

LASERS

Supersymmetric laser arrays

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Scaling up the radiance of coupled laser arrays has been a long-standing challenge in photonics. In this study, we demonstrate that notions from supersymmetry—a theoretical framework developed in high-energy physics—can be strategically used in optics to address this problem. In this regard, a supersymmetric laser array is realized that is capable of emitting exclusively in its fundamental transverse mode in a stable manner. Our results not only pave the way toward devising new schemes for scaling up radiance in integrated lasers, but also, on a more fundamental level, could shed light on the intriguing synergy between non-Hermiticity and supersymmetry.

Symmetries play a fundamental role in physical sciences, ensuring energy and momentum conservation while dictating the allowable dynamical laws that govern our world. The Lorentz invariance embodied in Maxwell's equations was crucial in developing the theory of relativity, and the exchange symmetry allows one to classify fundamental particles as either bosons or fermions. In high-energy physics, other overarching symmetries, such as charge, parity, and time reversal symmetry and supersymmetry (SUSY), have also emerged as a means to unveil the laws of nature (1, 2). The SUSY theory, first proposed within the context of particle physics as an extension of the Poincaré space-time symmetry, makes an ambitious attempt to provide a unified description of all fundamental interactions. In general, SUSY relates bosonic and fermionic degrees of freedom in a cohesive fashion, directly implying that each type of boson has a supersymmetric counterpart, a superpartner fermion, and vice versa (3). Even though the full ramifications of SUSY in high-energy physics is still a matter of debate that awaits experimental validation, supersymmetric techniques have already found their way into low-energy physics, condensed matter physics, statistical mechanics, nonlinear dynamics, and soliton theory, as well as stochastic processes and Bardeen-Cooper-Schrieffer-type theories (4–9).

Shortly after the discovery of semiconductor lasers, it was proposed that integrated arrays of such emitters may provide a viable avenue in scaling up the radiance (power per unit area per unit solid angle) while avoiding complications arising from nonlinearities and filamentation in broad-area devices (10). Unfortunately, such arrays tend to support multiple spatial modes (supermodes), an undesirable behavior that in turn degrades the quality of the emitted beam. This behavior has since fueled activities in search of strategies that enable the generation

of high-power and diffraction-limited coherent beams by enforcing the coupled laser array to operate in the fundamental (in-phase) mode. In this regard, several schemes have been developed, such as those using resonant leaky-wave coupling in antiguided arrangements (11–16). Alternatively, one may consider broad-area laser arrangements for high-power applications in which spatiotemporal dynamics are known to play a prominent role in the emission characteristics (17, 18). The development of fully integrated global approaches that apply to any

type of active arrays to enforce single-mode lasing in the fundamental transverse supermode will be beneficial. Here we report the realization of a supersymmetric laser array. This lattice emits in its fundamental mode in a stable fashion, as evidenced from far-field and spectral measurements. In this SUSY arrangement, the main array is paired with a lossy superpartner, whose role is to suppress all undesired higher-order modes while simultaneously enhancing the gain seen by the fundamental supermode of the primary lattice. In implementing such lasers, we made use of the SUSY formalism first proposed by Witten (19).

Within the context of nonrelativistic quantum mechanics, supersymmetric isospectrality can be established, provided that the Hamiltonian of the system, $H^{(1)}$, is factorized in terms of two operators A and $A^\dagger$; i.e., $H^{(1)} = A^\dagger A$ (5). Similarly, a superpartner Hamiltonian $H^{(2)}$ can be constructed via $H^{(2)} = AA^\dagger$ by exchanging the action of these two operators. If one now assumes that $|\varphi\rangle^{(1)}$ represents an eigenstate of $H^{(1)}$ with an eigenvalue $\lambda^{(1)}$ —i.e., $H^{(1)}|\varphi\rangle^{(1)} = A^\dagger A|\varphi\rangle^{(1)} = \lambda^{(1)}|\varphi\rangle^{(1)}$ —then it follows that $AH^{(1)}|\varphi\rangle^{(1)} = (AA^\dagger)A|\varphi\rangle^{(1)} = H^{(2)}A|\varphi\rangle^{(1)} = \lambda^{(1)}A|\varphi\rangle^{(1)}$. Hence, $A|\varphi\rangle^{(1)}$ is an eigenvector of $H^{(2)}$ with an eigenvalue $\lambda^{(1)}$. This immediately indicates that the two Hamiltonians are isospectral because they exhibit identical eigenenergies, i.e., $\lambda^{(2)} = \lambda^{(1)}$, and

