Fully Distributed Finite-Time Consensus of Directed Multiquadcopter Systems via Pinning Control

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***Abstract*—By using the terminal sliding-mode control (TSMC) and the pinning control methods, the fully distributed finite-time consensus problems are investigated for second-order multiagent systems (MASs) and multiquadcopter systems (MQSs) with directed topology. For the second-order MASs, a pinning control scheme is designed by analyzing the outdegree and indegree of nodes, and a TSMC protocol with the local information is proposed to achieve the finite-time consensus. Then, as an application of the MASs, the model of MQSs is constructed and its finite-time attitude consensus is discussed. Finally, the effectiveness of the proposed method is validated by two numerical examples.**

***Index Terms*—Distributed control, finite-time consensus, multiquadcopter systems (MQSs), pinning control, terminal sliding-mode control (TSMC).**

I. INTRODUCTION

IN THE past two decades, the consensus problem for the multiagent systems (MASs) has attracted tremendous attention due to its wide applications in various fields [1]–[3]. The key point of the consensus problem is to design a distributed consensus protocol, where all agents come to the same value only with the local information [4]–[6]. Research results for the distributed consensus problem are focused on convergence rate, pinning strategy, topology, etc. [7]–[12].

The most common results for the MASs are the asymptotic consensus, where the consensus can be achieved asymptotically in infinite time [13]–[17]. Due to the fast convergence rate and the strong robust ability, finite-time consensus results are more applicable. For the first-order MASs, many kinds of finite-time protocols are proposed [18]–[22]. By employing the theory of finite-time stability, if the interaction topology is connected and sufficiently large, the proposed protocols will solve the finite-time consensus problems for both the bidirectional interaction case and the unidirectional interaction case [19]. For the MASs with the continuous-time and discrete-time subsystems, the finite-time consensus can be achieved with the switching control method [20]. The binary consensus protocol is studied to obtain the finite-time consensus result [21]. The distributed robust fixed-time consensus result is presented, where the convergence time does not rely on the initial conditions [22]. For the second-order MASs, there are three typical methods to achieve the finite-time consensus [23]–[30]. The first method to solve the consensus problem is the homogeneous method, where the systems need to satisfy the homogeneous conditions [23]. Combined the homogeneous method with the sliding-mode control method, several protocols to achieve robust finite-time consensus are developed [24], [25]. The second method is the terminal sliding-mode control (TSMC) method, where each controller needs the neighbors' control information [26]. The third method is the Lyapunov method, where both the leaderless and leader–follower MASs with external disturbances are considered [27]–[30]. For the high-order MASs, by employing the Lyapunov method, the finite-time consensus is achieved for the leaderless and leader–follower structure [31]–[35]. The finite-time output consensus is achieved for higher-order MASs, and the active anti-disturbance control method is given to solve mismatched disturbances [32]. Actually, it is hard to design a fully distributed finite-time consensus controller for the undirected MASs.

The undirected network can be seen as a special directed network, research on the directed network is meaningful. With the nonlinear dynamics and directed network, the local and global asymptotic consensus protocols are studied for second-order MASs [36]. The finite-time containment control with multiple directed network dynamic leaders is investigated, where the bounded disturbances and unknown inputs are considered in [37]. The finite-time consensus protocol for directed second-order MASs is considered, and a new sliding-mode method is constructed. However, the controllers need to know the whole network's information [38]. Therefore, a simple distributed finite-time consensus protocol is needed for the directed second-order MASs.

Besides, it is hard to reach consensus under some fixed network topological structures [39]. An effective way to solve this problem is to impose additional controllers on some nodes. It is unrealistic to add additional controllers on all

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nodes [14], [39]–[49]. The pinning control is presented, where only a small fraction of nodes should be pinned [39], [40]. For the pinning control of MASs, it is better to pin the most highly connected nodes [40]. The pinned candidates are discussed in [39]–[41], where a small fraction of nodes is chosen to be pinned for different kinds of networks [41]–[44]. For the directed networks, the nodes whose outdegree are bigger than their indegree should be chosen as pinned candidates [41]. When the coupling strength is small, the nodes with low degree should be pinned first [43]. The auxiliary-system approach via pinning control is investigated for the two-layer complex networks, where different pinning strategies are listed [44]. By only pinning one node, the consensus can be achieved [45]. Aperiodically intermittent pinning controllers with logarithmic quantization are designed [46]. This article will point out which nodes should be pinned, and how large the pinning strength should be chosen.

Due to the ability to finish many complex tasks, low cost, and ease of operation, many studies have focused on quadcopter [50]–[52]. The quadcopter has six degrees of freedom, three degrees about the positions and three degrees about the attitudes. The quadcopter is a complex nonlinear dynamics system, which is hard to apply the advance control method. The most common method is the proportional–integral–derivative (PID) control method [53]. We have designed a new kind of quadcopter, which only has three degrees of attitudes. The details of the new quadcopter can be seen in Section IV.

The main contributions of this article can be summarized as follows.

1) A new fully distributed finite-time consensus method is proposed for the directed second-order MASs with disturbances.
2) The pinning strategy is introduced to make the whole system come to a consensus, where the nodes whose outdegree are no less than the indegree should be pinned, and the least pinning strength is selected.
3) The mathematical model of multiquadcopter systems (MQSs) with three degrees attitude is proposed, and the finite-time consensus of attitude MQSs can be achieved with the method devised in this article.

We have organized the remainder of this article as follows. Section II gives the preliminaries and problem statement. Section III proposes a distributed finite-time consensus tracking algorithm for directed MASs. The proposed algorithm has been used in the attitude consensus of MQSs in Section IV. Two examples are given to validate our results in Section V. Section VI gives the conclusion.

II. Preliminaries and Problem Statement

In this section, first, for directed networks, algebraic graph theory is introduced. Then, we have introduced some important lemmas, which will be used in the following section. Finally, we have pointed out the problem statement.

A. Preliminaries

Suppose the graph of the directed MASs with n agents is $\mathscr{G} = \{\mathscr{V}, \mathscr{E}, \mathscr{A}\}$, where the set of agents is $\mathscr{V} = \{v_1, \ldots, v_n\}$, v_i is the ith agent, the set of edges is $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$, $(v_i, v_j) \in \mathscr{E}$ is the directed edge from agent j to i, the agent i can get information from agent j, and the adjacency matrix is $\mathscr{A} = [a_{ij}] \in R^{n\times n}$. Define $a_{ii} = 0$, where $i \in \{1, \ldots, n\}$. If and only if there exists a directed edge (v_i, v_j) in $\mathscr{G}$, then $a_{ij} > 0$; otherwise, $a_{ij} = 0$ $(i \neq j)$. For the directed graph $\mathscr{G}$, define the Laplacian matrix as $L = [l_{ij}] \in R^{n\times n}$, where $l_{ii} = \sum_{j=1}^{n} a_{ij}$ and $l_{ij} = -a_{ij}$.

Define b_i as the connection strength between the leader and the ith agent. If there is connection, $b_i > 0$; otherwise, $b_i = 0$. Let $\mathbf{B} = \text{diag}[b_1, \ldots, b_n] \in R^{n\times n}$, $C = [\, C_1{}^T \ \ \cdots \ \ C_n{}^T \,]^T$, and $C = cL + \mathbf{B}$.

