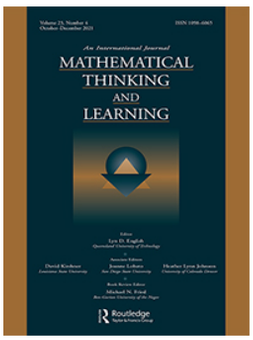

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Professional noticing coherence: exploring relationships between component processes

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ABSTRACT

Teacher noticing and related variants have ascended in prominence among the mathematics education research community. While the component processes of such noticing (e.g., attending, interpreting and deciding) have been cast as interrelated, capturing the relationships amongst the components has been more elusive. We focused on the component processes of teacher noticing with particular attention given to interrelatedness. Specifically, we were interested in how and the extent to which the component processes of professional noticing (attending, interpreting, deciding) are thematically connected when preservice elementary teachers are engaged in an assessment approximating professional noticing. We refer to this thematic linkage in this paper as coherence. Our findings suggest a complex interplay between the creation and continuation of themes when enacting professional noticing, and the quality of such noticing.

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Teacher noticing and related variants (e.g., professional vision, professional noticing of children's mathematical thinking) have captured the attention of the mathematics education research community for decades (Goodwin, 1994; Jacobs & Spangler, 2017; Mason, 1998, 2002, 2011; Schack et al., 2017; Sherin et al., 2011a). While the component processes of attending, interpreting, and deciding appear, intuitively and theoretically, to make sense as interrelated and essential to professional noticing of children's mathematical thinking, examining the relationships amongst the components has been more elusive.

Given the sustained focus upon teacher noticing, and the extent to which such noticing is framing practitioner guidance (Thomas et al., 2014) and shaping broader applications of noticing (Jackson et al., 2018) it is essential to fully understand connections amongst noticing component processes when such noticing is enacted. First and foremost, the extent that some common idea, feature or theme persists across noticing components is essential for considering the fundamental nature of their relationship. Put more simply, it is vital to understand the extent to which noticing enactment is internally coherent in order to better support the development of noticing activity among practitioners. At present, little is known about such internal coherence at the empirical level.

For this exploratory inquiry, we have focused on the nature of these component processes of teacher noticing with particular attention given to their theorized interrelatedness (Jacobs et al., 2010). *Specifically, we were interested in a) how and the extent to which the component processes of professional noticing (i.e. attending, interpreting, deciding) are thematically connected when preservice elementary teachers are engaged in an assessment approximating professional noticing as well as b) how such thematic connections interact with other evaluations of professional noticing quality*. Note, we refer to such thematic connections as coherence.

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Conceptual Framework

Foundations of Noticing

Mentioned earlier, there has been a growing focus on research related to teachers' noticing of aspects of the mathematics classroom including (but not limited to) children's activity, discourse, and positioning (Louie, 2018; Schack et al., 2017; Sherin et al., 2011a). Described as a "hidden core practice," noticing refers to "the act of focusing attention and making sense of situational features in a visually complex world" (Jacobs & Spangler, 2017, p. 771). Research on noticing is rooted in a long history of psychological examination of attention and processing in typical settings (Shiffrin & Schneider, 1977) including what is seen and not seen by individuals (Simons & Chabris, 1999). Mason (2011) provides an historical account of teacher noticing and describes noticing as a technique of "(a) *pre-paring* to notice in the moment, that is, to have come to mind appropriately and (b) *post-paring* by reflecting on the recent past to select what you want to notice or be sensitized to particularly, in order to *pare*, that is, to notice in the moment and so be enabled to act freshly rather than habitually" (pp. 36–37). As such, Mason presents noticing as a multifaceted process that is spontaneous and contextually situated, but also inextricable from ones' perspective and biases.

Regarding the examination of teacher noticing, the practice tends to be constructed around at least two primary processes, "attending to particular events in an instructional setting" and "making sense of events in an instructional setting"; further, these two processes are typically described as being interrelated (Sherin et al., 2011b, p. 5). In their influential work, Jacobs et al. (2010) referred to these processes as *attending* and *interpreting*. Building upon related constructs such as highlighting (Goodwin, 1994), attending is characterized by Jacobs et al. as the "extent to which teachers attend to a particular aspect of instructional situations: the mathematical details of children's strategies" (p. 172). Similarly, these scholars describe interpreting as the "extent to which the teacher's reasoning is consistent with both the details of the specific child's strategies and the research on children's mathematical development" (pp.172–173). Expanding on these components, van Es and Sherin (2021) also describe disregarding of certain classroom features and interactions as part of attending. This characterization of attending as the selection of noteworthy activity and along with the discard of (perceived) unimportant aspects of the mathematical moment is consistent with other noticing characterizations aimed at maximizing instructional opportunities (Stockero et al., 2017). Turning to interpreting, van Es and Sherin also posit the importance of an inquiry stance as part of this noticing component process. This stance of inquiry is organized around a search for meaning within mathematical activity and that the search for meaning itself is important to the process and holds intrinsic value.

Jacobs et al. (2010) also posited a third, interrelated component process, *deciding*, which connoted a teacher's intended response gleaned from one's interpretations of activity (which ostensibly flowed from one's attention to said mathematical activity). Building upon this perspective, we define the component processes, thusly,

> [a]ttending involves concentrating one's attention on the students' actions and verbalizations within a mathematical moment. For example, details worthy of attention might include a student's movement of manipulatives, finger counting, or voice level. Interpreting involves an analysis of the observed behaviors or verbalizations with the aim of making some determination regarding the mathematical understanding of a student. Deciding refers to the teacher's leveraging a particular interpretation to plan and enact a sound instructional or diagnostic course of action.. (Fisher et al., 2018, p. 211)

In practical terms, such noticing might appear as a teacher observing a child struggle to use a standard algorithm for addition, interpreting some underlying conceptual challenge, and then, in the moment, introducing a scaffold or model (e.g., empty number line, base-10 blocks, etc.) for the child to consider. For further illustrations of noticing in practice, see Thomas et al., 2014). In this tripartite characterization of noticing, we see how interactions amongst component processes ultimately influence

students' mathematical experiences in the classroom via deciding; thus, examinations of how these component processes connect and interact with one another is essential to understanding the practical impact of teacher noticing with respect to students.

