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# Hawkes processes in finance: market structure and impact

**Recto running head :** THE EUROPEAN JOURNAL OF FINANCE

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past efficiency, classical asset pricing theory and risk modelling have limitations in their ability to capture the new features of contemporary finance in today's markets. For instance, stock prices almost certainly do not evolve as a random walk and return distributions often deviate far from the normal.

Features observed in return time series such as fat tails, high peaks and extreme skewness are accepted stylized facts of financial markets in the post-crisis era. The occurrence of mini-flash crashes or so-called black swan events is no stranger to traders. They result in many practitioners seeking to neutralize their positions before the closing bell of a trading day. Price movements are also deeply influenced by news that often dominates investment sentiment and leads to either excessive risk-taking or contagion.

Recognizing the need for substantial further development of new techniques to model price movements in post-crisis financial markets, we set out to apply Hawkes processes to a variety of problems in finance. These processes, introduced by Alan Hawkes in his original works (Hawkes 1971a, b), are a family of stochastic point processes that model the arrival of events. The essence of Hawkes processes is that they describe the natural occurrence of events and trace their path. Hawkes processes posit that the historical record of a given event would increase the intensity of its recurrence. We can describe this as contagion, in the sense that the occurrence of cases of a contagious disease tends to lead to more cases. For example, the arrival of quotes for a specific stock tends to drive more quotes subsequently, with the increased probability of active quotes more likely result in final trades (Bowsher 2007). As an example of the usefulness of Hawkes processes, Yang et al. (2018) use a bi-variate model to demonstrate interesting and informative features of investors' behaviour. On the one hand, investors tend to chase positive returns, positive market sentiment also prolongs upward pressure on prices. On the other hand, when investors start to see negative returns, the market falls fast. Negative sentiment, once formed, tends to have contagious effects. More recently, inspired by Yang et al. (2018) using a bi-variate Hawkes process to model stock returns and investment sentiment, Chen et al. (2020) further introduce an entropy approach to identify whether the market trading is driven by return movements or news impacts. They find strong evidence that the financial market trading after the 2008 crisis is dominated by sentiment trading and often suffers from the contagion. This further confirms that, in comparison to classic Brownian Motion, the ability of Hawkes processes to model contagion would seem to be closer to the organic evolution of many financial problems in real markets. Hawkes processes are very flexible as they can accommodate a wide range of probability distributions, especially those suitable to describe extreme events or market conditions.

## 2. Background to the special issue

To further promote the use of Hawkes processes in financial economics, we organized a conference entitled Hawkes Processes in Finance. An important aim was to introduce this family of stochastic processes to finance researchers and practitioners in the belief that they may become part of the econometric toolbox for the analysis of contemporary finance problems. Knowledge transfer on a large scale, however, takes time and substantial efforts from finance researchers. The 'Hawkes Process in Finance Conference' was held at The Stevens Institute of Technology in Hoboken, New Jersey (<https://hawkes.stevens.edu/home>) on 7th and 8th June 2019. This special issue of The European Journal of Finance entitled 'Hawkes processes in finance, market structure and impact' contains a selection of nine papers that were presented at the conference and subsequently passed successfully through the journal's review process.

## 3. Articles in this special issue

Brownian motion, introduced by Bachelier (1900), has formed the backbone of many works in financial mathematics, especially the diffusion models that still prevail in finance. These fundamental works include, as Hawkes (2020) rightly points out, Samuelson (1972) for a Gaussian random walk, Black and Scholes (1973) for the Nobel Prize winning option pricing model and Merton's generalization in 1973. With consideration of financial jumps, Merton (1976) proposed Poisson jumps while Cont and Tankov (2004) comprehensively discussed Lévy jumps. Hawkes (2020), however, noticed that both processes do not explain the contagious nature of jump occurrence and thus suggests that a Hawkes jump process model that combines the diffusion and Hawkes processes can competently deal with a wide range of risk modelling and pricing issues even no matter of the market complexity. He gave a few examples of problems including trading across multiple markets, pricing and hedging, portfolio optimization and many more. The general conclusion is that Hawkes jump models are generally better than traditional diffusion or jump-diffusion models in terms of their fitness, forecasting performance or robustness. In addition, these prove that Hawkes processes are highly flexible and can be adapted to suit a good variety of finance problems.