Lemma 1 [54]: If $z_1 \in R$ and $z_2 \in R$, a and b are positive, then $|z_1|^a|z_2|^b \leq (a/[a+b])|z_1|^{a+b} + (b/[a+b])|z_2|^{a+b}$.

Lemma 2: Define $\Delta_i \triangleq (cm_2/[m_1+m_2])\sum_{j=1}^{n} a_{ij} - (cm_2/[m_1+m_2])\sum_{j=1}^{n} a_{ji} + b_i$, $\Delta \triangleq \text{diag}\{\Delta_1, \ldots, \Delta_n\}$. Then, $E^T C E^{(m_2/m_1)} \geq (E^{([m_1+m_2]/2m_1)})^T \Delta E^{([m_1+m_2]/2m_1)}$, where m_1 and m_2 are positive odd number. Furthermore, $E^T C \text{sign}(E) \geq (|E|^{(1/2)})^T \Delta |E|^{(1/2)} > 0$.

Proof: The proof here is simple, with the help of Lemma 1, we will show the main derivation process in the following:

$$
\begin{aligned}
&E^T C E^{\frac{m_2}{m_1}} \\
&= c\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}\left(e_i^{\frac{m_1+m_2}{m_1}} - e_j^{\frac{m_2}{m_1}} e_i\right) + \sum_{i=1}^{n} b_i e_i^{\frac{m_1+m_2}{m_1}} \\
&\geq c\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}\left(e_i^{\frac{m_1+m_2}{m_1}} - \frac{m_2}{m_1+m_2} e_j^{\frac{m_1+m_2}{m_1}}\right. \\
&\qquad\qquad \left. - \frac{m_1}{m_1+m_2} e_i^{\frac{m_1+m_2}{m_1}}\right) + \sum_{i=1}^{n} b_i e_i^{\frac{m_1+m_2}{m_1}} \\
&= c\frac{m_2}{m_1+m_2}\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} e_i^{\frac{m_1+m_2}{m_1}} + \sum_{i=1}^{n} b_i e_i^{\frac{m_1+m_2}{m_1}} \\
&\quad - c\frac{m_2}{m_1+m_2}\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} e_j^{\frac{m_1+m_2}{m_1}} \\
&= c\frac{m_2}{m_1+m_2}\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} e_i^{\frac{m_1+m_2}{m_1}} + \sum_{i=1}^{n} b_i e_i^{\frac{m_1+m_2}{m_1}} \\
&= \sum_{i=1}^{n}\left(c\frac{m_2}{m_1+m_2}\left(\sum_{j=1}^{n} a_{ij} - \sum_{j=1}^{n} a_{ji}\right) + b_i\right) e_i^{\frac{m_1+m_2}{m_1}} \\
&= \sum_{i=1}^{n} \Delta_i e_i^{\frac{m_1+m_2}{m_1}} \\
&= \left(E^{\frac{m_1+m_2}{2m_1}}\right)^T \Delta E^{\frac{m_1+m_2}{2m_1}}.
\end{aligned}
$$

From this lemma, we know if Δ is positive, $E^T C E^{(q/p)} \geq (E^{([p+q]/2p)})^T \Delta E^{([p+q]/2p)} > 0$. We can get the following result, $E^T C \text{sign}(E) \geq (E^{(1/2)})^T \Delta E^{(1/2)} > 0$, due to the proof is similar, we omitted it here. ■

Lemma 3 [55]: Suppose $x \in R^n$, $\dot{x} = g(x)$, $g(0) = 0$, $\delta \in (0, 1)$, and $\alpha > 0$, $V(x)$ is a continuous positive-definite function. If $\forall x$, there exists an open neighborhood of the origin, such that $\dot{V}(x) + \alpha(V(x))^{\delta} \leq 0$. Then, $V(x)$ reach to the origin in finite time. The setting time is no more than $(V(x(0))^{1-\delta}/[\alpha(1-\delta)])$.

In this article, we have omitted the independent variables. Suppose there is a vector $E \triangleq [e_1, \ldots, e_n]^T$, where e_i, $i = 1, \ldots, n$ are scalars, $\mathbf{1} \triangleq [1, \ldots, 1]^T$, $\mathbf{0} \triangleq [0, \ldots, 0]^T$, the sign function of e_i is $\text{sgn}(e_i)$, $\text{sig}(e_i)^\alpha = \text{sgn}(e_i)|e_i|^\alpha$, $\text{sig}^\alpha(E) \triangleq [\text{sgn}(e_1)^\alpha, \ldots, \text{sgn}(e_n)^\alpha]^T$, $E^\alpha = [e_1^\alpha \ \cdots \ e_n^\alpha]^T$, $\dot{E}^\alpha = [\dot{e}_1^\alpha \ \cdots \ \dot{e}_n^\alpha]^T$, and $\text{diag}(E^{\alpha-1}) = \text{diag}(e_1^{\alpha-1} \ \cdots \ e_n^{\alpha-1})$.

B. Problem Statement

Suppose there are n second-order agents. Each agent can be described as follows:

$$\begin{cases} \dot{x}_i = v_i \\ \dot{v}_i = u_i + f_i \end{cases} \tag{1}$$

where f_i is the known dynamic of the system, and u_i is the control input, $i = 1, \ldots, n$.

Suppose there is only one virtual leader, and the virtual leader is an isolated agent. The model of virtual leader is

$$\begin{cases} \dot{x}_0 = v_0 \\ \dot{v}_0 = g_0 \end{cases} \tag{2}$$

where x_0 and v_0 are the virtual leader's states, and g_0 is the unknown dynamic of the virtual leader's system.

The purpose of this article is to make all agents converge to x_0 and v_0 in finite time with distributed consensus protocol.

Suppose the coupling strength is c. Choose the following error functions:

$$\begin{aligned} e_{xi} &= c\sum_{j=1}^{n} a_{ij}(x_i - x_j) + b_i(x_i - x_0) = C_i(x - \mathbf{1}x_0) \\ e_{vi} &= c\sum_{j=1}^{n} a_{ij}(v_i - v_j) + b_i(v_i - v_0) = C_i(v - \mathbf{1}v_0). \end{aligned} \tag{3}$$

Let $E_x \triangleq [e_{x1}, \ldots, e_{xn}]^T$, $E_v \triangleq [e_{v1}, \ldots, e_{vn}]^T$, $x \triangleq [x_1, \ldots, x_n]^T$, $v \triangleq [v_1, \ldots, v_n]^T$, $\mathbf{B} \triangleq \text{diag}(b_1, \ldots, b_n)$, $f \triangleq [f_1, \ldots, f_n]^T$, and $u \triangleq [u_1, \ldots, u_n]^T$.

If $E_x = E_v = \mathbf{0}$, which means all $e_{xi} = e_{vi} = 0$, then $x_i = x_j = x_0$ and $v_i = v_j = v_0$, $i = 1, \ldots, n$. So, appropriate distributed controller is found such that E_x and E_v come to zero in finite time.