Returning to the component processes of noticing, it is important to note that descriptions of these components vary somewhat across orientation (Mason, 2011) and scope (Stockero et al., 2017) with some researchers using more colloquial phrasing such as "sizing up students' ideas and responding" (Ball et al., p. 453). There appears to be, though, some fundamental agreement that the practice of noticing is organized around the enactment of related skills or processes. Further, such noticing is "often conceptualized as focusing attention on and making sense of what students say or do before actually responding to them" (Jacobs & Spangler, 2017, p. 771). Indeed, "whether noticing is described as 'identifying what is noteworthy about a particular situation' and making connections between specific events and broad principles of teaching and learning (Van Es & Sherin, 2002, pp. 573–574) or a fluid enactment of attending, interpreting, and deciding, the presented research reflects a relatively shared understanding of what it means *to notice* [emphasis in the original]" (Thomas, 2017, p. 508). We argue that noticing is, fundamentally, the enactment of the component processes of attending, interpreting, and in some instances, deciding (Sherin et al., 2011a).

Over the past two decades, numerous empirical studies have been conducted. The investigation by Sherin & van Es (2005) regarding how participation in video clubs improved teachers' ability to notice classroom interactions as well as the work of Jacobs et al. (2010) were, notably, two studies that provided inertia to the study of teacher noticing. In their study of professional noticing (which also included a deciding component), Jacobs et al. found that teaching experience alone does not precipitate professional noticing capabilities. There is evidence, however, that noticing is a learnable skill (Jacobs & Spangler, 2017).

In the ensuing years, numerous empirical studies of noticing have been conducted ranging from the study of teacher noticing emerging from student-produced artifacts (Santagata, 2011) to the evaluation of peer-teaching video to improve noticing performance of middle grades and secondary teachers (Males, 2017). Sherin et al. (2011) described three types of noticing study: 1) focus on researcher-selected objects or artifacts; 2) retrospective noticing of teacher's own instruction; and 3) observation and interpretation of teachers' instructional activity. Examples of empirical studies aligned with each research type may be found in the edited volumes of Sherin et al. (2011a) and Schack et al. (2017). Each of these studies examined a component view of noticing, whether attending and interpreting, or attending, interpreting, and deciding.

Relationships among Component Processes of Noticing

A key aspect of noticing, as a construct, is the manner in which component processes are construed as interrelated in their enactment, and in the following sections, we present competing perspectives regarding linkages among such component processes. In describing her own examinations of teacher noticing, Sherin (2017) remarks,

> *Selective attention* [emphasis in the original] reflected the idea that teaching involves attending to some interactions while not attending to others, in other words, identifying key events. *Knowledge-based reasoning* [emphasis in the original] concerned the ways in which teachers interpret what they notice . . . like all perceptual processes, we argued that selective attention and knowledge-based reasoning interact in a dynamic manner. In some cases, selective attention may drive a teacher's knowledge-based reasoning . . . In other instances, a teacher's knowledge and experiences will influence what a teacher notices. (p. 403)

This notion of dynamically interrelated noticing components is reflected in the descriptive trajectory put forth by Van Es (2011) which, among other things, positions deciding as a natural outgrowth of more advanced interpretive capability.

Castro-Superfine et al. (2017) argue for a shift in perspective regarding the manner in which noticing is typically construed with respect to the temporal positioning of component processes (e.g., attending leads to interpreting). Rather, these authors describe a reflexive relationship between attending and interpreting, and note, similarly to Sherin (2017), that "one issue emerging from our research relates to the difficulty in distinguishing attending from interpreting" (Castro-Superfine et al., 2017, p. 421). They conclude with the assertion that "evidence from [preservice teachers] that include attending or interpreting do not necessarily suggest chronological order, rather it is possible that interpretations occur before attending in some cases and in others, attending likely occurs before interpretations are made" (Castro-Superfine et al., 2017, p. 423). In other words, their finding suggests that a temporal ordering of the component processes of noticing (i.e., attending → interpreting → deciding) is not entirely stable and that these processes may interact much more deeply and reflexively.

On this point, though, we note that many inquiries implied some temporal connections between the component processes by organizing the component processes in a particular order (Jacobs et al., 2010) or describing the manner in which one component flowed from another (Schack et al., 2013). However, more recent literature has critically examined such ordering (implied or otherwise) of attending, interpreting, and deciding, and such questions of temporality have not been entirely settled. For example, Sherin et al. (2011) remark that "teachers selectively attend to events that take place *and then* [emphasis added] draw upon their existing knowledge to interpret these noticed events" (pp. 80–81). While such questions of temporality are certainly important, they do not necessarily respond to other types of connections and relationships which may occur among the component processes of noticing – namely, thematic connections. Rather, we argue that temporal relationships among component processes is one possible lens for examination. Thematic coherence, the focus of this study, is another such lens.

In many portrayals, productive enactment of professional noticing involves the continuation of a common theme across the component processes of attending, interpreting, and deciding (Jacobs et al., 2010). For example, productive professional noticing centered on a child's perceptual counting activity (Steffe, 1992) might involve noting a count-from-one and physical interaction with counting materials (attending). From there, the teacher might posit that the child is operating from a perceptual counting scheme (interpreting). From this, the teacher might introduce arithmetic tasks featuring screened or concealed counting materials such that the child might move toward a figurative counting scheme and the development of quantitative mental imagery (deciding) (see Thomas & Tabor, 2012 for explication of this process). Central to this enactment of productive noticing is the continuation of a common theme (i.e., counting) through the component processes of attending, interpreting, and deciding. We note, however, that our conception of thematic noticing is distinct from other investigations of theme and noticing. Specifically, Dreher and Kuntze (2015) describe noticing as *theme-specific* when such noticing is directed toward particular aspects of a context or environment (i.e., the use of problematic representations). Such studies consider themes as they frame or situate noticing activity. Specifically, Dreher and Kuntze used vignettes to explore such thematic emergence in preservice and inservice teachers' professional noticing. Their findings suggest that the emergence of themes within noticing activity may signal some manner of increasingly sophisticated practice. Both inservice and preservice teachers did not always demonstrate an understanding of key mathematical features within their noticing activity; however, inservice teachers were more often able to identify pertinent themes and critical incidents within vignettes. Broadly, such themes may be related to students' mathematical thinking and/or equitable learning environments (Jacobs & Spangler, 2017). Walkoe (2015) provides one such example of thematic study with respect to students' mathematical thinking. In this study, preservice teachers' noticing of students' algebraic reasoning was examined to create a framework for development. Walkoe found that focused activity around rich video anchors could propel participants toward more sophisticated enactments of noticing of students' mathematical thinking. Regarding the noticing of equitable learning environments, Hand (2012) examined the activity of teachers, deemed equitable in nature by researchers, to determine what these individuals noticed in their classroom teaching. She found that these teachers tended focus their attention toward activities that would align

with the equity themes of power, status, and positioning. In summary, these are but a few examples of how theme emerges within the enactment of noticing; however, there remains much to learn with respect to the coherence of such themes across the component processes of noticing. Our study is concerned with the manner in which themes illustrate a linkage or relationship across the component processes of noticing and stands distinct from other conceptions of thematic noticing.

