

On Efficient Aggregation of Distributed Energy Resources

Zuguang Gao, Khaled Alshehri, and John R. Birge

Abstract—The rapid growth of distributed energy resources (DERs) is one of the most significant changes to electricity systems around the world. Examples of DERs include solar panels, small natural gas-fueled generators, combined heat and power plants, etc. Due to the small supply capacities of these DERs, it is impractical for them to participate directly in the wholesale electricity market. We study in this paper an efficient aggregation model where a profit-maximizing aggregator procures electricity from DERs, and sells them in the wholesale market. The interaction between the aggregator and the DER owners is modeled as a Stackelberg game: the aggregator adopts two-part pricing by announcing a participation fee and a per-unit price of procurement for each DER owner, and the DER owner responds by choosing her payoff-maximizing energy supplies. We show that our proposed model preserves full market efficiency, i.e., the social welfare achieved by the aggregation model is the same as that when DERs participate directly in the wholesale market.

I. INTRODUCTION

The increasing adoption of distributed energy resources (DERs) challenges fundamental assumptions in existing electricity markets design and operation, as end-consumers become proactive, and small-scale producers can have significant impacts on the power system [1]. The existence of DERs causes a fundamental shift, because electric power demand is largely assumed to be fixed from independent market operators' (ISOs) perspective [2]–[6]. This fundamental shift calls for the quantification of the impacts of DERs on wholesale markets design. Such quantification is not easy: ISOs do not have oversight over the distribution power network, and hence, cannot include DER owners as market participants. Furthermore, even if ISOs can oversee the distribution power system, it would be impractical to include DER supply directly into the wholesale electricity market operations through ISOs, due to the communicational, computational, and operational complexity.¹

Different models have been proposed to include DER supply into wholesale electricity markets [7]–[11]. The model

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¹The number of market participants would grow from hundreds to millions.

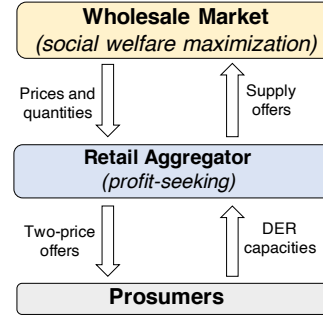


Fig. 1. Overall interactions in the proposed efficient aggregation model

we adopt here is to have an aggregating company whose role is to collect energy from DER owners, and participate in the wholesale market as a producer of electricity. See Fig. 1 for an illustration. This model has been adopted by California ISO and New York ISO [12], [13], and seems to be realistic for practical implementation, especially with FERC's recent Order No. 2222 in September, 2020 [14]. The aggregator here buys DER supply from their owners, and bids directly to the ISO, similar to generating companies. The relationship between the aggregator and prosumers² in the same geographical footprint is naturally monopolistic, where aggregators become price-making in retail electricity markets as they can send price offers to prosumers in order to collect DER supply. Such price offers need to be high-enough so that DER owners are attracted to sell, but small-enough so that the aggregator can maximize her profits. This profit-seeking behavior can impact the overall electricity market efficiency, but at the same time, due to the impracticality of direct DER participation into wholesale markets, aggregators are necessary and important players. This gives rise to the following important question: *In the presence of a profit-seeking and a monopolistic aggregator, is there an aggregation model that can attain a socially optimal (efficient) market outcome?* The presence of such a mechanism can in fact be significant. First, in reality, aggregators are mostly profit-seeking, and often monopolistic, which makes the markets prone to efficiency losses as demonstrated in [15], [16]. Second, it is infeasible for DER owners to participate in wholesale markets, so the presence of such intermediaries is inevitable. Third, if such a mechanism can be designed, it would address various debates surrounding whether or not DER aggregation need to be done by profit-making entities, or social intermediaries.

In this paper, we address the above question by propos-

²In this paper, we refer to consumers of electricity in the distribution grid who can sell their DER supply as prosumers.

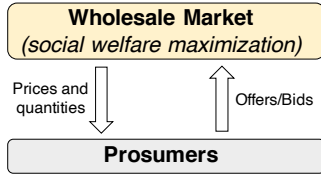


Fig. 2. Benchmark model

ing a DER aggregation model that yields efficient market outcomes, even when a DER aggregator $\mathcal{A}$ is monopolistic and profit-seeking. We show that with our proposed model, the optimal DER capacities being integrated through $\mathcal{A}$ are equivalent to a benchmark (ideal, not realistic) case in which DER owners can directly participate in the wholesale market.

