

# Time-Domain and Frequency-Domain Mappings of Voltage-to-Charge and Charge-to-Voltage in Capacitive Devices

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**Abstract**—In this work, we aim to show that there are generally four possible mapping functions that can be used to map the time-domain or frequency-domain representations of an applied voltage input to the resulting time-domain or frequency-domain electrical charge output; i.e. when the capacitive device is voltage-charged. Alternatively, there are four more possible combinations when the device is current-charged. The dual relationship between each pair of functions for the case of voltage or charge input are provided in terms of single or double Fourier transforms. All eight system functions coincide with each other if and only if a constant time- and frequency-independent capacitance is considered.

**Index Terms**—Capacitive devices, Circuit theory, Capacitance, Fourier transform

## I. INTRODUCTION

Ideal electrostatic capacitors are constructed from two conductors separated by a perfect insulator. The standard charge-voltage relation  $q = cv$  represents the linear relationship between the magnitude of the charge on each conductor with the difference of potential applied across the two conductors. It is true in both time and frequency domains for ideal capacitors having constant, time-independent and frequency-independent capacitances that can be only computed, and not measured, from voltage and charge.

However, other capacitive devices, such as electrochemical capacitors (including electric double-layer capacitors (EDLCs) and pseudocapacitors), cannot be characterized with a constant capacitance value [1]–[3]. Their electrodes are fractal and porous in nature, and are separated by ionic conductors leading to complex charge transports that are inherently different from ideal capacitors. Coupled ionic and electronic transports take place in these devices, which are ultimately controlled by diffusion dynamics and migration in the bulk electrolyte, and through their micro/nano-porous structured electrodes. The correlated disorder of the background electrolyte also has an impact on the overall charge transport process which makes it deviate from normal diffusion to subdiffusion, and hence internal dissipative processes [4], [5]. From a system-level point

of view, the spectral impedance phase angle of most -if not all- EDLCs and other similar energy storage devices deviate from the  $-90^\circ$  value expected for ideal capacitors, even at near-dc, sub-Hz frequencies [3], [6]. It has also been demonstrated experimentally that an EDLC charged with two different time-domain voltage waveforms to the same thermodynamic state of voltage and charge,  $(v, q)$ , exhibits different relaxation profiles when discharged into the same resistor [7]. This means that there are inherent memory effects in these devices [7]–[9], and that the dynamic (dis)charging paths should be taken into consideration when evaluating their performance metrics [10]–[12]. This is not the case with ideal capacitors whose relaxation depends only on the initial condition. In addition, several time-domain transient measurements conducted on EDLCs show in some form or another nonlinearities and power-law behaviors that prohibit the extraction of a constant capacitance from the system's response [11]–[14]. We note that the EDL structure for instance is ubiquitous in many other fields of fundamental and applied electrochemistry such as in intercalation materials, capacitive deionization and electrosorption, colloidal suspensions, bipolar membranes, etc., and therefore properly evaluating its electrical parameters is paramount for meaningful progress and development in these research areas.

Unfortunately, there is a conceptual misunderstanding, even in specialized literature, on the difference in computation of performance metrics between an ideal capacitor and other capacitive energy storage devices [15]. A classical example is the direct calculation of capacitance from the time-domain expression  $q = cv$ , but the same formulation of charge equal to the product of capacitance by voltage expressed in the frequency domain, and indirectly derived from impedance spectroscopy measurements, is used to characterize the same device, as if it was an ideal capacitor with constant capacitance (see [16]–[27] to cite a few). This is in direct contradiction with the convolution theorem [28], and clearly indicates that there is an urgent need for a unified methodology of system identification for such devices. Most recently, in [15] the authors studied the difference between the time-domain definition  $q_t(t) = c_t(t)v_t(t)$  and its frequency-domain counterpart defined as a convolution integral, i.e.  $Q_t(f) = (V_t \otimes C_t)(f)$ ,