Then, we have

$$\begin{aligned} E_x &= (cL + \mathbf{B})x - \mathbf{B}\mathbf{1}x_0 = (cL + \mathbf{B})(x - \mathbf{1}x_0) \\ E_v &= (cL + \mathbf{B})v - \mathbf{B}\mathbf{1}v_0 = (cL + \mathbf{B})(v - \mathbf{1}v_0). \end{aligned} \tag{4}$$

So

$$\begin{aligned} \dot{E}_x &= E_v \\ \dot{E}_v &= (cL + \mathbf{B})(u + f - \mathbf{1}g_0). \end{aligned} \tag{5}$$

Here is the assumption and the definition.

Assumption 1: Suppose there is a positive constant l_g, the unknown dynamic of the virtual leader's system g_0 satisfied the following condition, $|g_0| \le l_g$. Assume l_g is known to all nodes.

Definition 1: For any initial conditions, $\lim_{t\to\infty} x_i = x_j = x_0$ and $\lim_{t\to\infty} v_i = v_j = v_0$, $i, j \in \{1, \ldots, n\}$. The MASs (1) and (2) are said to achieve asymptotic consensus.

Definition 2: For any initial conditions x_{i0} and v_{i0}, if there is a constant $T_0 = T_0(x_0, v_0) > 0$, $\lim_{t\to T_0} x_i = x_j = x_0$, and $\lim_{t\to T_0} v_i = v_j = v_0$, and for all $t \ge T_0$, $x_i = x_j = x_0$, and $v_i = v_j = v_0$. The MASs (1) and (2) are said to achieve finite-time consensus.

III. Fully Distributed Finite-Time Consensus of Directed MASs

In this section, the pinning control method is investigated to ensure the finite-time consensus for the directed MASs.

Define the following functions as $\Phi = [\phi_1 \ \cdots \ \phi_n]^T$, $M = [\mu_1 \ \cdots \ \mu_n]^T$, $\mu_i = -\beta_1(q_1/p_1)e_{xi}^{q_1/p_1-1}e_{vi}$, $\phi_i = \text{sat}(\mu_i, u_s) = \begin{cases} u_s\text{sgn}(\mu_i) & \text{if}|\mu_i| > u_s \\ \mu_i & \text{otherwise} \end{cases}$, $i = 1, \ldots, n$, $u_s > 0$ is the threshold value of the saturation function.

Theorem 1: Under the condition of Assumption 1. The directed MASs (1) and (2) can achieve finite-time consensus under the following protocol:

$$\mathbf{u} = -f + \Phi - \beta_2 S^{q_2/p_2} - \alpha l_g \text{sgn}(S) \tag{6}$$

where $i = 1, \ldots, n$, $\Delta_i \triangleq (cq_1/[q_1 + p_1])\sum_{j=1}^n a_{ij} - (cq_1/[q_1 + p_1])\sum_{j=1}^n a_{ji} + b_i > 0$, $\alpha > 1$, $\alpha\Delta_i \ge b_i$, $\Delta_0 \triangleq \min(\Delta_i) > 0$, $\Delta \triangleq \text{diag}\{\Delta_1, \ldots, \Delta_n\}$ is positive matrix, l_g is the upper bound of g_0, I is identity matrix, $\beta_1 > 0$ and $\beta_2 > 0$ are constants, $p_1 > 0$, $p_2 > 0$, $q_1 > 0$, and $q_2 > 0$ are odd integers, $q_1 < p_1 < 2q_1$, and $p_2 > q_2$.

Proof: The terminal sliding surface can be selected as $S = E_v + \beta_1 CE_x^{q_1/p_1}$ and $S_i = e_{vi} + \beta_1 C_i E_x^{q_1/p_1}$.

From above, only when all $e_{xi} = e_{vi} = 0$, $S = E_v + \beta_1 CE_x^{q_1/p_1} = \mathbf{0}$, $i = 1, \ldots, n$. If $S = \mathbf{0}$, $\dot{E}_x = E_v = -\beta_1 CE_x^{q_1/p_1}$.

Choose the positive Lyapunov-candidate-function as $V_0 = (1/2)E_x^T E_x$, and then we have, $\dot{V}_0 = E_x^T \dot{E}_x = -\beta_1 E_x^T CE_x^{q_1/p_1}$.

By using Lemma 2, we can get

$$\dot{V}_0 \le -\beta_1 \left(E_x^{\frac{p_1+q_1}{2p_1}}\right)^T \Delta E_x^{\frac{p_1+q_1}{2p_1}} \le -\beta_1 \Delta_0 (2V_0)^{\frac{p_1+q_1}{2p_1}}.$$

From above, $E_x = \mathbf{0}$ in finite time and $E_v = \dot{E}_x = \mathbf{0}$ in finite time. So, if all the states of e_{xi} and e_{vi} on the slide surface, the states will reach to zero in finite time.

Select the protocol as (6)

$$\begin{aligned} \dot{S} &= \dot{E}_v + \beta_1 \frac{q_1}{p_1} C\text{diag}\left(E_x^{q_1/p_1-1}\right)E_v \\ &= C(u + f - \mathbf{1}g_0) + \beta_1 \frac{q_1}{p_1} C\text{diag}\left(E_x^{q_1/p_1-1}\right)E_v \\ &= C\left[-\mathbf{1}g_0 + \beta_1 \frac{q_1}{p_1}\text{diag}\left(E_x^{q_1/p_1-1}\right)E_v + \Phi \right.\\ &\qquad \left. - \beta_2 S^{q_2/p_2} - \alpha l_g \text{sgn}(S)\right]. \end{aligned} \tag{7}$$

Consider the positive Lyapunov-candidate-function as $V = 0.5S^TS$. Combining Assumption 1 and (7), we have

$$\begin{aligned}\dot{V} &= S^T\dot{S}\\ &= S^TC\Big(-\mathbf{1}g_0 + \beta_1\frac{q_1}{p_1}\mathrm{diag}\Big(E_x^{q_1/p_1-1}\Big)E_v + \Phi\\ &\qquad - \beta_2 S^{q_2/p_2} - \alpha l_g \mathrm{sgn}(S)\Big)\\ &= S^TC\Big(\beta_1\frac{q_1}{p_1}\mathrm{diag}\Big(E_x^{q_1/p_1-1}\Big)E_v + \Phi - \beta_2 S^{q_2/p_2}\Big)\\ &\quad - g_0 S^TC\mathbf{1} - \alpha l_g S^TC\mathrm{sgn}(S)\\ &\le S^TC\Big(\beta_1\frac{q_1}{p_1}\mathrm{diag}\Big(E_x^{q_1/p_1-1}\Big)E_v + \Phi - \beta_2 S^{q_2/p_2}\Big)\\ &\quad - g_0 S^Tb - \alpha l_g\Big(S^{\frac{1}{2}}\Big)^T\Delta S^{\frac{1}{2}}\\ &\le S^TC\Big(\beta_1\frac{q_1}{p_1}\mathrm{diag}\Big(E_x^{q_1/p_1-1}\Big)E_v + \Phi - \beta_2 S^{q_2/p_2}\Big)\\ &\quad - g_0\sum_{i=1}^{n} b_iS_i - l_g\sum_{i=1}^{n}\alpha\Delta_i|S_i|\\ &\le S^TC\Big(\beta_1\frac{q_1}{p_1}\mathrm{diag}\Big(E_x^{q_1/p_1-1}\Big)E_v + \Phi - \beta_2 S^{q_2/p_2}\Big).\end{aligned} \tag{8}$$

Due to $0 < q_1/p_1 < 1$, there is singularity problem for $\beta_1(q_1/p_1)\mathrm{diag}(E_x^{q_1/p_1-1})E_v$.

