Numerical Simulation of the effect of high confining pressure on the drainage behavior of liquefiable clean sand

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Abstract

This article presents numerical simulations investigating pore pressure buildup of a sand layer with a free drainage boundary at the top, under both low and high overburden pressures and subjected to earthquake base excitation. The numerical runs simulate two centrifuge experiments previously conducted and reported by the authors. In these tests, a 5m layer of clean Ottawa sand with relative density, D_r =45%, was tested under overburden pressures of 1 atm and 6 atm. The simulations were performed using Dmod2000, a non-linear effective stress numerical 1D site response analysis code. The tests had revealed that the response was partially drained rather than undrained, with much more partial drainage at 6 atm compared to 1 atm. The simulations correctly modeled this behavior, with very good agreement between simulated and measured centrifuge excess pore pressures. A key aspect of this good accord in the simulations was the correct selection in the simulations of the 1D drained volumetric stiffness of the sand, M' = $1/m_v$, as the coefficient of consolidation, c_v, is proportional to M'. Both c_v and M' were 2.5 to 3 times greater at 6 atm than at 1 atm in both centrifuge tests and simulations. Any future simulation of pore pressure response of sand under field drainage conditions needs to consider this large increase in volumetric stiffness at high

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overburden pressure. Good agreement was found between values of M' back figured from the centrifuge tests and from a consolidometer test on a different sand reported by Martin et al. (1975). The value of M' seems to increase approximately with the root square of the overburden pressure, and future simulations for high overburden and realistic field drainage conditions should account for this increase. The proper high pressure correction factor, K_σ, to be used in conjunction with liquefaction charts may be higher than one for some realistic field drainage conditions, due to this substantial decrease of sand compressibility under high overburden pressure.

Introduction

Earthquake-induced liquefaction effects can be damaging to a wide range of structures as well as dams and embankments. In order to assess the liquefaction potential of saturated cohesionless deposits, most engineers rely on the Simplified Method proposed by Seed and Idriss (1971). Based on this procedure, several liquefaction triggering charts have been developed that use either field measurements of shear wave velocity (V_s), or field penetration tools such as standard penetration (SPT) and cone penetration (CPT). Figure 1a shows a compilation of some of the most commonly used CPT-based liquefaction charts (Idriss and Boulanger 2008), while Fig. 1b presents the Andrus and Stokoe (2000) Vs- based liquefaction chart. Both charts in Fig. 1 are for clean sands, and the curves in Fig. 1a have been normalized to a field effective overburden pressure, σ'_{v0} = 1 atm. The original data points (case histories) used to generate the charts in Fig. 1 have been omitted for clarity. Two parameters are plotted in both Figs. 1a and 1b along the abscissa axis representing either the Cyclic Resistance Ratio, CRR which is the liquefaction resistance, or the Cyclic Stress Ratio, CSR, that represents the earthquake demand. The CRR corresponds to the curves in Figs. 1a and b, while the CSR is applied to the case histories and to the project where liquefaction is being evaluated. The data points and arrows in Fig. 1b are associated with CSR.

All case histories used to generate the liquefaction charts in Fig. 1 come from relatively shallow sites (less than 20 m) with $\sigma'_{v0} < 2$ atm (Idriss and Boulanger 2008). This makes the charts reliable tools for liquefaction assessment in most cases. However, in some important projects involving, for example, tall earth embankment dams, the critical liquefiable soil in the embankment or foundation soil may be subjected to σ'_{v0} much greater than 2 atm; up to or even higher than 10 atm (Gillette 2013).

The current state of practice for dealing with cohesionless deposits under such high overburden pressure is based on a proposal by Seed (1983). Undrained cyclic stress-controlled triaxial or simple shear tests at both 1 atm as well as the high $\sigma'_{v0} > 1$ atm, are conducted in order to generate a correction factor for that high pressure, $K_\sigma = (CRR)_{\sigma'v0} / (CRR)_1$, where $(CRR)_{\sigma'v0}$ and $(CRR)_1$ are the cyclic resistance ratios at the high pressure σ'_{v0} and at 1 atm, respectively. The CRR is the same parameter plotted for the curves along the abscissa axes in Fig. 1. The K_σ can be used either to multiply the CRR of the curves in Fig. 1, or, more conveniently, to divide the CSR of the case history or site under consideration. The latter method is adopted in this paper. A value of $K_\sigma <$ 1.0 will move the data point of the case history or project under consideration upward, while $K_\sigma >$ 1.0 will move the data point downward, as indicated by the arrows in Fig. 1b. As seen later herein, the current state of practice invariably recommends values of $K_\sigma < 1.0$ at high pressures, so the arrow in Fig. 1b always point upwards for $\sigma'_{v0} > 1$ atm.

In many projects, engineers rely on published state of practice K_σ charts rather than performing cyclic undrained tests at low and high pressures. Current state of practice is based on the sets of curves of K_σ versus relative density in Fig. 2, proposed by Youd et al. (2001) and Idriss and Boulanger (2008) for relative densities, D_r, between 40% and 80%, with K_σ ranging approximately between 0.5 and 0.85 at $\sigma'_{v0} = 6$ atm. Important publications with work on K_σ based on undrained testing, are: Seed and Harder (1990); Vaid and Thomas (1995); Hynes et al. (1999); Youd et al. (2001); Boulanger (2003); Boulanger and Idriss (2004); Idriss and Boulanger (2006; 2008); Montgomery et al. (2012) and Dobry and Abdoun (2015). In addition to the curves using relative density of Fig. 2, Idriss and Boulanger (2008) generated K_σ charts based on the CPT tip resistance, which generally follow the same trend of decreasing K_σ with confining pressure and are widely used in practice.

Ni et al. (2020) and Abdoun et al. (2020) recently performed a series of centrifuge experiments supplemented by extensive data analysis, aimed at examining the values of K_σ under simulated field conditions (field K_σ). Parallel regular undrained, stress-controlled cyclic triaxial tests were also conducted to obtain values of K_σ similar to those reported in the literature (laboratory undrained K_σ). In the centrifuge tests, 5m-thick clean Ottawa F65 sand layers with D_r ranging from 45% to 80% under either $\sigma'_{v0} = 1$ atm or $\sigma'_{v0} = 6$ atm – in both cases with a free draining boundary at the top - were subjected to 1D base earthquake shaking and developed similar high excess pore pressure ratios. This allowed determination of the field K_σ for $\sigma'_{v0} = 6$ atm. It was found that this field $K_\sigma > 1.0$, while the laboratory undrained K_σ from the undrained triaxial tests was 0.85, smaller than unity and consistent with the state of practice (Table 1). The discrepancy between field and laboratory undrained K_σ in Table 1 was attributed to the partially drained behavior of the sand layer under idealized field conditions in the centrifuge. This partial drainage behavior was more pronounced at 6 atm as compared with 1 atm, due to the decreased compressibility of the sand at the higher confining pressure. Abdoun et al. (2020) concluded that the current state of practice may be too conservative for some realistic field conditions, with a potential for significant savings in unnecessary liquefaction mitigation of tall embankment dams. They suggested additional experimental and numerical work to better understand the liquefaction behavior in the field under high confining pressure.

Scope of the paper

The main purpose of this paper is to calibrate and validate the nonlinear effective stress numerical code Dmod2000 (Matasovic 1993), for liquefaction evaluation at low and high effective overburden pressures. This is achieved by simulation of the two Ni et al. (2020) centrifuge experiments for the case of the sand layer having a $D_r = 45\%$, for which soil laboratory tests are also available to provide key input parameters. Most of these parameters used as input to the code were experimentally obtained, either back figuring them from the centrifuge experimental results (shear stiffness and compressibility), or calibrating them from the available laboratory cyclic triaxial testing (relation between excess pore pressure, cyclic shear strain and number of cycles in undrained condition). The permeability used in the code was also based on available small sample 1g tests on the same sand using constant head permeability tests.

The results of the two numerical simulations are compared to that of the centrifuge experiments conducted at $\sigma'_{v0} = 1$ atm and $\sigma'_{v0} = 6$ atm. The effect of partial drainage on the pore pressure behavior is further investigated by running additional hypothetical numerical simulations at 1 and 6 atm with perfect undrained condition at all elevations within the sand layer. Table 2 lists the four Dmod2000 runs discussed in the paper.

