice. The authors establish the following properties: basis property of the eigenfunctions, convergence of the Fourier series, the differentiability and monotonicity of the eigenvalues as the functions of frequency, the existence of the cut-off frequency, below which there exists only one propagating mode. They study dispersion relations for two maximal eigenvalues analytically, numerically, and asymptotically. The eigenvalues exceed the eigenvalues for the case of a waveguide with a free surface. For the given positive limits of the propagation speed, the authors find the minimum and maximum of the wavenumbers of two leading modes.