We can divided the ith space into the following two areas:

$$\begin{aligned}S_{i1} &= \left\{(e_{xi}, e_{vi})\middle|\beta_1\frac{q_1}{p_1}e_{xi}^{q_1/p_1-1}|e_{vi}| \le u_s\right\}\\ S_{i2} &= \left\{(e_{xi}, e_{vi})\middle|\beta_1\frac{q_1}{p_1}e_{xi}^{q_1/p_1-1}|e_{vi}| > u_s\right\}.\end{aligned} \tag{9}$$

The state S cross the area S_{i2} and lies in the area S_{i1}. In a finite time, the state S will reach to the point $[0 \quad 0]^T$.

1) When the states $[e_{xi} \quad e_{vi}]^T$ lie in S_{i1}, $\phi_i = \mu_i$. We get

$$\begin{aligned}\dot{V} &= S^T\dot{S}\\ &= S^TC\Big(-\mathbf{1}g_0 + \beta_1\frac{q_1}{p_1}\mathrm{diag}\Big(E_x^{q_1/p_1-1}\Big)E_v + \Phi\\ &\qquad - \beta_2 S^{q_2/p_2} - \alpha l_g\mathrm{sgn}(S)\Big)\\ &\le S^TC\Big(\beta_1\frac{q_1}{p_1}\mathrm{diag}\Big(E_x^{q_1/p_1-1}\Big)E_v + \Phi - \beta_2 S^{q_2/p_2}\Big)\\ &\le -\beta_2 S^TCS^{q_2/p_2}.\end{aligned} \tag{10}$$

Comply with Lemma 2, we know

$$\begin{aligned}\dot{V} &\le -\beta_2 S^TCS^{q_2/p_2}\\ &\le -\beta_2\Big(S^{\frac{p_2+q_2}{2p_2}}\Big)^T\Delta S^{\frac{p_2+q_2}{2p_2}}\\ &\le -\beta_2\Delta_0(2V)^{\frac{p_2+q_2}{2p_2}}.\end{aligned} \tag{11}$$

So, the states will enter S_{i2} or reach to the sliding surface $S = \mathbf{0}$ in finite time.

2) When the states $[e_{xi} \quad e_{vi}]^T$ lie in S_{i2}, $\phi_i = u_s\mathrm{sgn}(\mu_i)$.

We get

$$\begin{aligned}\dot{V} &= S^T\dot{S}\\ &= S^TC\Big(-\mathbf{1}g_0 + \beta_1\frac{q_1}{p_1}\mathrm{diag}\Big(E_x^{q_1/p_1-1}\Big)E_v + \Phi - \beta_2 S^{q_2/p_2}\\ &\qquad - \alpha l_g\mathrm{sgn}(S)\Big)\\ &\le S^TC\Big(\beta_1\frac{q_1}{p_1}\mathrm{diag}\Big(E_x^{q_1/p_1-1}\Big)E_v + \Phi - \beta_2 S^{q_2/p_2}\Big)\\ &= S^TC\Big(\beta_1\frac{q_1}{p_1}\mathrm{diag}\Big(E_x^{q_1/p_1-1}\Big)E_v + u_s\mathrm{sgn}(\mu_i) - \beta_2 S^{q_2/p_2}\Big).\end{aligned} \tag{12}$$

It is hard to guarantee the above equation is negative, another method is given to prove the result.

Two different cases are listed when the states lie in S_{i2}. Case 1, $e_{vi} > 0$, e_{xi} increases monotonously until it reaches the junction of the areas S_{i1} and S_{i2}. Case 2, $e_{vi} < 0$, e_{xi} decreases monotonously until it reaches the junction of the areas S_{i1} and S_{i2}. The analysis here is similar to the analysis in [56], we can get the result that the system can achieve $S = \mathbf{0}$ in finite time.

From above, only when $\Delta_i \triangleq (cq_1/[q_1+p_1])\sum_{j=1}^{n} a_{ij} - (cq_1/[q_1+p_1])\sum_{j=1}^{n} a_{ji} - b_i > 0$, the finite-time consensus can be reached. ■

From the above condition, we get the following results.

Theorem 2: In order to get the finite-time result, the nodes whose outdegree are no less than their indegree should be pinned, and the pinning strength should be larger than $-(cq_1/[q_1+p_1])(\sum_{j=1}^{n} a_{ij} - \sum_{j=1}^{n} a_{ji})$.

Proof: From the structure of the directed network, it is easy to know that $\sum_{j=1}^{n} a_{ij}$ is the indegree of the node i, $\sum_{j=1}^{n} a_{ji}$ is the outdegree of the node i. When $\sum_{j=1}^{n} a_{ji} < \sum_{j=1}^{n} a_{ij}$, $(cq_1/[q_1+p_1])(\sum_{j=1}^{n} a_{ij} - \sum_{j=1}^{n} a_{ji}) > 0$, this node does not need to be pinned. When $\sum_{j=1}^{n} a_{ji} \ge \sum_{j=1}^{n} a_{ij}$, $(cq_1/[q_1+p_1])(\sum_{j=1}^{n} a_{ij} - \sum_{j=1}^{n} a_{ji}) \le 0$, if we want $\Delta_i > 0$, the pinning control is needed, and the pinning strength b_i should satisfied that $b_i > -(cq_1/[q_1+p_1])(\sum_{j=1}^{n} a_{ij} - \sum_{j=1}^{n} a_{ji}) \ge 0$.

So, we can get the conclusion that the nodes whose outdegree are no less than their indegree should be pinned, and the pinning strength should be larger than $-(cq_1/[q_1+p_1])(\sum_{j=1}^{n} a_{ij} - \sum_{j=1}^{n} a_{ji})$. ■

Remark 1: If we want to achieve consensus as soon as possible, it is better to let Δ_0 as big as possible. Pinning a small part of nodes with huge strength does not always increase the convergence speed too much. The best way is to increase the minimum of Δ_i. What is more, in order to get the fast convergence rate, the nodes whose outdegree are less than their indegree should be pinned with appropriate pinning strength.

IV. Fully Distributed Finite-Time Attitude Consensus of Directed MQSs

In this section, we will introduce the attitude consensus of MQSs by using the method proposed above. Suppose the quadcopter system studied in this article can only move at the attitude angle of three degrees of freedom. This quadcopter

Fig. 1. Quadcopter platform.

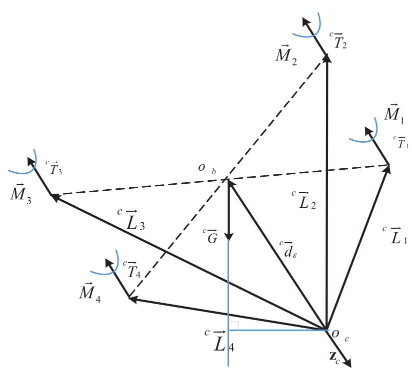

Fig. 2. Force analysis of the quadcopter.

platform is designed to verify the validity of the proposed protocol, seen from Fig. 1. The center of mass motion model is not analyzed here.