The following section summarizes the results of the centrifuge experiments as well as the strain-controlled undrained cyclic triaxial tests reported by Ni et al. (2020) and Abdoun et al. (2020); with the rest of the paper discussing the numerical simulations.

Centrifuge Experiments

Two centrifuge experiments were conducted in the 1-D laminar container and shaker of the geotechnical centrifuge facility at Rensselaer Polytechnic Institute (RPI). Figure 3 shows the centrifuge model configuration and setup used in Tests 45-1 and 45-6. Ottawa F65 sand, deposited by dry pluviation and having a relative density, $D_r = 45\%$, was used to build the liquefiable sand layer in both tests. The models were instrumented with accelerometers, pore pressure transducers, vertical LVDTs and bender elements, as shown in the figure. The names of the two experiments indicates the combination relative density of the sand (D_r) - effective overburden stress (σ'_{v0}) at the middle of the liquefiable sand layer in atm (e.g., in Test 45-1, $D_r = 45\%$ and $\sigma'_{v0} = 1$ atm). The prototype thickness of the liquefiable sand layer in both tests was about 5 m, with the layer having a free drainage boundary at the top. Additional details of the two tests as well as analyses and interpretations are presented by Ni et al. (2020) and Abdoun et al. (2020).

Centrifuge Experimental Results

Figures 4-7 show the time histories of acceleration, stress ratio (shear stress (τ) / σ'_{v0}), shear strain and excess pore pressure, measured in Tests 45-1 and 45-6, all in prototype units and reported by Ni et al. (2020). The input acceleration time histories in both experiments consisted of a 10-cycle uniform sinusoidal motion having a prototype frequency of 2 Hz. In both tests, the pore pressure ratio in the sand reached a maximum excess pore pressure ratio, $r_u = (r_u)_{max}$, of about 0.8 (Table 3). While the shape, frequency and duration of the input acceleration were the same in both tests, the acceleration amplitude was about an order of magnitude higher in Test 45–6, in order to achieve similar maximum pore pressure ratios.

Figure 8 shows the measured profiles of excess pore pressure, u, versus depth, z, for the instant at the end of shaking, when $u = u_{max}$ in the deepest pore pressure sensor. The depth, z, in Fig. 8 as well as subsequent figures, is measured from the top of the 5 m sand layer. As can be seen in Fig. 8, u increases with depth in both Tests 45-1 and 45-6, reaching the maximum $u = u_{max}$ at the deepest elevation. The slopes of the curves in Fig. 8 relate to the hydraulic gradients at the time of u_{max}, with steeper slopes corresponding to smaller gradients. The hydraulic gradient points upward in both tests, as expected, indicating that the pore fluid was flowing up toward the drainage boundary located at the top of the layer. Figure 9 shows the corresponding profiles of the pore pressure ratio, $r_u = u/\sigma'_{v0}$, also measured at the end of shaking. Figure 9 indicates that the maximum value of $r_u = (r_u)_{max}$ also occurred at the deepest elevation in both Tests 45-1 and 45-6. Table 3 includes the depths of the corresponding sensors as well as the values of $(r_u)_{max}$ measured by them in the two experiments. The System Identification analyses performed by Ni et al. (2020) indicated that for both tests the stress ratio and shear strain did not change much with depth within the layer, as illustrated by Figs. 5 and 6. Therefore, the variation of u and r_u with depth in the profiles of Figs. 8 and 9 is not due to differences in cyclic loading, as the cyclic loading varied little with depth within the sand layer during shaking.

Roles of cyclic triaxial stress-controlled and strain-controlled tests

Separate series of stress-controlled and strain-controlled cyclic triaxial undrained tests were conducted by Gecomp Corp on the same Ottawa sand used in the centrifuge experiments, at a similar relative density, and on soil specimens isotropically consolidated to effective confining pressures of 1 and 6 atm (Ni et al. 2020; Abdoun et al. 2020). The stress-controlled tests were mentioned earlier when discussing the laboratory undrained K_σ of Table 1. The strain-controlled tests are discussed in the following heading. It is important to note that the two series played two important but different roles:

- The stress-controlled tests were used to determine the conventional laboratory undrained value of K_σ for our sand in our Table 1, reproduced from Ni et al. (2020). This is consistent with the undrained testing approach used to produce the State-of-Practice recommendations of Fig. 2 (Youd et al. 2001; Idriss and Boulanger 2008).
- As discussed in the next heading, the strain-controlled tests were used to obtain the undrained pore pressure buildup model needed for the Dmod2000 numerical simulations (Fig. 10 and Table 4). Pore pressure buildup is determined more uniquely by cyclic strain than by cyclic stress, and use of strain-controlled tests result in additional insights and improved numerical modeling (Dobry and Abdoun 2015).

Strain-controlled cyclic triaxial tests

Two series of isotropically consolidated, cyclic triaxial <u>strain-controlled cyclic tests</u> were conducted in order to calibrate the undrained pore pressure model needed as input to Dmod2000. All triaxial tests were done by Geocomp Corp (Ni et al. 2020; Abdoun et al. 2020).

In these strain-controlled tests, samples of the same Ottawa F65 sand with a similar relative density (40%), were subjected to different levels of cyclic strain under 1 and 6 atm isotropic consolidation pressures. The excess pore pressure ratio, r_u, was recorded versus number of cycles up to full liquefaction ($r_u = 1.0$). A fresh sample was used for each cyclic shear strain level ($\boldsymbol{\gamma}_c$), with $\boldsymbol{\gamma}_c$ ranging from 0.003% to 0.3%. Figure 10 displays the relations between $\boldsymbol{\gamma}_c$ and pore pressure ratio, r_u, after 10 loading cycles for the two confining pressures. The plot shows that the 6 atm samples required a higher cyclic strain to reach the same level of r_u, as expected. The figure includes the best fit curves for these results at 1 atm and 6 atm confining pressures. These curves were used to generate the parameters for the pore pressure model in Dmod2000, as explained later herein. The figure also includes the range of undrained cyclic triaxial strain-controlled results compiled from the literature by Dobry in the 1980s and reproduced by Dobry and Abdoun (2015), for a number of clean and silty sands at confining pressures between 0.3 and 2 atm. Finally, Fig. 10 also includes the curve measured by Bhatia (1982) using cyclic direct simple shear tests (CDSS) and $\sigma'_{v0} \approx 2$ atm for their Ottawa sand.

Numerical simulations using Dmod2000

Several researchers have developed sophisticated constitutive soil models that aim at capturing the soil contractive and dilative behavior in 1D and 2D systems (Prevost 1977; Li et al. 1993; Mckenna and Fenves 2001; Elgamal et al. 2003; Dafalias and Manzari 2004; Gerolymos and Gazetas 2005). These models often require tens of parameters that are not typically available to practicing engineers. Alternative simplified non-linear 1D analysis codes, require less parameters and are easier to use, like Desra2 (Lee and Finn 1978), Dmod2000 (Matasovic 1993), and Deepsoil (Hashash 2009). Over the past few decades, significant efforts have been made toward validation and calibration of numerical codes simulating soil liquefaction in the free field and under structures. These efforts include VELACS (Arulanandan and Scott 1993) and the ongoing project LEAP (Manzari et al. 2017). The authors have participated in both efforts in various capacities.

In this paper, Dmod2000 was used to simulate both Tests 45-1 and 45-6 in order to calibrate and validate the software, as well as to gain additional insight into the pore pressure response of sand under a high overburden pressure. The authors took advantage of their experience in achieving successful Dmod2000 simulations of other RPI liquefaction centrifuge tests (Dobry et al. 2018). The Dmod2000 simulations of Tests 45-1 and 45-6 included in this paper are Type C predictions as defined in VELACS and LEAP, because the predictions were done after the events. That is, the centrifuge results were already available and were systematically used to calibrate the software. As listed in Table 2, four Dmod2000 runs were performed in this research (Run 45-1, Run 45-6, Run 45-1U, and Run 45-6U). Run 45-1 and Run 45-6 are Type C simulations of Tests 45-1 and 45-6. The two additional runs (Run 45-1U and Run 45-6U), used the same input parameters of Runs 45-1 and 45-6, except for the drainage condition. That is, in Runs 45-1U and 45-6U the drainage was turned off, in order to study the effect of drainage/no-drainage on the liquefaction behavior of sand under low and high effective overburden pressures. No-drainage is equivalent to have a permeability in the sand layer, $k \approx 0$.

