Hybrid Hierarchical Statistical Control of Robotic Manipulators

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Abstract: Here we present a hybrid hierarchical statistical control approach for the control of robotic manipulators. The bimodal dynamic imaging system considered in this paper utilizes two robotic manipulators to move the source and detector imaging modules. As this system contains both continuous and discrete dynamics, hybrid system control techniques are applied. The robotic arms used in this research are comprised of compliant joints, which have been shown to introduce process noise into the system. To address this, a full-state feedback statistical controller is developed to minimize joint angle variations for the system. The statistical controllers for the two robot arms are then coordinated using a hierarchical controller. Finally, the feasibility of the hybrid hierarchical statistical controller is demonstrated with numerical simulations.

Keywords: Hybrid hierarchical control, statistical control, stochastic systems, optimal control.

1. INTRODUCTION

Development of robotic and automated processes for the evaluation and classification of included masses is an ongoing area of research [18]. Mechanical systems designed for use around humans, called 'cobots' in industry, include additional safety features and systems to prevent harm to operators or damage to equipment, and therefore represent a good candidate system for the development of such medical systems. The Baxter Research Robot developed by Rethink Robotics is one such platform and was developed primarily for research in the cobot space.

The Baxter Cobot includes compliant actuators [17] in each joint as a safety measure. Should the arm collide with a human or piece of equipment during operation, the compliant actuator will 'yield' reducing the severity of the impact and improving the collision detection of the system. While this feature improves the safety of the robotic system, it also introduces noise to the joint angles for the systems [7]. This paper presents a statistical control approach to address the presence of this noise, and improve the tracking performance for the two-arm system.

Linear-quadratic-Gaussian (LQG) control [10], statistical control [21] and risk sensitive (RS) [5] control have been actively researched in literature. Statistical control is defined as the minimization of any finite or infinite linear combinations of the cost cumulants, thus LQG, statistical and RS control become special cases of statistical control. In classical LQG control, the first cumulant or the mean of the cost function is minimized. In statistical control, also known as minimal cost variance (MCV) control, the second cumulant, or the variance, of the cost function is minimized. And the minimization of all the "fixed weighted" linear combination of the cost cumulants corresponds to RS control [22].

MCV control is a special case of statistical control, where the second cumulant is optimized. In 1971 Sain and Liberty published an open loop result in minimizing the performance variance while keeping the performance mean close to a prespecified value [21]. The full state feedback second cumulant control problem is solved in [26], [22]. Nonlinear statistical control was investigated in [26]. More recently, the application of the statistical control concept to game theory has appeared in the literature [8].

In this paper, a statistical controller for the linearized robot manipulator is presented and evaluated for performance. Here, the intent is to minimize the joint angle variation so that the end-effector pose is less affected by system noise. Less variation in end-effector position will ensure that the laser and the camera will be in line-of-sight during image acquisition for the bimodal imaging experiment. The optimal design parameter for the statistical controller was developed using numerical simulations. *The contribution of this work consists of the application of statistical control to a robotic manipulator with complainant joints, as well as the implementation of statistical control in combination with a hybrid heirarchical control approach.*

In Section 2, Statistical Control is described. Then we discuss hybrid hierarchical statistical control architecture in Section 3. Finally, conclusions are presented in the last section.

2. FULL-STATE FEEDBACK STATISTICAL CONTROLLER

For stochastic dynamic systems, the presence of process and measurement noise causes the states and outputs of the system to become random variables. In optimal control applications, this also means the cost function

† Steve Lash is the presenter of this paper.

used for optimization also becomes a random variable. To address the stochastic nature of the system states and cost function, a statistical full-state feedback control method is presented here.

In statistical control, one or more cumulants of the stochastic cost function are minimized. In the particular configuration presented in this paper, all the states of the system (joint angles, velocities, torques, and end-effector positions) are available for measurement, so a full-state feedback approach is considered. We design separate statistical controllers for each of the robotic arms which will attempt to minimize the *variance* of the cost function comprised of weighted system states and inputs. A minimized variation in the cost function will ensure that the source (e.g. laser) and the detector (e.g. camera) will be in line-of-sight during image acquisition during experiment.

2.1. Statistical Control Preliminaries

We consider a stochastic linear time invariant dynamic system modeled on $[t_0, t_f]$. The dynamic process model is given by,

$$\begin{aligned} dx(t) &= Ax(t)dt + Bu(t)dt + Fdw(t), \\ x(t_0) &= x_0, \quad t \in [t_0, t_F] \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is an n-dimensional state vector at time t, $u(t) \in \mathbb{R}^m$ is an m-dimensional control vector at time t, x_0 is the initial condition. $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, and $F \in \mathbb{R}^{n\times p}$ are matrices of appropriate order. Here, $dw(t)$ is a Gaussian random process of dimension p with zero mean, covariance of $W(t)dt$. The random process is defined on a probability space $(\Omega_0, \mathcal{F}, \mathcal{P})$ where Ω_0 is a non-empty set, $\mathcal{F}$ is a σ-algebra of Ω_0, and $\mathcal{P}$ is a probability measure on (Ω_0, F). Also, we consider the output equation:

$$y(t) = Cx(t) + Du(t), \tag{2}$$

where, C and D are matrices of appropriate order. The quadratic random cost for the linear stochastic system is

$$\begin{aligned} J = \; & x'(t_F)Q_F x(t_F) + \\ & \int_t^{t_F} \left[x'(\tau)Qx(\tau) + u'(\tau)Ru(\tau)d\tau\right], \end{aligned} \tag{3}$$

where the state weighting symmetric matrix $Q \in \mathbb{R}^{n\times n}$ is positive semi-definite, the control effort weighting symmetric matrix $R \in \mathbb{R}^{m\times m}$ is positive definite, and the terminal penalty weighting symmetric matrix $Q_F \in \mathbb{R}^{n\times n}$ is positive semi-definite.

