Softmax Policy Gradient Methods Can Take Exponential Time to Converge

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Abstract

The softmax policy gradient (PG) method, which performs gradient ascent under softmax policy parameterization, is arguably one of the de facto implementations of policy optimization in modern reinforcement learning. For γ-discounted infinite-horizon tabular Markov decision processes (MDPs), remarkable progress has recently been achieved towards establishing global convergence of softmax PG methods in finding a near-optimal policy. However, prior results fall short of delineating clear dependencies of convergence rates on salient parameters such as the cardinality of the state space $\mathcal{S}$ and the effective horizon $\frac{1}{1-\gamma}$, both of which could be excessively large. In this paper, we deliver a pessimistic message regarding the iteration complexity of softmax PG methods, despite assuming access to exact gradient computation. Specifically, we demonstrate that the softmax PG method with stepsize η can take

$$\frac{1}{\eta}|\mathcal{S}|^{2^{\Omega\left(\frac{1}{1-\gamma}\right)}} \text{ iterations}$$

to converge, even in the presence of a benign policy initialization and an initial state distribution amenable to exploration (so that the distribution mismatch coefficient is not exceedingly large). This is accomplished by characterizing the algorithmic dynamics over a carefully-constructed MDP containing only three actions. Our exponential lower bound hints at the necessity of carefully adjusting update rules or enforcing proper regularization in accelerating PG methods. [1]

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