The following headings discuss the main constitutive parameters needed to run these 1D Dmod2000 free field simulations. The parameters and key equations are summarized in Table 4. A special emphasis is placed on the correct modeling of compressibility at low and high overburden pressures, following Abdoun et al.'s (2020) conclusion that the lower sand compressibility at 6 atm compared to 1 atm, was the main reason for the increased partial drainage during shaking at 6 atm observed in the centrifuge tests.

Shear stiffness and constitutive law

Bender element measurements in the centrifuge tests provided the shear wave velocity of the sand, V_s, which in turn allowed obtaining the initial shear modulus of the sand at very small strains, $G_{max} = \rho V_s^2$ (ρ = saturated total density of the soil). The constitutive shear stress-strain law assigned to each layer (backbone curve), follows the modified Kondner and Zelasko (1963) hyperbolic model (Matasovic 1993):

$$\tau = \frac{G_{max} \cdot \gamma}{1+\beta\left(\frac{G_{max}}{\tau_{mo}} \cdot \gamma\right)^{s}} \tag{1}$$

where τ is the shear stress corresponding to a shear strain, γ, τ_{mo} is shear strength, and β and s are constants estimated by fitting the secant shear modulus reduction curve (G/G_{max} vs. cyclic shear strain, γ_c), to that provided in Ni et al. (2020) based on the centrifuge results. Equation 1 is the initial, undegraded backbone stress-strain curve, which is degraded as excess pore water pressure increases in the layer, with G_{max} replaced by G_{mt} and τ_{mo} replaced by τ_{mt}, as shown in Table 4.

Pore water pressure buildup behavior due to undrained cyclic loading

The excess pore water pressure model implemented in Dmod2000 is based on the cyclic strain-based approach for undrained loading originally developed by Dobry et al. (1985) for sands, and updated by Vucetic and Dobry (1986, 1988) as follows:

$$r_u = \frac{P * f * N_c * F * (\gamma_c - \gamma_{tv})^{sp}}{1 + f * N_c * F * (\gamma_c - \gamma_{tv})^{sp}} \tag{2}$$

where *P, F, f* and s_p are fitting parameters that depend on the sand material and fabric. The strain variables, γ_c and γ_{tv}, are the uniform cyclic shear strain and the volumetric threshold shear strain, respectively. The results of the undrained cyclic strain-controlled triaxial tests performed by Geocomp at 1 and 6 atm (Ni et al. 2020 and Abdoun et al. 2020) were used to calibrate the pore pressure model in Eq. 2. Figure 10 shows the data points from the triaxial tests as well as the best fit curves based on the pore pressure model of Eq. 2, while Table 4 lists the numerical input parameters selected.

Roles of compressibility and permeability

Dmod2000 calculates the increase and decrease of excess pore pressures within the general framework of the classical Theory of Consolidation, originally developed for clays (e.g., Lambe and Whitman 1969). Specifically, the rate of change at a given elevation of the excess pore pressure, u, with time, t, is obtained as the sum of two terms:

$$\partial u/\partial t = (\partial u/\partial t)_{undrained} + c_v\,(\partial^2 u/\partial z^2) \tag{3}$$

where $(\partial u/\partial t)_{undrained}$ is obtained from the undrained pore pressure model (Eq. 2), and the second term, $c_v\,(\partial^2 u/\partial z^2)$, reflects the pore water flow at that elevation due to positive or negative volume changes throughout the sand layer (consolidation). During dissipation after the shaking, the first term of Eq. 3 is zero, and the equation becomes the familiar clay consolidation basic equation, where the excess pore pressure dissipation is completely controlled by the coefficient of consolidation, c_v, as well as by the drainage boundary conditions:

$$\partial u/\partial t = c_v\,(\partial^2 u/\partial z^2) \tag{4}$$

There could be some problems in applying Eqs. 3-4 to a sand layer which liquefies in part or all of its thickness, due to the presence of sand grains that are "floating" in the water and not in contact with each other, and hence settle (sediment) with time in a way that is different from that reflected in Eqs. 3-4 (Florin and Ivanov 1961; Scott 1986; Sharp and Dobry 2002). However, the condition $r_u = 1.0$ corresponding to liquefaction was not reached in the two centrifuge tests analyzed herein, and thus the application of Eqs. 3-4 used in Dmod2000 is clearly a valid approximation.

The coefficient of consolidation, c_v, is related to other soil parameters by the expression:

$$c_v = \frac{k}{\gamma_w m_v} \tag{5}$$

where γ_w = unit weight of water, k = permeability, and m_v = coefficient of volume change = $\partial\varepsilon_{vc}/\partial\sigma'_v$. In this expression for m_v, $\varepsilon_{vc} = s_{vc} / H$ is the consolidation vertical volumetric strain of the sand layer after shaking at a given time during dissipation; H = thickness of sand layer; and $\sigma'_v = \sigma'_{v0} - u$ is the effective overburden pressure. That is, m_v is a measure of the compressibility of the soil skeleton. For the Dmod2000 simulations, it is more convenient to work with the reciprocal of m_v, the constrained modulus, $M' = 1/m_v = \partial\sigma'_v / \partial\varepsilon_{vc}$. Therefore, c_v is redefined as:

$$c_v = \frac{k\,M'}{\gamma_w} \tag{6}$$

The volumetric stiffness M' is the parameter used in Dmod2000, with Table 4 including the corresponding expression, where M' is a function of both σ'_{v0} and $\sigma'_v = \sigma'_{v0}$ - u.

An important conclusion from Eqs. 3-4 and 6 is that from the viewpoint of excess pore pressure buildup and dissipation, neither the permeability, k, nor the volumetric stiffness, M', matter independently, but only through their effect on the coefficient of consolidation c_v, which is in turn proportional to the product k M'.

The next two headings discuss, respectively, the parameters M' and k used as input to the Dmod2000 simulations reported herein. The discussion related to sand compressibility is relatively involved due to its importance in partial drainage, especially because the Martin et al. (1975) formulation used by Dmod2000 and other numerical codes is not clearly explained elsewhere.

Sand compressibility

Assignment of the proper sand compressibility in the Dmod2000 runs is critical to a correct numerical simulation of the centrifuge tests. As discussed by Abdoun et al. (2020), pore pressure dissipation after shaking was much faster in Test 45-6 than in Test 45-1 (Fig. 7), and also much more partially drained during shaking in Test 45-6 (Figs. 8-9). This was correlated to the decreased compressibility of the sand in Test 45-6, associated with its 6 atm overburden pressure compared to the 1 atm applied in Test 45-1. As shown by Table 5 and by the two data points in Fig. 11, the volumetric stiffness modulus, $M' = u_{ave} / (\varepsilon_{vc})_{max}$, obtained from dissipation phase measurements, was 2.7 times greater in the 6 atm centrifuge test compared to the 1 atm test (Ni et al. 2020 and Abdoun et al. 2020). In Table 5 and Fig. 11, u_{ave} is the average excess pore pressure measured at the end of shaking throughout the sand layer, and $(\varepsilon_{vc})_{max}$ is the total volumetric strain starting from the end of shaking until the end of the excess pore pressure dissipation.

Martin et al. (1975) showed that the excess pore pressure during undrained cyclic loading, is related to the tendency to densification measured during drained cyclic loading on the same sand (see also Bhatia, 1982). A key aspect of this relationship is the volumetric stiffness modulus, $\bar{E}_r$, associated in first approximation to the unloading curve in 1D drained consolidometer tests. While Dmod2000 does not use Martin's model for pore pressure buildup, it retains the expression for $\bar{E}_r$ to relate pore pressure and volume changes during and after shaking (Table 4). Therefore, in principle, the volumetric stiffness M' obtained in centrifuge tests (Table 5), is the same parameter as $\bar{E}_r$ defined by Martin et al. (Table 4), $M' = \bar{E}_r = \Delta\sigma'_v / \Delta\varepsilon_v$.