The statistical control problem is to determine a control law such that the n-th cost cumulant of the cost function in (3) is minimized, while keeping the rest of the $(n-1)$-th cumulants at pre-specified levels.

2.2. Statistical Control Law

In this paper, we utilize the statistical control that minimizes the second cumulant (variance) of the cost function for a fixed first cumulant (mean). In case of the second cost cumulant minimization, the full-state-feedback linear statistical control law has the form [22], where the state weighting symmetric matrix $Q \in \mathbb{R}^{n\times n}$ is positive semi-definite, the control effort weighting symmetric matrix $R \in \mathbb{R}^{m\times m}$ is positive definite, and the terminal penalty weighting symmetric matrix $Q_F \in \mathbb{R}^{n\times n}$ is positive semi-definite.

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$$\begin{aligned} u(t) &= -R^{-1}B'(\mathcal{M} + \gamma\mathcal{V})x(t) \\ &= -K_{stat}x(t), \end{aligned} \tag{4}$$

where the positive semi-definite $\mathcal{M}$ and $\mathcal{V}$ are solutions of the coupled algebraic Riccati equations:

$$\begin{aligned} 0 = \; & A'\mathcal{M} + \mathcal{M}A + Q - \mathcal{M}BR^{-1}B'\mathcal{M} \\ & + \gamma^2\mathcal{V}BR^{-1}B'\mathcal{V}, \end{aligned} \tag{5}$$

$$\begin{aligned} 0 = \; & 4\mathcal{M}FWF^T\mathcal{M} + A'\mathcal{V} \\ & + \mathcal{V}A - \mathcal{M}BR^{-1}B'\mathcal{V} \\ & - \mathcal{V}BR^{-1}B'\mathcal{M} - 2\gamma\mathcal{V}BR^{-1}B'\mathcal{V}, \end{aligned} \tag{6}$$

with the boundary conditions $\mathcal{M}(t_F) = Q_F$ and $\mathcal{V}(t_F) = 0$ for a suitable Lagrange multiplier γ. We assume that $R > 0$, (A, B) is stabilizable, and $(\sqrt{Q}, A)$ is detectable.

Note that the statistical control with nonzero γ is known as 'Minimum Cost Variance' control, and the statistical control with $\gamma = 0$ is known as 'Linear Quadratic Gaussian' control.

For implementing the statistical control for non-zero reference tracking, we use the internal model principle to find a feedforward gain for a reference input [12]. The control law is rewritten as:

$$u = -K_{stat}x + \bar{N}r, \tag{7}$$

where

$$\bar{N} = N_u + K_{stat}N_x \tag{8}$$

is the feedforward controller gain. Note that we first design a state feedback gain K_{stat} using statistical control method such that $A - BK_{stat}$ is stable. Then, to obtain the values of N_x and N_u, we solve

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} N_x \\ N_u \end{bmatrix} = \begin{bmatrix} 0 \\ I \end{bmatrix}. \tag{9}$$

This feedforward control method is applicable for a slowly varying reference input and is valid for the proposed application detailed below.

2.4. Statistical Control Simulation of a Single Robot Manipulator

2.4.1. Baxter Robotic Manipulator Model

Baxter is a dual manipulator robot [18]. The linearized dynamics of the left manipulator taken around the operating point $[0, 0.7854, 1.5708, 0, 0.7854, 0]$ and zero joint velocity are formulated as:

$$\underbrace{\begin{bmatrix}\Delta\dot{q}_l\\ \Delta\ddot{q}_l\end{bmatrix}}_{14\times1} = \overbrace{\underbrace{\begin{bmatrix}0_{7\times7} & \mathbf{I_{7\times7}}\\ -\mathbf{A_{pl}^{-1}}(\mathbf{C_{pl1}}+\mathbf{C_{pl2}}) & -\mathbf{A_{pl}^{-1}}\mathbf{B_{pl}}\end{bmatrix}}_{14\times14}}^{\mathbf{A_l}} \underbrace{\begin{bmatrix}\Delta\mathbf{q_l}\\ \Delta\dot{\mathbf{q}}_l\end{bmatrix}}_{14\times1} + \overbrace{\underbrace{\begin{bmatrix}0_{7\times7}\\ \mathbf{A_{pl}^{-1}}\end{bmatrix}}_{14\times7}}^{\mathbf{B_l}} \underbrace{\Delta\tau}_{7\times1}. \tag{10}$$

