What role does professional noticing play? Examining connections with affect and mathematical knowledge for teaching among preservice teachers

Cindy Jong, Edna O. Schack, Molly H. Fisher, Jonathan Thomas & David Dueber

ZDM – Mathematics Education

ISSN 1863-9690

ZDM Mathematics Education
DOI 10.1007/s11858-020-01210-5

ONLINE FIRST

Springer

ZDM - Mathematics Education
https://doi.org/10.1007/s11858-020-01210-5

ORIGINAL PAPER

What role does professional noticing play? Examining connections with affect and mathematical knowledge for teaching among preservice teachers

Cindy Jong[1] · Edna O. Schack[2] · Molly H. Fisher[1] · Jonathan Thomas[1] · David Dueber[3]

Accepted: 1 December 2020

Abstract

This study examined the intersections of preservice elementary teachers' (PSET) professional noticing (PN) of children's mathematical thinking, two mathematical knowledge for teaching (MKT) domains: mathematics content knowledge (MCK) and mathematics pedagogical content knowledge (MPCK), and two affective domains: attitudes and dispositions toward teaching mathematics. An instructional module focused on PN of children's early algebraic thinking, as defined by Jacobs et al. (J Res Math Educ 41: 169–202, 2010) to include three components: attending, interpreting, and deciding, was implemented with 170 PSETs. The PSETs, who participated in the instructional module, the implementation group, showed significant positive growth in attending, interpreting, and attitudes toward mathematics. There was no significant change in dispositions toward teaching mathematics and a decrease was observed in PSET deciding and MKT. A comparison group of 126 PSETs enrolled in mathematics methods courses did not participate in the instructional module. PSETs in the comparison group showed a decrease across all measures including attending, interpreting, deciding, attitudes, dispositions, and MKT. Results showed statistically significant connections within and between some of the constructs; however, the limitations of the study call for further investigation.

Keywords Professional noticing · Mathematical knowledge for teaching · Affect · Attitudes · Dispositions · Preservice teachers

✉ Cindy Jong
cindy.jong@uky.edu

Edna O. Schack
e.schack@moreheadstate.edu

Molly H. Fisher
molly.fisher@uky.edu

Jonathan Thomas
jonathan.thomas1@uky.edu

David Dueber
david.dueber@uky.edu

1 University of Kentucky, 105 Taylor Education Building, Lexington, KY 40506, USA

2 Morehead State University, 304 Ginger Hall, Morehead, KY 40351, USA

3 University of Kentucky, 237 Dickey Hall, Lexington, KY 40506, USA

1 Introduction

Teacher noticing research has garnered much attention over the past two decades (Schack et al., 2017; Sherin et al., 2011; Stahnke et al., 2016), having been studied in relation to its impact on teacher practices (Jacobs et al., 2011; Stockero, 2008; van Es, 2011), teacher affect (Fisher et al., 2014), and teacher knowledge (Sturmer et al., 2013). Mathematics education research, simultaneously, has established strong connections between teacher content knowledge and teaching practices (Hill et al., 2008; Kaiser et al., 2015; Kunter et al., 2013). And, the influence of affect (e.g. attitudes, dispositions) on mathematics teaching practices (Jacobson & Kilpatrick, 2015; Philipp, 2007; Schoenfeld, 2015; Swars et al., 2018) also appears frequently in recent research.

Despite the growing interest in research on teacher noticing, our understanding of how teacher noticing intersects with mathematics knowledge for teaching (MKT) and affect to collectively impact teachers' instructional practices is not well-developed. Exploring the relationship among teacher

Published online: 19 January 2021

noticing, MKT, and affect seems an important step toward understanding if these constructs are interrelated and whether they do, or do not collectively influence teachers' instructional practices. Moreover, teacher noticing is a core practice that is learnable (Jacobs & Spangler, 2017) that, if additionally supported by improved MKT and/or affect, each might synergistically influence the other.

Blömeke et al. (2015), proposed a continuum of competence suggesting teacher noticing as the link between dispositions (cognition and affect) and teacher practices. Blömeke et al. (2015) provides the theoretical framework for the study herein in which we examined the intersection of preservice teachers' (PSETs') MKT, affect, and teacher noticing. We explored the following research questions:

1. To what extent can the implementation of a professional noticing module influence PSETs' professional noticing skills, affect toward teaching mathematics, and mathematical knowledge for teaching, in comparison to PSETs who did not participate in a professional noticing module?
2. In what ways do PSET professional noticing skills relate to their affect toward teaching mathematics, and mathematical knowledge for teaching?

2 Literature review

2.1 Professional noticing of children's mathematical thinking

Professional noticing of children's mathematical thinking (PN), a specific branch of teacher noticing defined by Jacobs et al. (2010), is widely referenced in the mathematics education community. It is important to note that Jacobs et al. considered deciding as the internal decision, not the actual enactment of the decision, thus their conceptualization of professional noticing of children's mathematical thinking is consistent with teacher noticing in that it is an internal, or hidden practice of teaching. Often, professional noticing of children's mathematical thinking is shortened to professional noticing, leading to conflation with the more general term, teacher noticing (Sherin et al., 2011) which generally includes only the attending and interpreting components, and sometimes only attending (Hanna, 2012). Teacher noticing is generally used as the umbrella term for the various conceptualizations of the construct. Jacobs and Spangler (2017) organize the varying conceptualizations found in the research into three categories: (1) attention only, (2) interrelated attention and interpretation, and (3) interrelated attention, interpretation, and decisions about next steps. The third conceptualization mirrors Jacobs et al.'s (2010) work and is the conceptualization addressed in this current study.

Other related, but nuanced terms for the components are employed in the literature. For example, in their systematic review of empirical research on teachers' perception, interpretation, and decision-making, Stahnke et al. (2016) used both "attending" and "perception" as search terms. Santagata and Yeh (2016) speak specifically to the terms perception and attending, using the two terms interchangeably. One can infer, then, that perception is often used to describe a skill similar to the attending component of PN.

Relevant to the definition of PN used herein, one-fourth of the 60 empirical studies reviewed by Stahnke et al. (2016) examined all 3 components of PN, attending (perception), interpretation, and decision-making. One finding of the review was that video-based interventions can support teachers to develop noticing skills. Schack et al. (2013) demonstrated that PN is a learnable skill through their study in which PSETs improved their responses to prompts related to attending, interpreting, and deciding over the course of a mathematics methods course embedded with video-based PN instruction and assessment.

Jacobs and Spangler (2017) construe teacher noticing as a core practice of teaching that is learnable, can positively impact student learning, and supports teacher learning. Teacher noticing research mirrors noticing research in fields beyond education, reinforcing the value of studying this construct. Noticing is "foundational for teachers' in-the-moment decision making" (Jacobs & Spangler, 2017, p. 772). Thus, conceptually, for research on understanding teaching practices and improving teacher practice, teacher noticing is important. In this current study, we focus on teacher noticing as a learnable skill, and explore the interrelationship of PSETs' PN with their affect and their MKT.

