Software Reliability Models with Bathtub-shaped Fault Detection

Maskura Nafreen, University of Massachusetts Dartmouth
Lance Fiondella, PhD, University of Massachusetts Dartmouth

Key Words: Software reliability, software reliability growth model, fault detection rate, conditional maximization algorithm

2021 Annual Reliability and Maintainability Symposium (RAMS) | 978-1-7281-8017-5/21/$31.00 ©2021 IEEE | DOI: 10.1109/RAMS48097.2021.9605800

SUMMARY & CONCLUSIONS

Researchers have proposed a multitude of software reliability growth models (SRGM), many of which possess complex parametric forms. In practice, SRGM should exhibit a balance between predictive accuracy and other statistical measures of goodness of fit, yet past studies have not always performed such balanced assessment. This paper proposes a framework for SRGM possessing a bathtub-shaped fault detection rate and derives stable and efficient expectation conditional maximization algorithms to fit these models. The illustrations compare multiple bathtub-shaped and classical models with respect to predictive and information theoretic measures. Our results indicate that SRGM possessing a bathtub-shaped fault detection rate outperformed classical models on both types of measures. The proposed framework and models may therefore be a reasonable compromise between model complexity and predictive accuracy.

1 INTRODUCTION

Many software reliability growth models have been proposed and several of the earliest models [1] were relatively simple, while recent models have become progressively more complex. Software practitioners advocate for simple models, but this has not deterred the proliferation of complex models. One valid criticism of complex models is their disregard for measures of statistical goodness of fit, including predictive accuracy. Moreover, some researchers inaccurately claim to employ predictive measures. Not considering a variety of measures is unethical because of the potential for harm to life and property if failures are underestimated in real systems.

Several software reliability researchers have developed mathematical frameworks to establish relationships between multiple models. Noteable examples include the work of Langberg and Singpurwalla [2] who showed how some models can be derived by assigning specific prior distributions in a Bayesian context. Miller [3] showed that several models are special cases of exponential order statistics models. Yamada et al. [4] proposed a two-step model fitting procedure, which first fit a curve to testing effort data followed by the mean value function of an NHPP SRGM to the fault discovery process. Gokhale et al. [5] demonstrated that several NHPP models with bounded mean value function are special cases of the enhanced NHPP possessing time-varying test coverage. Kuo et al. [6] proposed a framework to incorporate both testing effort and fault detection rate into SRGM capable of characterizing a wide range of possible fault detection trends. Huang et al. [7] presented an NHPP model and derived several existing models through a parametric family of power transformations. Kapur et al. [8] presented two Generalized Imperfect Non-homogeneous Poisson Process (GINHPP) models to account for imperfect debugging and error generation, and demonstrated existing NHPP SRGM as special cases. Inoue and Yamada [9] developed a generalized discrete software reliability growth model following a binomial process capable of considering program size.

This paper presents a framework for software reliability models possessing a bathtub shaped fault detection rate. Several bathtub hazard rates from the hardware reliability literature [10] are included. The three stages of the bathtub are adapted to the detection of software faults during testing, including (i) a burn-in phase characterized by the discovery and correction of superficial faults such as typos and elementary syntax errors; (ii) requirements verification, which exposes more complicated logical errors that require more detailed rework; and (iii) code comprehension characterized by a learning curve [11], where a significant amount of code has been tested, enabling testers to focus on improving code coverage in order to expose and correct remaining defects. To justify a bathtub-shaped fault detection rate, information theoretic and predictive measures of goodness of fit are computed. This analysis also considers reduced forms of the bathtub model, including classical SRGM [1], [12] and presents a visual taxonomy of the relationships between the models.

This paper extends [13], which presented a single software reliability growth model possessing a bathtub-shaped fault detection rate. Our primary contributions include (i) A framework for SRGM possessing bathtub-shaped fault detection rate and (ii) stable and efficient expectation conditional maximization (ECM) algorithms to enable application of these models. Our results indicate that, for the data sets considered, a software reliability growth model possessing a bathtub-shaped fault detection rate performed best with respect to both information theoretic and predictive measures of goodness of fit. The proposed framework coupled with efficient ECM algorithms 2 and goodness of fit assessment may therefore be beneficial to the software reliability assessment process.