and the frequency-domain definition  $Q_f(f) = C_f(f) V_f(f)$  and its time-domain equivalent  $q_f(t) = (c_f \circledast v_f)(t)$ . We denote by  $V(f)$  the Fourier transform of the input time-domain voltage  $v(t)$  and by  $Q(f)$  the Fourier transform of the output time-domain charge  $q(t)$  with the definitions  $\mathcal{F}\{g(t)\} = G(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi f t} dt$  and  $\mathcal{F}^{-1}\{G(t)\} = g(t) = (2\pi)^{-1} \int_{-\infty}^{\infty} G(f) e^{j2\pi f t} dt$ . In [15], we showed that the extraction of capacitance functions (rather than constant capacitance) from these two pairs of equations for the case of dispersive mono-fractional-order capacitors (known also as constant phase elements) subjected to sinusoidal voltage excitation do not lead to the same results. The same has been illustrated and verified for the case of linear voltage ramp and constant current input applied to a commercial supercapacitor device [10]. It is therefore one pair of equations or the other that can be used simultaneously, and the relation  $q = cv$  can only be applied on one-domain (time or frequency) but not the other. We recommended nonetheless the use of the multiplicative capacitance function in the frequency domain, i.e.  $Q_f(f) = C_f(f) V_f(f)$ , in order to be in line with the definition of impedance of linear time-invariant systems [15], [28].

This work is a direct extension of our previous studies [10], [15], and is primarily concerned with the system identification problem of capacitive devices from the two measurable quantities voltage and charge. The goal is to establish the different relationships between possible system capacitance functions *without* the prior assumption of any model. In Section II, we present the theoretical framework of this work while its implications are discussed further in Section III. It is important to note that there are no simulations or experimental results presented here since they have been readily presented in [10], [15] upon dealing with specific devices.

## II. THEORY

Consider the schematic illustration presented in Fig. 1 relating the input/output of a capacitive device in the time and frequency domains. It is evident that there are four possible operators that can map an input voltage (in the time or frequency domain) to an output charge (in the time or frequency-domain). When we refer to the frequency-domain, the time-domain quantity is dealt with indirectly through its Fourier frequency spectrum. We can write from the four signal representations  $v(t)$ ,  $V(f)$  and  $q(t)$ ,  $Q(f)$  the following relations [29], [30]:

$$q(t) = \mathcal{C}_{tt}\{v(t)\} \quad (1a)$$

$$Q(f) = \mathcal{C}_{ff}\{V(f)\} \quad (1b)$$

$$q(t) = \mathcal{C}_{tf}\{V(f)\} \quad (1c)$$

$$Q(f) = \mathcal{C}_{ft}\{v(t)\} \quad (1d)$$

where the four operators  $\mathcal{C}_{ij}$  ( $\{i, j\} = \{t, f\}$ ) map or transform the input signal in space  $j = \{t, f\}$  for time or frequency to space  $i = \{t, f\}$ . For example, the operator  $\mathcal{C}_{tt}$  in Eq. (1a) represents the mapping relationship between the voltage input  $v(t)$  (cause) and charge output  $q(t)$  (effect) both defined in the time domain, whereas the operator  $\mathcal{C}_{tf}$  in Eq. (1c) represents the mapping relationship between the

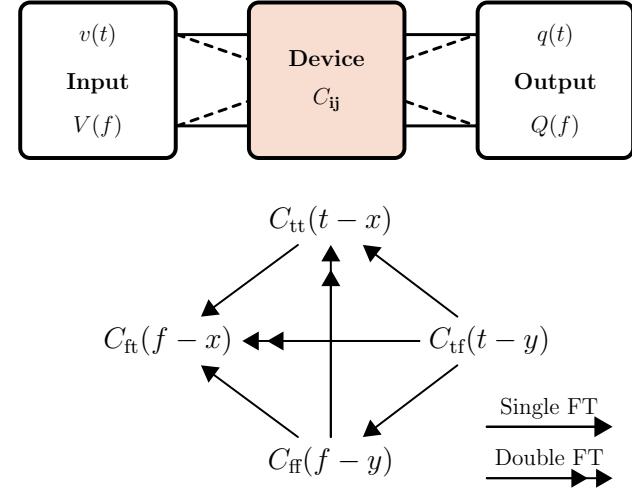


Fig. 1. Schematic diagram illustrating the two time-domain and frequency-domain representations of voltage input and the two time-domain and frequency-domain representations of the charge output in a capacitive energy device. The different relationships between the capacitance functions  $C_{ij}$  defined in Eq. 2 are also summarized (FT: Fourier transform).