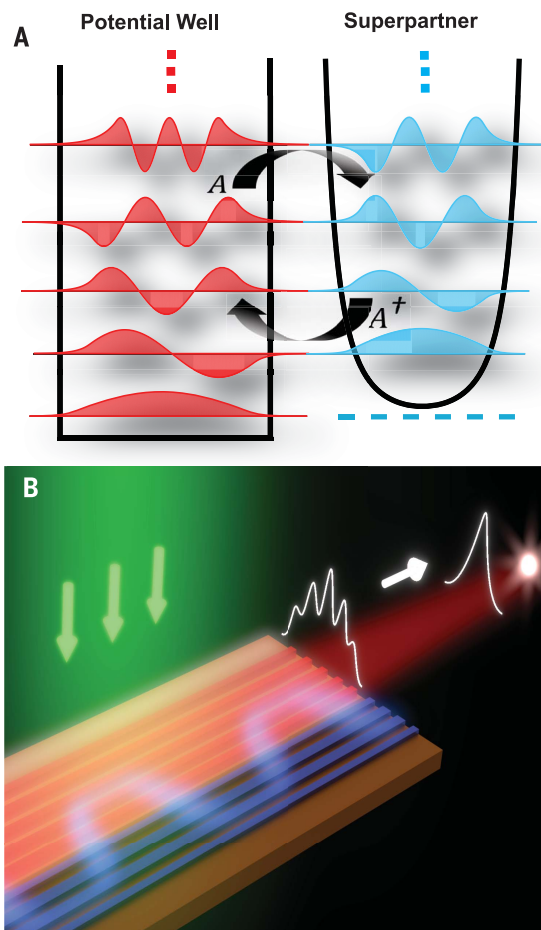


Fig. 1. Operation principle of SUSY laser array. (A) An infinite potential well and its superpartner in the unbroken SUSY regime. Apart from the ground state, all eigenvalues of the primary potential are exactly matched to those of the superpartner. The eigenfunctions of the primary potential and its supersymmetric counterpart are transformed into one another through the action of the operators A and $A^\dagger$. (B) Schematic representation of a SUSY laser array involving a primary active lattice (red) coupled to its lossy superpartner (blue). The SUSY laser emits exclusively in the fundamental in-phase mode.

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their eigenstates can be pairwise converted into one another through the action of the A and $A^\dagger$ operators: $|\varphi\rangle^{(2)} = A|\varphi\rangle^{(1)}$ and $|\varphi\rangle^{(1)} = A^\dagger|\varphi\rangle^{(2)}$. If the ground state of $H^{(1)}$ is annihilated by the action of the operator A , then the eigenenergy associated with the ground state of $H^{(1)}$ is zero, and therefore it will not have a corresponding state in $H^{(2)}$. In other words, all eigenvalues associated with the states of $H^{(1)}$ and $H^{(2)}$ are exactly matched except for the lowest-energy state of $H^{(1)}$. When this is the case, the SUSY is referred to as unbroken. Otherwise, if the ground state of $H^{(1)}$ has a counterpart in its superpartner $H^{(2)}$, with the same eigenvalue, the supersymmetry is broken. Using this approach in one-dimensional Schrödinger problems, one can always identify two SUSY potential

functions, $V^{(1)}(x)$ and the superpartner $V^{(2)}(x)$, that can be entirely isospectral except for the lowest-energy state of $V^{(1)}(x)$ (Fig. 1A) (5). In optics, SUSY can be introduced by exploiting the mathematical isomorphism between the Schrödinger and the optical wave equation (20). In this setting, the optical refractive index profile plays the role of the potential $V(x)$, which in the context of supersymmetry can be used for mode conversion (21, 22), transformation optics (23), design of Bragg gratings (24), and Bloch-like waves in random-walk potentials (25–28). However, the implications of SUSY isospectrality in active platforms, as well as its interplay with nonlinearity and non-Hermiticity, have so far remained unexplored. Our work lays the groundwork for such studies by demonstrating a SUSY-based laser.