The paper is organized as follows. We discuss the benchmark model in Section II. We then propose an efficient DER integration model, and state the main results of the paper in Section III. The paper concludes in Section IV. Due to space limitations, proofs and illustrative examples are omitted but can be found in an extended version of this paper [17].

Notation: We let $[z]^+ := \max\{z, 0\}$. We use boldface to distinguish a vector $\mathbf{x}$ from a scalar x . For an event $\mathcal{X}$, we let $\mathbb{1}\{\mathcal{X}\}$ denote the indicator function.

II. DIRECT PROSUMER PARTICIPATION MODEL

In this section, we introduce an ideal model where prosumers can participate directly in the wholesale market. This model, though not realistic, serves as a benchmark for evaluating the market efficiency of our following models. There are three parties in this model: prosumers who can buy and sell energy directly in the wholesale market, conventional generators who generate and sell electricity in the wholesale market, and an independent system operator (ISO) who clears the wholesale market. Fig. 2 provides an illustration.

A. Prosumer's Problem

Consider a power network with n nodes (locations). At each location k , there are n_k number of prosumers. We assume that each prosumer i at location k can purchase energy at the wholesale price λ^k . Furthermore, prosumer i at location k is endowed with a capacity $C_i^k \geq 0$ of power production.³ The power can be consumed locally by prosumer i or sold (partially) back to the wholesale market, again at the wholesale price λ^k . The prosumer observes λ^k and decides the amount of energy to buy/sell. Let u_i^k be prosumer i 's utility of power consumption. We make the following assumption on prosumers' utility of consumption.

Assumption 1. Each prosumer's utility of consumption u_i^k is increasing, strictly concave, and differentiable. Furthermore, the domain of u_i^k is $[0, Z]$ where $Z > C_i^k$ is some (large) upper bound of the amount of energy a prosumer can consume. We assume that $\frac{\partial u_i^k(z)}{\partial z} \rightarrow \infty$ as $z \rightarrow 0$, and $\frac{\partial u_i^k(z)}{\partial z} \rightarrow 0$ as $z \rightarrow Z$.

³ C_i^k can be zero for consumers with no DER supply, so the model includes those regular consumers.

We can then write prosumer i 's optimization problem as

$$\begin{aligned} \max_{z_i^k} \pi_i^k(z_i^k) &:= \lambda^k z_i^k + u_i^k(C_i^k - z_i^k) \\ \text{s.t.} \quad &C_i^k - Z \leq z_i^k \leq C_i^k, \end{aligned} \quad (1)$$

where z_i^k is the *net* amount of energy prosumer i sells. Since prosumers at location k can buy and sell energy at the same price λ^k , prosumer i essentially has z_i^k as the single decision to make in solving (1). For selling z_i^k at the wholesale price, prosumer i receives $\lambda^k z_i^k$; for buying $(-z_i^k)$ at the wholesale price, she is charged $\lambda^k(-z_i^k)$. The prosumer leaves with $C_i^k - z_i^k$ to consume and her utility from consumption would be $u_i^k(C_i^k - z_i^k)$. While Assumption 1 further imposes strict concavity of prosumers' utilities, our analysis throughout this paper remains largely applicable to generic concave utilities, but strict concavity allows us to derive unique analytical solutions and gain deep insights.

Lemma 1. Under Assumption 1, given any λ^k , there exists a unique optimal solution z_i^{k*} for the prosumer's problem (1) which satisfies

$$\left. \frac{\partial u_i^k(z)}{\partial z} \right|_{z=C_i^k - z_i^{k*}} = \lambda^k. \quad (2)$$

The above Lemma directly follows from the properties of problem (1). We denote by x_i^k the actual amount of energy prosumer i sells, and by d_i^k the actual amount of energy prosumer i buys. Without loss of generality, and for ease of exposition, we restrict our attention to the case in which at most one of x_i^k and d_i^k can be nonzero. We first solve (1) for z_i^{k*} . Then, we let $x_i^{k*} = [z_i^{k*}]^+$ and $d_i^{k*} = [-z_i^{k*}]^+$. We also use the notation $z_i^{k*}(\lambda^k)$ and $d_i^{k*}(\lambda^k)$ to denote the optimal response of prosumer i at location k for a given wholesale market price λ^k .