Noticing as Thematically Disconnected

While professional noticing is oft portrayed as thematically linked component processes, it is not clear that actual enactments of noticing are always connected in such a manner. Indeed, it is quite conceivable that a teacher could attend to and interpret one aspect of a mathematical moment and the subsequent decision be thematically unrelated. For example, one might attend to and interpret a child's perceptual counting activity and decide to engage in numeral identification activities (Wright et al., 2006). A lack of thematic link might also be evident between attending and interpreting. The disconnect sometimes might be due to contextual constraints (e.g., the need to move on, limited time, needing to focus on broader group needs) or some manner of overgeneralization. For example, in previous research, we observed preservice teachers attend to a child's direct modeling strategy leading to a correct answer but apply an overgeneralized interpretation relying on personal presumptions (Schack et al., 2013).

This manner of thematic disconnect between the component processes of attending, interpreting, and deciding forces the question of *whether thematically disconnected processes are professional noticing at all*. We argue, however, that such thematic connections should not be automatically ascribed to the practice of teacher/professional noticing – that enactment of professional noticing does not assume some basic degree of coherence. Jacobs and Spangler (2017) conceptualize the component processes of noticing as nested entities (i.e., attending nested within interpreting, interpreting nested within deciding – see p. 773). While this conceptualization assumes some fundamental interrelatedness between "what is noteworthy about a particular situation . . . [and] making connections between specific events and broader principles of teaching and learning" (Van Es & Sherin, 2002, pp. 573–574), there is broader disagreement regarding the sweep of the lens through which a teacher professionally notices a moment. Thomas (2017) states,

> Specifically, is noticing more appropriately focused on capturing and interpreting as much of the instructional landscape as possible including individual movements and postures? Or, should noticing processes be used as a filter to identify only the most impactful moments of a particular block of instruction (p. 508)?

Given the dynamic nature of professional noticing, the varying of purposes and perspectives with respect to such noticing opens the potential for a shift in one's orientation, *as one notices*, leading to the possibility of thematic disconnect across component processes. For example, a teacher may formulate an interpretation organized around a child's equitable participation in the moment, but posit a decision aimed at advancing the child's construction of unit (Steffe et al., 1988). As such, the thematic linkage between these two components would appear to be somewhat tenuous at best; however, as an enacted practice where such processes occur "almost simultaneously" (Jacobs & Spangler, 2017, p. 773), it is conceivable that an observable thematic connection may not always be present. This is not to suggest, in this example, that such thematic connections do not exist at all – perhaps they do in some subconscious space. Rather, we argue that thematic connections may not always be available to researchers (or even to the teacher him/herself) for inspection or consideration, and that the absence of such observable connection does not preclude the enactment of some manner of professional noticing. Reflective of such potential observable disconnects, researchers, at times, elect to focus solely upon one particular component process of noticing in order to examine a certain aspect of noticing practice (Males, 2017; Schack et al., 2013).

Thus, it is this focus on the thematic nature of relationships between attending, interpreting, and deciding that set the stage for our study. *For this study, we focused on a) how and the extent to which the component processes of professional noticing (i.e. attending, interpreting, deciding) are thematically connected when preservice elementary teachers are engaged in an assessment approximating professional noticing as well as b) how such thematic connections interact with other evaluations of professional noticing quality.*

In the following sections, we will present evidence supporting the proposed construct of coherence (i.e., thematic linkage) with respect to the component skills of professional noticing of children's mathematical thinking, abbreviated to professional noticing. Note, we use the term *professional noticing* in this section to describe a three-component perspective consistent with the work of Jacobs et al. (2010). In other sections, we use broader language (i.e., noticing, teacher noticing) to encompass perspectives that do not explicitly incorporate the deciding component process with respect to noticing.

Methods

This exploratory inquiry stems from a larger, previously reported, quasi-experimental study of Preservice Elementary Teacher (PSET) professional noticing capabilities within the context of early number and arithmetic operations (Fisher et al., 2018). Primarily, this section is organized around two distinct modes of analysis. The first centers on the quality of PSETs' professional noticing and we briefly describe our initial investigation in this area (i.e., participants, intervention, measure, analytic process) to provide context for the subsequent investigation of professional noticing coherence. While the initial evaluation of noticing quality centered on individual component processes of professional noticing (e.g., the quality of one's attending, interpreting, and deciding), analysis of coherence looked across the component processes of such noticing to determine the emergence and continuation of themes related to the mathematical activity of the moment.

Instructional Module

To study the development of PSETs' professional noticing capabilities, we constructed a module to be implemented in either mathematics pedagogy courses (i.e., methods courses) or mathematics content courses designed specifically for elementary preservice teachers.

The module consisted of five, one-hour segments which progressively nested (Boerst et al., 2011) the components of professional noticing (i.e., attending, interpreting, deciding) in its presentation and implementation. Organized around a detailed progression of children's numeracy referred to as the Stages of Early Arithmetic Learning (SEAL – Steffe, 1992; Steffe et al., 1988, 1983; Wright et al., 2006), the module relied upon video anchors which served as representations of practice (Grossman et al., 2009) by which PSETs practiced and developed capabilities with the components of professional noticing (Schack et al., 2013).

Specifically, PSETs engaged in approximations (Grossman et al., 2009) of professional noticing via video-recordings, and SEAL provided the interpretive and decision-making lenses for such noticing. For example, in a later session focused on attending and interpreting, PSETs watched a short video of a kindergarten student, pseudonym Angela, demonstrating a composite strategy to solve the task 9 + 6 – the student says, "I know nine and nine is eighteen, so I took away three [from the eighteen]. Then the answer is fifteen." After viewing the recording, PSETs identify key aspects of the student's strategy (attending) and consider which SEAL stage this strategy most closely typifies (interpreting). Through this process, emphasis was placed on centering one's noticing upon children's arithmetic reasoning (and associated counting schemes per Steffe et al., 1988) and sustaining this focus throughout the component processes of attending, interpreting, and deciding.