voltage input  $V(f)$  defined in the frequency domain and the charge output  $q(t)$  defined in the time domain. The mapping operators  $\mathcal{C}_{ij}$  are unique, and can be in general related to differential or integro-differential equations that govern the space-time charge distributions in a capacitive device.

For the case of devices which are assumed to be linear time-invariant wherein the input-output relationship is invariant under translation in time and the superposition principle holds, we can write the following linear integral equations of convolution [31]:

$$q(t) = \int C_{tt}(t-x)v(x)dx \quad (2a)$$

$$Q(f) = \int C_{ff}(f-y)V(y)dy \quad (2b)$$

$$q(t) = \int C_{tf}(t-y)V(y)dy \quad (2c)$$

$$Q(f) = \int C_{ft}(f-x)v(x)dx \quad (2d)$$

The four kernels  $C_{ij}$  ( $\{i, j\} = \{t, f\}$ ) can be viewed as kernel system functions as proposed by Bello [29], [30]. Note that when the limits of integration are not explicitly indicated in the equations they are to be taken from  $-\infty$  to  $\infty$ , and the integrals are assumed to exist. Also, all quantities are treated as normalized and thus dimensionless. It is important to clarify at this point that in [15] the focus was to establish the difference between assuming the correctness of the multiplicative equation  $q = cv$  in the time domain versus assuming its correctness in the frequency domain, which are both widely used definitions in the literature. This led us to define four different capacitance functions  $c_t(t)$ ,  $C_t(f)$  and  $C_f(f)$ ,  $c_f(t)$  by virtue of the convolution theorem as explained above. In this work, we approach the problem from a general system identification perspective as described by the set of Eqs. (2).

Therefore the functions  $C_{ij}$  ( $\{i, j\} = \{t, f\}$ ) should not be confused with  $c_t(t)$ ,  $C_t(f)$ ,  $C_f(f)$  or  $c_f(t)$ , which are not in conflict with the present analysis.

Equations (2a) and (2c) provide two different expressions for the same time-domain charge  $q(t)$ . With the use of Parseval's equation [32] we can write:

$$\int_{-\infty}^{\infty} f(t)G(t)dt = \int_{-\infty}^{\infty} F(\omega)g(\omega)d\omega \quad (3)$$

(under the assumption that  $f(t)$  and  $g(t)$  are absolutely integrable over  $-\infty < t < \infty$ ). The kernel functions  $C_{tf}(t-y)$  and  $C_{tt}(t-x)$  constitute a Fourier transform pair with  $t$  considered as a parameter:

$$\mathcal{F}\{C_{tf}(t-y)\} = C_{tt}(t-x) \quad (4)$$

In the same way, we can establish from equating the frequency-domain charge  $Q(f)$  given by both Eqs. (2b) and (2d) that:

$$\mathcal{F}\{C_{ff}(f-y)\} = C_{ft}(f-x) \quad (5)$$

with  $f$  being considered as a parameter. From Eqs. (2a), (2d) (after applying the Fourier transform to both sides of Eq. (2a)) and Eqs. (2c), (2b) (after applying the Fourier transform to both sides of Eq. (2c)), we can write:

$$C_{ft}(f-x) = \int e^{-j2\pi ft} C_{tt}(t-x)dt \quad (6)$$

and

$$C_{ff}(f-y) = \int e^{-j2\pi ft} C_{tf}(t-y)dt \quad (7)$$

respectively. Finally, from Eqs. (2a) and (2b) we can establish the following relation between  $C_{ff}(f-y)$  and  $C_{tt}(t-x)$ :