2.2 Affect in mathematics education

Affect, used here as a broad term including attitudes and dispositions, includes constructs that vary along the spectrum from emotion to cognition (Philipp, 2007). Attitudes are closer to emotion whereas dispositions are more cognitive in nature (Philipp, 2007), which is why we included both in our study. Schoenfeld (2015) points out the complexities of understanding affect in different contexts and while asserting that underlying beliefs influence practices, he emphasizes the importance of understanding how affect influences decisions and actions. He also urges researchers to examine changes in affective factors over time as he notes that, since it takes years to develop beliefs and practices then one would expect the same for substantive changes accompanied by support over time. Although there are affordances and drawbacks to various methods that examine affect in mathematics teaching, scholars agree that attitudes, dispositions, and beliefs matter (Aguirre & Speer, 1999; Jacobson & Kilpatrick, 2015; Pehkonen & Toerner, 1996; Roesken et al., 2011;

Wilkins, 2008). It is especially critical to examine methods for developing more positive attitudes toward mathematics among preservice elementary teachers who often have negative experiences that influence their productive disposition to teach mathematics (Ingram et al., 2018; Pourdavood & Liu, 2017). In this study, we focus on the two affective factors of attitudes and dispositions.

We draw on the Philipp (2007) description of affect as "a disposition or tendency or an emotion or feeling attached to an idea or object" (p. 259). He specifically defines attitudes as "manners of acting, feeling, or thinking… Attitudes, like emotions, may involve positive or negative feelings. …Attitudes are more cognitive than emotions, but less cognitive than beliefs" (p. 259). The main distinction between attitudes and dispositions is a feeling versus a tendency that might be more directly linked to an action. More recently, Jacobson and Kilpatrick (2015) described "productive disposition for teaching mathematics" as "mathematics teachers' malleable orientation toward—and concomitant beliefs, attitudes, and emotions about—their own professional growth, the subject of mathematics, and its teaching and learning that influences their own and their students' successful mathematics learning" (p. 402). Their definition includes a variety of observable traits and "rejects the notion…of a singular construct" but emphasizes the "adjective *productive*" (p. 403). We concur that there are links and overlap among affect, beliefs, attitudes, and dispositions. While dispositions are informed by beliefs, we use the term dispositions because the construct of interest is more about the ways in which preservice teachers intend to teach mathematics.

2.3 Mathematical knowledge for teaching

Shulman (1986), Ball et al. (2008), and Tatto et al. (2008) provide the foundation for thinking about the MKT explored in this study. Shulman (1987) identified the major categories of teacher knowledge as subject matter knowledge, pedagogical content knowledge (PCK), and curricular knowledge. He defines PCK as "the ways of representing and formulating the subject that make it comprehensible to others" (p. 9). PCK has remained the subject of much research over the past three decades (Ball et al., 2008; Depaepe et al., 2013; Hill et al., 2008) based on the premise that Shulman presented his theory as a heuristic method for studying the types and processes of knowledge needed by teachers.

Ball et al. (2008) assert that teaching "requires a special kind of content knowledge [that] is worth our collective investment and cultivation" (p. 394). Ball et al.'s work aimed to bring greater precision to the content knowledge needed by teachers through further clarifying Shulman's (1987) major categories of teacher knowledge and organizing into a map of the domains of mathematical knowledge for teaching (MKT). Like Shulman (1986) and Ball et al. (2008) perceive their map of MKT as a heuristic for further research to bring more clarity to the types of content knowledge needed by teachers.

Numerous studies in mathematics education have pursued the heuristic value of Shulman's (1986) and Ball et al.'s (2008) characterizations of content knowledge for teaching, researching one or more of the subdomains identified. While earlier studies of content knowledge for teaching often relied on measures such as number of courses taken or degrees or certifications acquired, more recent work has attempted to explore teacher content knowledge through quantitative or qualitative analysis. Hill et al. (2005) found content knowledge for teaching mathematics a significant predictor of first and third grade student gains. The work of Baumert et al. (2010) resulted in distinguishing content knowledge (CK) from PCK, asserting that insufficient CK cannot be redressed with increased emphasis on PCK in teacher education, but also, that CK alone is not as strong a predictor of instructional effectiveness as PCK. The results of these studies support the importance of domain-specific knowledge to instructional effectiveness. Charalambous et al. (2020), while contending that the distinguishability of CK and PCK has met with mixed results, still reaffirm the positive impact of teacher knowledge on student progress.

Tatto et al.'s (2008), large-scale international study, Teacher Education and Development Study in Mathematics (TEDS-M) developed The Knowledge for Teaching Mathematics instrument which assessed two domains of mathematics knowledge for teaching: mathematics content knowledge (MCK) and mathematics pedagogical content knowledge (MPCK) (Brese & Tatto, 2012). Selected items from the TEDS-M instrument formed the MKT assessment instrument of this study.

2.4 Connections: professional noticing, affect, and mathematical knowledge for teaching

As illustrated in the sections above, each construct, PN, affect toward mathematics, and MKT has its own rich history of research. In this study we are exploring not only the constructs independently but also potential relationships among the three constructs. As noted earlier in this article, Jacobson and Kilpatrick (2015) characterize the constructs of teacher disposition, knowledge, and practice as "necessarily entangled" (p. 402). Perhaps this entanglement is exactly what challenges researchers to both uniformly define the constructs and to attribute the impact of any one construct on another. Despite the challenging work, researchers have explored the interrelationships between or among PN, affect, and/or MKT.

Fisher et al. (2014) explored connections between PSETs' attitudes toward mathematics and their PN. Using the Attitudes Toward Mathematics Inventory (ATMI) developed by

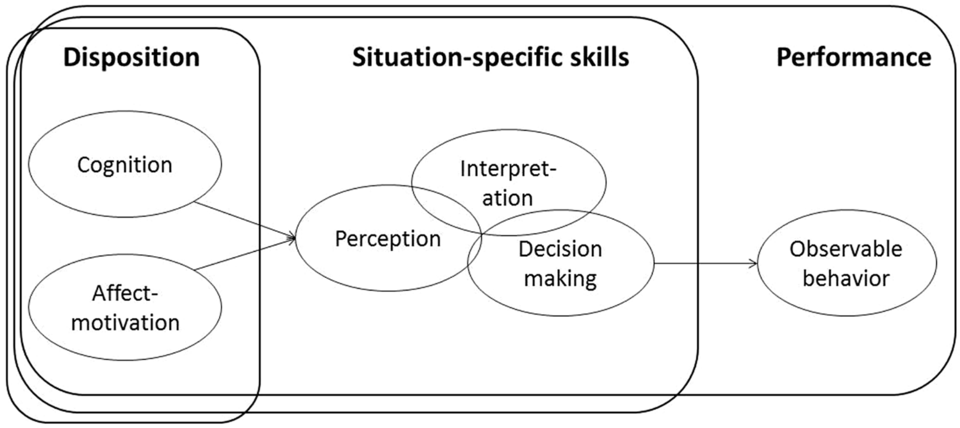


Fig. 1 Blömeke's (2015) Model of Competence as a Continuum. Reprinted from Blömeke et al., (2015)

Tapia and Marsh (2004) and adapted for preservice teachers by Schackow (2005) and Fisher et al. (2014) found significant change amongst participants from pre to post assessment on three of the four factors of the ATMI, including enjoyment, self-confidence, and motivation. Statistically significant positive changes were also observed in all three PN components; however, there were no significant correlations between ATMI factors and PN.