In the schematic of the proposed supersymmetric laser (Fig. 1B), the primary array in this SUSY arrangement is synthesized by coupling five identical ridge-waveguide cavities of length L . The individual waveguide elements are designed to support only the lowest-order transverse mode TE_0 . Consequently, each element on its own is expected to support resonances at angular frequencies $\omega_m = m\pi c/(Ln)$, where n represents the effective index associated with the TE_0 mode, c is the speed of light in vacuum, and m is an integer representing the longitudinal mode index. The evanescent coupling between the five waveguides causes every such resonant

frequency ω_m to split into a cluster of five frequencies, corresponding to the five supermodes of the active array. Optical supersymmetric strategies are then employed to build a superpartner index profile with propagation eigenvalues that match those of the four higher-order supermodes associated with the main (primary) array (29).

The SUSY laser arrays were realized on an InP wafer with InGaAsP quantum wells as the gain material. We used electron-beam lithography and plasma etching techniques to define the structures (Fig. 2A) (30). Finite element method simulations were performed to determine the modal content of these structures. Details on the design and simulation of these arrangements can be found in the supplementary materials (30). Figure 2B depicts the intensity profiles of all modes supported by this SUSY configuration. The performance of the SUSY laser was then assessed by means of a custom-made optical setup (30). The arrays were optically pumped at a wavelength of 1064 nm, emitted from a fiber laser. Spatial masks were used to selectively pump different regions. The coherent radiation (centered on 1450 nm) emerging from the cleaved facets of the lasers was monitored by both a spectrometer and an infrared camera, after remnants of the pump emission were blocked by means of a notch filter. The diffraction angles associated with the far-field emissions along the laser's slow axis were determined by raster scanning a rectangular

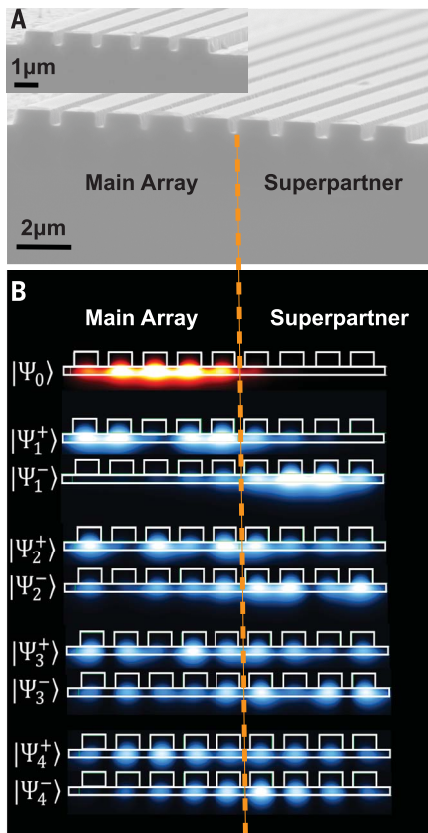


Fig. 2. Scanning electron microscope (SEM) image and modal field profiles of the SUSY laser array.

(A) SEM image of a fabricated SUSY lattice composed of a five-element primary array, positioned in close proximity (400 nm) to a four-element superpartner. The inset shows a stand-alone five-element laser array. (B) Intensity distributions associated with the eigenmodes supported by the SUSY arrangement, as obtained from numerical simulations. The fundamental mode of the five-element laser is only confined in the main array, whereas all higher-order modes are coupled to the lossy superpartner. The dashed line illustrates the boundary between the main array and the superpartner structures.

