Precision-Driven Partial Ambiguity Resolution Technique for Short to Medium Baseline Positioning

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Abstract—**GNSS carrier phase observations are fundamental for safety-critical applications where the requirements of accuracy and availability are stringent. Upon the deployment of new GNSS constellation and frequencies, the large number of observations can lead to a decreased probability of successfully mapping the real-valued carrier phase ambiguities to integer ones. Partial Ambiguity Resolution (PAR) relaxes the condition of fixing the complete set of ambiguities and finds instead the subset which maximizes the success rate or grants a low failure rate. This work introduces Precision-Driven (PD) PAR, a technique for which a constraint on the formal precision of the fixed solution is added, and the subset selection is realized based on the projection of the ambiguities into the position domain. The performance characterization is realized on a synthetic scenario, where observations from a triple-frequency, triple-constellation are employed and the estimation is realized on a snapshot (memoryless) manner.**

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1. INTRODUCTION

Global Navigation Satellite Systems (GNSS) has established as backbone for the provision of navigation and timing information for multiple applications. GNSS reliant services include power grid distribution or emergency response [1], as well as prospective autonomous vehicles [2], [3], [4]. Especially for the latter, the availability of precise and reliable positioning has become an imperative [5], [6]. The most stringent precision requirements can only be satisfied by the use of real-time kinematic (RTK), a relative positioning procedure which uses code and carrier phase observations to reach centimeter accuracy [7].

Unlike code observations, carrier phase pseudoranges may provide higher precision but are ambiguous, since only their fractional part is measured by the receiver [8]. The unknown number of integer cycles, commonly denoted as ambiguities, is to be determined jointly to the dynamical parameters of the tracked vehicle. The ambiguities are resolved in a four-step procedure known as Integer Ambiguity Resolution (IAR). In the first step, the integer nature of the ambiguity is disregarded and a standard least-squares adjustment is performed the so-called *float solution*. Secondly, the integer constraints on the float ambiguities are re-incorporated and these are mapped to an integer solution. Third, one is to decide whether the estimated integer ambiguities are accepted or not. A variety of tests have been proposed, with the ratio- and the fixed-failure rate tests probably being the most popular [9]. Finally, once the integer solution is found and accepted, *solution fixing* consists on correcting the remaining dynamical parameters by virtue of their correlation with the ambiguities [10].

The successful resolution of the complete set of ambiguities can be a challenging task, since a single bias or inaccuracy in a phase observation can completely spoil the estimation. Moreover, the probability of finding the correct set of ambiguities tends to decrease as the number of observations increases. Especially with the deployment of the new GNSS frequencies and constellations, one might be concerned about the IAR dimensional curse. The framework of Partial Ambiguity Resolution (PAR), introduced in [11], [12], allows to circumvent the IAR dimensional curse problem: instead of resolving the full set of observations, PAR identifies the subset of ambiguities which maximizes the probability of success rate or grants that a constraint on the failure rate is not violated [13], [14]. In his series of work, Brack set the keystone for PAR with the generalized IAR and proposed IA estimators for PAR [15], [16], [17], [18]. On a different note, the evaluation of all possible subsets constitute a NP-hard problem, most PAR approaches explore different heuristics for sorting the decorrelated ambiguities and avoid dealing with the large volume of subsets. Thus, one can aim at the largest possible upper bound on success rate [19], [20], highest signal-to-noise ratio [13], highest ADOP [21], minimum bias [22], etc. The aforementioned methods solely focus on the second and third step of IAR problem, i.e., PAR is approached only from the perspective of mapping the real-valued to the integer-valued ambiguities.

The projection of the estimated ambiguities into the position domain during the solution fixing has not been yet exploited to guide the PAR selection process. Provided that the en-

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tire purpose of using GNSS carrier phase observations is obtaining precise position estimates, this work introduces precision-driven PAR. The premise is straightforward: the covariance matrix of the fixed position solution is used to pose a constraint on the integer resolution minimization problem, based on a goal accuracy to achieve. Precision-guided PAR allows to efficiently work with large number of subsets, which makes the algorithm attractive for multi-GNSS multi-frequency applications with stringent availability and accuracy requirements. The experimental characterization comprises a comparison of the performance of Full Ambiguity Resolution (FAR), classical PAR based on sequential observation elimination and the proposed Precision-aided PAR. The evaluation employs multi-GNSS (GPS, Galileo and BeiDou) triple frequency observations in an snapshot (non-recursive manner) [23]. The performance for short and medium baselines is characterized, by simulating distances between base and rover positions from one to 30 km.

