

# Supramolecular interactions lead to remarkably high thermal conductivities in interpenetrated two-dimensional porous crystals

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## Abstract

The design of innovative porous crystals with high porosities and large surface areas has garnered a great deal of attention over the past few decades due to their remarkable potential for a variety of applications. However, heat dissipation is key to realizing their potential. We use systematic atomistic simulations to reveal that interpenetrated porous crystals formed from two-dimensional (2D) frameworks possess remarkable thermal conductivities at high porosities in comparison to their three-dimensional (3D) single framework and interpenetrated 3D framework counterparts. In contrast to conventional understanding, higher thermal conductivities are associated with lower atomic densities and higher porosities for porous crystals formed

11 from interpenetrating 2D frameworks. We attribute this to lower phonon-phonon scattering  
12 and vibrational hardening from the supramolecular interactions that restrict atomic vibrational  
13 amplitudes, facilitating heat conduction. This marks a new regime of materials design combining  
14 ultralow mass densities and ultrahigh thermal conductivities in 2D interpenetrated porous  
15 crystals.

16 **Keywords:** Metallic organic frameworks, porous crystals, phonon hardening, ultralow  
17 mass density, ultra high thermal conductivity

18 The wide range of structural design through adjustment of linker geometry, length and func-  
19 tional group in metal-organic frameworks (MOFs) and covalent-organic frameworks (COFs) make  
20 them one of the most highly sought after materials for targeted applications.<sup>1</sup> Interpenetration or  
21 the entwining of multiple lattices in these framework materials not only gives rise to fascinat-  
22 ing and intricate architectures but also has been leveraged to improve their physical properties  
23 and functionalities.<sup>2-5</sup> For example, the nonbonded, supramolecular interactions (such as van der  
24 Waals forces) between the individual networks endows interpenetrated MOFs (IMOFs) with en-  
25 hanced stability, added structural flexibility and higher gas-adsorption selectivity as compared to  
26 single MOFs.<sup>6,7</sup>

27 The possibility of interpenetration in porous framework materials originates from the large sol-  
28 vent accessible voids that allows for controllable entanglement between the single frameworks.  
29 In this regard, predictions from atomistic simulations have been used to identify candidate frame-  
30 work structures with high likelihood of interpenetration.<sup>8-10</sup> Experimentally, chemists have been  
31 successful in controlling the degree of interpenetration as well as in gaining unprecedented control  
32 over the modulation in the pore volume space in porous framework materials through the manipu-  
33 lation of external conditions such as choosing the appropriate solvents with the desired molecular  
34 structures or by changing the reaction temperature.<sup>11-16</sup> For instance, IMOFs with interweaving  
35 structures, minimal interpenetration and remarkably large pores ( $\sim 1.6$  nm) were fabricated by  
36 linking symmetrical secondary building units.<sup>17</sup> Whereas, on the other extreme, highly entangled  
37 systems with a record 54 interpenetrating networks have also been hydrothermally synthesized.<sup>18</sup>

38 Therefore, crucial for their incorporation in the applications such as in gas storage, catalysis, elec-  
39 trodes for supercapacitors and energy storage devices is the complete understanding of the influ-  
40 ence of interpenetration of porous frameworks on their thermal transport efficiencies.

41 Recently, thermal transport properties of porous framework materials have been of interest from  
42 a materials science standpoint as well as from an applied perspective.<sup>19-26</sup> Most of these works  
43 report glass-like thermal conductivities ( $\sim 0.3$  to  $1 \text{ W m}^{-1} \text{ K}^{-1}$ ) for porous framework materials,  
44 which can be further reduced via pore filling and infiltration of guest species through vibrational  
45 scattering mechanisms.<sup>19,24,27</sup> Recently, Sezginel *et al.*<sup>26</sup> have shown that interpenetration of 3D  
46 MOF with another guest 3D MOFs can lead to  $\sim 2 \times$  increase in the thermal conductivity of IMOF  
47 structure. They conclude that additional channels of thermal transport introduced through the  
48 interpenetration leads to a  $2 \times$  increase in thermal conductivity of their IMOFs, which can be  
49 predicted by a linear sum of the thermal conductivities of the two constituent 3D MOFs. This  
50 might be expected in view of the fact that the interpenetration leads to a reduction in porosity and  
51 a  $2 \times$  increase in mass density (that generally results in a concomitant increase in heat conduction  
52 in solids).<sup>28,29</sup>