The remainder of the paper is organized as follows. Section II proposes a framework for bathtub-shaped SRGM, while

Section III presents instances. Section IV describes parameter estimation methods. Section V reviews methods to assess model goodness of fit. Section VI compares alternative bathtub and classical models. Section VII provides conclusions and future research.

2 SOFTWARE RELIABILITY GROWTH MODELS

The nonhomogeneous Poisson process is a stochastic process [14] that counts the number of events that occur by time t. The expected value is characterized by the mean value function (MVF), which can take many functional forms. In software reliability, the NHPP counts the number of unique faults detected during testing. The MVF of several SRGM follow the general form

$$m(t) = \omega \times F(t) \tag{1}$$

where ω is the number of unique faults that would be detected as $t \to \infty$ and $F(t)$ is the cumulative distribution function (CDF) of a continuous probability distribution, characterizing the software fault detection process.

A framework for bathtub-shaped fault detection models with the following CDF is proposed

$$F(t) = 1 - e^{-b(t)} \tag{2}$$

where $b(t)$ is an arbitrary bathtub hazard function.

3 SOFTWARE RELIABILITY GROWTH MODELS

This section summarizes several bathtub hazard functions from the literature. Since many bathtub distributions simplify to increasing or decreasing trends, we also identify feasible simplifications and their relationship to other well-known software reliability growth models, including the Goel-Okumoto [1] and Weibull [12] SRGM. The section concludes with a visual summary of the relationships between the bathtub models and their simplifications. This taxonomy is also used in the illustrations, where it enables explicit comparison of the goodness of fit of bathtub and simpler SRGM to objectively assess if the additional complexity is justified.

3.1 Quadratic Model

Bain [15] and Gore et al. [16] considered low order polynomial functions. The quadratic hazard function

$$b(t) = \alpha + \beta t + \gamma t^2 \tag{3}$$

is bathtub-shaped when $-2(\alpha\gamma)^{\frac{1}{2}} \leq \beta < 0$ and $\alpha, \gamma \geq 0$. Substituting Equation (3) into Equation (2) produces the mean value function of the SRGM with bathtub-shaped fault detection rate characterized by the Quadratic Model

$$m(t) = \omega\left(1 - e^{-(\alpha+\beta\, t+\gamma\, t^2)}\right) \tag{4}$$

Parameters β, α, and γ respectively contribute to the three stages of the bathtub. Specifically, if the coefficient of the linear term β is decreasing, this trend can characterize a decreasing fault detection rate in the earlier stages of testing as simple problems are detected and removed with relative ease. The constant α represents the baseline fault detection rate associated with the second phase of the bathtub. Finally, the coefficient of the quadratic term γ contributes to the third phase, since this final term will eventually dominate the constant and linear terms. In the context of software fault detection, γ can characterize code comprehension as testers increase code coverage and narrow in on remaining sections of untested software, resolving logic issues to ensure the application conforms to requirements. A low value of γ may indicate that the software is difficult to comprehend or takes longer to achieve a high level of code coverage. In this final stage of testing, faults can no longer elude testers.

Setting α and γ to zero reduces the fault detection rate to $b(t) = \beta t$, indicating that the Goel-Okumoto model is a special case of the SRGM with bathtub-shaped fault detection rate characterized by the Quadratic Model.

3.2 Competing Risk Models

Hjorth [17] presented a distribution capable of exhibiting increasing, decreasing, constant, and bathtub-shaped rates

$$b(t) = \frac{\alpha}{1+\beta t} + 2\gamma t \tag{5}$$

The mean value function of the SRGM with bathtub-shaped fault detection rate characterized by Hjorth's competing risk model also contains the Goel-Okumoto model when $\alpha = 0$.

3.3 Modified Weibull (Lai) Model

Lai et al. [18] proposed a modified Weibull distribution possessing hazard rate

$$b(t) = a(\alpha + \lambda t)t^{\alpha-1}e^{\lambda\, t} \tag{6}$$

Substituting $\lambda = 0$ and $a = \frac{\lambda}{\alpha}$ reduces to the Weibull model, while substituting $\lambda = 0$, $\alpha = 2$, and $\beta = 2a$ produces the Goel-Okumoto model.