Kirchner and Vetter (2020) model limit order book (LOB) dynamics using a multivariate marked Hawkes process. Unlike the existing limit order book models such as Cont and de Larrard (2013) and Bacry, Jaisson, and Muzy (2016), the proposed

model defines events by selecting a set of non-zero excitement activities, the shape of the decay kernels, and the impact function nonparametrically. The event estimation methodology based on 'Hawkes skeleton' introduced by Embrechts and Kirchner (2017), offers a scalable approach to estimate a large number of financial events. Through defining high dimensional events using time stamps, order types and order sizes, the authors are able to identify rich features of the limit order book using Hawkes skeleton and Hawkes graph edges: (i) the market orders are the driving force among all the order book activities; (ii) the price impact function of market orders follows a linear function; (iii) bid-ask activities exhibit symmetric properties under the non-parametric approach; (iv) LOB imbalance provides a good future order arrival prediction. Overall, the authors introduce an alternative approach to model LOB dynamics using Hawkes processes, and new insights have been generated for a better understanding of the market price formation process and predicting future events.

Benzaquen, Fosset, and Bouchaud(2021) suggest the non-parametric calibration procedure for general Quadratic Hawkes models that are used to model the order book activities. The nature of the Hawkes processes enables us to capture the whole history of event occurrences like the order arrivals. But Benzaquen, Fosset, and Bouchaud(2021) apply the calibration in such a way that the full path of the price movements under the impact of the order book event arrivals can also be examined. Essentially, their model calibration shows that the effective quadratic kernel would be rank-one Zumbach kernel. It follows a power law and could indicate the true rate of the exogenous events like flash crashes or Covid 19 in proportion to the total order book events. This further enhances the model and makes it practically useful in finding the points where the system's capacity to deal with extreme conditions such as liquidity squeeze is challenged. They interpret this as the microstructural origin of the Zumbach effect and shows that the past trends tend to reduce the liquidity in the order book and subsequently cause the future realized volatility to increase.

Cai (2020) develops a Hawkes process model with a hidden marked process that represents extra random errors (ERE) caused by the data collection mechanisms and certain data cleaning procedures. Related to Kirchner and Vetter (2020) in estimating financial events, the author proposes a quantile approach to augment the basic Hawkes process in addressing noise data issues faced by financial modellers. The author introduces the hidden marked process to take account of the effect of ERE using the generalized Lambda distribution (GLD), and a MCMC-based method for parameter estimation is also provided for model calibration. Through thorough analyses of both simulation and empirical market data, Cai (2020) demonstrates a clear benefit of introducing a hidden marked process in the presence of ERE and shows that when ERE causes information of the underlying process to be lost, the intensity function may be underestimated. The proposed method can be generalized to address more complex Hawkes processes when ERE is a major concern for intensity underestimation.

Zhang et al. (2021) [Q4] present yet another extension of the basic Hawkes process where the market prices and investor sentiment are combined into a single Hawkes-Contact model. The Contact process is a stochastic process introduced by Zhang and Wang (2010) in modelling a disease-spreading mechanism. Zhang et al. (2021) introduce the Contact process into the Hawkes process to better fit the empirical data. The authors argue that while the news sentiment formation can be represented as a dynamic social Contact process but, when it is combined with the Hawkes-style event branching process, the model becomes more powerful in capturing richer statistical properties in returns such as probability density function (PDF); complementary cumulative distribution function (CCDF); and Lempel-Ziv Complex (LZC). The authors show that the real return distribution is often far from normal but the simulated returns through the proposed Hawkes-Contact model can achieve better fit to the real returns. More interestingly, the proposed model offers two weighting parameters which are able to show the strength of both the Hawkes contagion process and the finite range contact process, and altogether the full model can fulfil the goal of capturing multiple information flows that jointly drive the underlying price process.

Asset prices are characterized by occasional abrupt changes (jumps). Understanding how these jumps are distributed over time has recently gained the attention of researchers. The vast majority of the studies either adopt a univariate approach or consider the joint dynamics of jumps for assets within the same market. Few studies investigate these dynamics across different markets. It is this gap in the literature that the following papers contribute. Ferriani and Zoi(2020) examine, inter alia, whether jumps transfer between US and German stock markets, and thus whether contagion-type behaviour presents itself as abrupt price adjustments. Buccioli and Kokholm (2020) [Q5] take the 'flight-to-safety' behaviour as their central hypothesis and consider whether gold and stock markets are causally related in terms of their contribution to each other's jump propensity.