It is useful to clarify in more detail the exact relation between $\bar{E}_r$ and M', as well as their law of variation with the effective overburden pressures σ'_{v0} and σ'_v, reflected in the expression for $\bar{E}_r$ of Table 4. For example, $\bar{E}_r = M' = d\sigma'_v / d\varepsilon_v$ - defined in Table 4 and used in Dmod2000 - is a tangent modulus associated with very small changes in σ'_v and ε_v with time at a given elevation. On the other hand, $M' = u_{ave} / (\varepsilon_{vc})_{max}$ calculated in Table 5 is a secant modulus associated with the total changes in u_{ave} and ε_{vc} calculated for the whole sand layer during the whole time of dissipation in the centrifuge tests.

Figure 12a shows typical experimental results of a 1D consolidation test in dry sand involving initial loading as well as unloading and reloading curves (Seaman et al., 1963; Lambe and Whitman, 1969). Of interest to the discussion here are the first monotonic loading curve, OA, as well as the first unloading curve, AB, and subsequent reloading curve, BC. All three curves are concave upwards. Important observations related to each of the three curves are (Figs. 12 a, b & c):

- The <u>initial loading curve</u> OA has an equation of the form, $\sigma'_v = C\,(\varepsilon_v)^p$, with $p > 1$. Generally for sands it is $p \approx 2$ (Lambe and Whitman, 1969). This results in a tangent modulus, $d\sigma'_v / d\varepsilon_v$, proportional to $\sqrt{\sigma'_v}$. This square root law of variation of 1D modulus during initial monotonic loading is widely accepted and used for sands (Lambe and Whitman, 1969).
- The <u>first unloading curve</u> AB in Figs. 12a and 12b is the basis for the Martin et al. (1975) model for pore pressure buildup. The equation for the tangent $\bar{E}_r = d\sigma'_v / d\varepsilon_v$ in Table 4 was developed for such unloading curve.
- The <u>first reloading curve</u> BC in Fig. 12a is the most appropriate to the situation during the dissipation phase of centrifuge tests such as those simulated here, captured by the calculation of M' in Table 5. The increased pore pressure u_{ave} and corresponding decreased effective stress $\sigma'_v = \sigma'_{v0} - u_{ave}$ at the end of shaking (Table 5), corresponds in first approximation to Point B in the figure before dissipation starts. During dissipation, the pore pressure is reduced from u_{ave} to zero, with σ'_v increasing back to its initial value σ'_{v0} (Point C of Fig. 12a). That is, the dissipation corresponds to reloading. During the shaking phase before dissipation, either unloading or reloading may take place at different times and

locations within the sand layer due to partial drainage, depending if the soil is locally decreasing or increasing in volume with time. Therefore, both unloading curve AB and reloading curve BC in Fig. 12a may be needed. Fortunately, as illustrated by Fig. 12a, the two curves are very similar in most of their trajectory. In Dmod2000, the same expression for the tangent modulus M' = $\bar{E}_r$ = $\Delta\sigma'_v / \Delta\varepsilon_v$ shown in Table 4, is used irrespective of the soil being unloaded (pore pressure increasing) or reloaded (pore pressure decreasing). This seems reasonable in light of the measured behavior in Fig. 12a. In the rest of this paper, and in accordance with the way M' = $\bar{E}_r$ is used in Dmod2000, no difference will be made between the unloading and reloading value of this parameter.

The sketches in Figs. 12b and 12c illustrate the basic concept of the model proposed by Martin et al. (1975) for their unloading curve in sand during 1D drained confined compression, interpreted in Dmod2000 to be valid for both unloading and reloading. Curve OA of equation $\varepsilon_v = F\ (\sigma'_v)^{0.5}$ in Fig. 12b corresponds to initial loading. Curve AB in the same figure corresponds to unloading back from $\sigma'_v = \sigma'_{v0}$ to $\sigma'_v = 0$.

Curve OA' in Fig. 12b, passing by the origin, is identical and parallel to curve OA in the same Fig. 12b. That is, OA' is the unloading curve but with the permanent or slipping strain, ε_{vs0}, removed so the analysis can focus on the relation between σ'_v and the recoverable strain, ε_{vr}. During unloading from A' to O in Fig. 12b, σ'_v decreases from σ'_{v0} to zero, and ε_{vr} decreases from ε_{vr0} to zero.

This unloading curve, OA', is repeated in Fig. 12c, together with similar curves OA'', OA''', etc., corresponding to different values of σ'_{v0} and ε_{vr0}. In Fig. 12c, the "pseudo initial loading" curve OA'A"A''', is simply the locus of all points A', A'', A''', etc., corresponding to different values of σ'_{v0} and ε_{vr0}. Martin et al. defined the following set of expressions for the pseudo initial loading and unloading curves (see also Bhatia, 1982):

Pseudo initial loading curve:

$$\varepsilon_{vr0} = k_2 (\sigma'_{v0})^n \tag{7}$$

Unloading curve:

$$\varepsilon_{vr} = \varepsilon_{vr0} (\sigma'_v / \sigma'_{v0})^m \tag{8}$$

Figure 12d presents the original set of curves for ε_{vr0} and ε_{vr} originally presented by Martin et al. (1975), see also Bhatia (1982). The curves in Fig. 12d correspond to actual consolidometer experimental data on Crystal silica sand, with values of $k_2 = 0.0025$, $n = 0.62$ and $m = 0.43$ (stresses in psf, strains in percent). Note that in this consolidometer test, the maximum applied pressure was $\sigma'_{v0} = 4000$ psf ≈ 2 atm, significantly smaller than the maximum $\sigma'_{v0} = 6$ atm associated with our centrifuge tests and numerical simulations.

The expression for the tangent modulus $M' = \bar{E}_r$ along the unloading curve OA' of Fig. 12c was derived by Martin et al. (1975) from Eqs. 7-8:

$$\bar{E}_r = M' = \frac{(\sigma_v^{'})^{1-m}}{m\, k_2\, (\sigma_{v0}^{'})^{n-m}} \tag{9}$$

which is the same expression used by Dmod2000 and listed in Table 4. It is useful to convert Eq. 9 in terms of excess pore pressure, more directly relevant to the centrifuge tests and corresponding Dmod2000 simulations. In these, σ'_{v0} is the initial effective confining pressure before shaking starts, and $\sigma'_v = \sigma'_{v0} - u$ at any time during shaking and dissipation where u =

excess pore pressure. Even more conveniently, $\sigma'_v = \sigma'_{v0}\,(1 - r_u)$, where $r_u = u/\sigma'_{v0}$ = excess pore pressure ratio. Therefore:

$$M' = \frac{(1-r_u)^{1-m}}{m\,k_2}\left(\sigma'_{v0}\right)^{1-n} \tag{10}$$

For $r_u = 0$, M' is proportional to $(\sigma'_{v0})^{1-n}$. For the Crystal silica sand oedometer tests in Fig. 12d, n = 0.62, 1-n = 0.38, and for $r_u = 0$, M' is proportional to $(\sigma'_{v0})^{0.38}$. The initial tangent modulus corresponding to very small unloading or reloading increments, is also usually defined as the constrained modulus associated with compressional wave propagation (P-wave), generally accepted to be proportional to the root square of σ'_{v0} in sands (Lambe and Whitman 1969). This would suggest that in many sands, 1-n = 0.5, with n being closer to 0.5 than to 0.62.

In the Dmod2000 simulations presented herein, the exponent n was taken to be 0.5, and the parameters m and k_2 were best fitted to the experimental data points $(u)_{ave}$ and $(\varepsilon_{vc})_{max}$ in Table 5. The best fit corresponds to the solid curves of Fig. 11. As seen in Fig. 11, the slopes of the curves show a significantly higher M' at 6 atm as compared to 1 atm. Therefore, it was possible to fit to the 1 and 6 atm centrifuge results during dissipation (Table 5, data points in Fig. 11), a single set of parameters in Martin's equation that completely define the variation of M' for the Dmod2000 runs ($m = 0.4$, $n = 0.5$, and $k_2 = 0.006$; strain in percent and stresses in psf). These three parameters used in our simulations are listed in Table 4. The parameters $m = 0.43$, $n = 0.62$ and $k_2 = 0.0025$ obtained experimentally by Martin et al. from oedometer testing for Crystal silica sand (Fig. 12d) were also used to predict the compressibility curves in Fig. 11 (dashed lines). The figure shows that the two sets of curves are in remarkably good agreement, especially at 1 atm. This good agreement occurs despite the fact that the data correspond to two different sands tested using two different techniques (consolidometer and centrifuge tests). Furthermore, the dashed line for σ'_{v0} = 6 atm in Fig. 11 involves significant extrapolation, as the Martin's consolidometer results in Fig. 12d only cover up to about 2 atm. Figure 11 may open the door toward defining unique compressibility characteristics for a range of different clean sands deposited by dry pluviation, that could potentially be used by practitioners for numerical modeling in their projects, with these unique compressibility characteristics perhaps based on simple loading-unloading-reloading consolidometer experiments.