$$\begin{aligned} C_{ff}(f-y) &= \frac{1}{2\pi} \iint C_{tt}(t-x) e^{-j2\pi tf} e^{j2\pi yx} dt dx \\ &= \frac{1}{2\pi} \iint C_{tt}(t-x) e^{j2\pi(yx-tf)} dt dx \end{aligned} \quad (8)$$

and from Eqs. (2c), (2d) we have:

$$\begin{aligned} C_{ft}(f-x) &= \iint C_{tf}(t-y) e^{-j2\pi tf} e^{-j2\pi xy} dt dy \\ &= \iint C_{tf}(t-y) e^{-j2\pi(tf+xy)} dt dy \end{aligned} \quad (9)$$

Figure 1 summarizes the four different forms of system functions  $C_{ij}$  ( $\{i, j\} = \{t, f\}$ ) defined in Eq. (2) along with their pair relationships either as single or double Fourier transforms.

A simple example that demonstrates the correspondence between these system functions is the case in which the charge is a delayed copy input voltage  $v(t)$ , i.e. [31]:

$$q(t) = \mathcal{C}_{tt}\{v(t)\} = Av(t - \tau_0) \quad (A = 1) \quad (10)$$

Thus:

$$C_{tt}(t-x) = \delta(t-x-\tau_0) \quad (11)$$

which verifies Eq. 2a. From here the rest of the system functions in Eqs. 2b, 2c and 2d are

$$C_{ff}(f-y) = \delta(f-y)e^{-j2\pi y\tau_0} \quad (12)$$

$$C_{tf}(t-y) = e^{j2\pi y(t-\tau_0)} \quad (13)$$

$$C_{ft}(f-x) = e^{-j2\pi f(x+\tau_0)} \quad (14)$$

respectively. These functions can be transformed into one another as indicated in Fig. 1. Another example is when the resulting time-domain charge  $q(t)$  is a response to an input voltage  $v(t)$  given by [31]:

$$q(t) = \mathcal{C}_{tt}\{v(t)\} = v(t - \tau_0)e^{j2\pi f_0 t} \quad (15)$$

The four capacitance functions are then:

$$C_{tt}(t-x) = \delta(t-x-\tau_0)e^{j2\pi f_0 t} \quad (16)$$

$$C_{ff}(f-y) = \delta(f-y-f_0)e^{-j2\pi y\tau_0} \quad (17)$$

$$C_{tf}(t-y) = e^{j2\pi y(t+f_0)}e^{-j2\pi y\tau_0} \quad (18)$$

$$C_{ft}(f-x) = e^{-j2\pi(f-f_0)(x+\tau_0)} \quad (19)$$

Such types of functions are not only necessary in relating charge and voltage in capacitive devices but are also necessary in modeling communication channels for instance. As an example, the intrabody communication channel is predominantly capacitive and modeled by distributed networks of ideal capacitors, as shown in [33] for an in-vehicle scenario. The availability of frequency domain-based network analyzers forces the application of frequency-domain modeling techniques as seen for example in the scattering matrix results reported in [34].

The equation pairs presented above are all related to the case when a capacitive device is being voltage-charged. The electric charge on the device is thus a result (output). However, when using the electrical current instead of voltage to charge capacitive devices, this implies that the charge is readily created by the charging power supply since  $dq = \int i(t)dt$  [35] before being applied on the device. The input in this case is the electrical charge and the voltage measured is thus a result (output), as shown in Fig. 2. A new set of four equations similar to Eqs. (1) can be deduced for this case albeit the LHS must be the voltage and the RHS must be the charge, with the mapping functions ( $C_{ij}^*$ ) being inverse-capacitance functions. When a device is reciprocal, the output charge from Fig. 1 fed as an input in Fig. 2 should result in an output

