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Spontaneous Mathematical Moments Between Caregiver and Child During an Engineering Design Project

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Abstract

Research studies support caregivers' involvement in children's mathematical journeys, as foundational to their cognitive development and academic success as mathematical learners. The purpose of this intrinsic case study was to understand how a caregiver initiated and/or continually engaged their child in spontaneous mathematical moments during a family engineering project. Through the analysis of approximately 13.5 hours of video data, we identified three ways in which a caregiver guided, supported, and challenged their child through a shared endeavor of designing a remote-controlled delivery robot—discussing mathematical concepts and engaging in mathematical practices; encouraging use of physical objects, tools, and materials for mathematical application and visualization; questioning that invited mathematical exploration, contribution of ideas, and reasonableness of responses. These findings highlight caregivers' potential to engage their children in mathematics through guided participation in an out-of-school learning context not designed to elicit mathematical moments. Implications for how early childhood practitioners can develop opportunities and supports for caregivers and children to engage in mathematics in a variety of learning environments is included in the discussion.

Keywords Caregiver–child · Engineering · Mathematical moments · Out-of-school environment

Introduction

Engagement with mathematics happens everywhere—in school, on the playground (Gaudreau et al., 2021), through take-home math bags (Linder & Emerson, 2019), within the engineering design process (Firdaus et al., 2020), at museums (Pattison et al., 2018), and in the grocery store (Hanner et al., 2019)—and more often facilitated through and researched in designed or "engineered" learning environments (Vedder-Weiss, 2017). However, research that examines spontaneous and serendipitous mathematics moments, particularly between children and caregivers in unstructured and out-of-school contexts, are scant. Research on serendipitous learning and engagement in science highlights how it may enhance or alienate children from identifying with science (Vedder-Weiss, 2018), and provide playful opportunities for co-construction of knowledge (Crowley et al., 2001; Goodwin, 2007) and deep engagement in a particular topic of interest (Crowley & Jacobs, 2002), and afford time to engage in science practices such as argumentation (Vedder-Weiss, 2017). We hypothesize, as do others (e.g., Callanan & Braswell, 2006; Walkoe & Levin, 2020), that such spontaneous and serendipitous moments accumulate over time to support children's intuitive understanding of concepts in a variety of environments including school, libraries, homes, and other learning contexts (i.e., learning ecosystem), regardless of STEM field.

In our research, we aim to explore the following research question: How do caregivers engage their child(ren) in spontaneous mathematical moments within out-of-school

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learning environments? In this study, we address this question by presenting a single case study of how a caregiver[1] engaged their child in spontaneous mathematical moments within a project aimed at developing a program for integrating engineering design practices with an emphasis on emerging technologies (i.e., making, DIY electronics) into home environments of families. These mathematical moments are defined as a spontaneous experience to engage with and/or explore mathematical ideas and concepts in a learning environment not structured to elicit mathematical moments (i.e., designed or "engineered" learning environments), but arise within the situation (Cunningham, 2015).

At the core of this study is the discussion about what counts as mathematics. Oftentimes, mathematics in schools are perceived and privileged by many stakeholders, including caregivers, as the golden standard (e.g., Greiffenhagen & Sharrock, 2008; Nemirovsky et al., 2017). Although, school mathematics is a powerful form of learning that engages children from around the world in learning important concepts and skills, it strips the complexity and authenticity of engaging in mathematics, and often omits the cultural, home/community, and informal ways of knowing and doing mathematics (Civil, 2016; Nemirovsky et al., 2017). In return, this limits children's learning of mathematics, leads to a low sense of belonging and a negative view of self as a mathematics learner (e.g., Barbieri & Miller-Cotto, 2021). We argue that caregivers are able to engage children in situated and/or authentic mathematics within out-of-school learning contexts, particularly in situations where mathematical supports, such as prompt cards and signage, are not provided to caregivers (e.g., Hanner et al., 2019; Hassinger-Das et al., 2018).

The results from this study have implications for how early childhood practitioners consider, acknowledge, and improve mathematical learning for young children. The significance of understanding this particular phenomenon lies in "moving from the study of the probable to a close examination of the possible" (Wineburg & Wilson, 1991, p. 396), particularly within a broader narrative of supporting young children as learners in an ecosystem as opposed to spaces in which mathematics is bounded by standards and well-defined problems for example. We further contend that the results of this case study will support early childhood practitioners in developing opportunities and supports for caregivers and children to engage in mathematics in a variety of learning environments. These are detailed in the discussion.