There is research demonstrating relationships of MKT to PN or affect. Kersting (2008) found that in-service teachers with stronger mathematical content knowledge provided more sophisticated interpretations of teaching video clips. LaRochelle and Mammo's (2019) study also resulted in a significant correlation between MKT and PN. The results of their study, with middle and high school teachers, indicated a significant relationship between the PN deciding component and teachers' MKT, specifically, specialized content knowledge. Also investigating specialized content knowledge and PN, Dick (2017) observed that, as specialized content knowledge increased, PSETs engaged in increased professional noticing. Ingram et al. (2018) noted, though not through statistical analysis, similar parallel increases, in this case, PSETs' mathematics content knowledge and PSETs' affect toward mathematics over the course of three-year program. Kutaka et al. (2018) found positive changes in mathematics specialist program participants' MKT, attitudes, and beliefs in relation to non-participants but, similar to Ingram et al. (2018) did not statistically examine the connections among the constructs.

While there is evidence that MKT, affect, and PN can improve and some indication of relationships between one or more of these constructs, there is little research on the interrelationships of all three constructs. Research by Charalambous (2015) begins to foray into this interrelatedness through a multiple case study examining teacher knowledge, teacher beliefs, and teacher practices. The results indicate that beliefs and knowledge interact, often "mediating the effect of the other" (p. 427) on teacher practices. This reflects Blömeke et al.'s (2015) proposed model of continuum of teacher competence characterizing dispositions as the interaction of affect and cognition, mediated by situated specific skills (essentially PN), resulting in observable teacher practices.

We aim to further study the interrelatedness PSETs' PN, affect, and MKT during a semester in which PSETs are focused on developing PN. The authors of this study conducted earlier studies in which they examined relationships of PN to other constructs. This study extends that work by engaging preservice teachers in an intervention that includes complex classroom teaching situations and examining the interactions of PSET PN with affect, as measured by two domains, attitudes and dispositions, of the Mathematics Experiences and Conceptions Surveys (MECS) (Jong & Hodges, 2015), and MKT, as measured by selected items of the TEDS-M (Brese & Tatto, 2012).

3 Theoretical framework

As mentioned previously, Blömeke et al. (2015) proposed a framework that incorporates the three constructs researched in this study, suggesting they form a continuum of competence in which the situation-specific skills of perception, interpretation, and decision-making (PN, if you will) are the bridge between dispositional resources[1] (cognition and affect/motivation) and observable behaviors such as teacher practices (see Fig. 1). In essence, they are making a case that PN skills may bridge dispositions such as affect and cognition to teaching practices. Thus, Blömeke et al.'s (2015) framework is of particular note for the research described

[1] It is important to note that Blömeke et al. (2015) refer to dispositional resources as encompassing affect and cognition while the MECSD (Jong & Hodges, 2015) measures dispositions as defined by Philipp (2007).

Table 1 Detailed sessions from E^3A module

	Algebraic content focus	PN focus	Setting and content of video	Length of video (min)	Prompts for PN
Session 1	Equality; Patterns	Attending, Interpreting	1. Review of PN with 5th grader; 2. Whole class 2nd grade, 6–7-year olds	1. 0:51 2. 2:54	1. Attend, Interpret, Decide 2. Salient features attended to and interpret math understanding
Session 2	Patterns; functions	Attending, Interpreting, Deciding	1. Whole class 2nd grade, 6–7 year olds 2. Whole class 5th grade, 9–10 year olds 3. Whole class 2nd grade, 6–7 year olds	1. 2:05 2. 2:26 3. 3:30	1. Salient features attended to, interpret math understanding 2. Salient features attended to, interpret math understanding, decide next steps 3. Salient features attended to, interpret math understanding, decide next steps in-the-moment at video pauses
Session 3	Functions	Deciding	1. Whole class 5th grade, 9–10 year olds 2. Whole class 5th grade, 9–10 year olds	1. 7:16 (viewed for HW) 3:42	1. Critical incidents; student thinking; teacher decisions 2. Attend, Interpret, Decide

in this study in which we examine the impact of a module designed to develop PN skills and the relationship of PN with affect and MKT.

Considering teacher practices as observable behavioral competencies, we contend that examining PN, affect, and MKT individually as well their interactions can help us understand the "processes [PN, in this case] that mediate the transformations of dispositions into performance" (Blömeke et al., 2015, p. 13), furthering our knowledge of what dispositional resources are key to improving preservice teacher competence in the classroom.

4 Methodology

4.1 Participants and context

Participants were PSETs ($N=296$) enrolled in mathematics methods courses during one of five semesters at five universities (two urban, three rural) in the south-central United States. Some universities participated for one semester, others for multiple semesters. Not all PSETs consented to the study and thus, their data is not reported. Treatment group PSETs ($N=170$) participated in a module designed by the researchers to develop PN in the context of early algebraic thinking in a whole class setting.

The *Examining Essential Expressions in Algebra* (E^3A) module included three 60-min sessions focused on developing PN skills through the content of early algebraic reasoning, equality, patterns, and functions, with each session focused on one or more of these content areas. Video vignettes were used to prompt discussion about the three components of PN: attending, interpreting, and deciding to respond. The video vignettes were excerpts from whole class instruction in elementary mathematics classrooms in which one of the researchers taught the elementary students or co-taught with a classroom teacher. The elementary students were either 2nd graders (ages 6–7) or 5th graders (ages 9–10). Additional details on the E^3A sessions can be found in Table 1. Comparison group participants ($N=126$) completed the mathematics methods course "business as usual" without the integration of the three E^3A sessions.

4.2 Pre and post data collection

Participants completed pre and post assessments consisting of three instruments. PN was assessed using a video-based assessment ($N=268$). Regarding measurement in affective domains, we focused on the subscales of the MECS that measure attitudes (MECSA) and dispositions (MECSD) toward mathematics ($N=149$). Additionally, PSETs responded to selected TEDS-M items ($N=196$) chosen for their relation to MCK, items representing knowing or

Table 2 Sample sizes of pre and post completion across instruments

	Total	PN	MECS	TEDS-M
Treatment	170	147	116	149
Comparison	126	121	33	47
Total	296	268	149	196

applying, and MPCK, items representing enacting or planning in response to student conception, preconception or misconception. Not all participants completed all assessments, thus the varying sample sizes. The sample sizes in Table 2 include individuals who completed pre and post instruments.

4.2.1 Professional noticing: video-based assessment

We assessed PSET PN through a 74-s video clip of an authentic classroom in which children are engaging in the meaning of the equal sign in a whole group setting. The teacher presented a number sentence, $10+10=\_\_+5$ asking the children to determine what number to put in the blank to make the number sentence true. The full class setting requires PSETs to attend to multiple understandings as displayed by the various children's responses in close temporal proximity to each other, make sense of the responses, and make an instructional decision based on the responses. Specifically, the first responding child answers with "twenty-five" and then explains that he added all of the numbers in the problem. Another child responds with "four" and explains that if you "count by fives four times, it would be twenty". The video concludes with another child responding with "fifteen" and then explaining that he counted "fifteen, sixteen, seventeen, eighteen, nineteen, twenty". Throughout the video, the teacher records the children's strategies on a whiteboard.

After watching the video clip, the PSETs complete an assessment with prompts aligned to the three components of PN in the following order: (1) Pretend that you are the classroom teacher. What might you do next? Provide a rationale., (2) What mathematical thinking and actions did you observe?, and (3) What did you learn about the children's mathematical thinking that influenced your decision in question 1?. We intentionally prompted participants to decide first in an attempt to capture their in-the-moment thinking more closely, then to consider to what they attended and how they interpreted it, leading them to the decision they made in response to the first prompt. In a prior study (Schack et al., 2013), we asked the deciding question last. Often PSETs included deciding features in their response to the attending prompt, repeating their decision throughout all three prompts. Reversing the prompts was an attempt to allow them to decide, then reflect on what led to that decision. PSETs could replay the video as many times as they wished and there was no time limit for the assessment.