3.4 Weibull Extension (Lee) Model

Lee [19] proposed a three-parameter model with hazard rate

$$b(t) = \lambda\, \gamma\, t^{\gamma-1}e^{\phi\, t} \tag{7}$$

Substituting $\phi = 0$, $\lambda = \frac{\lambda}{\gamma}$ reduces to the Weibull model, while substituting $\phi = 0$, $\lambda = 2$ and $\beta = 2\,\lambda$ reduces to the Goel-Okumoto model.

3.5 Bathtub-shaped models' framework

Figure 1 depicts the relationships among the SRGM possessing a bathtub-shaped fault detection rate as well as simplifications that correspond to an existing model. For example Figure 1 illustrates how Lee's Weibull Extension Model reduces to the Weibull SRGM and further simplifies to the Goel-Okumoto model. Similarly, the quadratic and competing risk models also contain the Goel-Okumoto model as a special case. Furthermore, Lai's modified Weibull contains the Weibull and Goel-Okumoto models.

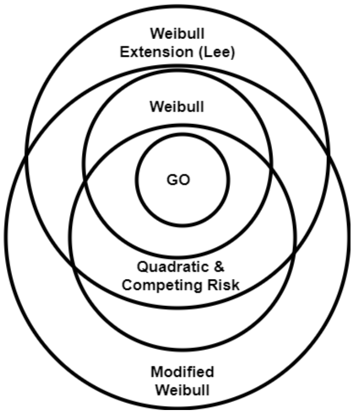

Figure 1 Relations among bathtub and traditional SRGM

4 PARAMETER ESTIMATION METHODS

This section describes various methods to estimate the parameters of a SRGM with the method of maximum likelihood estimation, including Newton's method [20] and the ECM [22] algorithm as well as initial parameter estimation with the EM algorithm [21].

4.1 Maximum Likelihood Estimation

Maximum-likelihood estimation maximizes the likelihood function or joint distribution of the failure data. Typically, the log-likelihood function is maximized. Failure count or grouped data consists of a vector of times $\mathbf{T} = \langle t_1, t_2, \ldots, t_n \rangle$ at which the intervals ended and failure counts $\mathbf{K} = \langle k_1, k_2, \ldots, k_n \rangle$ for these intervals

The log-likelihood function of a failure count dataset is

$$LL(\omega, \Theta|\mathbf{T}, \mathbf{K}) = \sum_{i=1}^{n} k_i \log(\omega) + \sum_{i=1}^{n} k_i \log\left(e^{-b(t_{i-1})} - e^{-b(t_i)}\right) - \omega\left(1 - e^{-b(t_n)}\right) - \sum_{i=1}^{n} \ln(k_i)! \quad (8)$$

where Θ is the vector of model parameters contained in $F(t)$.

The maximum likelihood estimate (MLE) of the numerical values of the parameters that best fit the data is found by numerically solving the system of simultaneous equation

$$\frac{\partial}{\partial \Theta} LL(\Theta) = 0 \quad (9)$$

Models of the form given in Equation (1) possess a closed form

$$\widehat{\omega} = \frac{\sum_{i=1}^{n} k_i}{F(t_n)} \quad (10)$$

which can be substituted into Equation (8) to reduce the set of simultaneous equations by one.

For example, the log-likelihood of the SRGM with bathtub-shaped fault detection rate characterized by the Quadratic Model is $LL(\omega, \alpha, \beta, \gamma|\mathbf{T}, \mathbf{K})$

$$= \sum_{i=1}^{n} k_i \log(\omega) + \sum_{i=1}^{n} k_i \log\left(e^{-(\alpha+\beta(t_{i-1}+\lambda t_{i-1}^2))} - e^{-(\alpha+\beta(t_{i-1}+\lambda t_i^2))}\right) - \omega\left(1 - e^{-(\alpha+\beta t_n+\lambda t_n^2)}\right) - \sum_{i=1}^{n} \log(k_i)! \quad (11)$$

with

$$\widehat{\omega} = \frac{\sum_{i=1}^{n} k_i}{1 - e^{-(\alpha+\beta t_n+\lambda t_n^2)}} \quad (12)$$

so that the reduced log-likelihood is $RLL(\alpha, \beta, \gamma|\mathbf{T}, \mathbf{K})$

$$= \sum_{i=1}^{n} k_i \log\left(\frac{\sum_{i=1}^{n} k_i}{1 - e^{-(\alpha+\beta t_n+\lambda t_n^2)}}\right) + \sum_{i=1}^{n} k_i \log\left(e^{-(\alpha+\beta(t_{i-1}+\lambda t_{i-1}^2))} - e^{-(\alpha+\beta(t_{i-1}+\lambda t_i^2))}\right) - \sum_{i=1}^{n} k_i - \sum_{i=1}^{n} \log(k_i)! \quad (13)$$