B. Generator's Problem

Let N_k denote the number of dispatchable conventional generators at location k . Generator j at location k chooses to supply $y_j^k \in [\underline{y}_j^k, \bar{y}_j^k]$. Let $c_j^k(y_j^k)$ denote the production cost. We have the following assumption.

Assumption 2. Each generator's cost function c_j^k is increasing, strictly convex, and differentiable in $[\underline{y}_j^k, \bar{y}_j^k]$. We let $\frac{\partial c_j^k(y_j^k)}{\partial y_j^k} \rightarrow 0$ as $y_j^k \rightarrow \underline{y}_j^k$ and $\frac{\partial c_j^k(y_j^k)}{\partial y_j^k} \rightarrow \infty$ as $y_j^k \rightarrow \bar{y}_j^k$.

By selling y_j^k , generator j earns a compensation $\lambda^k y_j^k$. Given a wholesale price λ^k , generator j maximizes its payoff by solving

$$\max_{y_j^k \in [\underline{y}_j^k, \bar{y}_j^k]} \hat{\pi}_j^k(y_j^k) := \lambda^k y_j^k - c_j^k(y_j^k). \quad (3)$$

Lemma 2. Under Assumption 2, given any λ^k , there exists a unique optimal solution y_j^{k*} for the generator's problem (3) which satisfies

$$\left. \frac{\partial c_j^k(y_j^k)}{\partial y_j^k} \right|_{y_j^k=y_j^{k*}} = \lambda^k. \quad (4)$$

The above Lemma directly follows from the properties of problem (3). We also use the notation $y_j^{k*}(\lambda^k)$ to denote the optimal response of generator j at location k at a given wholesale price λ^k .

C. The Economic Dispatch Problem

Many wholesale electricity markets in the United States and other countries are managed by independent system operators (ISOs) [18]. An ISO clears the market by matching supply and demand via social welfare maximization (in practice, this is often done by production cost minimization to meet fixed system demands [3]–[5]), while ensuring that the power flows satisfy the network and line capacity constraints. Specifically, let $X^k = \sum_{j \in [n_k]} x_j^k$ be the total power supply at node k from prosumers; let $Y^k = \sum_{j \in [N_k]} y_j^k$ be the total power supply at node k from conventional generators; and let $D^k = \sum_{j \in [n_k]} d_j^k$ be the total demand at node k . Furthermore, we let $\mathbf{B}\mathbf{h} \leq \mathbf{f}$ be the network constraints that are resolved from the DC approximation of the AC network, where $\mathbf{f}$ is the vector of capacities of transmission lines in the power network, and the system operator chooses a vector $\mathbf{h}$, where each element h^k is the net injection to node k . We have the following network constraints:

$$\mathbf{h} = \mathbf{D} - \mathbf{Y} - \mathbf{X}, \quad (5a)$$

$$\mathbf{1}^T \mathbf{h} = 0, \quad \mathbf{B}\mathbf{h} \leq \mathbf{f}. \quad (5b)$$

We note that (5a) ensures the total supply matches the total demand at each node; (5b) ensures that the total net injection by the system operator is zero over the power network (here, $\mathbf{1}$ is a vector of ones), and the total power transmission at each line does not exceed its capacity. In addition to the network constraints, the ISO needs to also consider all participant-specific constraints described earlier in problems (1) and (3):

$$\mathbf{C} - \mathbf{Z} \leq \mathbf{x} - \mathbf{d} \leq \mathbf{C}, \quad \underline{\mathbf{y}} \leq \mathbf{y} \leq \bar{\mathbf{y}} \quad (6)$$

The objective of the system operator is to maximize the social welfare, which includes the prosumer surplus (PS), generator surplus (GS), and merchandizing surplus (MS):

$$\begin{aligned} \text{PS} &:= \sum_{k \in [n]} \left(\sum_{i \in [n_k]} u_i^k(d_i^k - x_i^k + C_i^k) - \lambda^k(h^k + Y^k) \right), \\ \text{GS} &:= \sum_{k \in [n]} \sum_{j \in [N_k]} (\lambda^k y_j^k - c_j^k(y_j^k)), \quad \text{MS} := \sum_{k \in [n]} \lambda^k h^k, \end{aligned}$$

where we have imposed the relationship (5a) in deriving PS. The social welfare that the system operator optimizes is the sum of PS, GS, and MS. After canceling terms, the social welfare can be written as

$$\mathcal{W}_B := \sum_{k \in [n]} \left(\sum_{i \in [n_k]} u_i^k(d_i^k + C_i^k - x_i^k) - \sum_{j \in [N_k]} c_j^k(y_j^k) \right).$$