PSET responses were scored by research team members on a four-point scale using decision trees programmed in JavaScript for automated scoring. Briefly, the decision trees presented a response anonymously, followed by yes/no questions regarding the existence and quality of relevant information in the response. The responses for attending and interpreting were scored higher if it addressed multiple children's thinking and not just the correct thinking of one child, and the responses for deciding were scored higher if the response included a clear relationship to children's thinking and a rationale for the decision. The JavaScript program branched to a subsequent question based on the scorer's response. Results were stored in a spreadsheet that revealed the score for each component response. Two scorers scored each response. Discrepancies were discussed and resolved through a consensus coding process (Harry et al., 2005) to reach 100% agreement. The spreadsheet of decision tree data proved very helpful to pinpoint the discrepancies (see Schack et al., 2019 for details). Sample responses for each score are provided in Table 3.

4.2.2 Affect: mathematics experiences and conceptions surveys (MECS)

MECS is a set of instruments consisting primarily of six-point Likert-scale items (ranging from *strongly agree* to *strongly disagree*) designed to measure various affective factors related to teaching mathematics, such as attitudes, beliefs, dispositions, and self-efficacy over time (Jong & Hodges, 2015). A six-point Likert-scale was used to eliminate a neutral category and allow for more variation in responses. MECS uses the aforementioned Philipp (2007) definitions of affect. We were interested in the attitudes scale (MECSA) to examine PSETs' feelings toward mathematics and how positive they felt about teaching mathematics. MECSA consists of six items, such as "Mathematics is one of my favorite subjects." and "I look forward to teaching mathematics." We were also interested in the dispositions scale (MECSD) as a measure of PSETs' orientation toward teaching mathematics in more productive ways, knowing that such dispositions would also be informed by their underlying beliefs about mathematics teaching practices. MECSD consists of ten items, such as "I plan to engage students in mathematics discussions." and "I plan to encourage students to solve mathematics problems in more than one way."

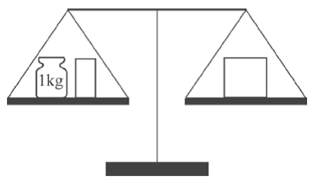
4.2.3 Mathematical knowledge for teaching: teacher education and development study in mathematics (TEDS-M)

The TEDS-M study, funded by the International Association for the Evaluation of Educational Achievement,

Table 3 Sample scores and responses

	Score 0	Score 1	Score 2	Score 3
Attending	The students attempted to answer the problem by coming up with different ways to solve the problem in their head. They then answered the problem and explained to the teacher how they came up with their answer	The children were trying to add the numbers they saw instead of trying to find a number that would make the equation true. The last student seemed to have been heading in the right direction, he decided to start at 15 and count up by 5 to get to 20, he needed to reach 25 so maybe with more time and thought he would have thought to start at 20 and count up by 5 to get the correct answer	There was a mix of answers. One student just added them all together and thought that was the answer. Another student was very confused in the sense of trying to explain 5 counting up to 4 and getting 20. I think he was trying to think along the lines of multiplication	The first child added $10+10+5$ and got 25 as his answer. The second child counted by 5 until her got to 20 and used a finger for each 5 to get 20. The third child knew that the total should be 20 so he started at 15 and counted up 5. This showed him his answer was 15
Interpreting	They do not understand the equation enough to complete it	I learned that most of the children did not understand that = signs separates equations into 2 separate parts. You cannot add all numbers to both sides of the equation because those two sides are separate	The first child can add numbers but doesn't understand how the number fit in an equation. The second child know how to add and multiply he just gets the symbols for each confused. The third child understand that you can simplify each side of the equation in order to find out what number is missing to make both sides equal one another	The first child has an understanding of addition but has not grasped basic equations such as the one shown yet. The second child has an understanding of multiplication, but again, misunderstood the problem. The third child understood that $10+10=15+5$, because both problems equal 20
Deciding	Pose a harder problem	I would use an easier problem to teach to the students who do not understand how to get the answer	I would have the last child, who came up with the right answer, explain to the class what he did and why he did it in more depth than he did in the video. Then I would try another practice problem with the class to test their new knowledge	I would have the student that answered it correctly stand up and show his work on the board. I would then give the other students a chance to ask him questions. I would also ask other students if they got to the same answer in a different way and have them show their work. My rationale is that students tend to do better when learning from one another rather than be lectured to. Giving them an opportunity to work it out and discuss it with one another will be better than if I tell them the correct answer and move on

Table 4 Sample TEDS-M items with accompanying follow-up questions

Content Question	Follow-up Question
Jeremy notices that when he enters 0.2 x 6 into a calculator his answer is smaller than 6, and when he enters 6 divided by 0.2 he gets a number greater than 6. He is puzzled by this and asks his teacher for a new calculator. What is Jeremy's most likely misconception? *(Score range: 0-2)*	If you were Jeremy's teacher, how would you address his misconception? *(Score range: 0-2)*
The objects on the scale make it balance exactly. On the left pan, there is a 1 kg mass and half a brick. On the right pan, there is one whole brick.  In the balance scale above, what is the mass of one whole brick? *(Score range: 0-1)*	Please explain how you solved the problem above. *(Score range: 0-2)*

examined how well preservice teachers were prepared to teach mathematics at both the primary and secondary levels in teacher education programs across 17 countries (Brese & Tatto, 2012). The TEDS-M contained MKT items under two domains, MCK and MPCK. To assess MKT in our study, we selected from the 22 released TEDS-M items in the primary level Number or Algebra Content Domains, four items from Number and four items from Algebra. We selected items from Number and Algebra Content Domains because we were interested in PSETs' MKT that was closely related to the content goals of the module. Five of the items represented MCK and three represented MPCK.

In addition to the eight TEDS-M released items, we wrote or adapted three follow-up constructed response items resulting in seven MCK and four MPCK items, a total of 11 items. We chose a limited number of items intentionally to keep the length of the assessments to a reasonable time frame for PSETs. In our selection process, we were also cognizant of the sub-domain, item format, and the international average score on each item. The sub-domains of the released items represented a mix of MCK Knowing, MCK Applying, MPCK Enacting, and MPCK Curriculum/Planning. One of the follow-up items was in the sub-domain of MPCK enacting, while the other two were MCK Reasoning. We opted for more constructed response (CR) than multiple choice (MC) items, providing a deeper view into PSET thinking. Finally, we included a range of difficulty levels, based on the reported international averages, from a low of 32% (20% full credit plus 12% partial credit) to a high of 97% (77% full credit plus 20% partial credit) and a median of 64%.

Responses were scored according to the TEDS-M scoring guide, and similar scoring guides for researcher-designed follow-up questions, with item scores ranging from 0 to 1 or 2, depending upon the problem. Two sample TEDS-M items and the related follow-up questions are in Table 4.