The rest of the paper is as follows. Section II introduces the basics of GNSS carrier phase positioning and the mixed real and integer parameter estimation. Then, a brief overview on Partial Ambiguity Resolution is presented in Section III. Section IV constitutes the main contribution of the work and introduces Precision-Driven PAR. Finally, Sections V and VI are the experimentation and outlook respectively.

2. MIXED REAL AND INTEGER ESTIMATION

Let us call the estimation problem where unknown parameters combine an integer-valued vector $\mathbf{a}$ and a real-valued vector $\mathbf{b}$ from a set of observations $\mathbf{y}$. The available measurements are described by

$$\mathbb{E}(\mathbf{y}) = \mathbf{A}\mathbf{a} + \mathbf{B}\mathbf{b} \tag{1}$$

$$\mathbb{D}(\mathbf{y}) = \mathbf{Q_y} \tag{2}$$

where $\mathbb{E}(\cdot)$ is the expectation operator and $\mathbb{D}(\cdot)$ the corresponding dispersion (i.e., covariance). The estimation of such parameters can be formulated as a regression problem with mixed real and integer vectors

$$\{\mathbf{a}, \mathbf{b}\} = \arg \min_{\mathbf{a}\in\mathbb{Z}^\mathbf{n}, \mathbf{b}\in\mathbb{R}^\mathbf{3}} \|\mathbf{y} - \mathbf{A}\mathbf{a} - \mathbf{B}\mathbf{b}\|^\mathbf{2}_{\mathbf{Q_y}} \tag{3}$$

for which a closed-form solution is not known. A rich literature on statistical performances for various estimators of the mixed model exist (see [24, Ch. 23] and therein). Additionally, a closed-form Cramér-Rao bound expression for the mixed estimation problem was recently presented [25]. In order to solve (3), a three-step decomposition is commonly applied

$$\min_{\mathbf{a}\in\mathbb{Z}^\mathbf{n}, \mathbf{b}\in\mathbb{R}^\mathbf{3}} \|\mathbf{y} - \mathbf{A}\mathbf{a} - \mathbf{B}\mathbf{b}\|^\mathbf{2}_{\mathbf{Q_y}} = \min_{\hat{\mathbf{a}}\in\mathbb{R}^n, \hat{\mathbf{b}}\in\mathbb{R}^3} \|\hat{\mathbf{e}}\|^2_{\mathbf{Q_y}} \tag{4a}$$

$$+ \min_{\mathbf{a}\in\mathbb{Z}^\mathbf{n}} \|\hat{\mathbf{a}} - \mathbf{a}\|^\mathbf{2}_{\mathbf{Q_{\hat{a}}}} \tag{4b}$$

$$+ \min_{\mathbf{b}\in\mathbb{R}^\mathbf{3}} \|\hat{\mathbf{b}}|\mathbf{a} - \mathbf{b}\|^\mathbf{2}_{\mathbf{Q_{\hat{b}|a}}} \tag{4c}$$

with $\hat{\mathrm{e}} = \mathbf{y} - \mathbf{A}\hat{\mathbf{a}} - \mathbf{B}\hat{\mathbf{b}}$, and (4a) describing a regular least squares (LS) where the integer nature of the problem is disregarded (i.e., notice that we look for $\hat{\mathbf{a}} \in \mathbb{R}^n$). The output of this solution is referred to as *float solution* with a joint distribution given by

$$\begin{bmatrix} \hat{\mathbf{a}} \\ \hat{\mathbf{b}} \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \hat{\mathbf{a}} \\ \hat{\mathbf{b}} \end{bmatrix}, \begin{bmatrix} \mathbf{Q_{\hat{a}}} & \mathbf{Q_{\hat{a}\hat{b}}} \\ \mathbf{Q_{\hat{b}\hat{a}}} & \mathbf{Q_{\hat{b}}} \end{bmatrix} \right). \tag{5}$$