Traditionally, the Newton-Raphson method [20] was employed to identify the MLE, but may not converge when the initial estimates are not close to the maximum.

4.2 Initial Parameter Estimation

The EM algorithm [23] provides a systematic calculus-based method to identify initial parameter estimates for some or all parameters of a model. For a mean value function of the form specified in Equation (1), the observed number of faults are an initial estimate of the number of faults such $\omega^{(0)} = n$. Initial estimates of the remaining parameters (Θ) can be determined by maximizing the log-likelihood function of the probability density function

$$\Theta^{(0)} = \sum_{i=1}^{n} \frac{\partial}{\partial \Theta} \log[f(t_i; \Theta)] = 0 \quad (14)$$

and solving to obtain closed-form expressions.

For example, the initial parameter estimates of the SRGM with bathtub-shaped fault detection rate characterized by the Quadratic Model are

$$\alpha = 0 \quad (15)$$

$$\beta = \sum_{i=1}^{n} \frac{1}{t_i} - 2\gamma \sum_{i=1}^{n} t_i \quad (16)$$

$$\gamma = \sum_{i=1}^{n} \frac{1}{t_i^2} - \beta \sum_{i=1}^{n} \frac{1}{2t_i} \quad (17)$$

The parameters β and γ can be estimated in multiple ways. The first is to solve Equations (16) and (17) as a pair of simultaneous equations. An alternative is to substitute the closed form solution for β on the right-hand side of Equation

(16) into Equation (17), solving for γ, and then substituting the estimate of γ into Equation (16) to determine β. A second alternative substitutes γ into Equation (16) and proceeds in a similar manner.

4.3 Expectation Conditional Maximization Algorithm

This section provides a brief overview of the expectation conditional maximization algorithm [24], which is an extension of the EM algorithm that simplifies computation by dividing a single M-step into p conditional-maximization (CM)-steps, where p denotes the number of model parameters. The CM-steps are the partial derivatives of the loglikelihood function $\frac{\partial LL}{\partial \Theta_i}$ or reduced log-likelihood function $\frac{\partial RLL}{\partial \Theta_i}$. The ECM algorithm updates one parameter at a time holding all others constant, reducing the maximum likelihood estimation process to p distinct 1-dimensional problems. Thus, in each CM-step, the ECM algorithm searches a single dimension of the parameter space to improve the loglikelihood monotonically. Successive CM-steps determine $\Theta_i^{(j)}$, which is the updated value of the i^{th} parameter in the j^{th} iteration.

Without loss of generality, the CM-step which updates the i^{th} parameter in the j^{th} iteration takes

$$\Theta^{jp+i} = \left\langle \Theta_1^{(j+1)}, \Theta_2^{(j+1)}, \ \ldots, \Theta_{i-1}^{(j+1)}, \Theta_i^{(j)}, \ldots, \Theta_p^{(j)} \right\rangle \quad (18)$$

as input, holds all values but $\Theta_i^{(j)}$ constant, and maximizes the partial derivative of the log-likelihood or reduced loglikelihood function with respect to Θ_i to produce $\Theta^{jp+(i+1)}$ containing Θ_i^{j+1}. Each CM-step monotonically improves the maximum likelihood estimate. After applying the CM-step for each parameter, a convergence criterion such as

$$\left|RLL_j - RLL_{j-1}\right| < \varepsilon \quad (19)$$

is tested, where $\varepsilon > 0$ is an arbitrarily small constant. If satisfied, the ECM algorithm terminates. For example, the CM-steps of the SRGM with bathtub-shaped fault detection rate characterized by the Quadratic Model are computed from Equation (13).