Participants

Module implementation participants (n = 213) were enrolled in either mathematics pedagogy or content (for elementary teachers) courses at one of five participating public universities (representing rural, urban, and suburban environments) in a U.S. south-central state. Comparison participants (n = 60) were enrolled in either mathematics pedagogy or content (for elementary teachers) courses at one of the implementing institutions by instructors who did not use the instructional modules. Note, comparison and implementation groups within a single institution were enrolled in different sections with different instructors. Decisions regarding module implementation were established at the onset of project design and proposal for external funding (see Acknowledgments). Our primary goal for these assignments was to ensure reasonably diverse demographic/geographic representation. Some comparison participants were enrolled in such courses at two additional institutions, demographically similar to the intervention institutions, within the same state (Fisher et al., 2018). In either case, the comparison participants were not exposed to the professional noticing module. There was no intentional sorting of participants at the individual level, and participation was a function of enrollment within a particular course section.

Lastly, we note that each of the instructors (implementation and comparison) were part of the same research team with similar levels of expertise regarding professional noticing. This positioning of researcher as implementer/practitioner is consistent with most research designs with respect to the study of noticing (Jacobs & Spangler, 2017; Schack et al., 2017; Sherin et al., 2011). Additionally, this cojoining of researcher and instructor roles is consistent with participatory action research approaches that position teachers as researchers of their own practice thus providing additional avenues for collaboration and innovation that may have gone undetected in more positivist research paradigms (Brydon-Miller & Maguire, 2009).

Measure of Professional Noticing Quality

A pre and post, video-based professional noticing assessment was administered to both implementation and comparison participants. At the onset and at the conclusion of the study, participants watched a brief (25s) video (embedded within an electronic survey) of a first-grade student, pseudonym David, and teacher interacting around a comparison difference-unknown task (Carpenter et al., 1999) – see Figure 1 for a transcript of the video segment.

While brief, this video segment contained prominent cues regarding David's thinking, but also included "nuanced details that might be missed by novices thus allowing for a range of scores" (Fisher et al., 2018, p. 219). Specifically, the video depicts David enacting a somewhat ambiguous strategy that may either be construed as consistent with a perceptual counting scheme (i.e. student needs to physically interact with the counting materials to construct units) or as consistent with a figurative counting scheme (i.e. student re-presents the physical materials to construct figural unit items) (Steffe, 1992; Steffe et al., 1988). For example, David's finger pattern of four, once attended to, could either be interpreted as a motor re-presentation (driving a construction of figural unit items) or as a perceptual replacement (e.g., a substitute physical unit item for the concealed seashells) (Steffe, 1992; Thomas & Tabor, 2012).

Having watched the video segment, participants responded to three prompts, each corresponding to one of the interrelated components of professional noticing – attending, interpreting, and deciding. The prompts were: (1) Please describe in detail what the child did in response to this problem, (2) Please explain what you learned about this child's understanding of mathematics, and (3) Pretend you are the teacher of this child. What problems or questions might you pose next? Provide a rationale for your answer. Given the assignation of an individual prompt with a particular professional noticing component, responses to such prompts were categorized as attending, interpreting, and deciding in nature. While participants, on occasion, addressed other components within a single response (e.g., proposing a decision within a response to an interpreting prompt), the explicit nature of the prompts

Transcript and Video Description of Assessment Video Clip (Schack et al., 2013)

Description	A teacher and a 1st grade (6-year-old) male student named David are working together on a math problem involving seashells and small, plastic counting-bears. The teacher is posing problem situations where each bear wishes to 'hold' one seashell. Below is the scripted exchange around one problem situation.
Teacher	"How about this one, so now I've got seven . . .you've got seven little bears, right? But now I have too many shells. I have eleven shells." *The teacher shows the eleven shells then covers them with his hand.* "How many shells am I going to have left over?"
David	*David briefly holds up seven fingers and glances at them.* "You've got eleven?" *David puts his fingers down and counts each of the seven bears, beginning at one, by touching each bear and whispering a counting sequence.* "one, two, three, four, five, six, seven." *While keeping his finger on the seventh bear, David raises four fingers on her other hand while whispering,* "eight, nine, ten, eleven". *David glances at his hand and says,* "Four! There is gonna be four shells left over."
Teacher	"Interesting, David . . . I was watching you work on that, but I am curious why it has to be four. Can you tell me more about your thinking?"
David	*Looks at his hand with four fingers raised.* "Well, it's gonna be four because that's how many is left over."

Figure 1. *Transcript and Video Description of Assessment Video Clip* (Schack et al., 2013).

themselves minimized this phenomenon. Nevertheless, this categorization of response type (i.e. attending, interpreting, deciding) as a function of the assessment structure limits somewhat, the nuance and complexity with which we might consider the interplay of professional noticing components as participants are responding to the prompts.

In our Schack et al. (2013) and Fisher et al. (2018) studies, we examined growth in PSETs' ability to professionally notice. We briefly describe the analysis process used for those studies because, in the findings, we will discuss the quality of PSET professional noticing (i.e. scores) in the context of coherence. Consistent with the analytic approach of Jacobs et al. (2010), and through an iterative, inductive coding process (Strauss & Corbin, 1994), a team of six researchers, working in pairs, identified emergent themes from samples of PSET responses which were then assimilated into scoring schemes for each professional noticing component (Fisher et al., 2018). For example, PSET responses to the attending prompt ("Please describe in detail what the child did in response to this problem.") were assigned a score of 1, 2, 3, 4 with 1 being the lowest quality response and 4 being the highest. The

interpreting and deciding scoring schemes resulted in three levels of quality (i.e., ranks 1, 2, 3). For example, for the attending component, the research team engaged in iterations of viewing/review of the video segment alongside sample responses from a single institution to identify emerging themes (Glaser & Strauss, 1967) with respect to each of the prompts. Examples of such themes (for the attending prompt) include,

> Identifying key, salient activity (i.e., " . . . the child counted the bears, and then counted
>
> up to the amount of shells on his fingers"); Identifying additional activity (i.e., " . . . he then looked to see how many fingers he had up"); Operational presumptions (i.e., " . . . he subtracted 11-7"); Purporting evidence that did not occur in the segment (i.e., " . . . the child counted back from 11 to 7"); Cognitive interpretations (i.e., " . . . the child lacks a sense of cardinality") (Schack et al., 2013, p. 387)

We identified and refined these themes based on observed patterns within the data and leveraged these themes to construct a set of benchmarks for each component process – (elaborate, salient, limited, subordinate), ranging from a high rank of four to a low rank of one. On this point, we note,

> The emergent themes with researcher-identified key features resulted in three response categories for the interpreting and deciding benchmarks, one fewer than the four natural ranks that resulted for the attending benchmarks. The fourth rank for the attending benchmarks illustrates a level of PSET response that went beyond the salient evidence as identified by the researchers. No pattern of "above and beyond" responses to the interpreting and deciding prompts emerged from the data.. (Schack et al., 2013, p. 388)

Again, the themes listed above refer to the earlier study aimed at determining professional noticing quality among PSETs; however, we list them here as they provide the context for the instrumentation and resulting data set repurposed for the current study – namely, the investigation of coherence across component processes of noticing.