where

$$A_{pl}^{-1} = \begin{bmatrix} -0.2525 & 1.2359 & -1.3411 & 0.2171 & -0.6623 & -1.8760 & -0.7642\\ 1.2359 & -0.0965 & -1.5473 & 0.1687 & -0.3324 & 0.1499 & -0.1311\\ -1.3411 & -1.5473 & 15.137 & 0.1357 & 8.8677 & 16.7180 & 7.3466\\ 0.2171 & 0.1687 & 0.1357 & 4.3447 & 8.6417 & 5.4724 & 4.0161\\ -0.6623 & -0.3324 & 8.8677 & 8.6417 & 94.2320 & 18.7970 & 13.8300\\ -1.8760 & 0.1499 & 16.7180 & 5.4724 & 18.7970 & 79.1160 & 46.2780\\ -0.76416 & -0.1311 & 7.3466 & 4.0161 & 13.8300 & 46.2780 & 1847.9 \end{bmatrix}, \tag{11}$$

$$\begin{aligned}\mathbf{B_{pl}} &= 0_{7\times7},\\ \mathbf{C_{pl1}} &= 0_{7\times7}\end{aligned} \tag{12}$$

and

$$C_{pl2} = \begin{bmatrix} 0 & -7.6493 & 0 & 0 & 0.0102 & 0 & 0\\ -7.6493 & -21.607 & 5.3508 & 5.1614 & -1.2548 & -1.1767 & 0\\ -7.6493 & -19.439 & 5.3508 & 5.1614 & -0.5213 & -1.1767 & -0.0035\\ 0 & 5.1614 & 0.1947 & 6.5152 & -0.9189 & -0.44536 & -0.0194\\ 0.01019 & -0.5213 & -0.0408 & -0.9189 & 0.1543 & -0.48784 & 0.0078\\ 0 & -1.1767 & 0.0399 & -0.4454 & -0.4878 & -1.322 & 0.0059\\ 0 & -0.00351 & -0.0013 & -0.0194 & 0.0078 & 0.0059444 & -0.0204 \end{bmatrix}. \tag{13}$$

The system is found controllable and observable with eigenvalues $\pm11.58i$, ±9.40, ±7.52, ±6.59, ±5.22, $-0.62 \pm 0.66i$, and $0.62 \pm 0.66i$. Due to the compliant joint construction used in the baxter robot [17], experimentation with the Baxter robot has shown additive process noise present in the joint angle measurements. To account for this additive noise, we add a zero-mean Gaussian white noise term dw with a covariance W to the model. The resulting stochastic linear model for the left robot manipulator is of the form:

$$\begin{aligned} dx_l(t) &= A_l x_l(t)dt + B_l u_l(t)dt + F_l dw_l(t),\\ x_l(t_0) &= x_{l0}, \quad t \in [t_0, t_F]. \end{aligned} \tag{14}$$

The control architecture used for the simulated set-point tracking is given in Fig. 1.

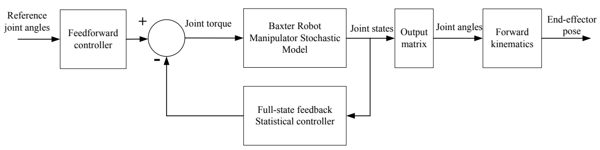

Fig. 1 Set-point tracking with statistical control simulation diagram.

The initial and refernece joint angles used in simulation are shown in Table 1.

Table 1 Initial and Reference joint angles.

	Initial Angles (rad)	Reference Angles (rad)
J1	-0.3927	0
J2	-1.1781	0.7854
J3	-1.9635	1.5708
J4	1.1781	0
J5	1.1781	0.7854
J6	0.3927	0
J7	0.3927	0

2.4.2. Statistical Design Parameter Selection

In order to select a suitable statistical design parameter γ, the joint angle tracking, and end-effector position and orientation tracking error were evaluated. The following two performance metrics were used to select a suitable value for γ:

- Tracking error and root-mean-squared tracking error: The tracking error is the difference between the set-points and the outputs. The root mean squared tracking error is defined in (15):

$$\mu_\varepsilon = \sqrt{\frac{\sum_{t=0}^{t=t_f} \varepsilon^2(t)}{N_\varepsilon}}, \tag{15}$$

where μ_ε is the root mean squared tracking error, ε is the error between the set-point and the current output value, N_ε is the number of samples.

- Standard deviation of tracking error: The standard deviation of the tracking error indicates the variation of the error from the mean error value. This is defined as:

$$\sigma_\varepsilon = \sqrt{\frac{\sum_{t=0}^{t=t_f} (\varepsilon(t)-\mu)^2}{N_\varepsilon - 1}}. \tag{16}$$

This formula is also referred as the corrected sample standard deviation. In this paper, we are interested in the steady-state pointing performance of the system. Therefore, we calculated these metrics on the steady-state part of the joint angle and end-effector pose response.

In this simulation, γ was varied from 0 to 1 using a step size of 0.1. For each γ the simulation was run 100 times. Each time a randomized seed was used to generate Gaussian white noise. The simulation duration was 10 seconds. Finally, we averaged the response. In this manner, we ensured the stochastic nature of the simulation. We calculated the root-mean-squared tracking error and standard deviation based on the response from $t = 3s$.