Sand permeability

The permeability, k = 1.2 E-4 m/sec, of Ottawa F65 sand with D_r = 45 %, was measured in a constant head test reported by El Ghoraiby et al. (2017). This value was used in the Dmod2000 Runs 45-1 and 45-6 reported herein, and is included in Table 4. Abdoun et al. (2020) studied the effect of effective confining pressure between 1 and 6 atm on the permeability of Ottawa F65 sand using a flexible wall permeability test in a triaxial cell, and concluded that the effect is negligible for practical purposes. This justifies using the same value of k for both 1 and 6 atm simulations, as shown in Table 4.

As discussed later herein, the authors found that this value of k = 1.2 E-4 m/sec, measured in the laboratory in regular permeability tests done at 1g, yields reasonable pore pressure response predictions for the two centrifuge tests when used without modification in Dmod2000. This is different from the findings of other researchers in the past including the authors, who reported that the permeability measured in the laboratory in permeability tests, had to be increased by a factor ranging from 4 to 25 in order to achieve successful simulations of centrifuge liquefaction tests (Ishihara 1993; Shahir et al. 2012; Dobry et al. 2018). Dobry et al. (2018) attributed this increased permeability during shaking and liquefaction to the lack of contact between particles due to the pore pressure generation. It may also be influenced by the opening of vertical drainage paths of much higher permeability by the upward flow of water.

The fact that no such permeability increase was necessary for the simulation herein of the centrifuge tests by the authors may be perhaps related to the following factors:

1. As mentioned before, from the viewpoint of the excess pore pressure buildup and dissipation calculated by Dmod2000, neither the permeability, k, nor the volumetric stiffness, M', matter independently, but only through their effect on the coefficient of consolidation c_v, which is proportional to the product k M' (Eq. 6). In the paper by Dobry et al. (2018), the total settlement during both shaking and dissipation was used to evaluate the value of M', instead of the more correct dissipation settlement only, as used now in Table 5 and Fig. 11. The use of a settlement value that was too large by Dobry et al. (2018), decreased the M', and thus required increasing the value of k to keep the product k M' and c_v at their correct levels.
2. No initial liquefaction (defined by a pore pressure ratio, $r_u = 1.0$), occurred in centrifuge Tests 45-1 and 45-6 discussed herein, with maximum values of $r_u \approx 0.8$ in both experiments (Table 3). This, together with the fact that in our tests the top of the sand layer <u>was not</u> the ground surface, may have minimized or eliminated the effects of loss of contact between particles and opening of vertical paths by piping. This is different from the previous centrifuge tests discussed above where k had to be increased, including VELACS and Dobry et al. (2018), where liquefaction did occur and the top of the sand layer coincided with the ground surface.

Numerical simulations and comparison with experimental results

Figures 4-7 show comparisons of time histories of several parameters, between those measured in the two centrifuge experiments and their corresponding Dmod2000 simulations. The figures include: accelerations, stress ratios, shear strains, and excess pore pressure ratios, r_u. The plots of pore pressure ratio in Fig. 7 include comparisons through both shaking and dissipation. Figures 4-7 show generally a very good match between time histories measured in the centrifuge, as previously reported by Ni et al. (2020), and those simulated by Dmod2000. Figures 8-9 present similar comparisons for the profiles of excess pore pressure and pore pressure ratio at the end of shaking. Table 3 lists the maximum pore pressure ratios, $(r_u)_{max}$, measured in Tests 45-1 and 45-6 as well as the corresponding r_u calculated at the same depth by the Dmod2000 runs. More details about these comparisons are discussed in the following subheadings.

Acceleration time histories

As shown in Fig. 4, there is a very good match between measured and computed acceleration histories, especially at the deeper elevations, which this being valid at both 1 and 6 atm. The computed accelerations start to deviate from the measured accelerations later in the shaking, especially at shallow elevations. This deviation may be related to the differences between measured and computed pore pressures during shaking at shallow elevations in Fig. 7.

Shear stress ratio and strain time histories

As shown in Figs. 5a and 6a, there is an excellent match between measured and computed shear stresses and strains at all elevations at 1 atm. It must be noted that the measured strains do not exceed about 0.1 to 0.15%, within the range of strains where the hyperbolic model used by Dmod2000 captures well the soil stress-strain response. The comparison at 6 atm is very good for shear stresses (Fig. 5b) and fair for shear strains (Fig. 6b). The reason for this deviation seems to be the higher pore pressure predicted by the numerical model compared to the experimental data (Fig. 7b).

<u>*Pore pressure time histories during and after shaking and pore pressure profiles at end of shaking*</u>

As shown in Fig. 7, there is a very good match between measured and computed pore pressure histories during the buildup phase, especially at deeper elevations, with the match deteriorating somewhat at shallower depths. Afterwards, during dissipation, the computed pore pressure is somewhat higher, dissipating more slowly than the measured excess pore pressure, especially at 1 atm. Figures 8 and 9, respectively, show the profiles of excess pore pressure and excess pore pressure ratio at the end of shaking. These profiles also exhibit a very good match, both in terms of values and trends, with $(r_u)_{max}$ occurring at the bottom in both measured and computed profiles.

Settlement after shaking

The vertical settlement time histories of the sand deposits in centrifuge Tests 45-1 and 45-6 were measured by vertical LVDTs placed at the surface of the sand layer, with the results reported by Ni et al. (2020). Figure 13 presents the time histories of the consolidation vertical volumetric strain of the sand layer after end of shaking, ε_{vc}, measured in Tests 45-1 and 45-6, growing from zero to their final values, $(\varepsilon_{vc})_{max}$. Each ε_{vc} was obtained by dividing the settlement measured by the LVDT at a given time after shaking on top of the sand layer, by the thickness of the layer.

Although the Dmod2000 code does not compute the settlement of the deposit, a procedure was adopted by the authors to predict consolidation settlement based on the pore pressure dissipation and the Theory of Consolidation (Dobry et al. 2018). For the purpose of this numerical simulation of the settlement, the 5m sand layer was divided into a number of sublayers. The contribution of each sublayer i to the total settlement of the layer, for a given small time period during dissipation is ΔH_i, given by the expression:

$$\Delta H_i = m_{vi}\, \Delta \boldsymbol{\sigma}'_i\, H_i \tag{11}$$

where m_{vi} is the compressibility coefficient of layer i during the time increment, H_i is the thickness of sublayer i, and $\Delta\boldsymbol{\sigma}'_i$ is the change in vertical effective stress at mid-depth of the sublayer i. $\Delta\boldsymbol{\sigma}'_i$ is equal (with opposite sign) to Δu, which is the excess pore pressure increment at the same depth, with Δu available in the output files of every Dmod2000 run for each sublayer i. By integrating (adding up) the values of ΔH_i for all sublayers within the sand layer, as well as over time during the dissipation phase, it was possible to extract the predicted settlement time histories shown in Fig. 13 for Runs 45-1 and 45-6. Additional details on the application of Eq. 11 can be found in Dobry et al. (2018).

These simulated time histories of vertical settlement strain after the end of shaking are compared in Fig. 13 with those measured in the centrifuge tests by the LVDTs. There is very good agreement between measured and calculated settlement time histories for Test 45-1 in Fig. 13a. However, there is a significant difference between measured and simulated records for Test 45-6, with Run 45-6 predicting 35% larger settlement (Fig. 13b). This is probably because the excess pore pressures at the beginning of dissipation predicted by the simulation are greater than for the experiment (Figs. 8b & 9b). It is not clear to the authors why there is more discrepancy between measured and computed pore pressure at 6 atm as compared to 1 atm.