To enhance rating reliability, we leveraged flow-process design principles (American Society of Mechanical Engineers (AMSE), 1947) to construct a series of rubrics featuring yes/no "decision points" to mediate the interpretive load of scoring each component (Schack et al., 2015). After developing the ranking schemes, we then engaged in expert review to establish both content and construct validity. Reviewing this process from our earlier study,

> Initially, the researchers worked in pairs to analyze and rank responses. All researchers
>
> discussed and refined the benchmarks used. After several iterations of scoring with small samples of data, the researchers developed a flowchart designed to strengthen interrater reliability. The benchmarks provided the foundation for yes/no style questions on the flowchart used to guide the raters' scorings. The resulting inter-rater reliability averaged 83% for all components across six scorers" (Fisher et al., 2018, pp. 220-221).

Note, each component process was scored independently of the others and blinded (pre/post and institution). Note, we abbreviate attending (A), interpreting (I), and deciding (D) in the following sections.

Determining Coherence across Professional Noticing Components

The data gathered from the professional noticing measure, initially designed to evaluate quality of such noticing, also allowed us to look across component processes for the presence of coherence. To ascertain this coherence, or relatedness, of PSET responses across components, two researchers inductively analyzed paired responses (i.e., PSETs' attending and interpreting responses (A-I); PSETs' interpreting and deciding responses (I-D), and PSETs' attending and deciding (A-D) to identify common themes across the paired responses (see Figure 2). As this study is situated within a framework of *professional noticing of children's mathematical thinking* and stemmed from an intervention emphasizing arithmetic reasoning, we focused our attention on emergent themes that were primarily arithmetically oriented, related to counting schema, or to mathematics pedagogy in these areas. Examples of such themes include, but were not limited to, operations, units construction, counting processes, representations, interactions with manipulatives, and finger movements. Further,

Attending Flow-Process Rubric

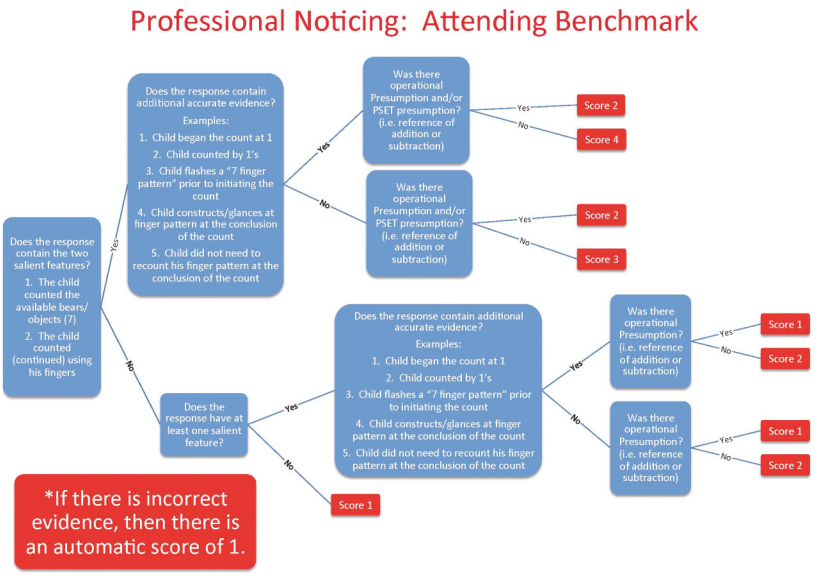

Figure 2. Attending Flow-Process Rubric.

while thematic coherence, as a construct, may encompass aspects of a particular moment beyond a participant's mathematical activity/reasoning, the directed nature of the professional noticing prompts (i.e., "Please describe in detail what the child did in response to this problem") necessarily resulted in responses organized around the mathematical activity of the moment – which is an intentional element of the larger investigation of professional noticing of children's mathematical thinking. Participant responses were reviewed by an individual researcher to develop these primarily arithmetically oriented themes. Responses were then evaluated, by the same individual researcher, for the presence of such themes and overlap of a theme across component processes. For example, references to counting within attending and interpreting responses was deemed coherent. The two researchers then compared their thematic analysis of responses and assignations of coherence across responses (A-I, I-D, A-D). In the instance of discrepancies, researchers negotiated to agree upon themes and coherence designations for response pairs. We elected to include the examination of paired attending and deciding (A-D) responses for our analysis although such pairings appear at odds with some temporal depictions of professional noticing – namely that deciding is mediated by interpreting, and interpreting is mediated by attending (e.g., Schack et al., 2013; Schoenfeld, 2011); we reasoned that PSETs may introduce a theme or idea in the attending response, discuss other ideas in an interpreting response, and revisit their initial theme or idea in the deciding response (see Figure 3). Further, as mentioned earlier, temporal assumptions regarding the component processes are increasingly being called into question (Castro-Superfine et al., 2017).

When evaluating paired responses for common themes, we considered only the presence (*coherent*) or absence (*not coherent*) of such themes in our determinations. As such, this initial and quite rudimentary definition of coherence refers to a thematic linkage between responses. Specifically, a thematic linkage refers to a common mathematical or pedagogical idea which manifests within two components of professional noticing (e.g., attending and interpreting). To operationalize this idea, a thematic linkage (or coherence) is ascribed when PSET responses to component items of a professional noticing measure feature a common theme or idea (see Figure 4 for examples). In the first example, the theme of counting (and counting on from composite) emerges in both the attending

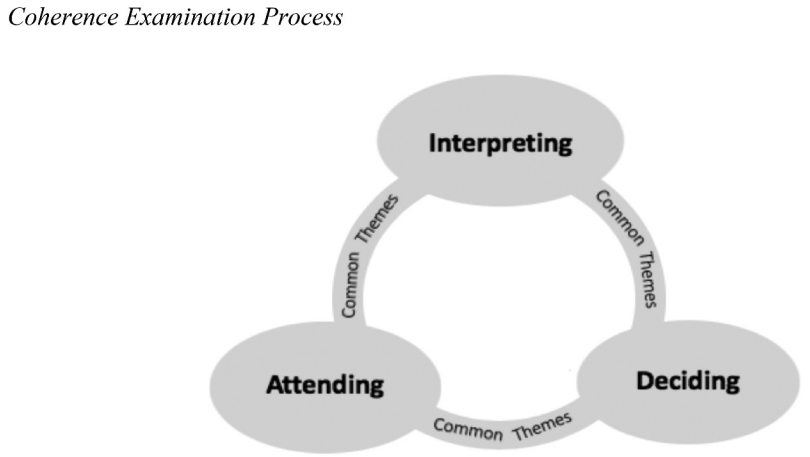

Figure 3. Coherence Examination Process.

and interpreting (A-I) responses. Conversely, another respondent's A-I responses do not feature a common theme. Rather, the attending response focuses on pairing and operations while the interpreting response focuses on key words. We conducted further exploratory analysis of coherent responses that appeared to increase in substance (e.g., building on a concept or idea), however, we were unable to disentangle the complexity of this analysis within the current study design and discuss this in more detail later in the paper. As such, our current analysis centers on a fundamental notion of coherence as thematic linkage across professional noticing components.