Table 2 RMS tracking error (μ) and variation (σ): $\gamma = 0$

$\gamma = 0$	μ	σ
Joint angle (rad)	0.02	0.0071
End-effector position (mm)	14.25	7.865
End-effector orientation (deg)	1.32	0.7372

Tables 2 through 4 show tracking performance for several values of $\gamma =$. We observe that the $\gamma = 0.3$ and

Table 3 RMS tracking error (μ) and variation (σ): $\gamma = 0.3$

$\gamma = 0.3$	μ	σ
Joint angle (rad)	0.0126	0.0043
End-effector position (mm)	8.86	4.63
End-effector orientation (deg)	0.84	0.45

Table 4 RMS tracking error (μ) and variation (σ): $\gamma = 0.9$

$\gamma = 0.9$	μ	σ
Joint angle (rad)	0.0139	0.004
End-effector position (mm)	4.31	4.2506
End-effector orientation (deg)	0.92	0.42

$\gamma = 0.9$ show better performance compared to $\gamma = 0$ case. $\gamma = 0.3$ produced the minimum tracking error in all cases (joint angles and end-effector pose). $\gamma = 0.9$ produced the minimum tracking error variation. Our control goal is to minimize the variation in the joint angle error, consequently, which leads to minimum end-effector position error. Therefore, we choose $\gamma = 0.9$ as the best design parameter value for the statistical control of the left arm of Baxter robot.

2.4.3. Statistical Controller Simulation

Figs. 2 show the time response of the end-effector position with the statistical controller of $\gamma = 0.9$. The parameter $\gamma = 0.9$ yielded better tracking error variation for the end-effector position. Therefore, we selected $\gamma = 0.9$ as the design parameter value for our statistical controllers.

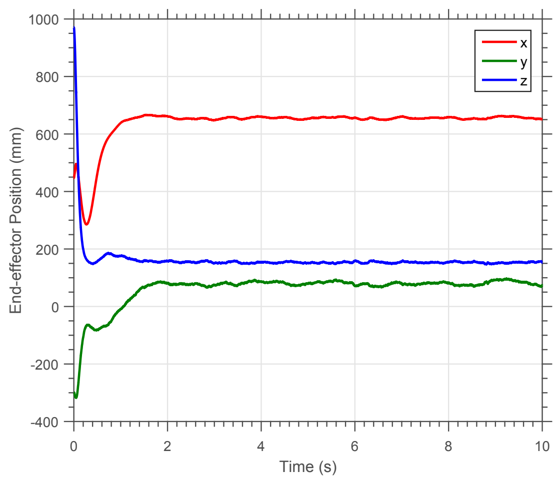

Fig. 2 End-effector position tracking with statistical control $\gamma = 0.9$.

3. HYBRID HIERARCHICAL STATISTICAL CONTROL ARCHITECTURE

A hierarchical hybrid agent control system architecture was selected to control the source and detector motion of the Baxter robotic arms for the desired scanning behaviour. The system consists of two tiers; The top tier consisting of the supervisory controller, and the bottom tier consisting of the agents. The supervisory controller generates commands for scanning, monitors the status of agents, and coordinates motion between them. Each of the Baxter arms is equipped with a different sensor and is considered an agent in this architecture. In this application, the supervisory control coordinates high-level motion, while the statistical controller determines low-level torque inputs for each joint.

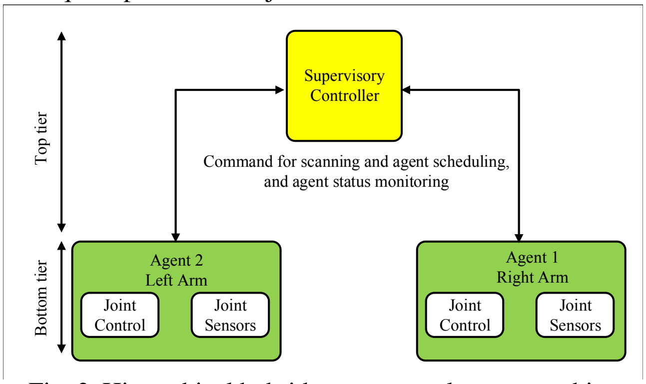

Fig. 3 Hierarchical hybrid agent control system architecture for a dual arm robot assisted bimodal dynamic imaging system.

3.1. Hybrid Automaton

The proposed hybrid hierarchical system includes both the continuous and discrete dynamics. The statistical controller developed for low-level joint position control is continuous, while the hierarchical controller used to coordinate motion between the two arms is discrete. As such, the system represents a hybrid automation system as defined in [16]. The hybrid automaton **H** can be described mathematically as

$$\mathbf{H} = (S, X, \Sigma_{in}, \Sigma_{out}, U, f, Init, \mathcal{D}, \mathcal{E}, G, \mathcal{R}) \qquad (17)$$

where S is the set of discrete states where the system is allowed to exist. X is the set that represents all of the continuous variables, possible in each of the states of S. The vector field, f, consists of time dependent functions such as differential equations that describe the time evolution of the variables in X. U denotes the tangent bundle of X. The initial states are the initial values of the continuous variables. $\mathcal{D}$ contains the range where each continuous variable and discrete event that are allowed to exist. $P(X)$ denotes the set of all subsets of X. Σ_{in} and Σ_{out} denote the set of all discrete inputs and discrete outputs. The set of edges, $\mathcal{E}$, describe what transitions are allowed to occur between states. The guard conditions, G, describe the events that must occur for a transition to take place. Finally, the reset map, $\mathcal{R}$, is the set of conditions that cause the system to enter its initial state.