The system operator's economic dispatch problem is then:

$$\begin{aligned} \max \quad & \mathcal{W}_B(\mathbf{h}, \mathbf{x} - \mathbf{d}, \mathbf{y}) \\ \text{subject to} \quad & (5) - (6). \end{aligned} \quad (8)$$

Assumption 3. *The system operator's economic dispatch problem (8) is feasible.*

Proposition 1 (Competitive Equilibrium). *Under Assumptions 1-3, there exists a unique optimal solution $(\mathbf{h}^*, (\mathbf{x} - \mathbf{d})^*, \mathbf{y}^*)$ to (8). Denote the optimal Lagrange multipliers of constraints (5a) by $\boldsymbol{\lambda}$. Then, the following statements are true:*

- $(\mathbf{x} - \mathbf{d})^*$ and $\boldsymbol{\lambda}$ satisfy (2).
- $\mathbf{y}^*$ and $\boldsymbol{\lambda}$ satisfy (4).

The above proposition states that solving the system operator's problem (8) leads to a competitive equilibrium. From the perspective of prosumer i at node k , this means that given the wholesale market price λ^k (the optimal Lagrange multiplier of (5a) for the same node), the corresponding solution to her problem, which satisfies (2), is the same as the optimal decision made by the ISO via solving (8). This is also true for all other prosumers and generators. Having all market participants being satisfied with the competitive equilibrium, it serves as a good benchmark for market efficiency.

III. EFFICIENT AGGREGATION MODEL

The direct participation model introduced in the previous section is a benchmark: prosumers are allowed to participate directly and sell their production in the wholesale market. In reality, the supply capacities of prosumers are typically too small for consideration in the wholesale market.⁴ Also, computing the dispatch and settlement for a large number of prosumers raises a computational burden on the system operator. The presence of DER aggregators brings benefits to the system, as they open the door for DER owners to participate and bring more flexibility to the grid. However, the profit-seeking nature of these aggregators can cause efficiency losses [15], [16]. To resolve this, we propose in this section an efficient aggregation model under two-part pricing. In this model, prosumers sell part of their DER supply productions to an aggregator $\mathcal{A}$, based on the price offers made by $\mathcal{A}$. The interactions between $\mathcal{A}$ and prosumers are modeled as a Stackelberg game [20]. The aggregator acts as a leader and announces a price pair (P_i^k, p_i^k) for each prosumer i at location k . The prosumer follows by choosing the amount of energy to sell. If the prosumer decides to sell a nonzero fraction of her capacity to the aggregator, she pays to the aggregator a participation fee P_i^k , and earns the price p_i^k for each unit of energy sold. The aggregator $\mathcal{A}$ then sells these procured capacity to the wholesale market at the wholesale price λ^k . The goal of the aggregator $\mathcal{A}$ is to choose prices (P_i^k, p_i^k) that maximize her profit, while anticipating how DER owners would respond. Note that DER owners only have access to one aggregator, so $\mathcal{A}$ is in fact monopolistic, which further signifies the importance of our mechanism as it yields socially-optimal outcomes.

We also note that differential pricing is allowed in this model, i.e., the price pair (P_i^k, p_i^k) can be set differently for

⁴California ISO requires a minimum of 0.5MW for a DER aggregator to participate [19], and New York ISO requires 0.1MW.

different prosumers. While it is reasonable to have varying prices depending on locations [21], there exist some debates on whether prices should be allowed to set differently for prosumers from the same location. We refer to [22] for alternative aggregation models with uniform pricing policies. In this paper, though we assumed differential pricing in the model, the equilibrium we obtain in the end has the same marginal price at each location, that is, $p_i^k = \lambda^k$, $\forall i \in [n_k]$. The participation fee can be differentiated by, for example, mailing different coupons to different prosumers to encourage their participations, which is arguably more justifiable. In the remainder of this section, we show that this aggregation model achieves the same socially-optimal market outcomes as in the direct participation model.