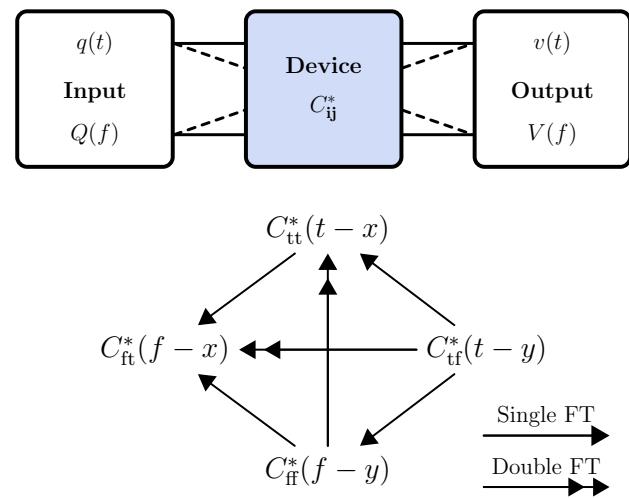


Fig. 2. Schematic diagram similar to Fig. 1 with the electrical charge being the input and the voltage being the output in a capacitive energy device.

voltage from Fig. 2 identical to that supplied as input to Fig. 1 [35]. However, this is not always necessarily true and hence the computation of a *capacitance* function is associated only with voltage excitation whereas the computation of *inverse capacitance* functions should be done in the case of current excitations.

### III. DISCUSSION

A capacitive device is treated here as a lumped electrical signal processing element at the link between voltage input and charge output or charge input and voltage output (single-input single-output device) as depicted in Figs. 1 and 2, respectively. The charge, being the time-integral of the current, can be the result of all current contributions taking place within the device and thus there is no real discrimination between what is effectively being used for the purpose of energy storage for future reuse, or what is being dissipated. In mathematical terms, the whole device can be thought of as an operator which transforms a certain input signal into an output signal, both of which can be described either in the time domain or frequency domain [29]. Cyclic voltammetry, constant current charge/discharge and impedance spectroscopy are all routine techniques for time-domain and frequency-domain measurements of capacitive energy storage devices [10], [12], [15], [36]. However, the relations presented above are actually valid for any arbitrary excitations as long as the linearity and time-invariance of the device are respected, noting that there are also identification methods for time-variant and nonlinear systems [30], [37], [38]. Furthermore, in reality the input and output signals can have superposed noises associated with measurement errors or other environmental conditions that should be taken into account when estimating the corresponding mapping system functions. This requires the use of efficient regularization methods and inverse problem computations and not simple multiplications and divisions as commonly done using the ideal capacitor formulae.

In fact, from the classical treatment inherited from ideal capacitors one can write interchangeably  $q = cv$  or  $v = q/c$  without worrying about which is the input variable (voltage or charge). This is clearly incorrect for the general case of capacitive functions. There is indeed a difference between the system functions  $C_{ij}$  and  $C_{ij}^*$  for a linear time-invariant capacitive device, and thus between voltage charging and current charging as illustrated in Figs. 1 and 2. The results further highlight that computing the capacitive (or inverse capacitive) functions is dependent on the form and type of the applied excitation, which is different from ideal capacitors. Applying a step function, linear ramp or sinusoidal excitation (voltage or current) will result in different capacitive functions. Therefore, one should specify clearly how the device is being excited and which capacitive (or inverse capacitive) function is being computed because these functions cannot be identical to each other except for the case of ideal capacitors (i.e.  $C_{ij} = C_{ij}^* = c$ ). We believe that this will have significant implications on the way capacitance (and similarly inductance) is computed in the future in an era where capacitive energy storage devices are on high demand in electrical vehicles, IoT systems and many other applications.

### IV. CONCLUSION

This work is an important companion to our recent work in [15]. We have shown here that there are several system functions that can map an arbitrary input voltage/charge, represented in time or frequency, to an output charge/voltage, represented in time or frequency for a capacitive device (see Figs. 1 and 2). From the estimation of one function (for example  $C_{ii}$  that maps voltage to charge when they are both defined in the time domain, one can deduct all other functions using single or double Fourier transforms. We emphasize that the functions  $C_{ij}$  and  $C_{ij}^*$  should be regarded as system functions and not capacitances, which are solely characteristics of ideal capacitors.

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