Findings

To provide some limited context for the coherence analyses that follow, we reiterate some of the findings reported in Schack et al. (2013) and Fisher et al. (2018). While these findings have been previously published, we find that revisiting them, briefly, in the following paragraph provides a productive lens through which to view the current study of professional noticing coherence.

In these prior studies, regarding the changing (pre/post) quality of PSET professional noticing, we detected shifts in PSET attending, interpreting, and deciding capabilities for both the implementation and comparison participants; however, the shifts (from lower rankings to higher rankings) are more pronounced among the implementation participants. As attending, interpreting, and deciding scores are not interval level, Wilcoxon signed ranks tests were conducted as a nonparametric alternative to paired t-tests. These tests revealed statistically significant differences, pre/post, across all three components of professional noticing at the implementation sites. The comparison sites, however, did not experience statistically significant changes across the component processes. Rather, only deciding was significant for this group (See Fisher et al., 2018, pp. 223–225 for statistical explication of these findings).

Moving to examinations of thematic coherence of PSET professional noticing responses, we note shifts toward coherence from pre/post among the implementation participants; however, there appear to be no such positive shifts among the comparison participants (See Tables 1 and 2).

Insert Tables 1 and 2

The data indicate the comparison participants' coherence between responses is more likely to either decrease or stay the same with respect to the implementation participants' response coherence. Note, *positive (pos.)* and *negative (neg.)* refer to respondents' change from lack of coherence to coherence (pre/post) or vice versa. *Same* refers to no change in respondents' coherence pre/post.

Examples of Responses by Type (Coherent, Lacking Coherence)

Attending-Interpreting		
Coherence	**PSET Attending Response**	**PSET Interpreting Response**
	"The child began by recounting the bears. Next, he counted on until he arrived at eleven using his fingers as place holders. He then counted to see how many fingers he was holding up."	"This child understands how to count on; however, he did start at one. He understood that he had to count on to eleven to see how many shells would be left over."
	Common Theme(s): Counting	
Lack of Coherence	"I am guessing that he was wanting to pair the shells to the red bears, so he subtracted seven from the total number of shells to get 4."	"He looks for key words."
	Theme(s): Pairing; Operations [subtraction]	*Theme(s): Key Words*
Interpreting-Deciding		
Coherence	**PSET Interpreting Response**	**PSET Deciding Response**
	"He used addition to solve a subtraction problem which can be an effective way to solve that type of problem. By realizing that he had 4 shells left over, he was able to conclude that the answer was 4."	"I would first explain to the child that he just completed a subtraction problem, then I would give him another problem using the shells and bears. I would take away one of the shells and ask him to repeat the problem to see if he really understands the concept. He needs to practice as much as possible."
	Common Theme(s): Operations [subtraction]	
Lack of Coherence	He knows how to count and how to identify the question.	Add another color choice of bear to see if the child can answer with other items.
	Theme(s): Counting	*Theme(s): Materials Adjustment*
Attending-Deciding		
Coherence	**PSET Attending Response**	**PSET Deciding Response**
	"This child first counted the bears individual[ly] by actually pointing to each bear. He then used his fingers to what seemed like count up to eleven from the actual number of bears he counted to get the number 4."	"I would next screen both objects and ask a similar type of question to see if he could solve the problem without actually physically counting the bears."
	Common Theme(s): Counting; Physical Interaction with Materials	
Lack of Coherence	"He counted how many bears there were, which were 7, then continued to count the leftovers until he reached 11. He used his fingers to count the difference between 7 and 11, which he said is 4."	"Can you draw a picture to help explain your answer? That way the student could draw out his thinking and explain whether or not he understands the finer concept of subtraction or addition."
	Theme(s): Counting; Finger Use	*Theme(s): Representation; Operations [addition; subtraction]*

Figure 4. Examples of Responses by Type (Coherent, Lacking Coherence).

Table 1. Score Distributions for Coherence Items.

		Attending-Interpreting Coherence (A-I)		Interpreting-Deciding Coherence (I-D)		Attending-Deciding Coherence (A-D)	
		Not Coherent	Coherent	Not coherent	Coherent	Not Coherent	Coherent
Implementation (n = 213)	Pre	33%	67%	62%	38%	68%	32%
	Post	13%	87%	39%	61%	43%	57%
Comparison (n = 60)	Pre	22%	78%	57%	43%	78%	22%
	Post	32%	68%	58%	42%	73%	27%

Table 2. Change Distributions for Coherence Items.

	Attending-Interpreting Coherence (A-I)			Interpreting-Deciding Coherence (I-D)			Attending-Deciding Coherence (A-D)		
	Neg.	Same	Pos.	Neg.	Same	Pos.	Neg.	Same	Pos.
Implementation (n = 213)	07%	66%	27%	14%	49%	37%	12%	51%	37%
Comparison (n = 60)	22%	67%	12%	20%	62%	18%	12%	72%	17%

We also aimed to determine if the implemented intervention was effective in regard to growth in coherence across PSET component responses. To determine the effectiveness of the implemented intervention, separate logistic regression analyses were performed on each of the six outcomes (A, I, and D (A/I/D) scores; A-I, I-D, A-D coherence). In each model, treatment condition (0 = comparison group, 1 = implementation group) was considered as a predictor and pretest score for that outcome was used as a covariate; thus, the effect of implementation condition was computed while controlling for pretest score. This strategy can be thought of as a logistic version of testing for a treatment effect using ANCOVA controlling for pretest score. The results for the treatment condition predictor, including odds ratios (an effect size for logistic regression), are reported in Table 3. For all items except attending, treatment condition had a statistically significant positive logistic regression coefficient; that is, the scores of the implementation group increased significantly more than the scores of the comparison group.