A. Prosumer's Problem

Consider prosumer i at location k . Upon seeing the prices (P_i^k, p_i^k) announced by $\mathcal{A}$, prosumer i decides if she would sell part of her capacity to $\mathcal{A}$. If she chooses so, she would pay a fee P_i^k to $\mathcal{A}$, and receives p_i^k for each unit of energy sold. We may write prosumer i 's payoff as

$$\pi_i^k(x_i^k, d_i^k) := \begin{cases} p_i^k x_i^k - P_i^k + u_i^k(d_i^k + C_i^k - x_i^k) - \lambda^k d_i^k, & \text{if } x_i^k > 0, \\ u_i^k(d_i^k + C_i^k) - \lambda^k d_i^k, & \text{if } x_i^k = 0, \end{cases}$$

Given $(P_i^k, p_i^k, \lambda^k)$, prosumer i solves:

$$\max_{x_i^k \in [0, C_i^k], d_i^k \in [0, Z - C_i^k + x_i^k]} \pi_i^k(x_i^k, d_i^k), \quad (9)$$

where d_i^k is the amount of energy prosumer i purchases at wholesale market price λ^k ,⁵ and x_i^k is the amount of energy she sells to the aggregator. For buying d_i^k at the wholesale price, prosumer i is charged $\lambda^k d_i^k$. If the prosumer does not sell ($x_i^k = 0$), she has a total of $d_i^k + C_i^k$ to consume, and her utility from consumption would be $u_i^k(d_i^k + C_i^k)$. If the prosumer chooses to sell $x_i^k > 0$ to $\mathcal{A}$, she is charged a participation fee P_i^k , and receives a compensation $p_i^k x_i^k$. The prosumer would have $d_i^k + C_i^k - x_i^k$ to consume in this case and her utility from consumption would be $u_i^k(d_i^k + C_i^k - x_i^k)$.

Let $x_i^{k*}(P_i^k, p_i^k, \lambda^k)$ and $d_i^{k*}(P_i^k, p_i^k, \lambda^k)$ denote the optimal response of prosumer i given aggregator's announced prices (P_i^k, p_i^k) and the wholesale market price λ^k .⁶ Note that if $p_i^k > \lambda^k$, the prosumer can arbitrage by buying at the price λ^k and selling at a higher price p_i^k . This will result in the prosumer earning infinite payoff and aggregator losing infinite profit, which would be avoided by the aggregator. Therefore, we may without loss of generality restrict our discussions to the case when $p_i^k \leq \lambda^k, \forall k \in [n], i \in [n_k]$. In the case $p_i^k = \lambda^k$, we may enforce that x_i^k and d_i^k cannot

⁵In real markets, end-use customers are represented in the wholesale market via utility companies. Here, for ease of exposition, we omit the analysis of the relationship between utility companies and end-use customers and assume that they represent the true preferences of their customers in the wholesale market without modifications. This allows us to focus on the strategic aggregation component of the problem. Nevertheless, our insights remain largely applicable if this assumption is relaxed.

⁶Sometimes, we drop arguments for simplicity.

both be nonzero. We then have the following lemma on the optimal response of prosumers.

Lemma 3. Consider an arbitrary prosumer i at location k . Let (z_1, z_2) be such that

$$\left. \frac{\partial u_i^k(z)}{\partial z} \right|_{z=z_1} = \lambda^k, \quad \left. \frac{\partial u_i^k(z)}{\partial z} \right|_{z=z_2} = p_i^k. \quad (10)$$

Then, under Assumption 1, both (z_1, z_2) exist and are unique. Furthermore, prosumer i 's optimal response can be described as follows.

- If $C_i^k \leq z_2$, then, we have $d_i^{k*} = [z_1 - C_i^k]^+$, $x_i^{k*} = 0$.
- If $C_i^k > z_2$, we have $x_i^{k*} = (C_i^k - z_2) \cdot \mathbb{1}\{\mathcal{X}\}$, $d_i^{k*} = 0$, where $\mathcal{X} := \{P_i^k \leq p_i^k (C_i^k - z_2) + u_i^k(z_2) - u_i^k(C_i^k)\}$.

The above Lemma states that if the capacity exceeds a certain value z_2 (note that z_2 is the value at which the marginal utility of consumption is equal to the aggregator's marginal price offer p_i^k) and the upfront fee P_i^k is not too high, then prosumers have an incentive to sell DER supply to the aggregator. If the capacity is too small or the upfront fee is too high, then prosumers would not sell DER supply and prefer to consume it locally.