Coefficient of consolidation (c_v) time history

The coefficient of consolidation, c_v, in the Theory of Consolidation was defined before in Eq. 6 in terms of the volumetric stiffness, M', with Eq. 9 providing the expression for M' fitted to the centrifuge data in Fig. 11 and used in the Dmod2000 runs.

Given the fact that k and γ_w in Eq. 6 are essentially constant with confining pressure, c_v is directly proportional to M'. Since $M'_{6\,atm}$ is significantly higher than $M'_{1\,atm}$ in the centrifuge tests (Table 5 and Fig. 11) - with this difference also reflected in the dependence of M' on σ'_{v0} in Eq. 9 used in the Dmod2000 simulations – it is expected that $c_{v6\,atm}$ should also be significantly higher than $c_{v1\,atm}$.

Equation 6 (c_v = k M' / γ_w), together with Eq. 9, were used in Runs 45-1 and 45-6 in conjunction with the calculated excess pore pressures, to compute the predicted time histories of c_v at different elevations within the sand layer. The results are shown by the curves in Fig. 14. The figure also includes the data points of the average experimental c_v for the whole layer during dissipation, backfigured by Abdoun et al. (2020) from centrifuge Tests 45-1 and 45-6. These c_v were calculated from the measured pore pressures using the basic consolidation formulation of Eq. 4 [$c_v = \partial u/\partial t \,/\, (\partial^2 u/\partial z^2)$]. Figure 14 demonstrates remarkable agreement between experimental and numerical simulations, enhancing the credibility of both sets of results.

Two trends in Fig. 14 deserve additional mention. The *first* is that for small excess pore pressures (at the beginning of shaking and end of dissipation), the numerical simulations indicate that $c_{v6\,atm}$ is almost three times greater than $c_{v1\,atm}$. This difference is a direct result of the much higher volumetric stiffness, M', in the sand at the higher pressure (Fig. 11), and explains both the greater partial drainage during shaking and faster dissipation at 6 atm, revealed by Figs. 7-9. This higher c_v at 6 atm is also present in the experimental data points of Fig. 14, as discussed by Abdoun et al. (2020). The *second* trend in Fig. 14 is the tendency for c_v to decrease during shaking as the excess pore pressures and r_u increase. This is a consequence of the reduction in M' as r_u increases, shown by Fig. 11 in which M' is the slope of the curves and also reflected in Eq. 10. That is, as the pore pressure builds up during shaking and approaches liquefaction, the sand becomes more compressible.

Undrained Numerical Simulations (Runs 45-1U and 45-6U)

The numerical simulations of Tests 45-1 and 45-6 were repeated using the same exact parameters of Runs 45-1 and 45-6, except for the drainage conditions, which were set to no-drainage. These were labeled Runs 45-1U and 45-6U (Table 2). Physically, it means that in these runs the permeability of the sand layer, $k \approx 0$. Therefore, as per Eq. 6, $c_v \approx 0$ also. The purpose of these two undrained runs was to compare the behavior of the actual partially drained deposit conditions to the idealized undrained conditions. That is, this allows to examine how "partially drained" the sand layers actually were during shaking in centrifuge Tests 45-1 and 45-6, as well as in their numerical counterparts, Runs 45-1 and 45-6. The excess pore pressure time histories and profiles calculated in these undrained simulations are included in Figs. 7-9, while the calculated values of r_u at the depths at which $(r_u)_{max}$ was measured in the centrifuge tests are listed in Table 3. The plots as well as Table 3 indicate that at the bottom of the sand layer, Runs 45-1U and 45-6U predict pore pressures slightly higher than, but very similar to those computed by Runs 45-1 and 45-6. This suggests that the pore pressure response up to the end of shaking in Runs 45-1 and 45-6 was close to undrained at the bottom of the layer, where the impervious boundary is located. As the elevation becomes shallower, the pore pressures in Runs 45-1 and 45-6 increasingly deviate from those of the undrained Runs 45-1U and 45-6U, demonstrating the significance of partial drainage in Runs 45-1 and 45-6, as well as in their corresponding experimental counterparts. It can be noted from Figs. 7-9 that this deviation from undrained is much more pronounced for Run 45-6 than for Run 45-1, indicating a more partially drained behavior at the higher overburden pressure. This confirms the previous conclusion that Test 45-6 was much more drained than Test 45-1, as also indicated by the higher c_v for Test 45-6 in Fig. 14.

Discussion of pore pressure response in Dmod2000 simulations

The discussion presented in this section focuses on the comparison between numerically predicted pore pressure response for the simulated field conditions (Runs 45-1 and 45-6), and assumed perfectly undrained conditions (Runs 45-1U and 45-6U). The discussion is also largely applicable to the experimental results of Tests 45-1 and 45-6, due to the good agreement already discussed between numerical and experimental excess pore pressure, presented in Table 3 and Figs. 7-9. Figure 8 includes the excess pore pressure profiles at the end of shaking, while Fig. 9 shows the corresponding pore pressure ratio profiles.

As shown in Fig. 8, the excess pore pressure profile for Run 45-6 has much greater hydraulic gradients than in the corresponding profile for Run 45-1, indicating that there is significantly higher upward water flow at σ'_{v0} = 6 atm than that at σ'_{v0} = 1 atm.

In Fig. 9, the excess pore pressure ratio profile for Run 45-1 is about constant, with $r_u = (r_u)_{max} \approx 0.7\text{-}0.8$ in the lower half of the sand layer (about 2.5 m), that is near liquefaction but not yet at liquefaction, which would correspond to $r_u = 1.0$. Run 45-1U shows a profile of r_u slightly higher than 0.8 in the bottom 2.5 m, showing that Run 45-1 is close to, but not completely undrained in the lower half of the sand layer. Figure 9 also indicates that in Run 45-6, which also reached

$(r_u)_{max} \approx 0.8$ at the bottom of the layer, the sand thickness having this high pore pressure ratio was much less than 1 m at the very bottom of the layer. This is, despite the fact that Run 45-6U showed $r_u \approx 0.8$ or more throughout the whole layer thickness, meaning that Run 45-6 corresponded approximately to undrained loading only for a fraction of 1 m at the bottom of the deposit.

In summary, in both Runs 45-1 and 45-6, the sand reached high pore pressure ratios close to, but slightly short of their undrained values at the bottom of the layer by end of shaking. However, in Run 45-6 the sand layer was draining and dissipating excess pore pressures significantly more than in Run 45-1 at higher elevations above the bottom. This resulted in a thickness of sand having a high pore pressure ratio ($r_u \approx 0.8$) at the bottom of the layer, that was significantly smaller in Run 45-6 than in Run 45-1. These conclusions are largely valid too for centrifuge Tests 45-1 and 45-6, also plotted in Figs. 8-9.

This difference in pore pressure responses between Run 45-6/Test 45-6 and Run 45-1/Test 45-1, seems to be explained by the higher c_v value at 6 atm compared to 1 atm, discussed earlier herein (Fig. 14). As discussed by Ni et al. (2020) and Abdoun et al. (2020), this higher c_v at 6 atm tended to make the sand layer deposit less prone to liquefaction in the 6 atm tests, resulting in the value of $K_\sigma > 1$ (Table 1), in a way that cannot be captured by cyclic undrained laboratory tests or undrained numerical simulations.

Overburden pressure factor, K_σ (Ni et al. 2020 and Abdoun et al. 2020)

As mentioned at the beginning of the paper, the current state of practice to evaluate liquefaction resistance at a high effective overburden pressure, $\sigma'_{v0} > 1$ atm, is based on the factor $K_\sigma = (CRR)_{\sigma'v0} / (CRR)_1$, where $(CRR)_{\sigma'v0}$ and $(CRR)_1$ are the cyclic resistance ratios, CRR, in the critical liquefiable layer under σ'_{v0} and 1 atm, respectively. This equation was developed assuming the same number of cycles to liquefaction failure ($r_u = 1.0$). The CRRs are typically obtained from cyclic undrained cyclic triaxial or simple shear tests. Using this approach invariably results in a laboratory undrained $K_\sigma < 1.0$ (Fig. 2).