The minimization problem in (4b) constitutes an integer least squares (ILS), for which the integer solution (i.e., estimation of $\mathbf{a}$ with the integer constraint) is found based on the float solution. A non-linear mapping $S(\cdot) : \mathbb{R}^n \mapsto \mathbb{Z}^n$ relates each float ambiguity estimate to an integer value:

$$\mathbf{a} = \mathbf{S}(\hat{\mathbf{a}}). \tag{6}$$

Integer estimators which include a validation step belong to the Integer Aperture (IA) estimation framework [11], [26], [27]. An IA estimator is characterized by its pull-in regions $\Omega_\mathbf{a}$, $\forall \mathbf{a} \in \mathbb{Z}^\mathbf{n}$, and it described by

$$S(\hat{\mathbf{a}}) = \sum_{\mathbf{a}\in\mathbb{Z}^\mathbf{n}} w_a(\hat{\mathbf{a}})\mathbf{a} + \left(\mathbf{1} - \sum_{\mathbf{a}\in\mathbb{Z}^\mathbf{n}} \mathbf{w_a}(\hat{\mathbf{a}})\right)\hat{\mathbf{a}} \tag{7}$$

with $w_a(\hat{\mathbf{a}})$ being the binary indicator

$$w_a(\hat{\mathbf{a}}) = \begin{cases} 1 & \text{if } \hat{\mathbf{a}} \in \Omega_\mathbf{a} \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

There are various alternatives to define the size or aperture of the pull-in regions [28], [29], [27]. This work considers the fixed-failure rate ratio test (FF-RT) [9], where the failure rate P_f is used as a tuning parameter.

Finally, the last minimization problem (4c) improves the positioning (i.e., real parameter vector $\mathbf{b}$) estimate upon the knowledge of the integer ambiguities $\mathbf{a}$, driving to a high-accurate position solution denoted as *fixed solution*. The mean $\mathbf{b}$ and covariance $\mathbf{Q_b}$ of the fixed estimate are based on the projection of the estimated integer ambiguities into the position domain as

$$\mathbf{b} = \hat{\mathbf{b}} - \mathbf{Q_{\hat{b}\hat{a}}}\mathbf{Q_{\hat{a}}}\left(\hat{\mathbf{a}} - \mathbf{a}\right), \tag{9}$$

$$\mathbf{Q_b} = \mathbf{Q_{\hat{b}}} - \mathbf{Q_{\hat{b}\hat{a}}}\mathbf{Q_{\hat{a}}^{-1}}\mathbf{Q_{\hat{a}\hat{b}}}, \tag{10}$$

where $\mathbf{Q_b} = \mathbf{Q_{\hat{b}|a}}$ refers to the covariance matrix of the position solution conditional on the estimated integer ambiguities. Unfortunately, the successful resolution of all the ambiguities might not always be possible. This would happen when the underlying observation model lacks sufficient strength. Moreover, as the number of observations grows, the multiplication of correctly integer mapping events leads to a decreasing probability of successful IAR estimation [27], [30]. Next, the framework of Partial Ambiguity Resolution is discussed.

3. PARTIAL AMBIGUITY RESOLUTION

Classical full ambiguity resolution (FAR) regards the complete set of ambiguities to be either fixed or not. The framework of Partial Ambiguity Resolution (PAR) relaxes this condition, allowing a subset of the ambiguities to be mapped to integers, while the complementary subset of ambiguities remain as real numbers.

Following the notation of [15], [14], let $\mathcal{I}$ be the index of the ambiguities mapped to integer values

$$\mathcal{I} \subseteq \{1, \ldots, n\},\ \mathcal{I} \in \mathfrak{J} \tag{11}$$

where $\mathfrak{J}$ denotes the set of all possible index sets $\mathcal{I}$ — i.e., there are two options per ambiguity, based on whether they are fixed or not—, whose cardinality is $|\mathfrak{J}| = 2^n$.

The complementary set to $\mathcal{I}$ (the non-fixed ambiguities) is denoted by $\bar{\mathcal{I}}$ and defined by

$$\mathcal{I} \cap \bar{\mathcal{I}} = \emptyset,\ \mathcal{I} \cup \bar{\mathcal{I}} = \{1, \ldots, n\}. \quad (12)$$

