Design and Evaluation of an Augmented Reality App for Learning Spatial Transformations and their Mathematical Representations

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ABSTRACT

There is a close relation between spatial thinking and mathematical problem-solving. This paper presents a newly developed educational Augmented Reality (AR) mobile application, BRICKxAR/T, to help students intuitively learn spatial transformations and the related mathematics through play. A pilot study with 7 undergraduate students evaluates students learning gain through a mental rotation and a math test on transformation matrices. The results show most students performed better with a higher score after learning with the app. Students found the app interesting to play and useful for learning geometric transformations and matrices.

Index Terms: Applied computing—Education—Interactive learning Environments

1 INTRODUCTION

Literature confirms the difficulties of learning/teaching spatial transformations and their mathematical reasonings with conventional methods [1]. Hence, there is a demand for novel technologies to address the problems in learning spatial transformations and their mathematical representations, such as vectors and matrices. Augmented Reality (AR) technology, with the ability to realize the interplay between digital content of abstract information and physical models, enable students to learn a subject through embodied learning [2]. Studies show that contextualizing mathematical concepts in a concrete setting and providing embodied exercises will help students improve learning mathematics [3].

AR technology has been investigated in literature as a learning tool to help students understand geometry and mathematics [4]. However, there is a lack of research on AR features for advancing students' math skills regarding spatial transformations and for a better understanding of geometric modeling and its mathematical logic.

In this paper we demonstrate BRICKxAR/T, an educational AR mobile application developed by the authors [5], to facilitate learning spatial transformations and their corresponding mathematics in a "Learning through Play" environment. BRICKxAR/T intends to help students better understand the mathematical logics behind geometric modeling through playing with the parameters of transformation matrices. The project employs the AR features of integrated physical-virtual interplay, embodied learning, and visualization to reveal the dynamic relationship between physical motions and the corresponding abstract math information, for intuitive learning of spatial transformations. The goal of the study is to provide an AR environment for spatiotemporal experiments in which students learn knowledge of spatial transformations that can be applied to computer-aided design, computer graphics, computer vision, etc. We have utilized the AR environment to support embodied learning in a 3D spatial environment and to contextualize mathematical concepts through synching them with the physical motions in real-time.

In the developed app, a LEGO® model is used as a physical manipulative for students to transform in the physical environment. In the meantime, an accurate marker-based AR registration between the virtual and physical LEGO models is used to enable the interplay between the physical LEGO model and its registered virtual model [6].

2 PROTOTYPE

BRICKxAR/T is developed based on the progressive learning method for learning spatial transformations in three levels of 'motions, mappings and functions' [7]. Using an AR intervention, physical interaction (motion), along with physical and virtual models' interplay (mapping) are supported in the application. Graphical illustrations are displayed in the 3D AR environment, representing the mapping operations. Mathematical functions are contextualized in the AR environment showing a synchronized relationship among motions, mappings, and functions.

The preliminary prototype is an RTS (Rotation, Translation, and Scale) game on AR-enabled iOS device to learn mathematical concepts of spatial transformations and to understand mathematical components of the transformations, i.e., variables, parameters, and functions. The mathematical functions match with the actual motions in real-time. The graphical representations (distance lines with dimensions, rotation arcs with angles) assist students to perceive the geometric reasonings behind transformation matrices. In this application students can play with a physical LEGO model, transform (translate and rotate) it and observe the corresponding translation and rotation matrices (first row matrices in Figure 1).

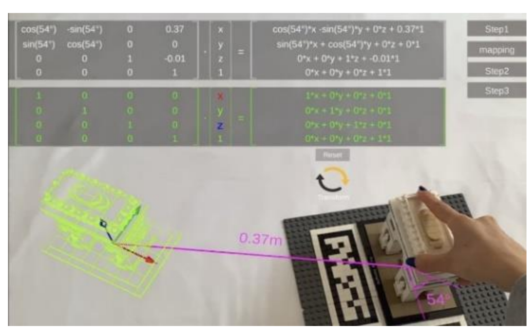

Figure 1: Transforming the physical model freely with hand in AR

While a coordinate system representation (axes) remains at the origin to visualize the relation between the pre-image (the virtual model) and image (the physical model) of the transformations, students can play with the virtual model and transform it using parameters of its matrix. The second-row matrices show transformations of the virtual model. Students can translate, rotate

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and scale the virtual model in x-, y- and z- axes through the corresponding sliders and track the changes in the corresponding matrices (Figure 2). The right-most vectors in both rows show the algebraic linear equations of matrix multiplication results.