5 Data analysis

As PN video-based assessment scores are not interval level data, Wilcoxon signed rank tests were performed to determine whether PN scores significantly changed from the pre to the post assessment. TEDS-M, MECSA, and MECSD scores are interval level data, thus a mixed design ANOVA was conducted on pre and post data for these scales to explore possible changes due to the treatment and repeated measures aspects of the design. In the event of a significant interaction, indicating a greater score change for the treatment group than the comparison group, a paired t test was performed to provide a significance test and Cohen's d effect size for the score change of the treatment group.

Next, in order to determine whether variables other than the treatment condition contributed to post-test scores, a multiple regression model was employed for each outcome of interest in which all measured variables at pre-test were used as predictors. Models were tested with and without interactions (e.g., PreAttending × Treatment), but none of the interactions had significant regression coefficients and these interaction models did not explain significantly more of the variability in the outcome than the main effects models. Therefore, only main effects models are reported. As not all measures were completed by all participants, missing data were handled using full information maximum likelihood, an approach which uses all available data and makes fewer assumptions about the nature of missingness than listwise or pairwise deletion techniques (Enders, 2010).

Psychometric properties (e.g. item and model fit, reliability) of the MECS and TEDS-M instruments were completed using WINSTEPS (Linacre, 2018) to inform subsequent analyses. At both pre and post assessment, the MECSA subscale exhibited one item with poor item fit according to INFIT and OUTFIT (Wright & Linacre, 1994) while the MECSD subscale exhibited two items with poor fit. Removal of these items results in a five item MECSA measure of attitudes (reliability of 0.93 pre and 0.91 post) and an eight item MECSD measure of dispositions (reliability of 0.86 pre and 0.84 post). All remaining items showed acceptable item fit, and principal component analysis of the Rasch residuals indicated each scale was strongly unidimensional.

Preliminary analyses of the TEDS-M items revealed excellent item fit according to INFIT and OUTFIT; however, the TEDS-M measure of MKT showed poor reliability (0.65 for pre and 0.55 for post) due to the limited number of items. This low reliability can reduce power in univariate significance testing such as *t* tests (Kanyongo et al., 2007), and make the results of multivariable statistical analyses such as multiple regression untrustworthy (Cole & Preacher, 2014). Thus, we present findings including TEDS-M scores with a great deal of caution.

Measurement error can create bias in multiple regression models in complex ways. While measurement error in the outcomes (i.e., post assessment variables) of a multiple regression model attenuate the strength of effects, measurement error in the predictors (i.e., pre assessment variables) can change both the strength and the pattern of results (Cole & Preacher, 2014). Due to substantial measurement error in TEDS-M data and modest measurement error in the MECSA and MECSD data, there is risk that the results of the multiple regression analyses were biased by that measurement error. Therefore, we performed a sensitivity analysis in which the original multiple regression results were compared to results in which TEDS-M, MECSA, and MECSD variables were modeled using a single indicator latent variable (SILV) (Hayduk, 1987) technique for correcting for measurement error in the TEDS-M variables. When the same models were estimated including SILV corrections for measurement error, all statistical decisions about significance were retained. Parameter bias was slight except for the coefficients for the TEDS-M predictor, which were all smaller in the observed model than the SILV model. Only the results of the more conservative observed model are reported and interpreted.

Table 5 Descriptive statistics of professional noticing, affect, and mathematical knowledge for teaching data

	Treatment		Comparison	
	Pre *M* (*SD*)	Post *M* (*SD*)	Pre *M* (*SD*)	Post *M* (*SD*)
Attending	1.03 (0.98)	1.34 (0.92)	0.96 (1.01)	0.83 (0.92)
Interpreting	0.77 (1.11)	1.22 (1.11)	0.88 (1.09)	0.66 (0.94)
Deciding	1.36 (0.81)	1.34 (0.88)	1.55 (0.98)	1.45 (0.99)
MECSA	21.85 (7.55)	24.09 (6.20)	22.73 (7.05)	22.49 (6.31)
MECSD	43.64 (3.95)	44.27 (3.35)	44.73 (2.91)	43.48 (4.47)
TEDSM	9.70 (2.75)	9.45 (2.13)	10.67 (2.33)	9.41 (2.05)

MECSA Mathematics Experiences and Conceptions Survey attitudes subdomain, *MECSD* Mathematics Experiences and Conceptions Survey dispositions subdomain, *TEDSM* items selected from the Teacher Education Development Study in Mathematics

6 Results

6.1 Changes in results on construct measures

Table 5 reports average pre and post scores for the three PN components, affect toward mathematics as measured by the MECSA (attitudes) and MECSD (dispositions), and MKT as measured by selected released TEDS-M items, of PSETs from both the treatment group and the comparison group, providing results related to research question 1. The treatment group, those participating in the E^3A module, exhibited an increase in attending and interpreting and a slight decrease in deciding, while the comparison group exhibited decreases for all three PN components. The treatment group showed increases from pre to post on attitudes and dispositions, but a decrease in MKT. The comparison group experienced a decrease in attitudes, dispositions, and MKT.

To determine the differences across groups and timepoints found in Table 5, tests of statistical significance were performed. As scores for the PN components cannot be considered interval level, non-parametric tests were performed. Wilcoxon signed-rank tests indicated that, for the treatment group, the positive change in attending was significant ($W = 0.31$, $p = 0.001$) and positive change in interpreting was significant ($W = 0.45$, $p < 0.001$), but the negative change in deciding was not significant ($W = -0.02$, $p = 0.701$). Alternatively, the comparison group experienced decreases in all three PN components, though none were statistically significant.

As MECSA, MECSD, and MKT scores are reasonably interval level, mixed-effect ANOVAs were performed and are reported in Table 6 with the left half of the table showing results of omnibus tests of interactions. All three

Table 6 Mixed-design ANOVA and post hoc tests

	Omnibus test of interaction effect				Treatment group paired *t* test				
	F	*df*	*p*	$\hat{\eta}_p^2$	Diff	*t*	*Df*	*p*	*d*
MECSA	11.38**	1, 146	0.001	0.072	2.235*	6.172	114	< 0.001	0.296
MECSD	6.322*	1, 146	0.013	0.042	0.635	1.762	114	0.081	0.161
TEDSM	6.124*	1, 162	0.014	0.036	-0.254	-1.271	117	0.206	0.092

Diff. is score difference between post-score and pre-score in the treatment group. d is Cohen's d

MECSA Mathematics Experiences and Conceptions Survey attitudes subdomain, *MECSD* Mathematics Experiences and Conceptions Survey dispositions subdomain, *TEDSM* items selected from the Teacher Education Development Study in Mathematics

Table 7 Standardized regression coefficients for observed analysis

Predictor	Outcome					
	PN			Affect		MKT
	PostATT	PostINT	PostDEC	PostMECSA	PostMECSD	PostTEDSM
	β	β	β	β	β	β
Treatment	0.27***	0.29***	− 0.00	0.21***	0.15	0.05
PreATT	0.27***	0.01	0.06	0.00	− 0.09	0.01
PreINT	0.16**	0.35***	− 0.01	0.05	− 0.04	− 0.06
PreDEC	− 0.00	0.06	0.06	0.03	− 0.07	0.01
PreMECSA	− 0.05	− 0.04	0.15	0.86***	− 0.01	0.11
PreMECSD	− 0.09	0.03	0.02	0.11*	0.46***	0.03
PreTEDSM	0.15	0.06	0.24*	0.02	0.11	0.58***
R^2	0.21***	0.21***	0.11*	0.77***	0.25***	0.27***