3.2. Supervisory Controller Model

The supervisory controller functions are as follows:

1. To generate and send commands to agents.
2. To schedule the scanning task command to control the agents.
3. To facilitate coordination between the agents so that they can act cooperatively.
4. To ensure safety in case of unexpected situations.

Supervisory controller automaton is modeled using discrete event systems. In our case, the supervisory controller is designed to execute its tasks sequentially. There are five discrete nodes, which constitute the discrete set $S = \{s_1, s_2, s_3, s_4, s_5\}$. Each nodes are defined as follows:

• **Idle Node** (s_1). In this node, the supervisory controller waits for the PowerON message from the operator. If the PowerON message is generated, the system jumps from node s_1 to node s_2. In the case, when the supervisory controller gets *UnexAg1Status = 1* or *UnexAg2Status = 1* message from node s_5 or *ScanCountFinish = 1* from node s_2, it sends the PowerOFF message to the operator.

• **Agent Scheduler Node** (s_2). In this node, the supervisory controller assigns the scanning task to Agent 1 and Agent 2 sequentially and checks for the scanning task status. If the scan count matches a pre-specified number, the system jumps from s_2 to s_1. Agent 1 sends back a scheduling success message via Ag1ScheduleFinish tag with 1 upon successful reception. Then the supervisory controller starts calculating task start and end time. The supervisory controller then jumps to s_3 to control Agent 1 to perform the task with a message tag *Ag1Active=1*. After a pre-specified time interval, the controller jumps back from s_3. The controller then jumps to s_4 with a message tag *Ag2Active=1*. The controller does the scheduling and time interval calculation for s_4. The scan count is incremented by 1 if Agent 2 sends back *AgTaskFinish = 1* tag. This sequence of jumping goes on until the scan is complete and s_2 generates a tag *ScanCountFinish = 1*.

• **AgentManeuver Node** (s_3 and s_4). In these nodes, the supervisory controller sends the control signal to permit pre-specified reference trajectory to the corresponding agent at the calculated task start time. The the supervisory controller will standby in s_3 or s_4 to wait for the task status feedback from Agent 1 or 2. After the supervisory controller receives the feedback, or if a TimeOut event is triggered, it will jump from s_3 to s_2 or from s_4 to s_2. The scanning task includes positioning of the robot arm joints at the desired position and capturing images. If the scheduled scanning task has been successfully performed by an agent, a feedback tag *AgTaskFinish = 1* is sent from the bottom tier to the top tier, and the supervisory controller jumps back to s_2.

• **UnexpectedStatusCheck Node** (s_5). In this node, the supervisory controller monitors for any unexpected situation. The unexpected situations can occur in different scenarios such as there is collision between the arms, there exist out of reach joint limit, or the operator is not satisfied with the scanning performance during the scanning. The generated message tag is *UnexAg1tatus* or *UnexAg2tatus*. If the tag is 1, the supervisory controller falls back to node s_1.

The finite set of continuous variable for the agent system is defined as
$X = \{q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$, where q_i, $i = 1, 2, \cdots, 7$ represents the joint angles of the robot arm. The continuous dynamics include the stochastic model of the robot arm with the state feedback statistical controller.

The finite set of discrete events has the input and output events. It can be represented as follows.

$$\Sigma_{in} = \begin{bmatrix} PowerOn \\ AgkTaskStatus \\ ScanCountFinish \\ UnexAgkStatus \end{bmatrix}', \qquad (18)$$

$$\Sigma_{out} = \begin{bmatrix} AgkTaskCMD \end{bmatrix};$$

Note that Agk denote $Ag1$ and $Ag2$. The discrete transitions in the system are:

$$\mathcal{E} = \begin{bmatrix} (s_1, s_2) \\ (s_2, s_1) \\ (s_2, s_3) \\ (s_3, s_2) \\ (s_2, s_4) \\ (s_4, s_2) \\ (s_3, s_5) \\ (s_4, s_5) \\ (s_5, s_1) \end{bmatrix}' \qquad (19)$$

The guard conditions are defined as:

$$\begin{bmatrix} G_1 \\ G_2 \\ G_3 \\ G4 \\ G5 \\ G6 \\ G7 \\ G8 \\ G9 \end{bmatrix} = \begin{Bmatrix} (s_1, s_2) \Rightarrow \{PowerON \neq \varnothing\} \\ (s_2, s_1) \Rightarrow \{ScanCountFinish = 1\} \\ (s_2, s_3) \Rightarrow \{(Ag1Active = 1) \wedge (ScanCountFinish = 0)\} \\ (s_3, s_2) \Rightarrow \{Ag1TaskFinish = 1\} \\ (s_2, s_4) \Rightarrow \{(Ag2Active = 1) \wedge (ScanCountFinish = 0)\} \\ (s_4, s_2) \Rightarrow \{Ag2TaskFinish = 1\} \\ (s_3, s_5) \Rightarrow \{UnexAg1Status = 1\} \\ (s_4, s_5) \Rightarrow \{UnexAg2Status = 1\} \\ (s_5, s_1) \Rightarrow \{(UnexAg1Status = 1) \vee (UnexAg2Stat = 1)\} \end{Bmatrix} \qquad (20)$$

The state transition diagram for the supervisory controller (top tier) is given in Fig. 4.