$$\alpha = \sum_{i=1}^{n} k_i \frac{\left(e^{\alpha} - e^{-(\chi-\alpha)}\right)\left(\frac{e^{2\chi-\alpha}(e^{-\sigma} - e^{-\tau})}{(e^{\chi-1})^2}\right)}{e^{-\tau} - e^{-\sigma}} \quad (20)$$

$$\beta = \sum_{i=1}^{n} k_i \,(1 - e^{-\chi}) \frac{\frac{e^{-\sigma} t_i - e^{-\tau} t_{i-1}}{1 - e^{-\chi}} + \frac{e^{\chi}(e^{-\tau} + e^{-\sigma}) t_n}{(e^{\chi-1})^2}}{e^{-\tau} - e^{-\sigma}} \quad (21)$$

$$\gamma = \frac{\sum_{i=1}^{n} k_i \,(1 - e^{-\chi}) \left(\frac{e^{-\sigma} t_i^2 - e^{-\tau} t_{i-1}^2}{1 - e^{-\chi}} + \frac{e^{\chi}(e^{-\sigma} - e^{-\tau}) t_n^2}{(e^{\chi-1})^2}\right)}{e^{-\tau} - e^{-\sigma}} \quad (22)$$

where $\chi = \alpha + \beta t_n + \gamma t_n^2$, $\tau = \beta t_{i-1} + \gamma t_{i-1}^2$, $\sigma = \beta t_i + \gamma t_i^2$. Thus, when the CM-step for α in Equation (20) is applied all instances of β and γ in χ, τ, and σ are held constant at their most recent estimates and the expression is solved for α. Similarly, the CM-step for β in Equation (21) holds all instances of α and γ constant and solves for β.

5 MODEL ASSESSMENT

Model assessment evaluates how well a model performs on a data set. Two complementary measures are the Akaike Information Criterion (AIC) and Predictive Mean Square Error (PMSE). The AIC is an information theoretic method to compare multiple models on a single data set, while PMSE measures the disagreement between a model's predictions and future observations. Ideally, a single model will perform best on both measures. When no single best model emerges, the user must make a subjective decision based on factors such as amount of data, stage of testing, and predictive horizon.

5.1 Akaike Information Criterion

The Akaike Information Criterion is an information theoretic measure of a model's statistical goodness-of-fit to a dataset. It is grounded in the concept of entropy, offering a relative measure of the information lost when a given model is applied. The AIC quantifies the tradeoff between a model's characterization of the observed data and the model's complexity. The AIC of model i is a function of the maximized log-likelihood and the number of model parameters, p.

$$AIC_i = 2p - 2LL\left(\hat{\theta}\middle|\mathbf{T}, \mathbf{K}\right) \quad (23)$$

The term $2p$ is a penalty function, which deters a model with an unnecessary number of parameters that may fits the observed data well, but compromise its predictive power.

5.2 Predictive Mean Square Error

The k-step predictive mean square error of the mean value function measures a model's predictive power. It is calculated by fitting a model to the data in the first $n - k$ intervals and computing

$$PMSE = \sum_{i=(n-k)+1}^{n} (K_i - \widehat{m}\,(t_i))^2 \quad (24)$$

which is the sum of squares differences between the cumulative number of faults observed $K_i = \sum_{j=1}^{i} k_i$ and the cumulative faults predicted by the fitted mean value function ($\widehat{m}\,(t_i)$) for the last k observations not used to fit the model.

6 ILLUSTRATIONS

This section illustrates the application of the ECM algorithm to an SRGM with bathtub-shaped fault detection rate. A comparative analysis of bathtub-shaped fault detection rate models and their simplified forms is then performed to assess these models with respect to information theoretic and predictive measures of goodness of fit.

6.1 Quadratic LL-ECM Application

This first example explains how the ECM algorithm is applied in the context of the SRGM with bathtub-shaped fault detection rate characterized by the Quadratic Model on the PL/I data set [27], which consists of over 1.3 million lines of code and exhibited 328 faults over nineteen ($n = 19$) weeks of testing. As noted in Section IV, the EM algorithm provides closed form expressions for the initial value of the parameters.