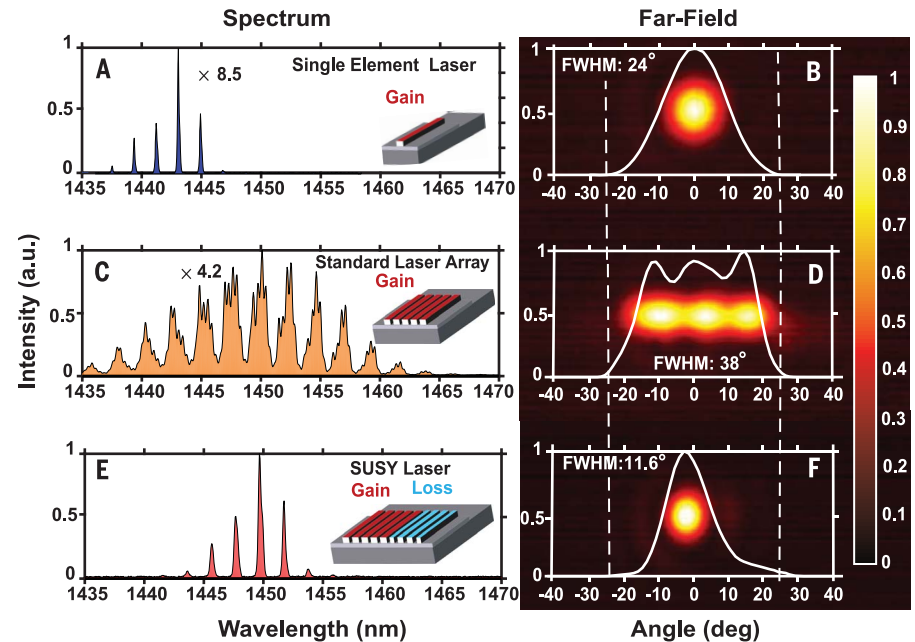


Fig. 3. Spectral and far-field characteristics of the SUSY laser array. (A, C, and E) Emission spectrum of (A) a single laser cavity, (C) a standard five-element laser array, and (E) a corresponding SUSY laser arrangement. The vertical axes are normalized to the spectrum of the SUSY laser. a.u., arbitrary units. Each longitudinal resonant frequency in the spectrum of the standard array splits into five lines corresponding to the five transverse supermodes. In contrast, the spectrum of the SUSY array is free from such undesired resonances, indicating that all higher-order modes are suppressed. (B, D, and F) Far-field diffraction patterns from the corresponding lasers. The measured diffraction angle associated with the SUSY laser (~5.8°) is smaller than that of the standard laser array (~19°) and a single waveguide laser (~12°). FWHM, full width at half maximum.