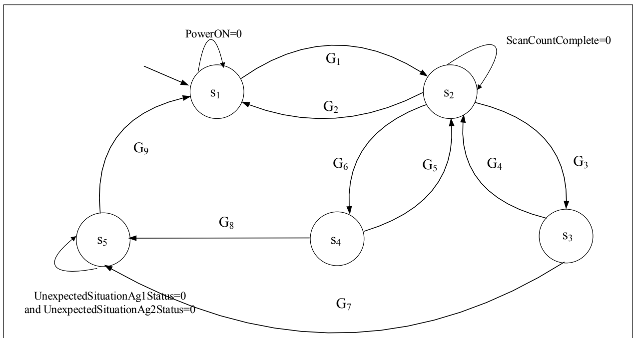

Fig. 4 State transition diagram for supervisory controller.

3.3. Agent Model

The agent functions are as follows:

1. To receive reference trajectory commands from the supervisory controller.
2. To perform the joint maneuver so that the poses of the two end-effector are in line-of-sight.
3. To send the task status feedback to the supervisory controller to notify the task success.

- **Idle node** (s_1). In this node, the agent is at the initial position with the initial pose. The agent waits for the *AgentTaskCMD* from the supervisory controller to check if the joint maneuver is required. After receiving *AgentTaskCMD*, the agent jumps to node s_2.
- **Joint maneuver node** (s_2). In this node, the agent changes its joint angles to the reference joint angles. If there are differences between the current and reference joint angles, then the agent adjusts its joint angles by applying required torque input. At this stage, a statistical controller is used to generate the required torque. If the differences between the current and reference joint angles are below certain thresholds, the agent jumps to node s_3.
- **Agent task status node** (s_3). In this node, the agent operates at the target joint angles with specified margins. The agent checks for the completion of the task assigned by the supervisory controller. The agent sends back a *AgentkTaskFinish* tag to the top tier to indicate that the joint maneuver is successful. If any unexpected situation occurs during the maneuver, the agent generates *UnexAgkStatus* tag to inform the top tier. Here k denotes 1 or 2.

The finite set of continuous variable for the agent system is defined as $X = \{q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$, where $q_i, i = 1, 2, \cdots, 7$ represents the joint angles of the robot arm. The continuous dynamics include the stochastic model of the robot arm with the state feedback statistical controller.

The finite set of discrete events has the input and output events. It can be represented as follows.

$$\begin{aligned} \Sigma_{in} &= \{AgentkTaskCMD\}; \\ \Sigma_{out} &= \{AgkTaskFinish, UnexAgkStatus\}; \end{aligned} \quad (21)$$

Note that $Agentk$ denote $Agent1$ and $Agent2$, and Agk denote $Ag1$ and $Ag2$. Initial conditions enforced on the system are:

$$Init = \{q_i = q_{i0}\}; i = 1, 2, \cdots, 7 \quad (22)$$

The discrete transitions in the system are:

$$\mathcal{E} = \{(s_1, s_2), (s_2, s_3), (s_3, s_1)\} \quad (23)$$

The guard conditions are defined as:

$$\begin{bmatrix} G_1 \\ G_2 \\ G_3 \end{bmatrix} = \begin{bmatrix} [l]\,(s_1, s_2) & \Rightarrow & \{AgentkTaskCMD \neq \varnothing\} \\ (s_2, s_3) & \Rightarrow & \{Jointanglediff < threshold\} \\ (s_3, s_1) & \Rightarrow & \{(AgkTaskFinish = 1) \vee (UnexAgkStatus = 1)\} \end{bmatrix} \quad (24)$$

The state transition diagram for the agents (bottom tier) is given in Fig. 5.

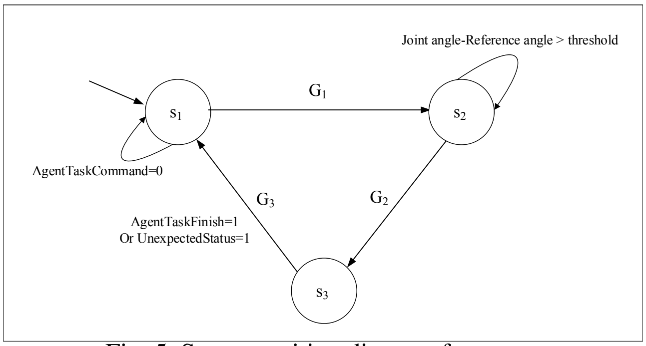

Fig. 5 State transition diagram for agents.

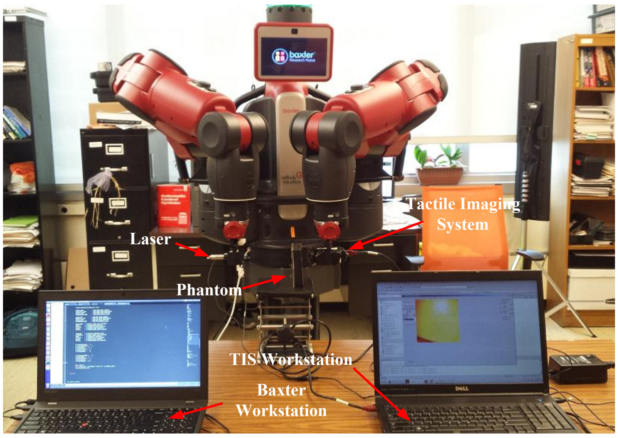

Fig. 6 System implemented with the Baxter robot.