B. Aggregator's Problem

The DER aggregator $\mathcal{A}$ collects power from prosumers and sells it in the wholesale market. By offering the prices (P_i^k, p_i^k) to each prosumer i at location k , $\mathcal{A}$ procures a total capacity of $\sum_{i \in [n_k]} x_i^{k*}(P_i^k, p_i^k)$ from location k . $\mathcal{A}$ then sells it at the wholesale market price λ^k . Given the wholesale market price λ^k , the aggregator's profit from prosumer i at location k is

$$\Pi_i^k(P_i^k, p_i^k) := P_i^k \mathbb{1}\{x_i^{k*}(P_i^k, p_i^k) > 0\} + (\lambda^k - p_i^k) x_i^{k*}(P_i^k, p_i^k).$$

Anticipating the response functions $x_i^{k*}(P_i^k, p_i^k)$, she seeks to maximize her overall profit:

$$\max_{\mathbf{P} \geq \mathbf{0}, \mathbf{p} \geq \mathbf{0}} \sum_{k \in [n]} \sum_{i \in [n_k]} \Pi_i^k(P_i^k, p_i^k), \quad (11)$$

Aggregator $\mathcal{A}$'s profit is composed of two parts; the total participation fees charged to prosumers who sell positive amount of energy and the profits earned by reselling the procured energy in the wholesale market. It should be clear that the Aggregator's problem (11) can be decomposed to optimizing (P_i^k, p_i^k) for each prosumer at each location:

$$\max_{P_i^k \geq 0, p_i^k \geq 0} \Pi_i^k(P_i^k, p_i^k). \quad (12)$$

Solving (12) for each $k \in [n]$ and $i \in [n_k]$ would lead to the vectors $(\mathbf{P}^*, \mathbf{p}^*)$, which constitute an optimal solution for (11). We have the following result on the optimal decisions of the aggregator.

Lemma 4. Under Assumption 1, consider an arbitrary prosumer i at location k , with z_1 as in (10), and wholesale

market price λ^k , When $z_1 < C_i^k$, the aggregator's optimal pricing decision is

$$p_i^{k*} = \lambda^k, \quad (13a)$$

$$P_i^{k*} = \lambda^k(C_i^k - z_1) + u_i^k(z_1) - u_i^k(C_i^k). \quad (13b)$$

When $z_1 \geq C_i^k$, prosumer i will not sell DER supply to $\mathcal{A}$, i.e., $\Pi_i^k(P_i^k, p_i^k) = 0$, for any $(P_i^k, p_i^k) \in \mathbb{R}_+^2$.

Lemma 4 provides an optimal solution (P_i^{k*}, p_i^{k*}) to (12), and the collection $(\mathbf{P}^*, \mathbf{p}^*)$ form an optimal solution to (11). We also note that the optimal solution may not be unique in general: $\mathcal{A}$ may deviate from (13) by further decreasing p_i^{k*} and increasing P_i^{k*} to earn the same profit, while keeping the response of the prosumer i being unchanged. This lemma states that there is an optimal pricing scheme which sets the marginal price p_i^k to be the wholesale market price λ^k , and all prosumers at the same location will be offered the same $p_i^k = \lambda^k$. The participation fees P_i^k , however, will be charged differently for prosumers with different utility functions. A keen reader would observe that in view of Lemma 1 and Lemma 3, the optimal DER supply to the wholesale market is the same with and without the aggregator $\mathcal{A}$, thus making this aggregation model economically efficient with socially optimal market outcomes.

C. Aggregator-Prosumers Interaction as a Stackelberg Game

Let $\mathcal{G}(\lambda)$ denote the Stackelberg game among the aggregator and the prosumers for a given vector of wholesale market prices λ . Aggregator $\mathcal{A}$ acts as a Stackelberg leader and sets the prices $(\mathbf{P}, \mathbf{p})$. The prosumer follows by responding with $x_i^{k*}(P_i^k, p_i^k, \lambda^k)$. We now define the equilibrium of the game.