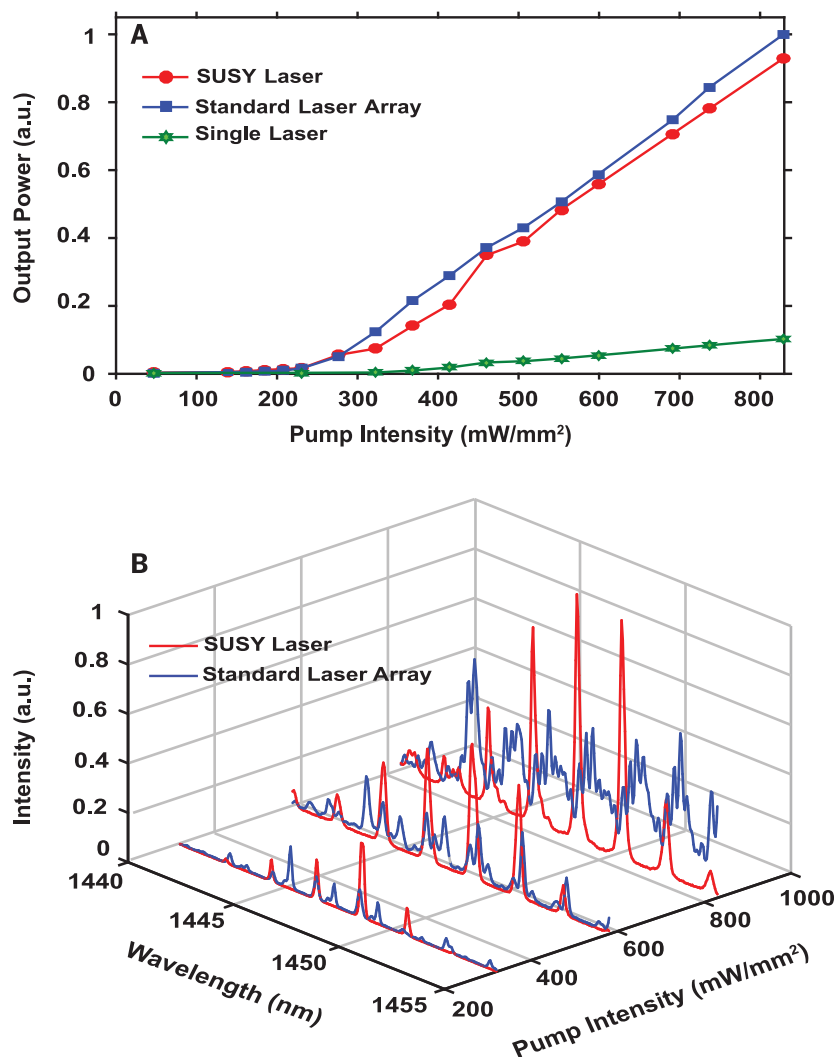


Fig. 4. Emission characteristics. (A) Light-light curves corresponding to a single laser element (green line), a five-element laser array (blue line), and the SUSY laser arrangement (red line). The output power and slope efficiency of the standard array and the SUSY laser are comparable and exceed that from a single cavity laser. (B) Comparison of spectral evolution behaviors of the standard and SUSY lasers. The standard laser array (blue line) is highly multimoded even at pump levels slightly above threshold, whereas the SUSY laser array (red line) remains transversely single-moded. Logarithmic scale plots are provided in the supplementary materials (30).

aperture placed in front of the array. At every step, the total emitted power from the laser was also measured by a photodiode.

The spectral response, far-field emission, and light-light characteristics are compared for three different configurations: (i) a single-ridge waveguide lasing element, (ii) a standard laser array involving five evanescently coupled ridge cavities, and (iii) a SUSY laser array composed of a primary active five-element lattice and its corresponding four-element lossy superpartner. In the latter configuration, the two lattices are fabricated in close proximity to each other and are therefore coupled. The system is designed so that SUSY is unbroken, which was achieved by appropriately varying the widths and spacings (equivalently, the corresponding effective refractive indices) of the ridge elements in the super-

partner array (Fig. 3, A, C, and E). The lasers are uniformly pumped at an average power density level approximately four times the threshold of the SUSY laser. Loss is introduced in the superpartner array by blocking the pump beam using a knife edge. Under these pumping conditions, the single-element cavity lases in a few longitudinal modes (in the TE_0 mode) at wavelengths around 1443 nm (Fig. 3A). When the five-element standard laser array is exposed to the same pump power density, each longitudinal mode is found to split into five lines corresponding to the resonances of the five supermodes involved (Fig. 3C). This multimode operation can lead to a substantial deterioration of the beam quality emitted by such a lattice. In contrast, when the SUSY laser array is illuminated at the same pump intensity level (while the super-

partner is blocked), the device emits in a single transverse supermode (Fig. 3E). Moreover, the peak intensity produced by this SUSY laser is now 4.2 times as high as that from the standard laser array (i.e., without superpartner) and 8.5 times as large as that from the single-element laser. These results clearly indicate that in a SUSY laser arrangement, all higher-order transverse modes are indeed suppressed in favor of the fundamental mode.