Starting from the initial estimate of $\alpha^{(0)} = 0$ and solving Equations (16) and (17) produces initial estimates for the remaining parameters, including $\beta^{(0)} = 0.010203$ and $\gamma^{(0)} = 0.000954$. The initial value of the log-likelihood function specified in Equation (11) is therefore 1328.43. The first iteration applies Equation (20), holding β and γ constant and solving for $\alpha^{(1)} = 0.012757$, which increases the loglikelihood value to 1334.94. Successive CM-steps update γ and β with Equations (22) and (21) respectively.

Figure 2 shows the CM-steps in the β and γ parameters superimposed on a contour plot of the log-likelihood function

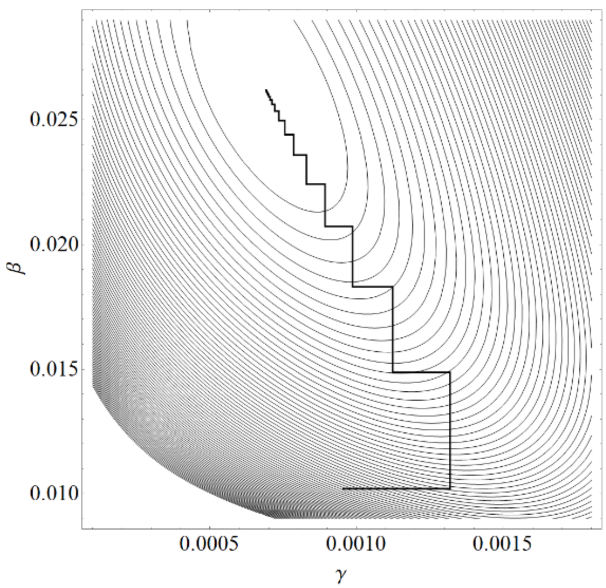

Figure 2 CM-steps superimposed on contour plot of log-likelihood function of SRGM with bathtub shaped fault detection rate

The 90° angle movements illustrate how only one parameter is updated at a time. The α parameter is also updated, but not shown here in order to present the process more clearly in two dimensions. Moreover, the value of α employed to produce the contour plot was the maximum likelihood estimate. Thus, the convergence of β and γ shown in Figure 2 is to the overall maximum likelihood estimate.

Figure 3 shows the monotonic improvements made by the ECM algorithm in each of the 44 iterations until convergence when Equation (19) is less than $\varepsilon = 10^{-15}$.

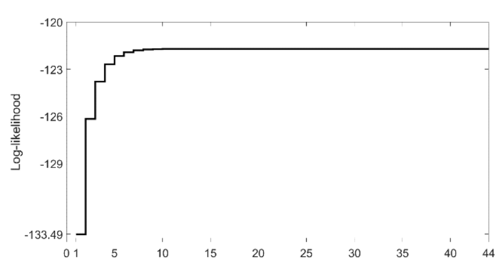

Figure 3 Improvement of Log-likelihood function in iterations of ECM algorithm for SRGM with bathtub-shaped fault detection rate

The resulting MLEs are $\hat{\alpha} = -0.024456$, $\hat{\beta} = 0.026199$, $\hat{\gamma} = 0.000691$, and the corresponding value of the log-likelihood value at these estimates is -133.494. Substituting the maximum likelihood estimates of α, β, and γ into Equation (12) produces the MLE of the initial number of faults $\hat{\omega} = 349.402$.

Figure 4 shows PL/I data as well as the plot of the mean value function produced by substituting the maximum likelihood estimates into Equation (4).

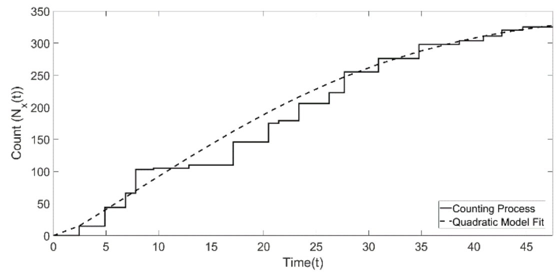

Figure 4 SRGM with bathtub-shaped fault detection rate characterized by Quadratic Model fit to PL/I data

6.2 Model Assessment

Table I summarizes the Akaike Information Criterion and the predictive sum of squares error computed with $k = 2$ of $n = 19$ weeks of data withheld from model fitting for each of the six bathtub-shaped distributions considered. Values in bold indicate models preferred by specific measures. The AIC prefers the SRGM with bathtub-shaped fault detection rate characterized by Lee's Weibull extension. However, PMSE prefers the SRGM with bathtub-shaped fault detection rate characterized by Lai's modified Weibull model.