Similarly to (13), now each set $\mathcal{I}$ relates to its pull-in region $\Omega_{\mathcal{I},a}$ and, as proposed in [15], the generalised IA estimator is given by

$$\begin{aligned} S(\hat{\mathbf{a}}) = &\sum_{\mathbf{a}\in\mathbb{Z}^{\mathbf{n}}} \operatorname{diag}\left[\mathbf{w}_{\mathbf{i},\mathbf{a}}(\hat{\mathbf{a}})\right]\mathbf{a} \\ &+ \left(\mathbf{I}_{\mathbf{n}} - \sum_{\mathbf{a}\in\mathbb{Z}^{\mathbf{n}}} \operatorname{diag}\left[\mathbf{w}_{\mathbf{i},\mathbf{a}}(\hat{\mathbf{a}})\right]\right)\hat{\mathbf{a}} \end{aligned} \quad (13)$$

with $i = 1, \ldots, n$. Thus, the binary indicator function becomes a vector

$$\mathbf{w}_{\mathbf{i},\mathbf{a}}(\mathbf{x}) = \begin{cases} 1 & \text{if } \mathbf{x} \in \mathbf{\Omega}_{\mathcal{I},\mathbf{a}} \text{ and } \mathbf{i} \in \mathcal{I} \\ 0 & \text{otherwise.} \end{cases} \quad (14)$$

In plain words, given $\hat{\mathbf{a}} \in \Omega_{\mathcal{I},a}$, ambiguities whose index $i \in \mathcal{I}$ are fixed to integer numbers, while the remaining (those belonging to $\bar{\mathcal{I}}$) are kept as real numbers. One can distinguish among three cases for the PAR estimation: i) success: whenever ambiguities with index $i \in \mathcal{I}$ are correctly fixed; ii) failure: whenever a single ambiguity is wrongly estimated; iii) undecided: when no ambiguity is fixed:

$$\begin{aligned} &\text{Success:} \quad \hat{\mathbf{a}} \in \Omega_{\mathcal{I},a}, \mathcal{I} \in \mathfrak{J}\backslash\emptyset, \mathbf{a} \in \mathbb{Z}^{\mathbf{n}} | \mathbf{a} = \mathbf{a_{true}} \\ &\text{Failure:} \quad \hat{\mathbf{a}} \in \Omega_{\mathcal{I},a}, \mathcal{I} \in \mathfrak{J}\backslash\emptyset, \mathbf{a} \in \mathbb{Z}^{\mathbf{n}} | \mathbf{a} \neq \mathbf{a_{true}} \\ &\text{Undecided:} \ \hat{\mathbf{a}} \in \Omega_{\emptyset} \end{aligned} \quad (15)$$

PAR methods are generally classified in two categories: *model-* and *data-driven* schemes. Model-driven methods base the subset selection $\mathcal{I}$ only on $\mathbf{Q}_{\hat{\mathbf{a}}}$, while data-driven approaches integrate also the float ambiguities $\hat{a}$. Nonetheless, the effect of projecting the resolved integer ambiguities into the position domain has not yet been considered as criteria for the subset selection. Next Section introduces the notion of Precision-Aided PAR.

4. PRECISION-DRIVEN PAR

The successful ambiguity integer estimation serves to constrain and enhance the position estimates, as shown in (9)-(10). Since $\mathbf{Q}_{\mathbf{b}}$ encodes the accuracy of the fixed solution, we might consider a minimum precision α to be achieved:

$$\operatorname{tr}\left(\mathbf{Q}_{\mathbf{b}}\right) \leq \alpha^2, \quad (16)$$

where $\operatorname{tr}(\cdot)$ denotes the trace operator. The geometry matrix $\mathbf{B}$ is assumed to be projected into a local tangent plane (such as North-East-Down or East-North-Up), without loss of generality, so that the position deviation matches the horizontal and vertical planes. Given the float solution as input and adding the precision requirement (16), the following minimization problem is obtained

$$\begin{aligned} &\min_{\mathbf{a}\in\mathbb{Z}^{\mathbf{n}}} \|\hat{\mathbf{a}} - \mathbf{a}\|^2_{\mathbf{Q}_{\hat{\mathbf{a}}}} + \min_{\mathbf{b}\in\mathbb{R}^3} \|\hat{\mathbf{b}}|\mathbf{a} - \mathbf{b}\|^2_{\mathbf{Q}_{\hat{\mathbf{b}}|\mathbf{a}}}, \\ &\text{s.t. } \operatorname{tr}\left(\mathbf{Q}_{\mathbf{b}}\right) \leq \alpha^2, \end{aligned} \quad (17)$$

where the constraint function acts as condition to whether an integer solution shall or not be estimated, in case the precision does not comply with the minimum required accuracy. Such case would occur solely on scenarios with limited satellite visibility. Instead, the bottleneck of FAR relates to presenting low success rates or not passing the required ambiguity ratio test.