53 In this work, we show that 3D networks formed from 2D layered frameworks possess thermal  
54 conductivities that are more than an order of magnitude higher as compared to a single 3D frame-  
55 work at similar mass densities and are also higher in comparison to 3D interpenetrated frameworks  
56 (formed from two individual 3D frameworks with  $2 \times$  reduced porosities). We show that the mas-  
57 sive increase in thermal conductivity holds for a variety of idealized interpenetrated frameworks  
58 with a wide range of interatomic interaction parameters that result in stable structures with low  
59 densities. This is counterintuitive to the general notion that increased atomic densities lead to  
60 higher thermal conductivities in nonmetallic solids. We mostly attribute this to phonon hardening  
61 in the interpenetrated frameworks due to the supramolecular interactions between the 2D layers  
62 that restricts the motion of the atoms. We apply our proof of concept to a realistic framework  
63 that is designed based on the 2D COF-1 structure. Our results show that these interpenetrated  
64 frameworks can possess more than an order of magnitude higher thermal conductivity compared

65 to the prototypical 3D COF-300 structure at similar mass densities and porosities. This highlights  
 66 the prowess of interpenetrated frameworks made from 2D layers as structural materials with ul-  
 67 tralight weight characteristics accompanied by superior thermal transport properties that separate  
 68 them from any other class of materials.

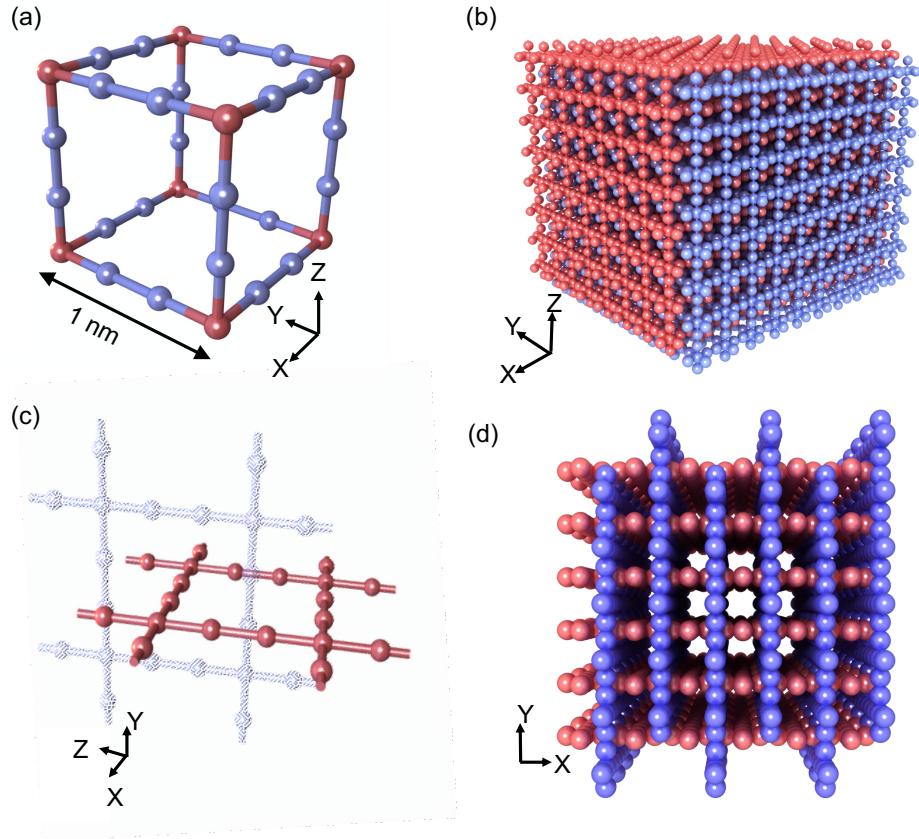


Figure 1: (a) Schematic illustration of the unit cell for our 3D idealized porous crystals. (b) The atoms are displaced by 5 Å in all three principle directions to form an interpenetrated 3D porous crystal as shown in the schematic of the 2D→3D. (c) Similarly, our interpenetrated 2D→3D structures are formed by interpenetrating 2D orthogonal layers. (d) Schematic cross-sectional view of our computational domain for the interpenetrated 2D→3D idealized porous crystals. We use computational domain sizes of  $80 \times 80 \times 80 \text{ \AA}^3$  for all our structures.