Table 1: Goodness of fit of bathtub-shaped models

Model	AIC	PMSE
Quadratic	251.41	2439.23
Competing risk	430.43	2132.20
Modified Weibull (Lai)	250.12	476.23
Weibull extension (Lee)	248.33	1369.00
Goel-Okumoto	264.63	5821.29
Weibull	254.17	3013.01

7 CONCLUSIONS AND FUTURE RESEARCH

This paper proposed a framework for SRGM possessing a bathtub-shaped fault detection rate and derived stable and efficient expectation conditional maximization algorithms to fit these models. The illustrations compared multiple bathtub-shaped and classical models with respect to predictive and information theoretic measures. Our results indicated that SRGM possessing a bathtub-shaped fault detection rate outperformed classical models on both types of measures. The framework and models may therefore be a reasonable compromise between model complexity and predictive accuracy.

Future research will generalize the framework to accommodate additional bathtub-shaped distributions and seek to identify the causes of the bathtub shape such as test procedures and application architecture.

ACKNOWLEDGEMENT

This material is based upon work supported by the National Science Foundation under Grant Number (#1749635).

REFERENCES

1. Goel and K. Okumoto, "Time-dependent error-detection rate model for software reliability and other performance measures," IEEE Transactions on Reliability, vol. R-28, no. 3, pp. 206–211, Aug 1979.
2. N. Langberg and N. Singpurwalla, "A unification of some software reliability models," SIAM Journal on Scientific and Statistical Computing, vol. 6, no. 3, pp. 781–790, 1985.
3. D. Miller, "Exponential order statistic models of software reliability growth," *IEEE Transactions on Software Engineering*, vol. SE-12, no. 1, pp. 12–24, Jan 1986.
4. S. Yamada, H. Ohtera, and H. Narihisa, "Software reliability growth models with testing-effort," *IEEE Transactions of Reliability Engineering*, vol. R-35, no. 1, pp. 19–23, april 1986.
5. S. Gokhale, T. Philip, P. Marinos, and K. Trivedi, "Unification of finite failure non-homogeneous poisson process models through test coverage," in *Proceedings of International Symposium on Software Reliability Engineering*, Oct 1996, pp. 299–307.
6. S. Kuo, C. Huang, and M. Lyu, "Framework for modeling software reliability, using various testing-efforts and fault-detection rates," *IEEE Transactions on Reliability*, vol. 50, no. 3, pp. 310–320, 2001.
7. C. Huang, M. Lyu, and S. Kuo, "A unified scheme of some nonhomogenous poisson process models for software reliability estimation," *IEEE Transactions on Software Engineering*, vol. 29, no. 3, pp. 261– 269, 2003.
8. P. Kapur, H. Pham, S. Anand, and K. Yadav, "A unified approach for developing software reliability growth models in the presence of imperfect debugging and error generation," *IEEE Transactions on Reliability*, vol. 60, no. 1, pp. 331–340, March 2011.
9. S. Inoue and S. Yamada, "Generalized discrete software reliability modeling with effect of program size," *IEEE Transactions on Systems, Man, and Cybernetics - Part A: Systems and Humans*, vol. 37, no. 2, pp. 170–179, March 2007.
10. C. Lai and M. X. R. Barlow, *Stochastic Ageing and Dependence for Reliability*. Berlin, Heidelberg: Springer-Verlag, 2006.
11. R. Hou, S. Kuo, and Y. Chang, "Efficient allocation of testing resources for software module testing based on the hyper-geometric distribution software reliability growth model," in *Proceedings of International Symposium on Software Reliability Engineering, White Plains, NY, USA*, 1996, pp. 289–298.
12. S. Yamada and S. Osaki, "Reliability growth models for hardware and software systems based on nonhomogeneous poisson processes: A survey," *Microelectronics Reliability*, vol. 23, no. 1, pp. 91–112, 1983.