*p < 0.05. **p < 0.01. *** p < 0.001. The coefficient for the treatment variable only standardizes the outcome, in order to provide a standardized difference between the treatment and comparison groups while controlling for the other predictors. All other coefficients are completely standardized

PreATT Pre-attending, *PreINT* Pre-interpreting, *PreDEC* Pre-deciding, *PostATT* Post-attending, *PostINT* Post-interpreting, *PostDEC* Post-deciding, *MECSA* Mathematics Experiences and Conceptions Survey attitudes subdomain, *MECSD* Mathematics Experiences and Conceptions Survey dispositions subdomain, *TEDSM* items selected from the Teacher Education Development Study in Mathematics

interactions are significant, indicating that treatment group scores for affect (attitudes and dispositions) and MKT improved relative to comparison group scores. The results of post-hoc paired t-tests on the treatment group are on the right half of Table 6. Concerning affect, exposure to the E^3A module was associated with a significant positive change in attitudes, and a non-significant positive change in dispositions. MKT exhibited a non-significant decrease following the treatment module. Note that dispositions and MKT exhibited a treatment effect according to the ANOVA interaction term, but no significant improvement in scores for the treatment group. The significance of the treatment effect was therefore at least partially due to statistically significant score decreases in the comparison group. Furthermore, the low effect sizes of the treatment effect (partial $\eta^2 < 0.05$, $d < 0.2$) for dispositions and MKT suggest that the E^3A module does not substantially influence these variables.

6.2 Uncovering connections between or among constructs

In regard to research question 2, results of multiple regression models for all outcome variables are presented in Table 7; each column represents a separate multiple regression model. All models had a significant R^2. The E^3A module (Treatment), was seen to be a significant positive predictor of post-attending, post-interpreting, and post-attitudes, consistent with an earlier study reporting changes in PSET results on the various constructs (Fisher et al., 2019). The treatment, however, was not a significant predictor of post-deciding, post-dispositions, nor post-MKT. Within the construct of PN, pre-interpreting significantly predicted post-attending. Similarly, within the construct of affect, pre-dispositions significantly predicted post-attitudes. The only significant predictors for post-dispositions and MKT was the pre-score of the same variable.

Looking across constructs, pre-MKT significantly predicted post-deciding, suggesting a connection between the two constructs of MKT and PN. There was no other significant prediction of a post measurement of PN, affect, or MKT by a different construct.

7 Discussion

In this study we explored whether participation in the E^3A module within a mathematics methods course would influence PSETs' PN, affect toward mathematics (attitudes and dispositions), and MKT, and examined the connections among constructs. Like Jacobs and Spangler (2017), we consider PN a core practice of teaching. The theoretical framework for our study, Blömeke et al.'s (2015) continuum of teacher competence, considers PN as a situation-specific bridge that transforms one's internally-held traits into observable teacher practices. Pulling these two perspectives of PN together, it makes sense that PN can be both a situation-specific skill and a core teaching practice. As such, PN is the hidden skill that enables a teacher to integrate their latent, or not directly observable, traits, with the specifics of the situation, say student thinking, then carry out the decision in observable teacher practices. We were interested in PSET development and on which traits we can effect change that will ultimately improve teacher practices.

Our results indicate that the E^3A module did effect significant positive change on two PN components, attending and interpreting, and on one aspect of affect, attitudes. The findings regarding impact on PN was similar to prior research (Schack et al., 2013; Stahnke et al., 2016) as was the impact on attitudes (Fisher et al., 2014; Ingram et al., 2018). Further, participants in the treatment group made significant gains in attending and interpreting, unlike the comparison group where decreases were found in all PN components. This supports the argument that "noticing expertise is not something that teachers routinely possess, but research has consistently shown...is a learnable practice" (Jacobs & Spangler, 2017, p. 772). While these are promising results for the value of the E^3A module, there remain questions as to what treatments might have a broader impact on teacher traits and practices to further improve effective teacher development.

The interactions of PN components were also of interest for this research. Graphically, Blömeke et al. (2015) illustrate the components of situation-specific skills in a three-way overlapping Venn diagram, inferring an interrelationship of these components. Our analysis of the PN components for the treatment group showed several significant relationships. Post-scores of attending and interpreting, but not deciding, were significantly predicted by the pre-score of the same component. Pre-interpreting significantly predicted post-attending. This interpreting–attending finding within the PN construct supports the conceptualized interrelated nature of professional noticing components (Blömeke et al., 2015; Jacobs & Spangler, 2017) though it could also be a function of both items assessed through the same video-based assessment.

We were also interested in interactions of dispositions and attitudes. Our analysis revealed pre-dispositions significantly predicted post-attitudes. This may indicate that a more productive disposition toward teaching mathematics leads to an increase in attitudes in combination with participation in the module embedded in the methods class. This result is promising in light of Philipp's (2007) assertion that dispositions are more cognitive in nature than attitudes, and thus dispositions scaffold changes in attitudes. To affirm this, further research is needed, perhaps to better define the boundaries of these constructs (Sherin, 2017) supporting the forward movement of our understanding of teacher traits.

Given Blömeke et al.'s (2015) proposed continuum, one would expect to find a relationship between dispositional resources and situation-specific skills, that ultimately impact teacher practices. The only significant prediction between or among the constructs studied was found in pre-MKT to post-deciding. Both pre to post MKT and deciding resulted in lower scores and it might be the case that the regression analysis picked up on these similar slopes. The significance, though, may indicate a connection between MKT and the deciding component of PN, similar to LaRochelle and Mammo (2019) finding a connection between teachers' deciding and specialized content knowledge. Ours is a tenuous claim due to the similar regression analysis slopes of MKT and deciding and the MKT assessment reliability issues.

Reliability of our MKT instrument was compromised by the few numbers of items as well as the split between types of items, that is, some items were characterized as MCK and some as MPCK. Generally, released items present limitations of scope and quality, however, Brese and Tatto (2012) describe an intentional process when selecting the released items that began with "a stratified random sample of the items, stratif[ied] on both proportion correct and…MCK and MPCK" in an effort to represent "the full range of difficulty, content, and item format" (p. 3) thus, the limitation of these released items appear to be less than typical. Fisher et al. (2018) assessed preservice teachers using Learning Mathematics for Teaching (LMT) (Hill et al., 2004) but results were limited due to the participants being PSETs rather than inservice teachers for whom the test was designed. Such instrument limitations underscore the repeated call for an instrument that is more reliable in assessing MKT, and its subdomains, of preservice teachers. With the recent change in options to access the full range of TEDS-M items (G. Kaiser, personal communication, May 31, 2020), further research on connections of PN, affect, MCK, and MPCK might employ a more robust set of items for both MCK and MPCK.

Measuring PN is challenging because of its hidden nature and its conceptualization as three interrelated components. We approached the measurement of PN as a decomposition of the three components (i.e., individual prompts for attending, interpreting, deciding) (Grossman et al., 2009). In authentic practice, the enactment of the components of PN is thought to be near instantaneous (Jacobs et al., 2010) to the extent that it is difficult to establish any temporal axis regarding these components (Castro Superfine et al., 2017). Further, the extent to which the component skills are nested within one another complicates measurement schemes which treat each component skill as somewhat discrete. Nevertheless, the measurement approach of this study may be considered an approximation of practice (Grossman et al., 2009) and is consistent with other studies of this construct (see, e.g., Floro & Bostic, 2017; Jacobs et al., 2010; Krupa et al., 2017).