69 To systematically investigate the effect of varying interactions between the single frameworks,  
 70 we perform atomistic simulations on idealized porous structures as opposed to realistic materials  
 71 since we are more interested in the qualitative insight on the effect of interpenetration of 2D lay-  
 72 ered frameworks on thermal transport as opposed to their material specific properties. Figure 1a

73 shows our idealized cubic structure with a pore volume of  $\sim 1 \text{ nm}^3$ . Similar idealized structures  
74 have been used to glean significant insight into the thermal transport properties of MOFs.<sup>19,23,26</sup>  
75 Our interpenetrated structures based on 3D (Fig. 1b) and 2D (Fig. 1c) frameworks are generated  
76 by creating copies and translating the idealized structures. An example of the computational do-  
77 main formed through interpenetration of idealized 2D frameworks (2D $\rightarrow$ 3D) is shown in Fig. 1d.  
78 We base the interatomic potential for our idealized crystals on the COMPASS force field<sup>30</sup>, with  
79 the nonbonded interactions between interpenetrating frameworks defined by a Lennard-Jones (LJ)  
80 potential,  $V(r) = \varepsilon[2(\sigma/r)^9 - 3(\sigma/r)^6]$ , where  $r$  is the interatomic separation, and  $\sigma$  and  $\varepsilon$  are the  
81 LJ length and energy parameters, respectively. A range of  $\sigma$  and  $\varepsilon$  values are chosen to understand  
82 the effect of interframework interactions; as the value of  $\sigma$  determines an individual frameworks'  
83 mobility, we can systematically control the positioning of the frameworks relative to each other  
84 in our interpenetrated structures (see Supporting Information for details regarding the force field  
85 parameters). We note that depending on the linker shape and size, the porosity and the van der  
86 Waals dimension (that would fit in the pores without touching the pore walls) can be methodically  
87 varied in realistic framework materials.<sup>31</sup> For example, in the isorecticular MOF series, depending  
88 on the size of the ligands controlled through pore functionalization, the percent free volume in the  
89 crystals can be systematically varied from  $\sim 56 \%$  to  $91 \%$ .<sup>31</sup> For interpenetrated structures, this  
90 also allows for the control of an individual frameworks' mobility.<sup>26</sup>

91 Figure 2a shows the thermal conductivities of our idealized structures as a function temperature,  
92 which are calculated based on the Green-Kubo (GK) approach. (see Supporting information for  
93 more details). Also shown are the respective  $1/T$  fits highlighting the strong role of anharmonic  
94 Umklapp scattering processes in all of our structures.<sup>32,33</sup> The more notable result shown in Fig. 2a  
95 is the dramatic increase in thermal conductivity (by almost an order of magnitude) of the structure  
96 formed from the interpenetration of 2D frameworks into a single 3D framework (2D $\rightarrow$ 3D). This  
97 is much higher in comparison to the two fold increase in thermal conductivity of the structure  
98 formed by the interpenetration of 3D frameworks (3D $\rightarrow$ 3D) as shown in the figure. It is also  
99 interesting to note that while the density of the 3D $\rightarrow$ 3D interpenetration is twice as high as the