4. SIMULATION

The hybrid hierarchical statistical controller system was implemented using MATLAB Simulink for the supervisory controller, statistical joint controller, and physics simulation. In addition, two feedforward controllers were added to achieve set-point tracking.

The simulation was repeated 20 times with variations in the seed value used to generate the additive Gaussian white noise values. A summary of the simulation performance is given in Table 5. When both the agents reached steady-state positions, the system started acquiring images. At t=16s, Agent 1 switched to a new position after being provided with the new reference position. This movement was repeated until 112s.

Table 5 Simulation Example Points.

time	Agent 1			Agent 2		
(s)	X (mm)	Y (mm)	Z (mm)	X (mm)	Y (mm)	Z (mm)
0	554.80	-84.80	100.4	603.60	197.20	74.20
1	827.10	8.56	296.50	603.60	197.20	74.20
2	669.8	-77.30	171.10	603.60	197.20	74.20
8	669.8	-77.30	171.10	603.60	197.20	74.20
9	669.8	-77.30	171.10	939.90	1169.00	414.50
10	669.8	-77.30	171.10	655.30	76.90	155.10
...	...	...	...	...	...	...
112	618.80	-78.30	150.20	631.20	73.80	153.50

Fig. 9 shows the trajectory tracking error of Agent 1 and Agent 2. Enlarging a section of the plot from $t = 17s$ to $t = 24s$ reveals the tracking error did not settle to zero, but rather oscillated around the zero value. This was due to the presence of the Gaussian white noise in the system. The largest error for both agents was observed during the initial motion, however measured mean and standard deviations of the agents after reaching steady state values were (mean±standard deviation) was 1.49± 0.86 mm for Agent 1 and 1.50± 0.86 mm for Agent 2. When both the agents reached steady-state positions, the system started acquiring images.

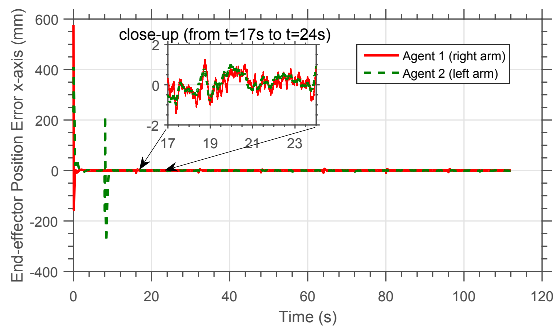

Fig. 7 End-effector X-axis trajectory tracking error.

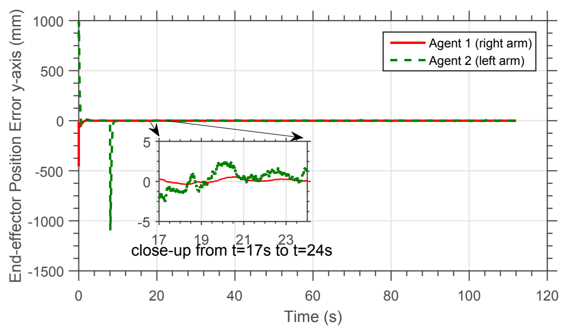

Fig. 8 End-effector Y-axis trajectory tracking error.

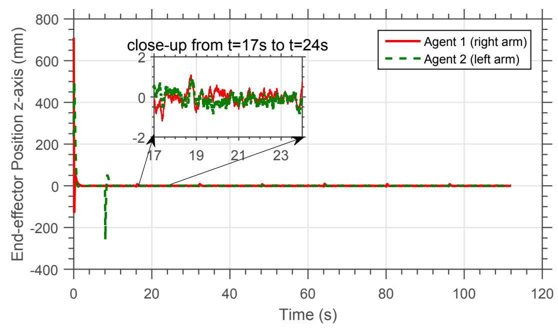

Fig. 9 End-effector Z-axis trajectory tracking error.

Note that the experiment for collecting reference joint angles took 112 seconds. The images were taken at 15 positions throughout the motion over a row length of 30mm. Assuming a square sample of 30mm per side, a full matrix scan of the sample area would take approximately 30-35 minutes. For this test, position dwell time at each sample point was set to 8 seconds as the images were captured manually at these points. Integration of the shutter command with the controller would allow for shorter dwell times, thus reducing the capture time required for a full 30mm square sample.

5. CONCLUSIONS

This paper presented a hybrid hierarchical statistical control architecture for the tracking control of a dual-agent stochastic system, based on the Baxter research robot. A full-state feedback controller was designed for low-level joint control of each robotic arm, while high-level motion tracking was implemented using a hybrid hierarchical supervisory controller. The results of the implementation in a MATLAB simulation environment yielded a tracking error of approximately 2mm over a series of 20 trials. A comparison of the statistical joint controller to other more conventional controllers such as LQR and PID control is left for future work.

REFERENCES

[1] E. G. Alberkht, "On the Optimal Stabilization of Nonlinear Systems", *PMM - Journal of Applied Mathematics and Mechanics*, Vol. 25, pp. 1254-1266 1961.

[2] L. Arnold, *Stochastic Differential Equations: Theory and Applications*, John Wiley & Sons, 1974.