Precision-driven PAR relaxes the search for the complete set of ambiguities while retaining the accuracy constraint on the resulting positioning solution. Thus, the resulting optimization leads to

$$\begin{aligned} &\min_{\mathbf{a}_{\mathcal{I}}\in\mathbb{Z}^{|\mathcal{I}|}} \|\hat{\mathbf{a}}_{\mathcal{I}} - \mathbf{a}_{\mathcal{I}}\|^2_{\mathbf{Q}_{\hat{\mathbf{a}}_{\mathcal{I}}}} + \min_{\mathbf{b}\in\mathbb{R}^3} \|\hat{\mathbf{b}}|\mathbf{a}_{\mathcal{I}} - \mathbf{b}\|^2_{\mathbf{Q}_{\hat{\mathbf{b}}|\mathbf{a}_{\mathcal{I}}}}, \\ &\text{s.t. } \operatorname{tr}\left(\mathbf{Q}_{\hat{\mathbf{b}}\hat{\mathbf{a}}_{\mathcal{I}}}\mathbf{Q}^{-1}_{\hat{\mathbf{a}}_{\mathcal{I}}}\mathbf{Q}_{\hat{\mathbf{a}}_{\mathcal{I}}\hat{\mathbf{b}}}\right) \geq \operatorname{tr}\left(\mathbf{Q}_{\hat{\mathbf{b}}}\right) - \alpha^2, \end{aligned} \quad (18)$$

where the precision requirement has been reformulated, since the trace of the float solution covariance $\mathbf{Q}_{\hat{\mathbf{b}}}$ remains unchanged during the ILS and solution fixing procedures. The subscript $\mathcal{I}$ indicates the subset of ambiguities to be fixed and their associated covariance matrices (e.g., subtracting from the vector $\hat{\mathbf{a}}$ elements belonging to $\bar{\mathcal{I}}$ and similarly for the rows and columns of matrices $\mathbf{Q}_{\hat{\mathbf{a}}_{\mathcal{I}}}$ and $\mathbf{Q}_{\hat{\mathbf{b}}\hat{\mathbf{a}}_{\mathcal{I}}}$).

Notice that solving (18) implies a high computation burden, since the subset $\mathcal{I}$ is not known a priori. The bootstrapped upper bound on the success rate can be applied on the $Z-$transformed ambiguities [31], so that subset selection corresponds to the maximum number of ambiguities satisfying the failure rate constraint [14]. In that case, the precision constraint would become an additional validation test. An interesting procedure consists on evaluating first the formal precision of a subset and, if passed, then performing the $Z-$decorrelation solely with the $\mathcal{I}$ elements for the failure rate test. Despite being computationally intensive, the above-mentioned approach might lead to a better $Z-$decorrelation (due to a lower dimensional problem [32]) and improved ILS estimates. Whenever multi-constellation multi-frequency is used and as the size of $\bar{\mathcal{I}}$ increases, one might consider alternative heuristics to avoid analyzing the rapidly growing number of hypothesis (for instance, satellite selection via convex geometry [33] or sorting $\mathbf{Q}_{\hat{\mathbf{a}}}$ based on the associated $\mathbf{Q}_{\mathbf{y}}$). Subset selection can also be formulated in analogy with ILS search strategies [34]: i) initial minimum number of observations and consequently add new ones when the failure and precision criteria are respected (search with enumeration); ii) from the complete set of observations, the subset can be reduced by progressively eliminating observations until the precision metric is achieved (search and shrink).

Notice that [35] discusses the use of the determinant of the position covariance matrix to the power of $1/6$ as representation of the positioning precision $\det(\mathbf{Q}_b)^{1/6}$. Although this metric allows accounting for the cross-correlated effects in the covariance, the metric evaluation increases the computational needs. Given that covariance matrices are positive semi-definite and based on the inequality of arithmetic and geometric means [36], the trace and the determinant of a generic positive semi-definite matrix M are related based on the following inequality:

$$\det(\mathbf{M})^{1/p} \leq \frac{\operatorname{tr}(\mathbf{M})}{p}. \quad (19)$$

Thus, the trace of the covariance matrix $\mathbf{Q}_{\mathbf{b}}$ can be considered a proper precision metric and an upper bound for the volume of the position confidence ellipsoids proposed in [35].