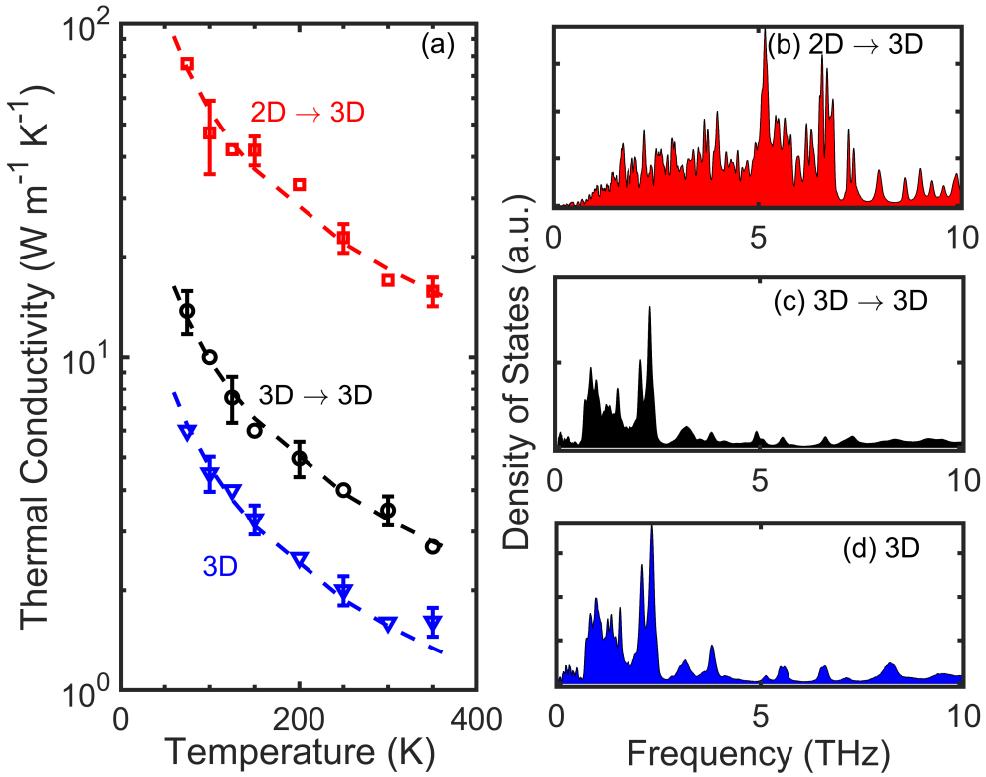


Figure 2: (a) Temperature dependent thermal conductivities of our idealized 3D, interpenetrated 2D → 3D and interpenetrated 3D → 3D porous crystals. The dashed lines represent the  $1/T$  temperature trends highlighting the role of anharmonic phonon-phonon scattering in these idealized structures. The interpenetrated 2D → 3D structure shows drastically enhanced thermal conductivities throughout the temperature range in comparison to both the 3D and the interpenetrated 3D → 3D structures; although the mass density of the interpenetrated 3D → 3D structure is twice that of the 3D case, our interpenetrated 2D → 3D has a similar low mass density as that of the 3D structure all the while possessing an order of magnitude higher thermal conductivity. Vibrational density of states of our (b) interpenetrated 2D → 3D, (c) interpenetrated 3D → 3D and (d) 3D structures calculated from molecular dynamics simulations.

100 single 3D framework, the density of the 2D→3D structure is comparable to that of the individual  
101 3D framework; for crystalline solids in general, an increase in density is usually accompanied by  
102 an enhancement in their thermal and mechanical properties.<sup>34</sup> This suggests that the mechanism  
103 behind the dramatic increase in heat conduction in our 2D→3D frameworks could be different  
104 in comparison to the 3D→3D case, where it has been previously shown by Sezginel *et al.*<sup>26</sup> that  
105 additional channels of heat transfer are introduced by the interpenetration of two 3D frameworks.

106 To understand the mechanism leading to the dramatic thermal conductivity increase in our  
107 2D→3D structures, we first compare the vibrational density of states (DOS) of our frameworks as  
108 shown in Figs. 2b-d. The DOS for the 3D and 3D→3D cases are similar, while the DOS for the  
109 2D→3D shows pronounced phonon hardening as compared to the other two cases. Moreover, the  
110 heat carrying vibrations (as predicted by the spectral heat flux calculations detailed in the Support-  
111 ing Information) in the 3D and 3D→3D cases are also similar, reinforcing the suggestion that ad-  
112 dditional heat transfer pathways are introduced in the same frequency range due to interpenetration  
113 of two 3D frameworks without affecting the spectral nature (see Fig. S6). In contrast, spectral heat  
114 flux calculations show that heat carrying vibrations in the 2D→3D case encompasses a broader  
115 spectral range (see Figs. 2b-d). Normalizing the frequency range for our structures and calculating  
116 the per mode contribution to the total heat flux shows that in the 2D→3D case, the proportion of  
117 heat carrying vibrations are shifted to comparatively higher frequencies (in a per-mode-basis).