To further verify the anticipated SUSY response, we collected the far-field radiation from these three laser systems along with the diffraction profiles in the direction of the slow axis (parallel to the wafer). These measurements are correspondingly displayed in Fig. 3, B, D, and F. A comparison between these three radiation patterns reveals a pronounced difference in the way a SUSY laser operates. As opposed to the standard laser array, whose far field exhibits a multilobe profile with a diffraction angle of $\sim 19^\circ$ (Fig. 3D), the far field of the SUSY array displays a single bright spot, having instead a much smaller divergence angle of $\sim 5.8^\circ$ (Fig. 3F). This low-divergence behavior is a characteristic attribute of a laser array operating only in its in-phase lowest-order mode (31). The slight asymmetry observed in the diffraction pattern (Fig. 3F) is attributed to the optical attraction exerted by the partner array. In addition, in the standard array system, we observed a multilobe far-field pattern that changes with pump intensity (30). Even more importantly, the beam spot size associated with the SUSY laser is narrower than that of a single laser element ($\sim 12^\circ$), as shown in Fig. 3B, indicating a higher brightness associated with the SUSY arrangement. These experimentally obtained diffraction patterns are in good agreement with numerical simulations (30).

Finally, in the light-light curves corresponding to these three lasers and the evolution of their spectra, both SUSY and standard laser arrays outperform the single-element laser in terms of output power (Fig. 4A). When the overall output power is compared, the two arrays (standard and SUSY) are found to exhibit similar thresholds and slope efficiencies. On the other hand, Fig. 4B provides valuable information as to the lasing onset for higher-order supermodes. As the pump power is gradually increased above the threshold, the higher-order modes of the standard laser array start to successively emerge in the spectrum (blue lines in Fig. 4B), whereas the SUSY array still lases in its fundamental transverse mode with larger spectral peaks (red lines in Fig. 4B). These observations confirm that in a SUSY laser, all undesired higher-order modes are effectively eliminated via coupling to the lossy superpartner, giving the fundamental mode the opportunity to prevail. Our results also show that the loss of the superpartner array has no effect on the efficiency of the SUSY laser.

The output power as well as the radiance emerging from a single-mode SUSY laser array are expected to scale up with the number of elements involved. Nevertheless, to achieve high power levels in the individual elements, one

must consider the role of nonlinearities that can lead to spatiotemporal effects. The resilience of these arrays to fabrication imperfections and errors is discussed in the supplementary materials (30), in which a sensitivity analysis clearly showing that the SUSY transformations are robust against first-order perturbations is described. This feature, to some degree, relaxes the requirements for phase-locking in the presence of fabrication and environmental errors, as well as variations of index due to nonlinearities or temperature because of intensity inhomogeneities.

By harnessing notions from supersymmetry, we provided the first realization of an integrated supersymmetric laser array. Our results indicate that the existence of an unbroken SUSY phase in conjunction with a judicious pumping of the laser array can promote the in-phase supermode, thus producing a high-radiance emission. This mechanism of phase-locking is resilient to first-order deviations in fabrication and provides a global approach that can be systematically applied to a wide range of coupled active lattices. Our results may have practical implications for designing high-brightness single-mode laser arrays from ultraviolet to mid-infrared sources while introducing a platform to study, at a fundamental level, the interplay between non-Hermiticity and supersymmetry.

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SUPPLEMENTARY MATERIALS

www.sciencemag.org/content/363/6427/623/suppl/DC1
Materials and Methods
Figs. S1 to S8
References

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Stability through symmetry

A common route to get more light out of a laser system is to couple multiple lasers to form an array. However, instabilities owing to cross-talk and interference between different modes of individual cavities is generally detrimental to performance and could ultimately be damaging to the laser cavities. Hokmabadi *et al.* applied notions derived from supersymmetry, a theory developed in high-energy physics to describe the make-up and properties of particles, to design a stable array of semiconductor lasers (see the Perspective by Kottos). Based on symmetry arguments, the method is scalable and could provide a practical platform to design and develop complex photonic systems.

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