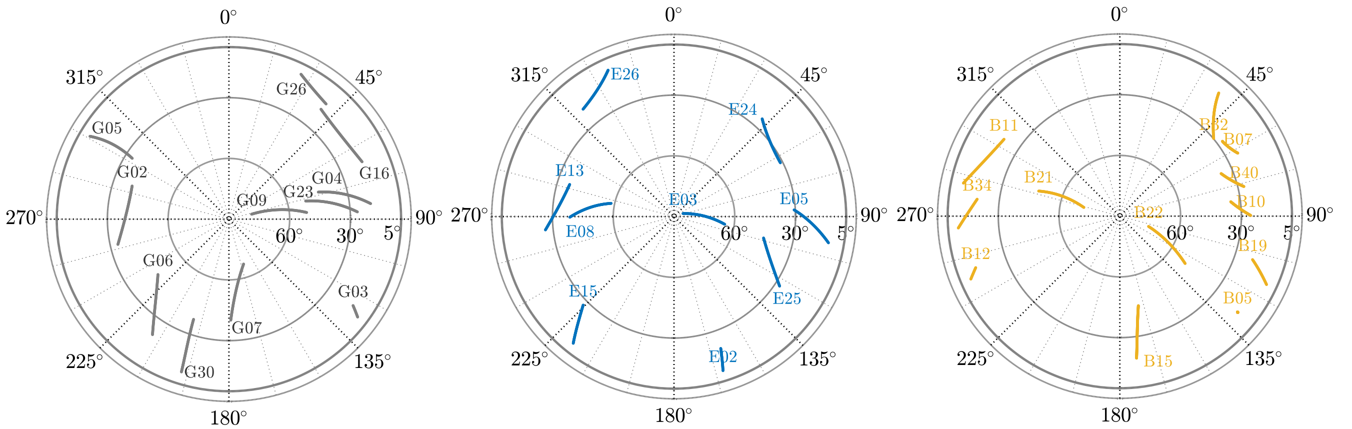

Figure 1. **Sky plot for the GPS tracked satellites (left), Galileo (middle) and BeiDou (right).**

Table 1. **Wavelengths and code noise standard deviations for GPS, Galileo and BeiDou.**

Constellation		**Code Noise** [cm]	**Wavelength** [cm]
GPS	L1	37	19.03
	L2	27	24.42
	L5	25	25.45
Galileo	E1	25	19.03
	E5a	20	25.48
	E5b	20	24.83
BeiDou	B1	31	19.20
	B2	30	24.83
	B3	25	25.63

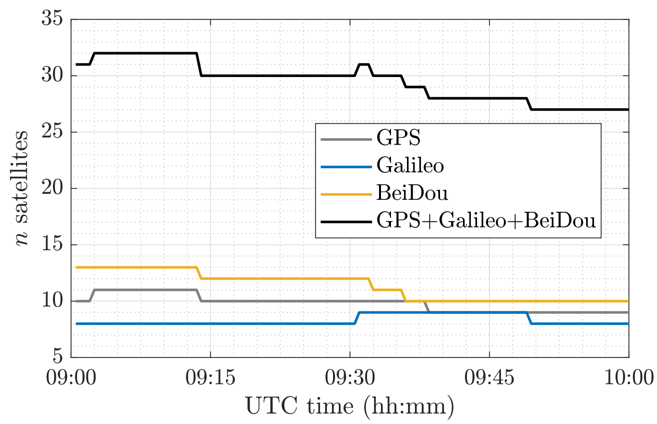

Figure 2. **Number of observations along time for GPS, Galileo and BeiDou constellations over time.**

5. Numerical Results

The performance characterization for FAR, PAR and Precision-Aided PAR is realized on two hours of simulated GNSS measurements collected at the DLR site in Neustrelitz, Germany for the 6th January 2020 (DOY 06, 09:00-10:00 UTC) with a sampling time of 30 seconds. FAR performs an ILS estimate and the acceptance is based on the FF-RT. For DD-PAR, ILS is performed on the subset which maximizes the number of ambiguities satisfying a success rate of $1 - P_f$ (based on the open source PAR [34]) plus an additional FF-RT for the solution acceptance. It is considered a multi-GNSS scenario, where GPS (L1, L2, L5), Galileo (E1, E5a, E5b) and BeiDou (B1, B2, B3) observations are tracked over three different frequencies with an elevation mask of 10°. Fig. 1 depicts the skyplot for the three constellations, while Fig. 2 illustrates the number of satellites for every constellation along the one hour duration of the experimentation. The undifferenced (zenith-referenced standard deviation) code noise are based on the least-squares variance component estimation study in [37], [38]. In all cases, the undifferenced phase noise is considered to be 0.2 centimeters. As in [14], the differential ionospheric delays are modelled based on a distance dependent function $\sigma_L = 0.8$ mm/km. Thus, the covariance matrix of the observations $\mathbf{Q_y}$ is composed by the sum of the measurement noise and the uncertainty of the ionospheric delay. Both the differential ionospheric delays and undifferenced code and phase noises are scaled with the elevation dependent function $1/\sin(el)$. The fixed failure rate is set to $P_f = 0.1\%$ and the precision metric $\alpha = 5$ cm. Each experiment is composed by 1000 Monte Carlo runs.