118 From the above discussion, we observe that vibrational hardening can lead to a significantly  
119 enhanced thermal conductivity in interpenetrated structures formed by 2D frameworks. The ef-  
120 fect on thermal conductivity of interframework interactions between the interpenetrated 2D layers,  
121 however, needs to be fully understood in order to design frameworks with superior thermal trans-  
122 port properties. Therefore, we investigate the changes in the thermal conductivity, mean squared  
123 displacement (MSD) and density for our 2D→3D structures across a wide range of values of  $\varepsilon$  and  
124  $\sigma$  (see Figure 3). As shown by the darker shaded regions in Fig. 3a, even though a high value of  
125  $\varepsilon$  represents better coupling between the frameworks, it does not guarantee a high thermal con-  
126 ductivity since the value of  $\sigma$  also affects thermal transport in our interpenetrated structures;  $\sigma$  can

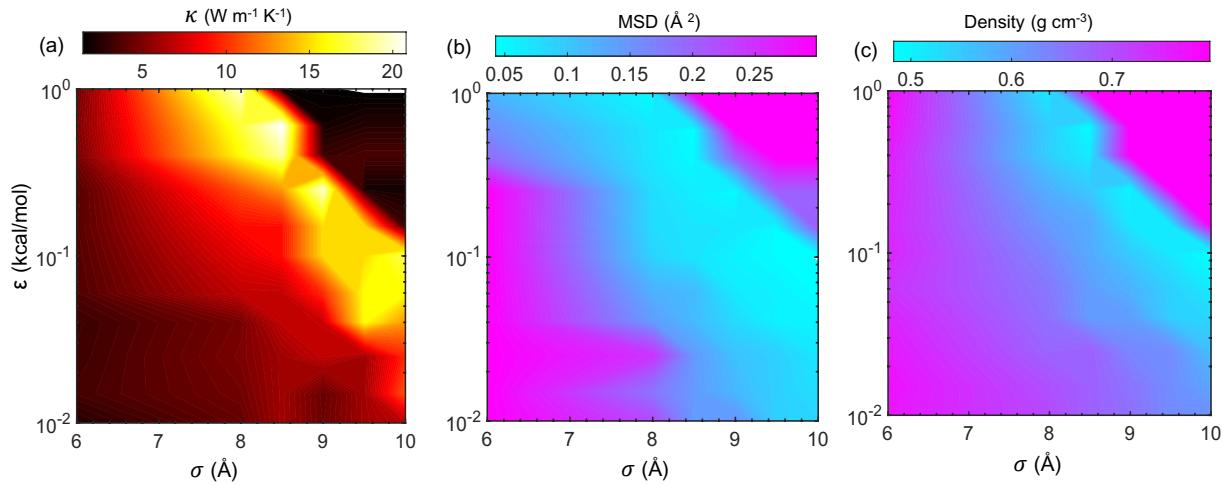


Figure 3: Effect of framework interaction on (a) thermal conductivity, (b) mean square displacement (MSD) of the atoms, and (c) mass density. The combinations of energy ( $\varepsilon$ ) and length ( $\sigma$ ) parameters that result in high thermal conductivities are associated with low mass densities and MSDs of the frameworks. Usually for crystalline solids, higher mass densities are associated with higher thermal conductivities. For interpenetrated 2D $\rightarrow$ 3D porous crystals, however, the further apart the 2D layers are with respect to each other, the higher the thermal conductivities. Similarly, 2D layers that are “locked-in” demonstrate comparatively higher thermal conductivities, which can be attributed to lower phonon-phonon scattering.

127 have a major influence on the framework's mobility with a high value allowing structural stability  
128 for the 2D frameworks, whereas a low value results in the translation of the 2D layers relative to  
129 each other as demonstrated in Fig. 3b. Comparing Figs. 3a and 3b, it becomes apparent that a lower  
130 MSD (where the frameworks are more likely to be "locked in" their equilibrium positions) leads  
131 to higher thermal conductivities.

132 The values of  $\varepsilon$  and  $\sigma$  can also affect the mechanical properties and structural integrity of the  
133 2D $\rightarrow$ 3D interpenetrated frameworks. As shown in Fig. 3c, lower densities are associated with  
134 similar combinations of the force field parameters that also lead to higher thermal conductivities  
135 (Fig. 3a). As noted above, this is contrary to the general understanding of crystalline solids where  
136 higher densities usually lead to higher thermal conductivities.<sup>34</sup> For these 2D interpenetrated struc-  
137 tures, the combination of force field parameters that result in lower densities also lead to higher  
138 Young's modulus and higher stress response as calculated from additional simulations of uniaxial  
139 tension on the interpenetrated frameworks (see Fig. S9). We note that as higher Young's modulus  
140 is generally associated with higher sound velocity in solids, the increasing Young's modulus is  
141 also indicative of higher velocities for Debye-like phonons in the interpenetrated structures with  
142 the combination of force field parameters that result in higher thermal conductivities.