The literature is replete with studies exploring the constructs influencing teacher practice. Mired within this research are multiple challenges, not the least of which is PN's "hiddenness" (Jacobs et al., 2013, p. 723). Affect and MKT are similarly hidden, and thus can only be measured through inference. One strength of this study is our reliance on both analytic measures, MECSA, MECSD, and TEDS-M, as well as the more holistic PN assessment. As Blomeke et al. (2015) state, "Using combinations of approaches, we may also be able to cover the *processes* mediating the transformation of dispositions into performance." (p. 12) which aligns with our goal to determine which traits we can positively effect that will lead to improved teaching performance.

The results of our study invite serious consideration of further research directions. Further studies should revisit the subdomains of MKT (Ball et al., 2008) to explore how the component skills of PN relate to each of the subdomains, using a more robust instrument. Such work might reveal certain subdomains of MKT mediate and influence the practices of PN suggesting that PN is, in some form, a manifestation, albeit hidden, of knowledge (Thomas et al., 2017). Another potential outcome might be that PN and MKT operate somewhat reflexively with knowledge and noticing informing one another. A third outcome might result in a better understanding of the transformative action of PN supported by dispositional resources, both cognitive and affective, as proposed by Blömeke et al. (2015). Any of these results could lead to more intentional design of teacher education programs.

Acknowledgements This research was supported by a collaborative grant from the National Science Foundation IUSE Awards: 1431098, 1431133, and 1431631. The opinions expressed herein are those of the authors and do not necessarily reflect views of the National Science Foundation.

References

Aguirre, J., & Speer, N. M. (1999). Examining the relationship between beliefs and goals in teacher practice. *The Journal of Mathematical Behavior, 18*(3), 327–356.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389–407. https://doi.org/10.1177/0022487108324554.

Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., et al. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal, 47*(1), 133–180. https://doi.org/10.3102/0002831209345157.

Blömeke, S., Gustafsson, J.-E., & Shavelson, R. J. (2015). Beyond dichotomies: Competence viewed as a continuum. *Zeitschrift für Psychologie, 223*(1), 3–13.

Brese, F., & Tatto, M. T. (2012). *User guide for the TEDS-M international database*. Amsterdam: International Association for the Evaluation of Educational Achievement (IEA).

Castro Superfine, A., Fisher, A., Bragelman, J., & Amador, J. M. (2017). Shifting perspectives on preservice teachers' noticing of children's mathematical thinking. In E. O. Schack, M. H. Fisher, & J. Wilhelm (Eds.), *Teacher noticing: Bridging and broadening perspectives, contexts, and frameworks* (pp. 409–426). Cham: Springer.

Charalambous, C. Y. (2015). Working at the intersection of teacher knowledge, teacher beliefs, and teaching practice: A multiple-case study. *Journal of Mathematics Teacher Education, 18,* 427–445. https://doi.org/10.1007/s10857-015-9318-7.

Charalambous, C. Y., Hill, H. C., Chin, M. J., & McGinn, D. (2020). Mathematical content knowledge and knowledge for teaching: Exploring their distinguishability and contribution to student learning. *Journal of Mathematics Teacher Education, 23,* 579–613. https://doi.org/10.1007/s10857-019-09443-2.

Cole, D. A., & Preacher, K. J. (2014). Manifest variable path analysis: Potentially serious and misleading consequences due to uncorrected measurement error. *Psychological Methods, 19*(2), 300–315.

Depaepe, F., Verschaffel, L., & Kelchtermans, G. (2013). Pedagogical content knowledge: A systematic review of the way in which the concept has pervaded mathematics educational research. *Teaching and Teacher Education, 34,* 12–24.

Dick, L. K. (2017). Investigating the relationship between professional noticing and specialized content knowledge. In E. O. Schack, M. H. Fisher, & J. Wilhelm (Eds.), *Teacher noticing: Bridging and broadening perspectives, contexts, and frameworks* (pp. 339–358). Cham: Springer.

Enders, C. K. (2010). *Applied missing data analysis*. New York: Guilford Press.

Fisher, M. H., Schack, E. O., Thomas, J., Jong, C., Eisenhardt, S., Tassell, J., & Yoder, M. (2014). Examining the relationship between preservice elementary teachers' attitudes toward mathematics and professional noticing capacities. In J.-J. Lo, K. R. Leatham, & L. R. Van Zoest (Eds.), *Research trends in mathematics teacher education* (pp. 219–237). Berlin: Springer International.

Fisher, M. H., Thomas, J., Schack, E. O., Jong, C., & Tassel, J. (2018). Noticing numeracy now!: Examining changes in preservice teachers' noticing, knowledge, and attitudes. *Mathematics Education Research Journal, 30*(2), 209–232.

Fisher, M. H., Thomas, J., Jong, C., Schack, E. O., & Dueber, D. (2019). Comparing preservice teachers' professional noticing skills in elementary mathematics classrooms. *School Science and Mathematics, 119*(3), 142–149.

Floro, B., & Bostic, J. (2017). A case study of middle school teachers' noticing during modeling with mathematics tasks. In E. O. Schack, M. H. Fisher, & J. Wilhelm (Eds.), *Teacher noticing: Bridging and broadening perspectives, contexts, and frameworks* (pp. 73–89). Cham: Springer.

Grossman, P., Compton, C., Igra, D., Ronfeldt, M., Shahan, E., & Williamson, P. W. (2009). Teaching practice: A cross-professional perspective. *Teachers College Record, 111*(9), 2055–2100.

Hanna, W. G. (2012). Examining what inservice elementary teachers notice when presented with instances of students mathematical thinking. *Proceedings of the 34rd Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Kalamazoo, MI: Western Michigan University.

Harry, B., Sturges, K. M., & Klingner, J. K. (2005). Mapping the process: An exemplar of process and challenge in grounded theory analysis. *Educational Researcher, 34*(2), 3–13.

Hayduk, L. A. (1987). *Structural equation modeling with LISREL: Essentials and advances*. Baltimore: Johns Hopkins University Press.

Hill, H. C., Schilling, S. G., & Ball, D. L. (2004). Developing measures of teachers' mathematics knowledge for teaching. *Elementary School Journal, 105,* 11–30.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371–406. https://doi.org/10.3102/00028312042002371.

Hill, H. C., Blunk, M. L., Charalambous, C. Y., Lewis, J. M., Phelps, G. C., Sleep, L., & Ball, D. L. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Cognition and Instruction, 26*(4), 430–511. https://doi.org/10.1080/07370000802177235.

Ingram, N., Linsell, C., & Offen, B. (2018). Growing mathematics teachers: Pre-service primary teachers' relationships with mathematics. *Mathematics Teacher Education and Development, 20*(3), 41–60.

Jacobs, V. R., & Spangler, D. A. (2017). Research on core practices in K-12 mathematics teaching. In J. Cai (Ed.) *Compendium for research in mathematics education* (pp. 766–792. National Council of Teachers of Mathematics.

Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education, 41,* 169–202.

Jacobs, V. R., Lamb, L. L. C., Philipp, R. A., & Schappelle, B. P. (2011). Deciding how to respond on the basis of children's understandings. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 97–116). New York: Routledge.

Jacobs, V. R., Sherin, M. G., & Philipp, R. A., (2013). Mathematics teacher noticing: A hidden skill of teaching. *Work Group presented at the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education,* 723–729. Chicago, IL.