Since the characterization of the precision-aided PAR performance is of the most relevant for this contribution, the analysis is realized in a snapshot (non-recursive) manner, to avoid getting gains from a Kalman-filter styled algorithm. Fig. 3 describes the three cases for an integer aperture estimator: success, failure and undecided rates. On the ordinate axis, the baseline length in km is shown. It becomes clear that, as the distance between stations grows, the differential ionospheric delays play an essential role, and eventually fixing the ambiguities becomes too complicated of a task. Fig. 3 (top) depicts the integer ambiguity regions for FAR, (middle) shows the conventional data-driven PAR (DD-PAR), while (bottom) corresponds to the proposed PD-PAR. For baselines longer than 20 km, both FAR and DD-PAR are unable to perform any successful ambiguity fixing, while PD-PAR manages to obtain a successful fix rate of over 80%. On the other hand, the percentage of failure rate increases, leading to think than conventional aperture test shall be re-visited for PAR approaches. Somehow astonishing appears the performance of conventional DD-PAR, which offers a rather high failure rate for baselines of 20 km and it is unable to successfully fix ambiguities over that baseline length.

Fig. 4 illustrates the positioning performance of the fix position solution against the baseline length. Constant along the baseline length, the dash-dot line represents the precision criteria $\alpha = 5$ cm for this work. Such value can be adapted for the specifics of a particular application. The solid black