143 Finally, to show that our results are applicable for realistic organic-based framework materials,  
144 we design 2D $\rightarrow$ 3D interpenetrated frameworks by orthogonally combining 2D layers based on the  
145 COF-1 structures as shown in the schematic of the computational domain in Fig. 4a. Along with  
146 the parameters for the Polymer consistent force-field describing the interatomic interactions, the  
147 schematic of the unit cell of our COF-1 structure are given in the Supporting Informations. We  
148 note that there have been several similar 3D networks generated from 2D layers reported in prior  
149 literature.<sup>2,35-41</sup> For comparison, we also calculate the thermal conductivity for a 3D COF-300  
150 structure with a similar mass density (the schematic of the domain is shown in Fig. 4b). Fig-  
151 ure 4c shows the temperature dependent thermal conductivities of the two structures highlighting  
152 the more than an order of magnitude higher thermal conductivity of our 2D $\rightarrow$ 3D structure based  
153 on COF-1. Moreover, the interpenetrated framework also shows a more pronounced  $1/T$  trend

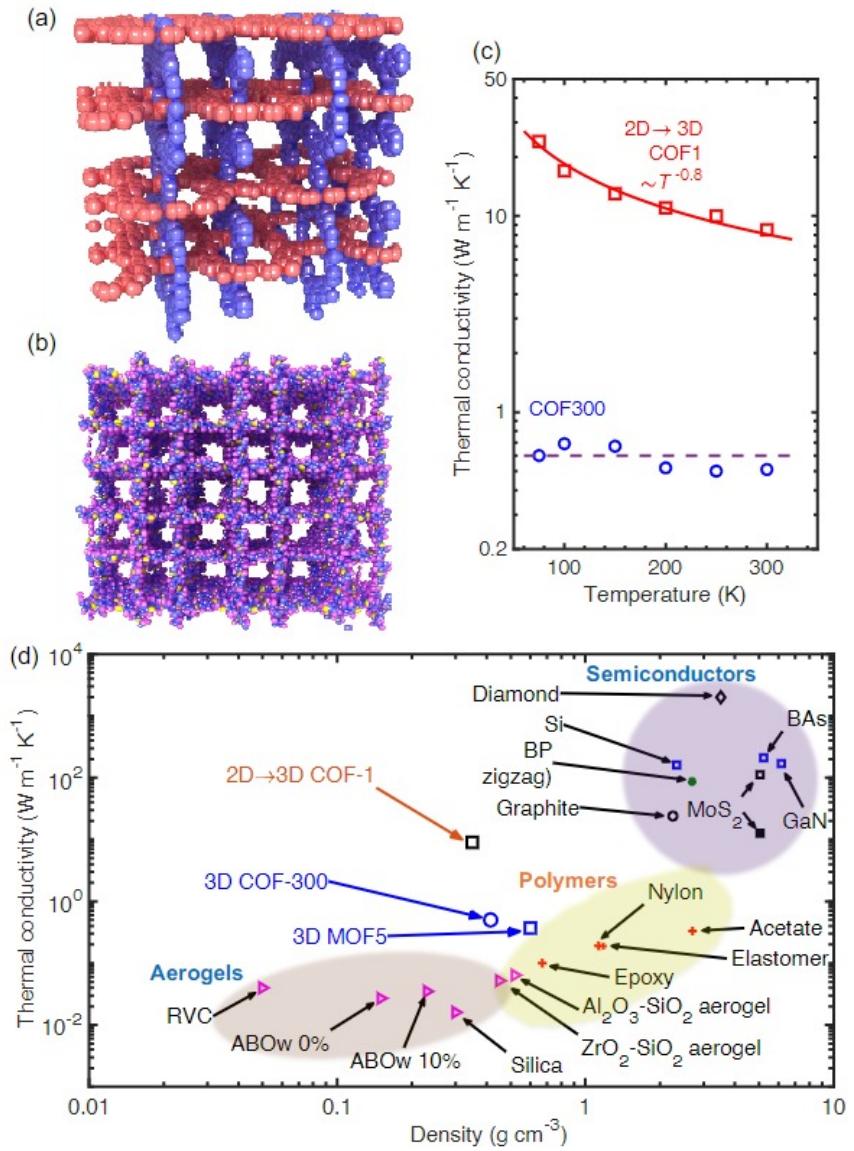


Figure 4: Schematic representations of our (a) interpenetrated 2D→3D porous framework based on the realistic material COF-1 and (b) 3D porous crystal based on COF-300. (c) The thermal conductivity of the interpenetrated structure is remarkably higher in comparison to the 3D counterpart even though the mass densities are similar for the two structures. The temperature dependence (or the lack thereof for the 3D COF-300) structure suggests that vibrational scattering at the pore walls dictates thermal conductivity in the 3D COF-300 case, whereas anharmonic phonon scattering is more dominant in the interpenetrated structure. (d) Thermal conductivity as a function of mass density for different classes of materials. Semiconductors with relatively higher mass densities have higher thermal conductivities, whereas porous solids such as metal organic frameworks, covalent organic frameworks and aerogels possess low thermal conductivities. Our interpenetrated 2D→3D structure based on COF-1 marks a new regime of materials design combining ultralow densities and ultrahigh thermal conductivities.

154 (see Fig. 4c) compared to 3D COF-300, which is usually attributed to Umklapp scattering or an-  
155 harmonic phonon-phonon scattering dominated processes dictating heat conduction. For defect  
156 free, crystalline solids these multiple phonon scattering processes create thermal resistance and  
157 lead to the typical  $1/T$  trend with temperature.<sup>32,33</sup> In contrast to our COF-1 based interpenetrated  
158 framework, the thermal conductivity for our COF-300 structure is independent of temperature for  
159 the temperature range studied in this work, which can mostly be attributed to vibrational scattering  
160 at the pore walls in the 3D structure.

161 To gauge the thermal performance of our 2D→3D interpenetrated frameworks, we compare the  
162 thermal conductivity of our interpenetrated COF-1 structure with that of other classes of materials  