A. Mathematical Model

The following assumptions are listed.

Assumption 2: The quadcopters are central symmetrical rigid body.

Assumption 3: The resistance and gravity of the quadcopters are not affected by the flight environment and other factors, and will remain unchanged.

Assumption 4: The rotational inertia of the quadcopters remains unchanged.

As shown in Fig. 2, force analysis of the quadcopter is carried out. ${}^{c}\overrightarrow{T}_i = \left[0 \;\; 0 \;\; -c_T\varpi_i^2\right]^T$ is the force vector produced by the propeller. ${}^{c}\overrightarrow{L}_1 = \left[(\sqrt{2}/2)d \;\; (\sqrt{2}/2)d \;\; -d_\varepsilon\right]^T$, ${}^{c}\overrightarrow{L}_2 = \left[-(\sqrt{2}/2)d \;\; (\sqrt{2}/2)d \;\; -d_\varepsilon\right]^T$, ${}^{c}\overrightarrow{L}_3 = \left[-(\sqrt{2}/2)d \;\; -(\sqrt{2}/2)d \;\; -d_\varepsilon\right]^T$, and ${}^{c}\overrightarrow{L}_4 = \left[(\sqrt{2}/2)d \;\; -(\sqrt{2}/2)d \;\; -d_\varepsilon\right]^T$ are the position of tension operating points in the coordinate system of rotating platform, respectively. ${}^{c}\overrightarrow{G} = {}^{b}_{e}\mathbf{R}\,{}^{e}\overrightarrow{G} = \left[-mg\sin\theta \;\; mg\cos\theta\sin\phi \;\; mg\cos\theta\cos\phi\right]^T$ is the gravity vector, m is the quality of the quadcopter, and g is the local acceleration of gravity. ${}^{c}\overrightarrow{d}_\varepsilon = \left[0 \;\; 0 \;\; -d_\varepsilon\right]^T$ is the position of the center of mass in the frame of rotating platform.

From above, we know the total reverse torque produced by the propeller ${}^{c}\tau_M$ can be seen as follows:

$$ {}^{c}\tau_M = \vec{M}_1 + \vec{M}_2 + \vec{M}_3 + \vec{M}_4 = \begin{bmatrix} 0 \\ 0 \\ c_M\left(\varpi_1^2 - \varpi_2^2 + \varpi_3^2 - \varpi_4^2\right) \end{bmatrix}. $$

From Fig. 2, when the quadcopter is doing attitude tilt, the resultant moment is torque generated by propeller tension, reverse torque, and gravity. We have the following result:

$$ {}^{c}\tau_G = {}^{c}\vec{G} \times {}^{c}\vec{L}_G = \begin{bmatrix} -mg\sin\theta \\ mg\cos\theta\sin\phi \\ mg\cos\theta\cos\phi \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ d_\varepsilon \end{bmatrix} \tag{13} $$

where ${}^{c}\vec{L}_G$ is the arm of gravity. So, the resultant external moment of the quadcopter is

$$ \begin{aligned} {}^{c}\tau &= {}^{c}\tau_M + {}^{c}\vec{G} \times {}^{c}\vec{L}_G + \sum_{i=1}^{4} {}^{c}\overrightarrow{T}_i \times {}^{c}\overrightarrow{L}_{T_i} \\ &= \begin{bmatrix} 0 \\ 0 \\ c_M\left(\varpi_1^2 - \varpi_2^2 + \varpi_3^2 - \varpi_4^2\right) \end{bmatrix} + \begin{bmatrix} mg\cos\theta\sin\phi d_\varepsilon \\ mg\sin\theta d_\varepsilon \\ 0 \end{bmatrix} \\ &\quad + \begin{bmatrix} \frac{\sqrt{2}}{2}c_T\left(\varpi_1^2 + \varpi_2^2 - \varpi_3^2 - \varpi_4^2\right) \\ \frac{\sqrt{2}}{2}c_T\left(-\varpi_1^2 + \varpi_2^2 - \varpi_3^2 + \varpi_4^2\right) \\ 0 \end{bmatrix} \end{aligned} \tag{14} $$

where ${}^{c}\overrightarrow{L}_{T_i}$ is the arm of lift.

The attitude dynamics equation in the rotating platform coordinate system is as follows:

$$ J \cdot {}^{c}\dot{\omega} = -{}^{c}\omega \times \left(J \cdot {}^{c}\omega\right) + {}^{c}\tau \tag{15} $$

where $J \in {}^{3\times 3}$ is the moment of inertia of the experimental platform.

Combining (14) and (15), we can get

$$ \begin{cases} \dot{\omega}_{x_c} = \omega_{y_c}\omega_{z_c}\left(\frac{J_2 - J_3}{J_1}\right) + \frac{\frac{\sqrt{2}}{2}c_T\left(\varpi_1^2+\varpi_2^2-\varpi_3^2-\varpi_4^2\right)+mg\cos\theta\sin\phi d_\varepsilon}{J_1} \\ \dot{\omega}_{y_c} = \omega_{x_c}\omega_{z_c}\left(\frac{J_3 - J_1}{J_2}\right) + \frac{\frac{\sqrt{2}}{2}c_T\left(-\varpi_1^2+\varpi_2^2-\varpi_3^2+\varpi_4^2\right)+mg\sin\theta d_\varepsilon}{J_2} \\ \dot{\omega}_{z_c} = \omega_{x_c}\omega_{y_c}\left(\frac{J_1 - J_2}{J_3}\right) + \frac{c_M\left(\varpi_1^2-\varpi_2^2+\varpi_3^2-\varpi_4^2\right)}{J_3}. \end{cases} \tag{16} $$

Due to the limitation of the platform, the angular velocity is smaller when flying. So, the following equation is correct: $\omega_{x_c} = \dot{\phi}$, $\omega_{y_c} = \dot{\theta}$, and $\omega_{z_c} = \dot{\psi}$. We have the following result:

$$ \begin{cases} \ddot{\phi} = \dot{\theta}\dot{\psi}\left(\frac{J_2 - J_3}{J_1}\right) + \frac{\frac{\sqrt{2}}{2}c_T(\varpi_1^2+\varpi_2^2-\varpi_3^2-\varpi_4^2)+mg\cos\theta\sin\phi d_\varepsilon}{J_1} \\ \ddot{\theta} = \dot{\phi}\dot{\psi}\left(\frac{J_3 - J_1}{J_2}\right) + \frac{\frac{\sqrt{2}}{2}c_T(-\varpi_1^2+\varpi_2^2-\varpi_3^2+\varpi_4^2)+mg\sin\theta d_\varepsilon}{J_2} \\ \ddot{\psi} = \dot{\phi}\dot{\theta}\left(\frac{J_1 - J_2}{J_3}\right) + \frac{c_M(\varpi_1^2-\varpi_2^2+\varpi_3^2-\varpi_4^2)}{J_3}. \end{cases} $$