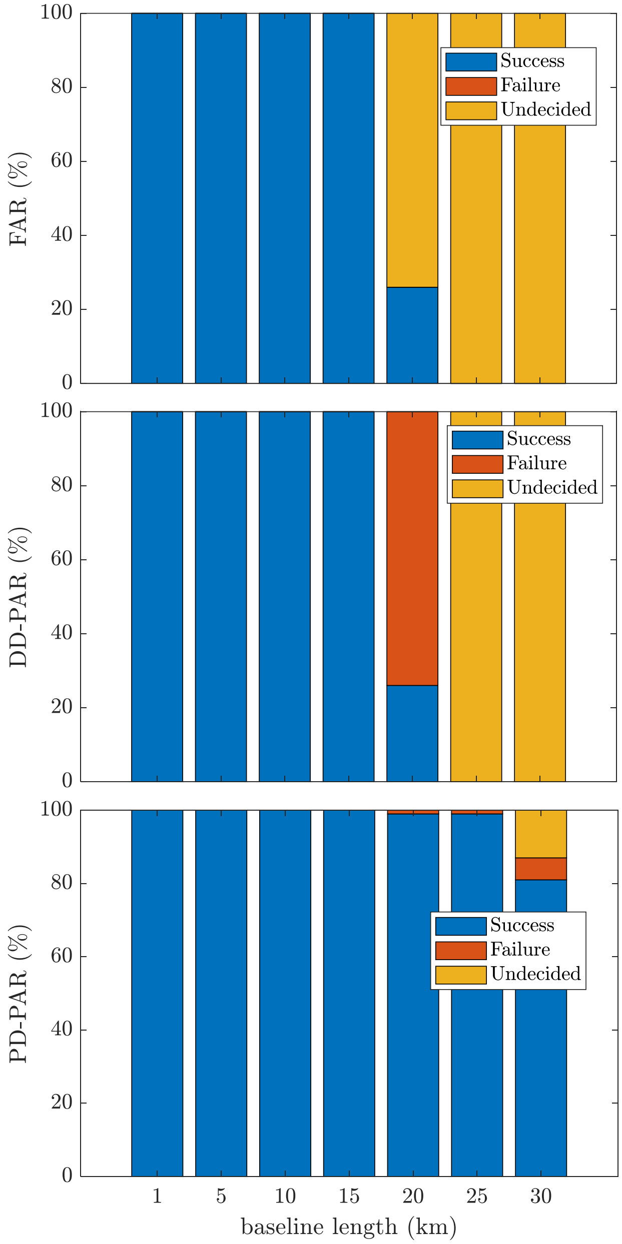

Figure 3. **Comparison of the success, failure and undecided rates.**

line corresponds to the trace of the covariance solution for the FAR solution (reachable only when the complete set of ambiguities solution are found). Similarly, the dash line corresponds to the covariance of the fixed position solution of PD-PAR. One can barely observe any difference between PAR and PD-PAR for short baselines, since the removal of satellites is barely necessary. On the contrary, for baseline lengths over 20 km, the elimination of a higher number of satellites make evident the difference between FAR and PD-PAR. Notice that, the actual root mean squared error (RMSE) of an estimator attains the covariance for the fixed position solution only when all the ambiguities are successfully estimated. Hence, due to the failure rate of PD-PAR for a 30 km baseline, the wrongly fixed ambiguities lead to a worsen positioning solution. This corresponds to the worse case scenario, for which an integer solution is considered valid although it does not correspond to the true one.

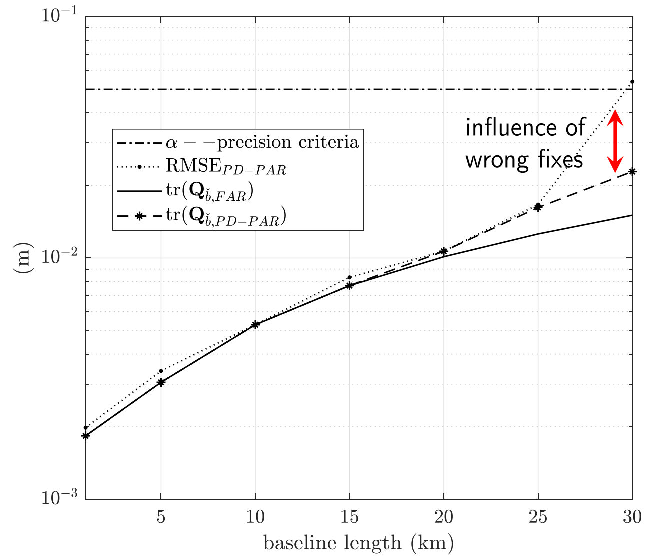

Figure 4. **Comparison of the estimated precision metric α against the root mean positioning error for PD-PAR, as well as the trace of the covariance matrices associated to the FAR and PD-PAR fixed solutions.**

6. Outlook and Future Work

GNSS carrier phase observations are fundamental for safety-critical applications where the requirements of accuracy and availability are stringent. Upon the deployment of new GNSS constellation and frequencies, the large number of observations can lead to a decreased probability of successfully mapping the real-valued carrier phase ambiguities to integer ones. Partial Ambiguity Resolution (PAR) relaxes the condition of fixing the complete set of ambiguities and find instead the subset which maximizes the success rate. Conventional PAR solutions are designed to maximize the probability of success at the ILS problem, which often leads to fixing an extremely low number of satellites. Thus, the associated fixed positioning solution lacks the required accuracy. This work introduces Precision-Driven PAR (PD-PAR), a constrained alternative on PAR for which a minimal precision criteria is added to the mixed real and integer parameter estimation problem. Then, the selection for the subset of ambiguities to be fixed is based upon their projection on the position domain. The performance characterization is realized on a synthetic scenario, where observations from a triple-frequency, triple-constellation are employed and the estimation is realized on a snapshop manner. It is shown that the proposed PD-PAR extensively overperform FAR and the conventional data-driven PAR. While the later two are unable to find an integer solution for baselines over 20 km on an instantaneous manner, PD-PAR manages to obtain a high success rate for baselines of even 30 km. The negative note relates to the appearance of certain chance of failure rate, for the test of alternative ILS tests shall be considered as future work. In summary, PD-PAR represents an appealing alternative to conventional RTK and PAR solutions, able to provide instantaneous precise localization even for medium baseline lengths at a cost of a slight degradation on the positioning performance.

Acknowledgments

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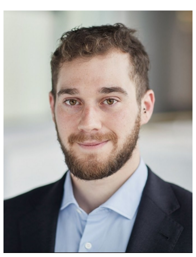
BIOGRAPHY



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