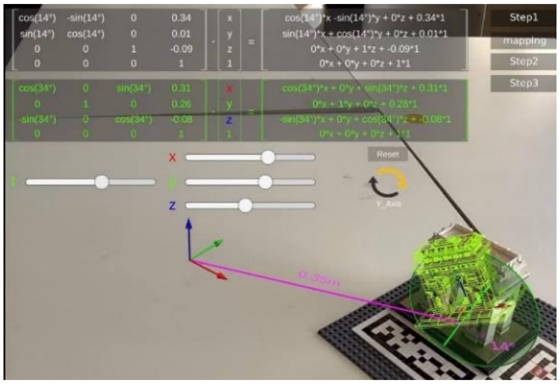

Figure 2: Student interacting with parameters of the matrix on the screen of the AR device (iPad) to transform the virtual model

3 Pilot User Study

We conducted a pilot user study on a group of 7 undergraduate students with at least college-level knowledge in algebra and geometry and moderate familiarity with AR. Students first took pre-tests on the Purdue Visualization of Rotation Test (PVRT) (20 questions), and a math test (13 questions) on transformation matrices designed based on the Khan Academy's learning materials. Within a 1- to 5-week time period, without disclosure of the tests' answers, students participated in remote workshops held through Zoom meetings due to the COVID-19 pandemic. Each student was provided with a LEGO set and an iPad with BRICKxAR/T installed. In the workshop, students watched a video lecture on geometric transformations and a demo on how to play with the app. Students then used BRICKxAR/T for 30- to 40-miniute to practice the described learning activities and then took the post tests (the PVRT and the math test) and a questionnaire. We evaluated students learning gains about spatial and math skills by comparing their scores in the pre- and post-sessions. The questionnaire and iPad screen recordings of students playing with the app are used for further analyses.

4 Analysis and Results

Due to the small sample size in the pilot study, data visualization and non-parametric statistical analysis are the focus of our evaluation (i.e., finding the difference between pre- and post-sessions' results). Figure 3 shows the students' scores in pre- and post-sessions for the PVRT (left) and math test (right).

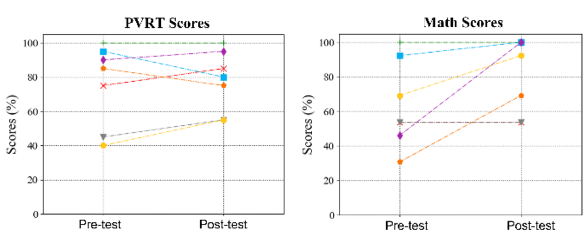

Figure 3: Students' scores in pre- and post- sessions in the PVRT (left) and math test (right).

Figure 3 illustrates that students' scores have improved in the post-sessions (shown as positive slopes) in most samples. The mean scores of the students in the PVRT ($mean_{pre\text{-}test}$ =75.7, $mean_{post\text{-}test}$ = 77.9) and math test ($mean_{pre\text{-}test}$ = 63.74, $mean_{post\text{-}test}$ = 81.32) have increased after the workshop, and more substantially in the math tests (27.59%) than PVRT (2.8%).

Our results showed that the students could answer math and, to a lesser extent, PVRT questions more correctly after completing the AR workshop, meaning that BRICKxAR/T might provide a way of improving students understanding and learning spatial transformations and their matrices. The results of the questionnaire showed that students were interested in playing with BRICKxAR/T and thought that the app was useful for learning transformation matrices. Most students claimed that they used visual imagery to visualize the concepts they learned in the workshop to answer the math test.

5 Observations and Conclusion

The screen recordings showed that students were seated and dedicated a limited space to play, hence, faced a lack of space to play with physical models although they were supposed to move around the physical models and explore the AR environment through different perspectives. Also, students played with the physical models only for a short period of time which could be due to the limitations of the AR registration method in this experiment (the markers need to be attached to the models). The researchers believe that most of the observed problems may improve in a face-to-face workshop, with better communication while dedicating more space and time to the workshop and improving the registration method in the future. This study reveals the potentially significant impacts of AR on understanding spatial transformations and their mathematical representations, which contribute to learning STEM (Science, Technology, Engineering, and Mathematics) disciplines. Our pilot study showed a promising result; however, a larger sample size and a comparison with a control group are required for more reliable statistical results.

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