163 as a function of their mass densities in Fig. 4c. In general, the increase in mass density usually  
164 leads to higher thermal conductivities as exemplified by the relatively higher thermal conductivities  
165 for metals and semiconductors. Similarly, fully dense 2D materials such as BP and MoS<sub>2</sub> also  
166 possess high thermal conductivities. The thermal conductivity of MOF-5 and COF-300 show  
167 enhanced thermal conductivity compared to aerogels and other porous materials with similar mass  
168 densities. However, our interpenetrated COF-1 structure with more than a ten-fold increase in  
169 thermal conductivity compared to the 3D organic frameworks marks a new regime of materials  
170 design that combines ultrahigh thermal conductivity with ultralow mass density. This regime can  
171 particularly be useful for structural materials that demand superior physical properties such as high  
172 thermal conductivity.<sup>42,43</sup>

173 In summary, our systematic atomistic simulations show that porous framework solids made  
174 from the interpenetration of 2D layers are endowed with enhanced thermal transport properties  
175 that derive from the supramolecular interactions between the frameworks. We show that inter-  
176 penetration of 2D layers have superior thermal conductivities as compared to individual and inter-  
177 penetrated 3D frameworks. We ascribe this to the enhanced phonon hardening and shift of heat  
178 carrying vibrations to higher frequencies in the interpenetrated 2D porous crystals. In contrast to  
179 the conventional understanding, our results from extensive MD simulations show that for interpen-  
180 etrated 2D frameworks, high thermal conductivities are associated with higher porosities. With the

181 growing interest for organic framework materials, 2D→3D interpenetration in COFs and MOFs  
182 paves a new strategy in attaining “user-defined” physical properties in these materials through con-  
183 trol over the level of interpenetration between the individual 2D layers. These physical properties  
184 could potentially be further manipulated by the introduction of guest molecules inside the pores.  
185 In this regard, our interpenetration scheme could also increase gas storage capacities by increas-  
186 ing adsorption sites and also leading to better thermal management of the large amounts of heat  
187 generated during adsorption.

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## 192 **Competing Interests**

193 The authors declare no competing interests.

## 194 **Data Availability**

195 The data supporting the present work is available from the corresponding authors upon reasonable  
196 request

## 197 **Supporting Information Available**

198 The Supporting information is available free of charge at .

199 • Details of the computational domain setup, equilibrium molecular dynamics approach, non-  
200 equilibrium molecular dynamics approach, interatomic potentials, spectral heat flux, me-

201 chanical properties.

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325 **TOC Graphic**

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