[3] R. Beard, G. Saridis and J. Wen, "Sufficient Conditions for the Convergence of Galerkin Approximations to the Hamilton-Jacobi Equation," *Automatica*, vol. 33, no. 12, p. 2159-2177, 1997.

[4] R. W. Beard, G. N. Saridis and J. T. Wen, "Approximate Solutions to the Time-Invariant Hamilton-Jacobi-Bellman Equation," *PMM - Journal of Optimization Theory and Applications*, vol. 96, no. 3, March, p. 589-626, 1998.

[5] A. Bensoussan and J. H. van Schuppen, "Optimal Control of Partially Observable Stochastic Systems with an Exponential-of-Integral Performance Index," *SIAM Journal on Control and Optimization*, Volume 23, pp. 599–613, 1985.

[6] T. Chen, F. L. Lewis and M. Abu-Khalaf, "A Neural Network Solution for Fixed-Final Time Optimal Control of Nonlinear Systems," *Automatica*, Vol. 43, pp. 482-490, 2007.

[7] S. Cremer, L. Mastromoro, D. Popa, 'On the perfomance of the Baxter Resarch Robot', *2016 IEEE International Symposium on Assembly and Manufacturing (ISAM)*, Fort Worth, TX, USA, August 21-24, 2016

[8] R. W. Diersing, "H_∞, Cumulants, and Control", *Ph.D. Dissertation, Department of Electrical Engineering, University of Notre Dame*, August 2006.

[9] B. A. Finlayson, *The Method of Weighted Residuals and Variational Principles.*,New York: Academic Press, 1972.

[10] W. H. Fleming, and R. W. Rishel, *Deterministic and Stochastic Optimal Control*, New York: Springer-Verlag, 1975.

[11] W. H. Fleming, and H. M. Soner, *Controlled Markov Processes and Viscosity Solutions*, New York: Springer-Verlag, 1992.

[12] B. A. Francis and W. M. Wonham, "The internal

model principle of control theory", *Automatica*, Vol. 12, No. 5, pp. 457-465, 1976.

[13] G. W. Gardiner, *Handbook of Stochastic Methods for Physics, Chemistry, and the Natural Sciences*, Second Edition, Springer-Verlag, 1997.

[14] B. Kang and C. Won, 'Nonlinear Second Cost Cumulant Control using Hamilton-Jacobi-Bellman Equation and Neural Network Approximation.,' *Proc. of the American Control Conference*, Baltimore, MD, 2010.

[15] A. E. Lim and X. Y. Zhou "Risk-Sensitive Control with HARA Utility", *IEEE Transactions on Automatic Control*, Volume 46, Number 4, pp. 563–578, 2001.

[16] J. Lygeros and K. H. Johansson and S. N. Simic and Jun Zhang and S. S. Sastry, "Dynamical properties of hybrid automata", *IEEE Transactions on Automatic Control*, Vol. 48, No. 2, pp. 2-17, 2003.

[17] M.W. Matthew, 'Series Elastic Actuators'. Master's Thesis, Massachusetts Institute of Technology, Cambridge, MA, USA, 1995.

[18] F. Saleheen "Bimodal Dynamic Imaging System for Tumor Characterization using Hybrid Hierarchical Statistical Control," doctoral dissertation, Temple University, August 2017.

[19] M. K. Sain "Control of Linear Systems According to the Minimal Variance Criterion—A New Approach to the Disturbance Problem", *IEEE Transactions on Automatic Control*, Volume AC-11, Number 1, pp. 118–122, January 1966.

[20] M. K. Sain, "Performance Moment Recursions, with Application to Equalizer Control Laws," *Proceedings 5th Annual Allerton Conference on Circuit and System Theory*, pp. 327-336, 1967.

[21] M. K. Sain and S. R. Liberty "Performance Measure Densities for a Class of LQG Control Systems", *IEEE Transactions on Automatic Control*, Volume AC-16, Number 5, pp. 431–439, October 1971.

[22] M. K. Sain, C.-H. Won, B. F. Spencer Jr.,and S. R. Liberty, "Cumulants and Risk-Sensitive Control: Cost Mean and Variance Theory with Application to Seismic Protection of Structures, *Advances in Dynamic Games and Applications, Annals of the International Society of Dynamic Games*, Volume 5, pp. 427-459, Boston: Birkhauser, 2000.

[23] P. J. Smith, "A Recursive Formulation of the Old Problem of Obtaining Moments from Cumulants and Vice Versa," *The American Statistician*, vol. 49, no. 2, pp. 217-218, 1995.

[24] A. Stuart and J. K. Ord, *Kendall's Advanced Theory of Statistics*, Volume 1 "Distribution Theory", Fifth Edition, Oxford University Press, New York, 1987.

[25] C. H. Won, "Comparative Study of Various Control Methods for Attitude Control of a LEO Satellite," *Aerospace Science and Technology*, No. 5, pp. 323-333, 1999.

[26] C.-H. Won, "Nonlinear n-th Cost Cumulant Control and Hamilton-Jacobi-Bellman Equations for Markov Diffusion Process" *Proceedings 44th IEEE Conference on Decision and Control*, Seville, Spain, December, 2005.

[27] C. Won, R. W. Diersing and B. Kang, "Statistical Control of Control-Affine Nonlinear Systems with Nonquadratic Cost Function:HJB and Verification Theorems, *Automatica*, Vol. 46 (10), pp. 1636-1645, 2010.,