Abdoun et al. (2020) tested different sand samples of Ottawa sand of $D_r = 45\%$ consolidated to isotropic consolidation pressures, σ'_c, of 1 atm and 6 atm in undrained cyclic triaxial stress-controlled tests. The excess pore pressure ratio, r_u, was recorded versus time and number of stress cycles. The tests resulted in $K_\sigma = 0.85$ for $\sigma'_c = 6$ atm and ten cycles to failure defined by $r_u = 1.0$. An almost identical $K_\sigma = 0.84$ was obtained when failure was defined by $r_u = 0.8$. Both values of this laboratory undrained K_σ are listed in Table 1.

A similar procedure was used to calculate the field K_σ from centrifuge experiments -where partial drainage may occur during shaking - by Ni et al. (2020) and Abdoun et al. (2020). That is, in the newly defined field K_σ, the restriction to purely undrained loading is removed. In the centrifuge tests, $r_u = (r_u)_{max} = 0.8$ after ten cycles of shaking rather than $r_u = 1.0$ was used as failure criteria for CRR, because of the difficulty of reaching $r_u = 1.0$ in exactly ten cycles of shaking in these experiments. This was justified and verified based on the K_σ from the undrained tests, which is rather insensitive to the exact value of r_u being 0.8 or 1.0 (Table 1).

For each of the centrifuge Tests 45-1 and 45-6, the median CSR value of the cycles before start of stress-strain degradation due to increased excess pore pressure, was used to calculate the field K_σ at $\sigma'_{v0} = 6$ atm. This field K_σ corresponds to $\sigma'_{v0} = 6$ atm, with number of cycles, N = 10 cycles, and $r_u = (r_u)_{max} = 0.8$, as this was the maximum value of r_u reached in the two centrifuge tests. Using this approach, Ni et al. (2020) and Abdoun et al. (2020) obtained a field $K_\sigma = 1.28$ for $\sigma'_c = 6$ atm at $r_u = 0.8$, as listed in Table 1.

By definition, centrifuge experiments are a better representation than undrained testing of the pore pressure response for the specific set of field conditions modeled by them. Therefore, the field $K_\sigma = 1.28$ of Table 1 also represents better the situation for the field conditions modeled in the centrifuge, than the laboratory undrained $K_\sigma = 0.84$ or 0.85. It is important to clarify the reason for the discrepancy. The discussion in previous sections already provided considerable insight into the reason for the difference: increased partial drainage, and hence deviation at high overburden pressure from the undrained loading condition assumed by the State of Practice (SoP). This increased partial drainage is in turn related to the increased volumetric stiffness and increased coefficient of consolidation at a high overburden pressure, compared with the situation for an overburden pressure of 1 atm (Table 5 and Fig. 11).

Andrus and Stokoe (2000) Field Liquefaction Charts

A main objective of this section is to link the State of Practice with the centrifuge and numerical results at 1 atm and 6 atm discussed in previous sections. The field liquefaction triggering chart developed by Andrus and Stokoe (2000) using the soil shear wave velocity, V_s, was selected for this purpose (Fig. 1b). This chart could be used because V_s was actually measured by bender elements in the sand layer in both centrifuge Tests 45-1 and 45-6. Also, previous work has shown that the Andrus and Stokoe (2000) chart does an excellent job in separating liquefaction and no liquefaction of recent uncompacted clean and silty sandy fills, both in the centrifuge and in the field during actual earthquakes (Dobry et al., 2013; Abdoun et al. 2019).

Figure 15 is an enlargement of a small part of the Andrus and Stokoe (2000) chart of Fig. 1b, with the data points of centrifuge Tests 45-1 and 45-6 and Dmod2000 Runs 45-1 and 45-6 plotted on the chart as four data points. Following usual convention, they are open data points, because in all cases there was no liquefaction ($r = (r_u)_{max} \approx 0.8 < 1.0$, see Table 3). Table 6 lists the values of V_{s1} and $(CSR)_{7.5}$ needed to plot these four data points. The V_{s1} for the sand layer were obtained from the bender element measurements in the two centrifuge tests before the shaking, as previously listed in Table 4. The values of cyclic stress ratios, CSR, for both the centrifuge and Dmod2000, were obtained as the median of the undegraded stress ratio history peaks, at the beginning of the four stress ratio time histories of Fig. 5. This follows the general approach implemented for centrifuge tests by Abdoun et al. (2013) and Dobry et al. (2013). All shakings had a duration of 10 cycles, corresponding approximately to an earthquake moment magnitude, M_w = 7. Therefore, in order to bring each CSR to a magnitude, $M_w = 7.5$, the Magnitude Scaling Factor, MSF, specified by Andrus and Stokoe 2000 was applied:

$$(CSR)_{7.5} = CSR/MSF \quad (14)$$

where $MSF = (M_w /7.5)^{-2.56}$. That is, for $M_w = 7.0$, $MSF = (7/7.5)^{-2.56} = 1.19$, and $(CSR)_{7.5}$ = CSR/1.19. The $(CSR)_{7.5}$ was further reduced by 10% to account for the 1D shaking in the centrifuge experiments, as compared to the 2D shaking in the field case histories used to generate liquefaction curves such as those in Figs. 1b and 15 (Seed, 1979; Dobry et al., 2013)

The $(CSR)_{7.5}$ and V_{s1} obtained this way in Table 6, were used to plot the data points of Fig. 15. It is important to note that <u>no K_σ correction factor</u> was applied to any of these data points of Fig. 15. The plot indicates that Test 45-1 and Run 45-1 plot very close to each other, as expected given the very good comparison between measured and simulated responses in Figs. 4-7. Test 45-6 and

Run 45-6 also plot reasonably close to each other, confirming the good comparison between measured and simulated responses in Figs. 4-7.

A *first* observation from Fig. 15 relates to the location with respect to the Andrus and Stokoe curve, of the two data points for Test 45-1/Run 45-1. All field case histories of liquefaction and no liquefaction from actual earthquakes originally used to calibrate the curve of Fig. 15, were associated with effective overburden pressures less than 2 atm, so the curve is directly applicable to Test 45-1/Run 45-1, corresponding to $\sigma'_{v0} = 1$ atm. The two data points for Test 45-1/Run 45-1, plot at some distance below the curve, correctly as they are associated with $r_u = 0.8$, compared with $r_u = 1.0$ on the curve. This comparison confirms again the validity of the Andrus and Stokoe chart for recent uncompacted fills in the field and centrifuge.

A *second* observation for Fig. 15 is that the two points for Test 45-6/Run 45-6 plot above the points for Test 45-1/Run 45-1, suggesting the need for a $K_\sigma > 1.0$ in order to lower these 6 atm data points to the same level of the 1 atm data points (since they have about the same r_u). This is consistent with $K_\sigma = 1.28 > 1.0$ obtained by Abdoun et al. (2020) and Ni et al. (2020), and listed in Table 1 for the 6 atm centrifuge simulation!

Finally, the locations of the data points in Fig. 15 relative to each other and to the curve, suggest that calibrated 1D numerical simulations - in conjunction with existing field liquefaction charts - have the potential to become useful practical tools in projects involving high overburden pressures. This calibration includes proper accounting of the decrease of sand compressibility as the overburden increases.

Summary and Conclusions

The paper presents numerical simulations of two centrifuge tests performed at high (σ'_{v0} = 6 atm) and low (σ'_{v0} = 1 atm) effective overburden pressures testing the effect of overburden pressure on the pore pressure buildup and liquefaction response of a sand layer due to base shaking. The 5m-thick layer had a free drainage boundary at the top. The simulations was performed using Dmod2000, a non-linear effective stress 1D site response analysis code. Hypothetical undrained simulations were also conducted at 1 and 6 atm to gain additional insight on the degree of partial drainage in the centrifuge tests. The simulations were Class C predictions, as the results of the centrifuge tests were used to backfigure some of the input soil parameters.