To investigate the relationship between PSET attending (A), interpreting (I), and deciding (D) performance with their coherence across the component processes, we examined polychoric correlations on nine item pairs (A/I/D each crossed with A-I coherence/I-D coherence/A-D coherence) for the pre-implementation assessment for all participants (intervention and comparison, n = 273). A/I/D are very weakly correlated with each other and A/I/D are weakly to moderately correlated with A-I/A-D/I-D (see Table 4). Note, the statistically significant negative correlation between I and I-D is somewhat puzzling and worthy of additional attention. As some of the study participants were enrolled in mathematics content courses (rather than mathematics teaching methods courses), it is possible that these individuals' instructional repertoire was less developed, thus influencing findings related to deciding performance.

Particularly noteworthy are the observed significant relationships between AI-coherence and attending and interpreting as well as AD-coherence and interpreting and deciding. In these instances, the determination of a theme when attending and the continuation of that theme through either the interpreting and/or deciding component processes is positively correlated with professional noticing quality in all three of the components. These findings suggest complex interplay between the selection (within the attending process) and continuation of themes during the approximation of professional noticing, and the quality of such noticing responses.

Table 3. Logistic Regression Coefficients for Treatment Condition on each posttest Dependent Variable, Controlling for pretest Score of that Variable.

Dependent Variable	Logistic Regression Coefficient	Standard Error	p	Odds Ratio
Attending	0.261	0.239	.275	1.298
Interpreting	0.674	0.252	.007**	1.962
Deciding	1.136	0.288	<.001***	3.114
Attending-Interpreting (A-I) Coherence	1.256	0.355	<.001***	3.510
Interpreting-Deciding (I-D) Coherence	0.811	0.297	.006**	2.250
Attending-Deciding (A-D) Coherence	1.298	0.322	<.001***	3.553

Note: n = 273; * = significant at .05, ** = significant at .01, *** = significant at .001;

Table 4. Pretest Correlations Between Noticing Components and Coherence Types.

	Attending	Interpreting	Deciding	Attending-Interpreting Coherence (A-I)	Interpreting-Deciding Coherence (I-D)	Attending-Deciding Coherence (A-D)
Attending						
Interpreting	.109					
Deciding	.080	.025				
Attending-Interpreting Coherence (A-I)	.253**	.476***	.175			
Interpreting-Deciding Coherence (I-D)	.148	.177	−.169*	.144		
Attending-Deciding Coherence (A-D)	.065	.298**	.273	.302**	.525***	

Note: $n = 273$; * = significant at .05, ** = significant at .01, *** = significant at .001

Discussion

Building upon studies of teacher noticing/professional noticing which describe the practice as an assemblage of interrelated component processes, we argue that coherence is an important consideration for productive enactment of such noticing. As researchers examine the reflexivity and temporal connections between these component processes (Castro-Superfine et al., 2017; Sherin, 2017), there is an undergirding assumption that the processes are related in some fashion. Toward this end, this exploratory investigation of thematic coherence provides one such window into the nature of such relationships.

Noticing Performance and Coherence

One of the vexing issues regarding the study of professional noticing relates to measurement (Jacobs & Spangler, 2017; Thomas, 2017). Broadly speaking, measurement approaches may focus on categorization, scoring/ranking, or relating to a standard (Stockero & Rupnow, 2017). Common to many approaches, though, is attention to the nature or quality of practice *for a particular component process* (Jacobs et al., 2010; Krupa et al., 2017; Schack et al., 2013). We note, however, that not all researchers adopt a strict component-driven measurement posture, for example, Leatham et al. (2015) focused more broadly on the noticing of mathematical opportunities within a complex instructional context.

Stockero and Rupnow (2017) examined attending and interpreting in concert while considering preservice teachers' decision-making as a separate process. Nevertheless, the component processes of noticing seem to drive, or at least inform, many considerations of measurement. We argue that, in addition to considering the quality of individual component processes, coherence across component processes may provide further insight into one's noticing performance.

One example of how coherence may illuminate our understanding of professional noticing relates to the manner in which novices' (e.g., preservice teachers) initially approximate (Grossman et al., 2009) noticing. While Van Es (2011) explicated characteristics of baseline noticing including "attend[ing] to the whole class environment, behavior, learning, and teacher pedagogy" and "form[ing] general impressions of what occurred" (p. 139), little attention is given to the connective tissue between one's attending and interpreting (i.e., forming impressions). When attempting to notice within complex environments, the quality of novices' enactment of component processes may be uniformly low; however, coherence provides a lens through which they are beginning to organize such processes in concert with one another – specifically, *the extent to which novices are able to sustain an idea or theme across the component processes of noticing*. Whereas thematically disconnected attempts at professional noticing may reflect some approximation or incipient form of noticing, the continuation of a common theme (see Figure 2) across components may signal some initial noticing proficiency (i.e., thematic reasoning) that would otherwise

go undetected in more component-oriented measures of noticing (Fisher et al., 2018; Schack et al., 2013). Quite possibly, such thematic coherence may be an initial first step toward more productive forms of noticing. Moreover, it is plausible that teaching to prioritize careful thematic construction in the context of attending (e.g., identifying the key mathematical feature of a given moment) may be a productive strategy for improving responsive teaching practices across all components. In particular, the significant relationships of A-I coherence with attending and interpreting response quality as well as of A-D coherence with interpreting and deciding response quality lend credence to such claims.

On this point, though, it is important to grapple with consequences stemming from component-oriented measurement schemes (Krupa et al., 2017; Males, 2017; Schack et al., 2013). The isolation of individual components of noticing via individual assessment prompts for each process (attending, interpreting, deciding) likely results in data which look quite different than other, more synthetic approaches which combine these processes in some manner. This speaks to one of the many measurement challenges associated with professional noticing (Jacobs & Spangler, 2017; Thomas, 2017). Specifically, professional noticing is, simultaneously, theorized in terms of its components (i.e., attending, interpreting, deciding) and performance thereof as well as in gestalt – the ways in which such components are interrelated, reflexive and enacted dynamically and fluidly. Exacerbating this tension is the manner in which most (if not all) measures of professional noticing are, to some extent, proximal to the actual practice. For example, watching a video-recorded excerpt of an instructional interaction and then responding to written prompts is, arguably, quite distant from actual professional noticing in one's own classroom. Such activity is, however, a viable decomposition of practice (Grossman et al., 2009) that provides, arguably, a productive platform to develop professional noticing capacities. Nevertheless, we suggest that findings in this manuscript (and many other examinations of noticing) be interpreted with an eye for context and limitation.