Let

$$ \begin{cases} U_1 = \frac{\sqrt{2}}{2}c_T(\varpi_1^2 + \varpi_2^2 - \varpi_3^2 - \varpi_4^2) \\ U_2 = \frac{\sqrt{2}}{2}c_T(-\varpi_1^2 + \varpi_2^2 - \varpi_3^2 + \varpi_4^2) \\ U_3 = c_M(\varpi_1^2 - \varpi_2^2 + \varpi_3^2 - \varpi_4^2) \end{cases}, \text{ we have} $$

$$\begin{cases} \ddot{\phi} = \dot{\theta}\dot{\psi}\left(\frac{J_2-J_3}{J_1}\right) + \frac{U_1+mg\cos\theta\sin\phi d_\varepsilon}{J_1} \\ \ddot{\theta} = \dot{\phi}\dot{\psi}\left(\frac{J_3-J_1}{J_2}\right) + \frac{U_2+mg\sin\theta d_\varepsilon}{J_2} \\ \ddot{\psi} = \dot{\phi}\dot{\theta}\left(\frac{J_1-J_2}{J_3}\right) + \frac{U_3}{J_3}. \end{cases} \tag{17}$$

Define $a_{\theta 1} = ([J_3 - J_1]/J_2)$, $a_{\theta 2} = (1/J_2)$, $a_{\theta 3} = (mgd_\varepsilon/J_2)$, $a_{\phi 1} = ([J_2 - J_3]/J_1)$, $a_{\phi 2} = (1/J_1)$, $a_{\phi 3} = (mgd_\varepsilon/J_1)$, $a_{\psi 1} = ([J_1 - J_2]/J_3)$, and $a_{\varphi 2} = (1/J_3)$, we have the final attitude dynamics equation

$$\begin{cases} \ddot{\phi} = a_{\phi 1}\dot{\theta}\dot{\psi} + a_{\phi 2}U_1 + a_{\phi 3}\cos\theta\sin\phi \\ \ddot{\theta} = a_{\theta 1}\dot{\phi}\dot{\psi} + a_{\theta 2}U_2 + a_{\theta 3}\sin\theta \\ \ddot{\psi} = a_{\psi 1}\dot{\phi}\dot{\theta} + a_{\psi 2}U_3. \end{cases} \tag{18}$$

There are eight quadcopters in our laboratory, the final attitude dynamics systems for the i-quadcopters system can be described, $i = 1, \ldots, 8$

$$\begin{cases} \ddot{\phi}_i = a_{\phi 1i}\dot{\theta}_i\dot{\psi}_i + a_{\phi 3i}\cos\theta_i\sin\phi_i + a_{\phi 2i}U_{1i} \\ \ddot{\theta}_i = a_{\theta 1i}\dot{\phi}_i\dot{\psi}_i + a_{\theta 3i}\sin\theta_i + a_{\theta 2i}U_{2i} \\ \ddot{\psi}_i = a_{\psi 1i}\dot{\phi}_i\dot{\theta}_i + a_{\psi 2i}U_{3i}. \end{cases} \tag{19}$$

B. Fully Distributed Finite-Time Attitude Consensus of Directed MQSs

The analysis of the three degrees attitude angles is similar, so we only prove the pitch angle, and the analysis of other angles is similar to the pitch angle.

Define $f_i = a_{\theta 1i}\dot{\phi}_i\dot{\psi}_i + a_{\theta 3i}\sin\theta_i$, $\mathbf{a} \triangleq \text{diag}(a_{\theta 21}, \ldots, a_{\theta 2n})$, $\mathbf{f} \triangleq [f_1, \ldots, f_n]^T$, $E_\theta \triangleq [e_{\theta 1}, \ldots, e_{\theta n}]^T$, $E_{\dot{\theta}} \triangleq [e_{\dot{\theta}1}, \ldots, e_{\dot{\theta}n}]^T$, $\theta \triangleq [\theta_1, \ldots, \theta_n]^T$, $\dot{\theta} \triangleq [\dot{\theta}_1, \ldots, \dot{\theta}_n]^T$, and $\mathbf{U}_2 \triangleq [U_{21}, \ldots, U_{2n}]^T$.

We have the following attitude equation:

$$\ddot{\theta} = \mathbf{f} + \mathbf{a}\mathbf{U}_2. \tag{20}$$

Suppose there is a virtual leader of the pitch angle. The model of virtual leader is

$$\ddot{\theta}_0 = g_0 \tag{21}$$

where θ_0 is the virtual leader's states, and g_0 is the virtual leader's dynamic.

The purpose of this article is to make all agents come to the virtual state with the distributed consensus protocol.

Choose the following error functions:

$$e_{\theta i} = c\sum_{j=1}^{n} a_{ij}\left(\theta_i - \theta_j\right) + b_i(\theta_i - \theta_0) = C_i(\theta - \mathbf{1}\theta_0)$$
$$e_{\dot{\theta}i} = c\sum_{j=1}^{n} a_{ij}\left(\dot{\theta}_i - \dot{\theta}_j\right) + b_i\left(\dot{\theta}_i - \dot{\theta}_0\right) = C_i\left(\dot{\theta} - \mathbf{1}\dot{\theta}_0\right) \tag{22}$$

where c is the coupling strength.

We will prove $E_\theta = E_{\dot{\theta}} = \mathbf{0}$ in finite time.

We have

$$E_\theta = (cL + \mathbf{B})\theta - \mathbf{B}\mathbf{1}\theta_0 = (cL + \mathbf{B})(\theta - \mathbf{1}\theta_0)$$
$$E_{\dot{\theta}} = (cL + \mathbf{B})\dot{\theta} - \mathbf{B}\mathbf{1}\dot{\theta}_0 = (cL + \mathbf{B})\left(\dot{\theta} - \mathbf{1}\dot{\theta}_0\right). \tag{23}$$

So

$$\dot{E}_\theta = E_{\dot{\theta}}$$
$$\dot{E}_{\dot{\theta}} = C(\mathbf{f} + \mathbf{a}\mathbf{U}_2 - \mathbf{1}g_0). \tag{24}$$

Define the following function:

$$\mu_i = -\beta_1\frac{q_1}{p_1}e_{\theta i}^{q_1/p_1-1}e_{\dot{\theta}i}$$
$$\phi_i = \text{sat}(\mu_i, u_s) = \begin{cases} u_s\text{sgn}(\mu_i) & \text{if}|\mu_i| > u_s \\ \mu_i & \text{otherwise} \end{cases}$$

$u_s > 0$ is the threshold value of the saturation function, $i = 1, \ldots, n$.

Define $M = [\,\mu_1 \;\; \cdots \;\; \mu_n\,]^T$ and $\Phi = [\,\phi_1 \;\; \cdots \;\; \phi_n\,]^T$.