13. L. Fiondella and S. Gokhale, "Software reliability model with bathtubshaped fault detection rate," in *Proceedings of Annual Reliability and Maintainability Symposium*, Jan 2011, pp. 1–6.
14. S. Ross, *Introduction to Probability Models, 8th ed.*, New York, NY, USA, 2003.
15. L. Bain, "Analysis for the linear failure-rate life-testing distribution," *Technometrics*, vol. 16, no. 4, pp. 551–559, 1974.
16. A. Gore, S. Paranjape, M. Rajarshi, and M. Gadgil, "Some methods for summarizing survivorship data in nonstandard situations," *Biometrical Journal*, vol. 28, no. 5, pp. 577–586, 1986.
17. U. Hjorth, "A reliability distribution with increasing, decreasing, constant and bathtub-shaped failure rates," *Technometrics*, vol. 22, no. 1, pp. 99–107, 1980.
18. C. Lai, M. Xie, and D. Murthy, "A modified weibull distribution," *IEEE Transactions on Reliability*, vol. 52, pp. 33–37, 2003.
19. L. Lee, "Testing adequacy of the weibull and log linear rate models for a poisson process," *Technometrics*, vol. 22, no. 2, pp. 195–199, 1980.
20. R. Burden and J. Faires, *Numerical Analysis*, 4th ed., ser. The Prindle, Weber and Schmidt Series in Mathematics. Boston: PWS-Kent Publishing Company, 1989.
21. H. Okamura, Y. Watanabe, and T. Dohi, "An iterative scheme for maximum likelihood estimation in software reliability modeling," in *Proceedings of International Symposium on Software Reliability Engineering*, 2003, pp. 246–256.
22. X. Meng and D. Rubin, "Maximum likelihood estimation via the ECM algorithm: A general framework," *Biometrika*, vol. 80, no. 2, pp. 267– 278, 1993.
23. A. Dempster, N. Laird, and D. Rubin, "Maximum likelihood from incomplete data via the em algorithm," *Journal of the Royal Statistical Society. Series B (Methodological)*, vol. 39, no. 1, pp. 1–38, 1977.
24. V. Nagaraju, L. Fiondella, P. Zeephongsekul, C. Jayasinghe, and T. Wandji, "Performance optimized expectation conditional maximization algorithms for nonhomogeneous poisson process software reliability models," *IEEE Transactions on Reliability*, vol. 66, no. 3, pp. 722–734, 2017.
25. Y. Sakamoto, M. Ishiguro, and G. Kitagawa, *Akaike information criterion statistics*. Tokyo: KTK Scientific Publishers; Dordrecht; Boston: D. Reidel; Hingham, MA: Sold and distributed in the U.S.A. and Canada by Kluwer Academic Publishers, 1986.
26. S. Narula, "Predictive mean square error and stochastic regressor variables," *Journal of the Royal Statistical Society. Series C (Applied Statistics)*, vol. 23, no. 1, pp. 11–17, 1974.
27. M. Ohba, "Software reliability analysis models," *IBM Journal of Research and Development*, vol. 28, pp. 428–443, 1984.

BIOGRAPHIES

Maskura Nafreen
Department of Electrical and Computer Engineering
University of Massachusetts - Dartmouth
285 Old Westport Road
North Dartmouth, MA 02747, USA

e-mail: mnafreen@umassd.edu

Maskura Nafreen is a PhD student in the Department of Electrical and Computer Engineering at the University of Massachusetts Dartmouth (UMassD). She received her BS.c. (2018) in Electrical and Electronics Engineering from Ahsanullah University of Science and Technology in Bangladesh.

Lance Fiondella, PhD
Department of Electrical and Computer Engineering
University of Massachusetts - Dartmouth
285 Old Westport Road
North Dartmouth, MA 02747, USA

e-mail: lfiondella@umassd.edu

Lance Fiondella is an associate professor in the Department of Electrical & Computer Engineering at UMassD. He received his PhD (2012) in Computer Science & Engineering from the University of Connecticut. Dr. Fiondella's papers have been the recipient of twelve conference paper awards, including the 2015 R.A. Evans/P.K. McElroy Award from the Reliability and Maintainability Symposium (RAMS).