Jacobson, E., & Kilpatrick, J. (2015). Understanding teacher affect, knowledge, and instruction over time: An agenda for research on productive disposition for teaching mathematics. *Journal of Mathematics Teacher Education, 18*(5), 401–406.

Jong, C., & Hodges, T. E. (2015). Assessing attitudes toward mathematics across teacher education contexts. *Journal of Mathematics Teacher Education, 18*(5), 407–425.

Kaiser, G., Busse, A., Hoth, J., König, J., & Blömeke, S. (2015). About the complexities of video-based assessments: Theoretical and methodological approaches to overcoming shortcomings of research on teachers' competence. *International Journal of Science and Mathematics Education, 13,* 369–387.

Kanyongo, G. Y., Brook, G. P., Kyei-Blankson, L., & Gocmen, G. (2007). Reliability and statistical power: How measurement fallibility affects power and required sample sizes for several parametric and nonparametric statistics. *Journal of Modern Applied Statistical Methods, 6*(1), 81–90.

Kersting, N. (2008). Using video clips of mathematics classroom instruction as item prompts to measure teachers' knowledge of teaching mathematics. *Educational and Psychological Measurement, 68*(5), 845–861.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. Washington: National Academy Press.

Krupa, E. E., Huey, M., Lesseig, K., Casey, S., & Monson, D. (2017). Investigating secondary preservice teacher noticing of students' mathematical thinking. In E. O. Schack, M. H. Fisher, & J. Wilhelm (Eds.), *Teacher noticing: Bridging and broadening perspectives, contexts, and frameworks* (pp. 49–72). Cham: Springer.

Kunter, M., Klusmann, U., Baumert, J., Richter, D., Voss, T., & Hachfeld, A. (2013). Professional competence of teachers: Effects on instructional quality and student development. *Journal of Educational Psychology, 105*(3), 805.

Kutaka, T. S., Ren, L., Smith, W. M., Beattie, H. L., Edwards, C. P., Green, J. L., et al. (2018). Examining change in K-3 teachers' mathematical knowledge, attitudes, and beliefs: The case of Primarily Math. *Journal of Mathematics Teacher Education, 21*(2), 147–177.

Larochelle, R., & Mammo, B. (2019). Correlations between professional noticing of students' mathematical thinking and specialized content knowledge. In S. Otten, A. G. Candela, Z. de Araujo, C. Haines, & C. Munter (Eds.), *Proceedings of the forty-first annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. St Louis: University of Missouri.

Linacre, J. M. (2018). *Winsteps®* (Version 4.3.1) [Computer Software]. Beaverton, Oregon: Winsteps.com.

Pehkonen, E., & Toerner, G. (1996). Introduction to the theme: Mathematical beliefs. *Zentralblatt für Didaktik der Mathematik, 28,* 99–100.

Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester, Jr. (Ed.) *Second handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics*, *1*, 257–315. Information Age.

Pourdavood, R. G., & Liu, X. (2017). Pre-service elementary teachers' experiences, expectations, beliefs, and attitudes toward mathematics teaching and learning. *International Journal of Learning, Teaching and Educational Research, 16*(11), 1–27.

Roesken, B., Pepin, B., & Toerner, G. (2011). Beliefs and beyond: Affect and the teaching and learning of mathematics. *ZDM, 43*(4), 451.

Santagata, R., & Yeh, C. (2016). The role of perception, interpretation, and decision making in the development of beginning teachers' competence. *ZDM, 48*(1), 153–165. https://doi.org/10.1007/s11858-015-0737-9.

Schack, E. O., Fisher, M. H., Thomas, J., Eisenhardt, S., Tassell, J., & Yoder, M. (2013). Prospective elementary school teachers' professional noticing of children's early numeracy. *Journal of Mathematics Teacher Education, 16,* 379–397. https://doi.org/10.1007/s10857-013-9240-9.

Schack, E. O., Fisher, M. H., & Wilhelm, J. (Eds.). (2017). *Teacher noticing: Bridging and broadening perspectives, contexts, and frameworks*. New York: Springer.

Schack, E. O., Dueber, D., Jong, C., Thomas, J., & Fisher, M. H. (2019). Computer-programmed decision trees for assessing teacher noticing. Paper presented at the *Annual Meeting of the American Educational Research Association.* Toronto, Canada.

Schackow, J. B. (2005). Examining the attitudes toward mathematics of preservice elementary school teachers enrolled in an introductory mathematics methods course and the experiences that have influenced the development of these attitudes. *Graduate Theses and Dissertations*. Retrieved from http://scholarcommons.usf.edu/etd/853.

Schoenfeld, A. H. (2015). What counts, when? Reflection on beliefs, affect, attitude, orientations, habits of mind, grain size, time scale, context, theory, and method. In B. Pepin & B. Roesken-Winter (Eds.), *From beliefs to dynamic affect systems in mathematics education* (pp. 395–404). Berlin: Springer International.

Sherin, M. G. (2017). Exploring the boundaries of teacher noticing: Commentary. In E. O. Schack, M. H. Fisher, & J. Wilhelm (Eds.), *Teacher noticing: Bridging and broadening perspectives, contexts, and frameworks* (pp. 401–408). Cham: Springer.

Sherin, M. G., Jacobs, V. R., & Philipp, R. A. (Eds.). (2011). *Mathematics teacher noticing: Seeing through teachers' eyes*. New York: Routledge.

Shulman, L. S. (1986). Those who understand?: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4–14.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*(1), 1–23.

Stahnke, R., Schueler, S., & Roesken-Winter, B. (2016). Teachers' perception, interpretation, and decision-making: A systematic review of empirical mathematics education research. *ZDM, 48,* 1–27.

Stockero, S. L. (2008). Using a video-based curriculum to develop a reflective stance in prospective mathematics teachers. *Journal of Mathematics Teacher Education, 11,* 373. https://doi.org/10.1007/s10857-008-9079-7.

Swars, S. L., Smith, S. Z., Smith, M. E., Carothers, J., & Myers, K. (2018). The preparation experiences of elementary mathematics specialists: examining influences on beliefs, content knowledge, and teaching practices. *Journal of Mathematics Teacher Education, 21*(2), 123–145.

Tapia, M., & Marsh, G. E., II. (2004). An instrument to measure mathematics attitudes. *Academic Exchange Quarterly, 8,* 16–21.

Tatto, M. T., Schwille, J., Senk, S., Ingvarson, L., Peck, R., & Rowley, G. (2008). *Teacher Education and Development Study in Mathematics (TEDS-M): Policy, practice, and readiness to teach primary and secondary mathematics. Conceptual framework.* East Lansing, MI: Teacher Education and Development International Study Center, College of Education, Michigan State University.

Thomas, J., Jong, C., Fisher, M. H., & Schack, E. O. (2017). Noticing and knowledge: Exploring theoretical connections between professional noticing and mathematical knowledge for teaching. *The Mathematics Educator, 26*(2), 3–25.

van Es, E. (2011). A framework for learning to notice student thinking. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 97–116). New York: Routledge.

Wilkins, J. L. (2008). The relationship among elementary teachers' content knowledge, attitudes, beliefs, and practices. *Journal of Mathematics Teacher Education, 11*(2), 139–164.

Wright, B. D., & Linacre, J. M. (1994). Reasonable mean-square fit values. *Rasch Measurement Transactions, 8*(3), 370.

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.