Theorem 3: Under the condition of Assumptions 1–4. The directed MQSs (20) and (21) can achieve finite-time consensus under the following protocol:

$$\mathbf{U}_2 = \mathbf{a}^{-1}\left(-\mathbf{f} + \Phi - \beta_2 S^{q_2/p_2} - \alpha l_g\text{sgn}(S)\right) \tag{25}$$

where $i = 1, \ldots, n$, $\Delta_i \triangleq (cq_1/[q_1 + p_1])\sum_{j=1}^{n} a_{ij} - (cq_1/[q_1 + p_1])\sum_{j=1}^{n} a_{ji} + b_i > 0$, $\alpha > 1$, $\alpha\Delta_i \geq b_i$, $\Delta_0 \triangleq \min(\Delta_i) > 0$, $\Delta \triangleq \text{diag}\{\Delta_1, \ldots, \Delta_n\}$ is positive matrix, I is identity matrix, $\beta_1 > 0$ and $\beta_2 > 0$ are constants, $p_1 > 0$, $p_2 > 0$, $q_1 > 0$, and $q_2 > 0$ are odd integers, $q_1 < p_1 < 2q_1$, and $p_2 > q_2$.

Proof: Select the terminal sliding surface as $S = E_{\dot{\theta}} + \beta_1 C E_\theta^{q_1/p_1}$.

We know when $S = \mathbf{0}$, E_θ and $E_{\dot{\theta}}$ will get to zero in finite time.

From above

$$\begin{aligned} \dot{S} &= \dot{E}_{\dot{\theta}} + \beta_1\frac{q_1}{p_1}C\text{diag}\left(E_\theta^{q_1/p_1-1}\right)E_{\dot{\theta}} \\ &= C(\mathbf{f} + \mathbf{a}\mathbf{U}_2 - \mathbf{1}g_0) + \beta_1\frac{q_1}{p_1}C\text{diag}\left(E_\theta^{q_1/p_1-1}\right)E_{\dot{\theta}} \\ &= C\Big[-\mathbf{1}g_0 + \beta_1\frac{q_1}{p_1}\text{diag}\left(E_\theta^{q_1/p_1-1}\right)E_{\dot{\theta}} + \Phi \\ &\qquad - \beta_2 S^{q_2/p_2} - \alpha l_g\text{sgn}(S)\Big]. \end{aligned} \tag{26}$$

Choose the positive Lyapunov-candidate-function as $V = 0.5S^T S$. Combining (25) with (26), one has

$$\begin{aligned} \dot{V} &= S^T\dot{S} \\ &= S^T C\Big[-\mathbf{1}g_0 + \beta_1\frac{q_1}{p_1}\text{diag}\left(E_\theta^{q_1/p_1-1}\right)E_{\dot{\theta}} + \Phi \\ &\qquad - \beta_2 S^{q_2/p_2} - \alpha l_g\text{sgn}(S)\Big] \\ &\leq S^T C\left(\beta_1\frac{q_1}{p_1}\text{diag}\left(E_\theta^{q_1/p_1-1}\right)E_{\dot{\theta}} + \Phi - \beta_2 S^{q_2/p_2}\right). \end{aligned}$$

The following proof is similar to the proof in Theorem 1, so we omit here. The fully distributed finite-time attitude consensus of directed multiquadcoppters systems is achieved. ■

V. Numerical Simulations

To illustrate the effectiveness of the proposed algorithms, two simulation results are presented.

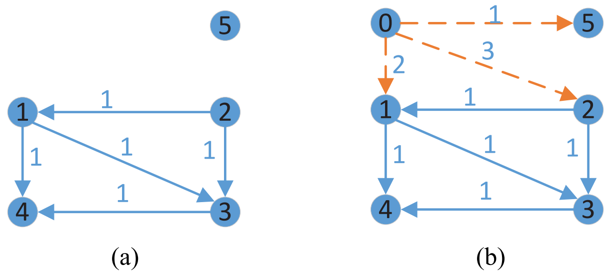

Fig. 3. Topology of the MASs. (a) Without pinning. (b) With pinning.

A. Finite-Time Consensus of Directed MASs

As can be seen from Fig. 3(a), the topology of the directed MASs with five agents is presented.

The leader dynamic is described as

$$\begin{cases} \dot{x}_0 = v_0 \\ \dot{v}_0 = 0.02\sin(x_0) \end{cases} \tag{27}$$

the dynamics of the ith follower are

$$\begin{cases} \dot{x}_i = v_i \\ \dot{v}_i = u_i + x_i \end{cases}, i = 1, \ldots, 5. \tag{28}$$

Suppose the leader's initial condition is $[x_0(0) \ \ v_0(0)]^T = [2 \ \ 3]^T$, and the five agents' initial condition are $x(0) = [4 \ -1 \ \ 1 \ \ 3 \ -2]^T$ and $v(0) = [1 \ \ 0 \ -2 \ -1 \ \ 2]^T$.

From Theorem 2, we need to pin the agents 1, 2, and 5, as shown in Fig. 3(b). The adjacent matrix is

$$A = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

the Laplacian of the follower system can be written as

$$L = \begin{bmatrix} 2 & 0 & -1 & -1 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and the interconnection relationship between the leader and the followers is $\mathbf{B} = \mathrm{diag}(2 \ \ 3 \ \ 0 \ \ 0 \ \ 1)$.

Select the terminal sliding surface, $S = E_v + \beta_1 C E_x^{q_1/p_1}$.

The control is selected as, $\mathbf{u} = \Phi - \beta_2 S^{q_2/p_2} - \alpha l_f \mathrm{sgn}(S)$, where $l_f = 1$, $\alpha = 2$, $p_1 = p_2 = 5$, $q_1 = q_2 = 3$, $\beta_1 = \beta_2 = 1$, $\Phi = [\phi_1 \ \cdots \ \phi_n]^T$, $\mu_i = -\beta_1(q_1/p_1)e_{i1}^{q_1/p_1 - 1}e_{i2}$, $\phi_i = \mathrm{sat}(\mu_i, u_s) = \begin{cases} u_s \mathrm{sgn}(\mu_i) & \text{if} |\mu_i| > u_s \\ \mu_i & \text{otherwise.} \end{cases}$

Figs. 4 and 5 show the results of the proposed protocol in (6). Within finite time, all the states reach the virtual leader's states.

B. Finite-Time Consensus of Directed MQSs

In this simulation, we have five quadcopters. Use the same topology as above, as shown in Fig. 3.

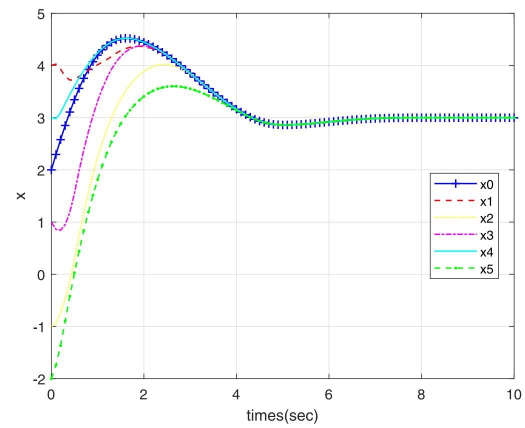

Fig. 4. States of x for the MASs.