Theorizing Variants of Coherence

Thus far, we have confined our examination of coherence to thematic connections which manifest across PSET responses in an approximated professional noticing context. However, during the course of our study, we informally explored another variant of coherence. Specifically, we attempted to define a type of coherence which we will refer to as *augmented coherence*, that drew upon the notion of building on concepts related to students' mathematical thinking (Leatham et al., 2015). While this manuscript focuses on our results on thematic coherence, our efforts to evaluate augmented coherence continue. See Figure 5 for an example of augmented coherence.

In this example, the PSET introduces themes of counting and finger patterns (among others) in the attending response, and the interpreting response builds on these themes by theoretically framing the child's counting strategies via perceptual counting (Steffe, 1992; Steffe et al., 1988) and noting the extent to which such counting is predicated on physical materials. Returning to the first example of Figure 3 and its thematic coherence (i.e., counting), the example in Figure 4 illustrates the potential for individuals to not only follow a theme across the component processes, but to leverage that theme and braid it with more

Example of Augmented Coherence

PSET Attending Response	PSET Interpreting Response
I think that the child counted the bears, starting from one, up to the seventh and then began counting on his fingers up to eleven. He was then holding up four fingers in which he replied his answer. He had to start from one instead of seven and then had to use his fingers to keep track of the leftovers.	He still uses perceptual counting and counting-from-one strategies. He had to actually count the bears by touching next to them and then had to also use his fingers to count up to find the leftovers. He is in the second stage of the SEAL stages which means that he is still using count-from-one strategies and needs perceptual or manipulatives to count up.

Figure 5. Example of Augmented Coherence.

complex theoretical ideas regarding the mathematical moment at hand. We argue that such responses are augmented in that one component process substantively extends or builds upon another. In the example above, the theme of counting, initially raised in the attending response, is further developed in the interpreting response via the overlay of the PSET's knowledge of mathematical development in this area. We note, though, that such augmented coherence may be distinct from other portrayals of noticing development which layer increasingly sophisticated and elaborated means of attending and interpreting, but do not explicitly ascribe thematic connection across the component processes of noticing (Van Es, 2011). In the *Learning to Notice* framework put forth by van Es, one indicator of highest level of noticing (extended) is "on the basis of interpretations, propose alternative pedagogical solutions" (p. 139). Imagine a practitioner who attends to arithmetic activity of a small group of students and makes some interpretation regarding the students' varying units construction. Perhaps, the teacher's decision, though, is aimed at introducing a context, transitioning to the next task group or lesson agenda item, etc. Indeed, explicit thematic connections may become somewhat tenuous in these instances even though the teacher is ostensibly leveraging some interpretation of the moment to formulate a decision (in accordance with higher levels of noticing). While such portrayals of noticing in the extant literature may imply some manner of coherence across the component processes of noticing, such connections are not made entirely explicit. Whereas augmented coherence rests on the idea that, fundamentally, there is some explicit and observable thematic connection across component processes that is developed or elaborated upon in some way.

Ultimately, we did not include this variant in our analyses as we struggled to establish reliable boundaries for such coherence. Nevertheless, we argue that other variants of coherence exist beyond thematic coherence and ascertaining the nature of such variants will do much to illuminate relationships between the component processes of teacher noticing.

Implications for Professional Learning

Stipulating that considerations of thematic coherence, as one potential variant, are of value with respect to teacher noticing, our data suggest that such coherence may develop in conjunction with an intervention focused on professional noticing. As we observed improvement in participants' attending, interpreting, and deciding capabilities (Schack et al., 2013), it could be that this positive change in coherence was a natural byproduct of such growth. Alternately, some portion of the intervention could have directly promoted development in this area. Given the emphasis on a detailed numeracy framework (Steffe, 1992) within the professional noticing intervention, such promotion of continued themes (i.e., arithmetic counting schemes) was fundamental to the experience. Findings put forth by Jacobs et al. (2010) regarding the deliberate and explicit practice required for growth in noticing suggest that, perhaps, coherence (as an aspect of noticing) may also be intentionally developed. In any event, professional learning leaders may begin to experiment with thematic coherence as valuable connective tissue for the component processes of noticing.

Development of coherence among practitioners (particularly novice practitioners) is not a trivial aspect of mathematics teaching. For example, attending to key aspects of a child's counting, deriving some deeper meaning from such activity (i.e. units construction), and acting upon that interpretation in a manner that directly connects to counting development is a remarkably responsive instructional sequence. Essential to this sequence is the extent to which the practitioner is able to sustain a common theme (i.e., counting) throughout the component processes of noticing. Additionally, thematic coherence is also essential for productive noticing in equitable domains of mathematics teaching. For example, professional coaching experiences aimed at developing participants' noticing for equity (Baldinger, 2017) may explicitly examine how certain themes (i.e., social organization of classroom, student compliance) move across the component processes and influence teachers' attention, reasoning, and decision-making in a given moment.

Limitations and Conclusion

This study is limited by the constrained nature of the professional noticing measure. Specifically, the use of a single video anchor and prompts for pre/post situates noticing performance within a somewhat narrow context (i.e. child's units construction activity). Additionally, the association of the component processes with particular measure prompts does not reflect evolving perspectives regarding the interplay between such processes (Castro-Superfine et al., 2017). A more complex measurement approach (potentially featuring multiple videos) would likely offer a more comprehensive portrayal of participants' noticing and connections therein. Further, the rating inconsistencies between attending and interpreting/deciding (i.e. 4-point scale attending; 3-point scale interpreting and deciding), along with our relatively rudimentary approach to reliability (i.e., negotiated agreement) potentially creates a mismatched portrayal of noticing performance within the individual components. This level of measurement sophistication may be related to our finding of a statistically significant negative correlation between I and I-D which was incongruent with other findings and somewhat puzzling.

Lastly, our inability to establish reliable boundaries for augmented coherence limits the scope of this work as we are primarily confined to comment on more rudimentary variants of thematic coherence. Despite these limitations, we argue that our study provides important initial terrain to consider and examine linkages between the component processes of PSETs' noticing.

This work represents a first foray into the nature of coherence and the manner in which coherence may illuminate the dynamic and reflexive relationships between the component processes of teacher noticing. We fully stipulate that the findings here cast only faint candlelight on this topic; nevertheless, we optimistically find much promise in the questions that may be drawn from this inquiry including those related to different types of coherence, and how such variants may enlarge our understanding of teacher noticing as a practice. Toward this end, research that further explores the nature of noticing and generates discourse across projects is needed. Our intent with this investigation is to create some initial space for just such a conversation.

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