Ferriani and Zoi (2020) consider a generalized Hawkes process in which the evolution of intraday jump intensity in the US and German stock markets is allowed to be determined by jump size and volatility (in addition to allowing for self and cross-excitation). Using five-minute returns to the S&P500 and Euro Stoxx 50 indices observed between 13/09/2007 and 30/04/2014, their results show that contagion effects do not appear to be present in jumps – a finding consistent with that

found by Corradi, Distaso, and Fernandes(2012). Their analysis also finds, contrary to intuition, that jump intensity and volatility are inversely related (implying that large price changes during stress periods are driven by continuous volatility and not jumps). Thus, the importance of jump risk (either via jump occurrence or jump contagion across markets) may have been previously overstated.

Buccioli and Kokholm (2020) make use of a Hawkes process-driven model in their study of the joint dynamics of stock and gold prices. Their model allows for jumps in each market such that both self and cross-excitation are permitted. The maintained assumption of 'flight-to-quality' is imposed on the model by focusing on the dynamics of negative jumps in the stock market and positive jumps in the gold market. The direction of causality in this jump-structure is left unrestricted. Using a long span of daily returns to the S&P 500 index and gold (observed in the London Bullion Market) observed between 02/09/1976 and 13/09/2018, their results confirm the presence of a significant flight-to-quality effect with negative stock market price jumps leading to positive gold price jumps. Moreover, they find that this effect is present for approximately 20 days after the stock market price fall. Additional analysis even shows that their jump intensity model can be used to predict future intensity levels, with forecast accuracy superior to those based on GARCH and VIX-based model predictions. Their findings underline the importance of incorporating flight-to-safety effects within an asset pricing and risk management context.

Empirically, Yang et al. (2021) take advantage of a multivariate Hawkes process model to examine the jump contagion behaviour across energy ETFs and other representative assets. Using five-minute high-frequency trading data, authors first detect return jumps by applying the bi-power variation jump detection method as in Barndorff-Nielsen and Shephard (2004, 2006), then model these jumps under a multivariate Hawkes process framework. Their main findings include: negative jumps tend to self-contagion more than positive ones across all assets. Within the energy ETF group, the ETF with Master Limited Partnership (MLP) segment exhibits less negative self-contagion and stronger positive self-contagion than other ETFs. In addition, the influence of negative jumps on ETFs from both the equity index and the energy future index is stronger than that of the positive jumps.

How much price variation is produced within the market, i.e. endogeneity, is an interesting and important research question in the literature on market microstructure (Shiller 1980; LeRoy and Porter 1981; Bouchaud 2009). Mark, Sila, and Weber (2020) fit a Hawkes process model with two parametric kernel specifications to the high-frequency mid-prices of Bitcoin traded in one of the largest cryptocurrency exchanges. The authors then construct a reflexivity index to quantify the extent of endogeneity in the Bitmex market. By comparison to traditional assets, authors find that the Bitcoin market performs similar to fiat-currency markets with the mean reflexivity value being around 80%, and that the Bitcoin mid-prices exhibit long-memory properties. Another contribution of the article is that the authors pinpoint a way of identifying the optimal estimation horizon to deal with the trade-off problem between estimation accuracy and robustness to regime changes.

#### 4. Directions for future research

With the studies reported here, we express the hope that finance researchers and practitioners will appreciate the significance of a modelling approach that more closely describes how the real markets and the associated financial time series behave. Hawkes processes can be effectively and efficiently applied to a variety of finance problems. As Hawkes (2019) points out, the classic diffusion models in finance need to be updated. New phenomena such as financial jumps, contagion, endogeneity, non-Gaussian return and other phenomena can be effectively dealt with by Hawkes processes. Going forward, the booming of the fintech sector and technological advancement in the finance sector surely means that questions in finance about risk will become more prominent and more complex. We believe that Hawkes processes will play a substantial role in developments in the years ahead.

To conclude, we thank all those researchers who have contributed to this special issue: presenters at the conference; authors whose papers appear; the referees; those who participated in the conference; and particularly those at the Stevens Institute, who organized it. Finally, we express our thanks and gratitude to Alan Hawkes not only for his pioneering work, but for being an active participant in and supporter of our efforts.

#### Note

<sup>1</sup> The Dodd-Frank Wall Street Reform and Consumer Protection Act is commonly referred to as Dodd-Frank. It is a United States federal law that was enacted on 21 July 2010. Apart from the US Congress, it is also available at the Commodity Futures Trading Commission (CFTC).

[https://www.cftc.gov/sites/default/files/idc/groups/public/@swaps/documents/file/hr4173\\_enrolledbill.pdf](https://www.cftc.gov/sites/default/files/idc/groups/public/@swaps/documents/file/hr4173_enrolledbill.pdf). 

## Disclosure statement

No potential conflict of interest was reported by the author(s) [Q6].

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