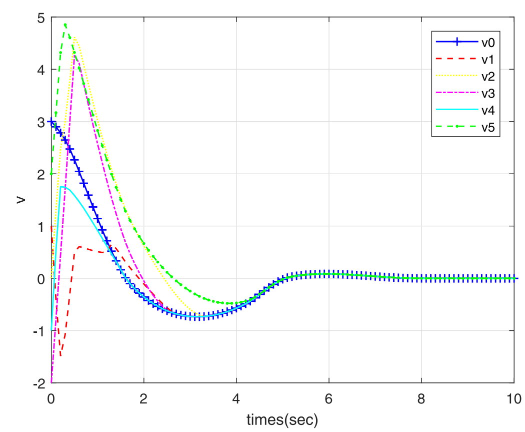

Fig. 5. States of v for the MASs.

TABLE I
MODEL PARAMETER OF MQSS

Parameter name	Symbols	Value
Mass of one quadcopter	m	1.975 kg
Distance between motor shaft and center of gravity	L	0.23 m
Distance between the center of gravity and the center of rotation	d_ε	0.10 m
Moment of inertia 1	J_1	0.027 $kg \cdot m^2$
Moment of inertia 2	J_2	0.027 $kg \cdot m^2$
Moment of inertia 3	J_3	0.027 $kg \cdot m^2$

The leader dynamic is described as

$$\begin{cases} \ddot{\phi}_0 = a_{\phi 20} U_{10} \\ \ddot{\theta}_0 = a_{\theta 20} U_{20} \\ \ddot{\psi}_0 = a_{\psi 20} U_{30} \end{cases} \tag{29}$$

and the dynamics of the ith follower are described as

$$\begin{cases} \ddot{\phi}_i = a_{\phi 1i}\dot{\theta}_i\dot{\psi}_i + a_{\phi 3i}\cos\theta_i \sin\phi_i + a_{\phi 2i} U_{1i} \\ \ddot{\theta}_i = a_{\theta 1i}\dot{\phi}_i\dot{\psi}_i + a_{\theta 3i}\sin\theta_i + a_{\theta 2i} U_{2i} \\ \ddot{\psi}_i = a_{\psi 1i}\dot{\phi}_i\dot{\theta}_i + a_{\psi 2i} U_{3i}. \end{cases} \tag{30}$$

The model parameter can be selected as in Table I.

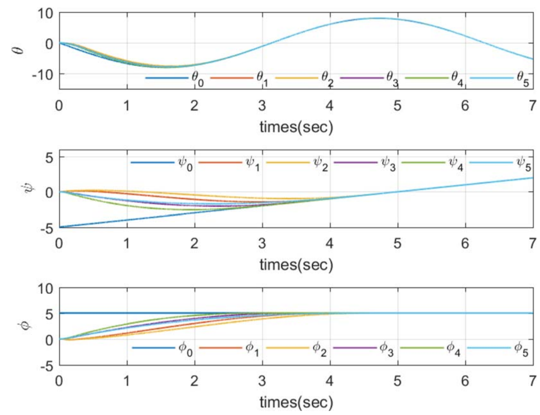

Fig. 6. States of angle for the MQSs.

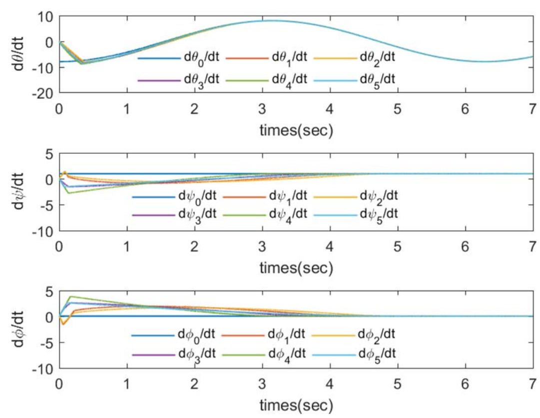

Fig. 7. States of angular velocity for the MQSs.

So, we get the following parameters, $a_{\theta 1i} = a_{\phi 1i} = a_{\psi 1i} = 0$, $a_{\theta 20} = a_{\phi 20} = a_{\psi 20} = a_{\theta 2i} = a_{\phi 2i} = a_{\psi 2i} = 37$, and $a_{\theta 3i} = a_{\phi 3i} = 71.7$, $i = 1, \ldots, 5$.

Suppose the leader's initial condition is $[\,\theta_0(0)\;\; \dot{\theta}_0(0)\,]^T = [\,0\;\; -10\,]^T$, $[\,\psi_0(0)\;\; \dot{\psi}_0(0)\,]^T = [\,0\;\; -10\,]^T$, and $[\,\phi_0(0)\;\; \dot{\phi}_0(0)\,]^T = [\,0\;\; -10\,]^T$. When we turn on the quadcopters' electric power, all the quadcopters will come to a horizontal arrangement, and all the five quadcopters' initial conditions are zero.

Choose the control protocol as

$$\mathbf{U}_2 = \mathbf{a}^{-1}\Big(-\mathbf{f} + \Phi - \beta_2 S^{q_2/p_2} - \alpha l_g \mathrm{sgn}(S)\Big) \tag{31}$$

and the TSM surface as

$$S = E_2 + \beta_1 C E_1^{q_1/p_1} \tag{32}$$

where $l_f = 20$, $\alpha = 2$, $p_1 = p_2 = 5$, $q_1 = q_2 = 3$, and $\beta_1 = \beta_2 = 1$.

When $S = \mathbf{0}$, that is, $S = E_2 + \beta_1 C E_1^{q_1/p_1}$. So, within finite time, E_1 and E_2 will get to zero.

The states of angle for the MQSs are shown in Fig. 6. No matter the leader's states are constant function, linear function, or trigonometric function, all the followers' states will reach to the leader's states in finite time. From Fig. 7, we found all the followers' angular velocity will converge to the leader's states in finite time. The control signals for the MQSs are shown in Fig. 8.

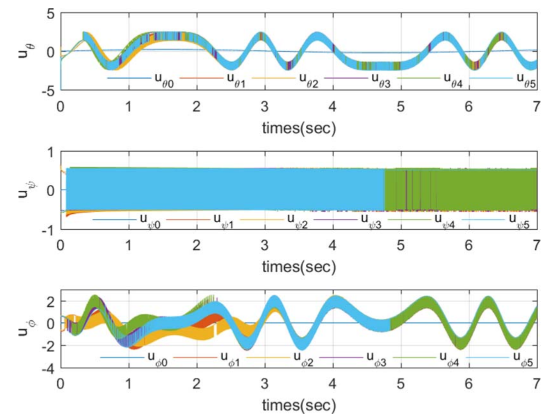

Fig. 8. Control signals for the MQSs.

VI. Conclusion

In this article, the fully distributed finite-time consensus of directed second-order MASs has been designed, where each control only needs its neighbors' state information. The pinning consensus strategy is studied for the second-order MASs with unknown dynamic virtual leader. It is pointed out the nodes whose outdegree are no less than the indegree should be pinned, and the least pinning strength has been calculated in Theorem 2. Furthermore, the proposed method has been used into finite-time consensus of MQSs.

It is worth mentioning here that some related literature about group consensus, duplex networks, switched coupled neural networks, and optimal control must be known [11]–[13], [33], [34], [44], [46]. Therefore, in our consecutive study, we will work on developing the distributed protocol about group consensus of duplex networks and switched coupled neural networks.

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