Definition 1. $(\mathbf{P}^*, \mathbf{p}^*, \mathbf{x}^*(\mathbf{P}^*, \mathbf{p}^*))$ constitutes a Stackelberg equilibrium of the game $\mathcal{G}(\lambda)$ if

- Prosumers: For any $x_i^k \in [0, C_i^k]$, we have that

$$\pi_i^k(x_i^{k*}(P_i^k, p_i^k), P_i^k, p_i^k, \lambda^k) \geq \pi_i^k(x_i^k, P_i^{k*}, p_i^{k*}, \lambda^k),$$

for all $k \in [n]$ and $i \in [n_k]$.

- Aggregator: For all $\mathbf{P} \geq \mathbf{0}, \mathbf{p} \geq \mathbf{0}$, we have that

$$\hat{\Pi}(\mathbf{P}^*, \mathbf{p}^*, \mathbf{x}^*(\mathbf{P}^*, \mathbf{p}^*), \lambda) \geq \hat{\Pi}(\mathbf{P}, \mathbf{p}, \mathbf{x}^*(\mathbf{P}, \mathbf{p}), \lambda).$$

We then have the following Stackelberg equilibrium for the game $\mathcal{G}(\lambda)$, which follows directly from the prosumer's optimal response (Lemma 3) and the aggregator's optimal pricing (Lemma 4).

Proposition 2. Under Assumption 1, the game $\mathcal{G}(\lambda)$ admits a Stackelberg equilibrium that satisfies

$$P_i^{k*} = [\lambda^k(C_i^k - z_i^k) + u_i^k(z_i^k) - u_i^k(C_i^k)]^+, \quad (14a)$$

$$p_i^{k*} = \lambda^k, \quad (14b)$$

$$x_i^{k*}(P_i^{k*}, p_i^{k*}) = [C_i^k - z_i^k]^+, \quad (14c)$$

for each prosumer i at each location k , where z_i^k satisfies $\frac{\partial u_i^k(z)}{\partial z} \Big|_{z=z_i^k} = \lambda^k$, and $d_i^k = [z_i^k - C_i^k]^+$.

We note that the game $\mathcal{G}(\lambda)$ may admit other Stackelberg equilibria, but Proposition 2 provides the most economically efficient one. In case of non-uniqueness, this economically efficient equilibrium corresponds to the case where prosumers slightly prefer participation, i.e., if selling x_i^k amount of energy earns the prosumer i the same π_i^k as not selling, then she chooses to sell this x_i^k .⁷ The above equilibrium is quite intuitive, $\mathcal{A}$ passes the location marginal price λ^k obtained from wholesale market outcomes as is to prosumers, so $\mathcal{A}$ has no marginal profits from $\mathbf{p}^*$. Instead, $\mathcal{A}$ makes all of the profits from the upfront participation fees $\mathbf{P}^*$.

D. Generator's Problem

For a given wholesale market price λ^k , the conventional generators solve the same problem as described in Section II-B, and the result of Lemma 2 still applies under the current aggregation model.

E. The Economic Dispatch Problem

The system operator solves an optimization problem similar to that in the direct participation model as described in Section II-C. The network constraints (5) remain valid. Besides, the ISO also considers participant-specific constraints:

$$\mathbf{0} \leq \mathbf{x} \leq \mathbf{C}, \quad \mathbf{0} \leq \mathbf{d} \leq \mathbf{Z} - \mathbf{C} + \mathbf{x}, \quad \underline{\mathbf{y}} \leq \mathbf{y} \leq \bar{\mathbf{y}} \quad (15)$$

The objective of the system operator is to maximize the social welfare, which now includes the prosumer surplus (PS), aggregator surplus (AS), generator surplus (GS), and merchandizing surplus (MS):

$$\begin{aligned} \text{PS} &:= \sum_{k \in [n]} \sum_{i \in [n_k]} \left(u_i^k(d_i^k - x_i^k + C_i^k) - \lambda^k d_i^k \right. \\ &\quad \left. + p_i^k x_i^k - P_i^k \mathbb{1}\{x_i^k > 0\} \right) \\ \text{AS} &:= \sum_{k \in [n]} \sum_{i \in [n_k]} \left(P_i^k \mathbb{1}\{x_i^k > 0\} + \lambda^k x_i^k - p_i^k x_i^k \right) \\ \text{GS} &:= \sum_{k \in [n]} \sum_{j \in [N_k]} (\lambda^k y_j^k - c_j^k(y_j^k)), \quad \text{MS} := \sum_{k \in [n]} \lambda^k h^k. \end{aligned}$$