The main conclusions of the paper are:

- The soil response was partially drained rather than undrained, with much more partial drainage at 6 atm compared to 1 atm. The simulations correctly modeled this behavior, with very good agreement between simulated and centrifuge excess pore pressures. Very good accord was also found for other parameters such as soil accelerations, shear stress ratios and shear strains. The undrained simulations revealed that for both 1 and 6 atm, the pore pressures at the bottom of the layer in the centrifuge was close to undrained, with this response becoming much more partially drained at shallower elevations in the 6 atm compared with the 1 atm test.
- It was found that the key soil property, that needed to be modeled correctly to achieve this good agreement, was the 1D drained volumetric stiffness of the soil for unloading/reloading, $M' = 1/m_v$, as the coefficient of consolidation of the sand, c_v, is proportional to M'. Good agreement was found between M' results, backfigured from the centrifuge tests and those from a consolidometer test on a different sand reported by Martin et al. (1975).

- Both c_v and M' were 2.5 to 3 times greater at 6 atm than at 1 atm in centrifuge tests and simulations, with this consistency between experimental and simulation compressibility parameters paving the road towards explaining the success of the simulations. Future simulations of pore pressure response of sand under field drainage conditions should consider this large increase in volumetric stiffness at high overburden pressure.
- The value of M' for a given excess pore pressure ratio seems to be approximately proportional to $\sqrt{\sigma'_{v0}}$. This was substantiated for two sands by: (i) the centrifuge settlement measurements after shaking; and (ii) results of a consolidometer test including unloading by Martin et al. (1975).
- The proper K_σ at high σ'_{v0} to be used in field liquefaction design charts, may be greater than one for some realistic field drainage conditions, due to this lower compressibility and higher c_v at high overburden. As shown in this paper, one example of such field condition is a liquefiable soil layer underlain by impervious clay or bedrock, and overlain by a pervious non-liquefiable soil layer such as an engineered filter. In this respect, the Dmod2000 simulations confirmed the conclusion obtained directly from the centrifuge experiments by Ni et al. (2020) and Abdoun et al. (2020).
- Properly calibrated 1D numerical simulations, in conjunction with field liquefaction charts, have the potential to become useful practical tools in projects involving high overburden pressures.

Data Availability Statement

Some or all data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request.

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Table 1 Values of $K_{\sigma} = (CRR)_6 / (CRR)_1$ at 6 atm measured in cyclic triaxial and centrifuge tests on Ottawa sand of D_r = 40-45%, and values proposed in SoP charts (Ni et al. 2020)

<table>
<tr><th>Experimental Method</th><th>Pore Pressure Ratio, r_u, after 10 cycles</th><th>CRR_6/ CRR_1</th><th>K_{σ} from Experim-ents</th><th>SoP K_{σ} proposed by Youd et al. (2001), Fig. 2</th><th>SoP K_{σ} proposed by Idriss and Boulanger (2008), Fig. 2</th></tr>
<tr><td>Stress-controlled undrained cyclic triaxial tests at σ'_c = 1 and 6 atm</td><td>0.8
1.0</td><td>0.172/0.204
0.174/0.206</td><td>0.84
0.85</td><td rowspan="2">0.70</td><td rowspan="2">0.85</td></tr>
<tr><td>Centrifuge model tests at σ'_{v0} = 1 and 6 atm</td><td>0.8(*)</td><td>0.114/
0.089</td><td>1.28</td></tr>
</table>

(*) $r_u = (r_u)_{max} = 0.8$ measured at the bottom of the sand layer at the end of shaking in centrifuge Tests 45-1 and 45-6

Table 2 Simulations using nonlinear effective stress code Dmod2000

Run	σ'_{v0} (atm)	Excess pore pressure calculated?	Drainage allowed?	Soil stress-strain properties allowed to degrade as pore pressure builds up
45-1	1	yes	yes	yes
45-1U	1	yes	no	yes
45-6	6	yes	yes	yes
45-6U	6	yes	no	yes

Table 3 Measured maximum pore pressure ratios, $(r_u)_{max}$, in centrifuge Tests 45-1 and 45-6, and calculated r_u at similar depth and time in Dmod2000 runs [$r_u = (r_u)_{max}$ for Tests 45-1 and 45-6 from Ni et al., 2020]

Centrifuge experiment or Dmod2000 numerical simulation	σ'_{v0} (atm)	Depth, z (m)	r_u
Test 45-1	1	3.88	0.80
Run 45-1	1	3.96	0.80
Run 45-1 U	1	3.96	0.88
Test 45-6	6	4.88	0.76
Run 45-6	6	4.35	0.78
Run 45-6 U	6	4.35	0.82

Table 4. Summary of analytical models and input parameters used in numerical simulation: Run 45-1 and Run 45-1U as well as Run 45-6 and Run 45-6U

Parameters	Experiment	
	Run 45-1/ Run 45-1U	Run 45-6/ Run 45-6U
Average vertical stress at the middle of the layer	1 atm	6 atm
Average normalized shear wave velocity, V_{s1}	166 m/sec	167 m/sec
Rayleigh damping	0.5%	
Constitutive stress-strain model (initial, undegraded stress-strain backbone curve)	$\tau = \dfrac{G_{max} \cdot \gamma}{1 + \beta \, (\frac{G_{max}}{\tau_{mo}} \cdot \gamma)^s}$	
	$\beta = 2.55$, s = 0.6	$\beta = 4.5$, s = 0.7
Stress-strain degradation model due to increased pore pressure	$\tau = \dfrac{G_{mt} \cdot \gamma}{1 + \beta \, (\frac{G_{mt}}{\tau_{mt}} \cdot \gamma)^s}$	
	$G_{mt} = G_{max} * \sqrt{1 - r_u}$, $\tau_{mt} = \tau_{mo} * (1 - {r_u}^v)$	
	$v = 1$	$v = 10$
Unit weight	19.5 kN/m^3	19.5 kN/m^3
Pore pressure model under undrained cyclic loading	$r_u = \dfrac{P \, . f . N_c . F . (\gamma_c - \gamma_{tv})^{s_p}}{1 + f . N_c . F . (\gamma_c - \gamma_{tv})^{s_p}}$	
	$P = 1.3$, $f = 1$ $F = 2$ $\gamma_{tv} = 10E - 4 = 0.01\%$, $s_p = 1$	$P = 1.1$, $f = 1.5$ $F = 2.1$ $\gamma_{tv} = 10E - 4 = 0.01\%$, $s_p = 1.6$
Compressibility during and after shaking	$\bar{E}_r = M' = \dfrac{1}{m_v} = \dfrac{(\sigma_v')^{1-m}}{m \, k_2 \, (\sigma_{v0}')^{n-m}}$	
	$m = 0.4$, $k_2 = 0.006$ $n = 0.5$ (stresses in psf; as strains in in/in are assumed by Dmod2000, $k_2 =$ 0.00006 actually used. When strains are in %, $k_2 = 0.006$).	
Coefficient of permeability, k, during and after shaking	1.2 E -4 m/sec in Runs 45-1 & 45-6 Zero in Runs 45-1U & 45-6U	

Table 5 Volumetric stiffness during dissipation phase in centrifuge tests (Abdoun et al. 2020)

Test number	σ'_{v0} (kPa)	$(u)_{ave}$ (kPa)	$(r_u)_{ave} = (u)_{ave} / \sigma'_{v0}$	$(\varepsilon_{vc})_{max}$ (%)	Secant $M' = u_{ave} / (\varepsilon_{vc})_{max}$ (Mpa)	M'_6 / M'_1
Test 45–1	98.6	64.03	0.65	0.085	75.3	2.7
Test 45–6	600.4	223.5	0.37	0.110	203.2	

Table 6 Values of CSR and Vs1 used for data points in the field liquefaction chart of Fig. 15

Effective overburden pressure,	Test/Run	V_{s1} (m/s)	CSR[(1)]	$(CSR)_{7.5}$ = 0.756 CSR[(2)]
1 atm	Test 45-1	166	0.089	0.067
	Run 45-1	166	0.082	0.062
6 atm	Test 45-6	167	0.114	0.086
	Run 45-6	167	0.103	0.078

[(1)] CSR = τ_c / σ'_{v0} in the experiment, where σ'_{v0} is the effective overburden vertical pressure before shaking, and τ_c is the representative cyclic shear stress.

[(2)] $(CSR)_{7.5}$ = (0.9/1.19) CSR = 0.756 $(CSR)_{7.5}$, with $(CSR)_{7.5}$ including a 10% reduction in CSR due to the 2D character of the shaking in the field.