The social welfare is the sum of the above four terms. By the supply-demand balance $\mathbf{h} = \mathbf{D} - \mathbf{Y} - \mathbf{X}$, and after canceling terms, we write the social welfare as

$$\mathcal{W}_A := \sum_{k \in [n]} \left(\sum_{i \in [n_k]} u_i^k(d_i^k + C_i^k - x_i^k) - \sum_{j \in [N_k]} c_j^k(y_j^k) \right),$$

which is the same as $\mathcal{W}_B$. The system operator's economic dispatch problem is then:

$$\begin{aligned} &\max \quad \mathcal{W}_A(\mathbf{h}, \mathbf{x}, \mathbf{d}, \mathbf{y}) \\ &\text{subject to} \quad (5), (15). \end{aligned} \quad (17)$$

Assumption 4. The system operator's economic dispatch problem (17) is feasible.

⁷Alternatively, one can impose a slight perturbation of P_i^{k*} to $P_i^{k*} - \epsilon$, where $\epsilon > 0$, to ensure maximum DER supply.

As the system operator solves (17), the wholesale market prices λ are given by the optimal Lagrange multiplier of the constraint (5a). We then have the following proposition.

Proposition 3 (Competitive Equilibrium). *Under Assumptions 1, 2, 4, there exists an optimal solution $(\mathbf{h}^*, \mathbf{x}^*, \mathbf{d}^*, \mathbf{y}^*)$ to (17). Let λ denote the corresponding optimal Lagrange multipliers of constraints (5a). Then, the following statements are true:*

- $(\mathbf{x}^*, \mathbf{d}^*)$ are consistent with Lemma 3, given $(\mathbf{P}^*, \mathbf{p}^*)$ and λ .
- $(\mathbf{P}^*, \mathbf{p}^*)$ are consistent with Lemma 4, given $\mathbf{x}^*$ and λ .
- $\mathbf{y}^*$ satisfies (4), given λ .

We now present the following theorem, which states that our proposed aggregation model achieves the same market efficiency as the benchmark direct participation model.

Theorem 1 (Main Result). *Let $\mathcal{W}_A^*$ be the optimal social welfare of (17), and let $\mathcal{W}_B^*$ be the optimal social welfare of (8). Then, we have that $\mathcal{W}_A^* = \mathcal{W}_B^*$. Further, we have that the optimal $\mathbf{x}^*, \mathbf{d}^*, \mathbf{y}^*$ (from Proposition 3) solving (17) are the same as those solving (8).*

In summary, under the proposed aggregation model, the aggregator procures energy from prosumers using two-part pricing, the aggregator would optimally pay the wholesale market price to the prosumers for each unit of energy procured, while the participation fee is differently charged to each prosumer as the additional consumer surplus when she sells those energy compared with not selling. Theorem 1, together with Proposition 3, implies that under the aggregator's two-part differential pricing scheme, the prosumers' optimal decisions are exactly the same as those in the direct participation model. As a result, the social welfare achieved under the aggregation model matches that of the direct participation model, and there is no loss of market efficiency.

Remark 1. *The significance of Theorem 1 follows from the fact that via our proposed aggregation model, DER aggregation through a profit-seeking aggregator $\mathcal{A}$ is equivalent to solving a socially-optimal economic dispatch model where $\mathcal{A}$ is absent. Hence, the potential efficiency loss due to presence of a monopolistic profit-seeking aggregator $\mathcal{A}$ is off-set by two-part pricing. This is not possible via one-part pricing, as demonstrated in [15], [16], where efficiency loss arises from the profit-seeking behavior of $\mathcal{A}$.*

IV. CONCLUSIONS

To include DER capacities in the wholesale electricity market, we proposed an efficient DER aggregation model. The aggregator announces a two-part price offer to each prosumer (DER owner): a participation fee and a per-unit price paid to the prosumer for the energy procured. This interaction is modeled as a Stackelberg game, which admits an efficient equilibrium. Under this equilibrium, the per-unit price is equal to the wholesale market price, and prosumers choose to sell the same amount of energy as if they were participating directly in the wholesale market (ideal case). As

a result, the social welfare achieved under this aggregation model is the same as the social welfare achieved when prosumers can sell directly, i.e., there is no efficiency loss even with a monopolistic aggregator.

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