Research on caregiver–child interactions and conversations in out-of-school STEM contexts (e.g., museums, zoos) highlights the potential for caregivers to guide and bolster children's STEM understandings, interest in and positive dispositions toward STEM, recall of STEM information, and greater likelihood of pursuing a STEM-degree and career (e.g., Acosta et al., 2021; Callanan et al., 2020; Haden et al., 2014; Milner-Bolotin & Marotto, 2018). Within such contexts, caregivers enact varying roles as they negotiate how to support their child and leverage their child's strengths from the beginning to the end of a STEM activity or exhibit (e.g., Sadka & Zuckerman, 2017; Simpson et al., 2021a). As such, the uniqueness of caregiver–child conversations lies in a caregiver's ability to build on their children's abilities, experiences, and prior knowledge (e.g., Sun & Moreno, 2021; Umphress, 2016; Uscianowski et al., 2020), as well as a caregiver's ability to attend (i.e., observe and/or listen) to their children engaged in a STEM activity or exhibit and respond accordingly (Callanan & Braswell, 2006). Engaging in the teaching and learning of mathematics within out-of-school contexts (e.g., homes, playgrounds) is often framed by caregivers as a shared family experience and tends to include budgeting, home improvement projects, games, gardening, cooking, and verbal exchanges during mealtime (e.g., Esmonde et al., 2012; Pea & Martin, 2010). In what follows, we include relevant scholarship that highlights caregiver's mathematical practices and talk, tool use, and questioning with their child(ren) when engaging in mathematical moments in out-of-school learning environments.

First, in this study, mathematical practices are not monolithic, mutually exclusive, or homogenous practices defined as those strategies, approaches, and activities undertaken by "experts" in the field of mathematics or defined by curriculum and standards documents (Moschkovich, 2013). Instead, we consider mathematical practices as cultural, social, and cognitive ways of engaging with mathematics when the need arises within a familial context (e.g., home, school, forest) (e.g., Moschkovich, 2013). For example, Goldman and Booker (2009) and Takeuchi (2015) described family mathematical practices as engagement in distributed problem solving and reasoning around situations such as budgeting for the prom, engaging in baseball records and statistics, or calculating international currency conversions. Additional scholarship on family mathematical practices highlighted the role of caregivers in modeling particular practices—ways of thinking about a math task or how to use a diagram for information (Parks & Bridges-Rhoads, 2018). Moreover, research has shown that caregivers tend to focus more on number concepts (e.g., counting. reading numbers), geometry concepts (e.g., shapes), and operations (e.g., addition) in their everyday and out-of-school experiences with their young children than other mathematical concepts such as representations, fractions, and measurement (e.g., Galindo et al., 2019; Ramani et al., 2015; Zippert & Rittle-Johnson, 2020).

[1] We use caregiver to represent an adult who consistently provided care and informal education to their child(ren) in our study.

Second, mathematical tools and manipulatives encompass a variety of forms and mediums including semiotic tools (e.g., signs, symbols), digital tools (e.g., geogebra, online manipulatives), and physical or concrete tools (e.g., rulers, unifix cubes). Meta-analyses show a positive effect of mathematical tools on children and adolescents' achievement, retention, and attitudes toward math when utilized within a school context (e.g., Carbonneau et al., 2013; Hillmayr et al., 2020; Peltier et al., 2020). The focus here is on any physical tool or object that children can feel, touch, move, manipulate, or rotate to create, apply, perceive, and/or formalize mathematical ideas and concepts in out-of-school contexts. As an example, Goldman and Booker (2009) noted how families utilized appropriate tools (e.g., calculators, paper and pencil) depending on the situation at hand. One family used a ruler to determine the distance from home to school on a city map. One could argue that such tool use is more common to tool use in workplace environments such as a pilots' method for finding crosswinds (Noss et al., 2000) or hand drawn-sketches to install carpet (Masingila, 1994). Cultural and familial contexts may also involve the use of everyday materials (i.e., non-mathematical tools) for mathematical use (e.g., Masingila et al., 2011; Sanderson, 2017; Simpson et al., 2021b). For instance, Owens and Kaleva (2007) described how sticks with a mark was employed as a measurement tool for the construction of a canoe.

Third, questioning can serve many purposes such as to seek information, promote reasoning and critical dispositions, stimulate curiosity and exploration, and generate multiple explanations (e.g., Osborne & Reigh, 2020; Walker & Nyhout, 2020). As stated by Osborne and Reigh (2020), "questioning is one of the most important epistemic cognitive acts" (p. 281). Research consistently shows that children gain an understanding of science and mathematics concepts, engaged in problem solving, enhanced their language development, and participated in more STEM-talk when a caregiver understood the goal of the activity, made connections to STEM concepts and/or prior knowledge, and posed open-ended wh-questions, as well as high cognitively challenging questions (e.g., Chandler-Campbell et al., 2020; Crowley et al., 2001; Duong, 2021; Franse et al., 2020; Haden et al., 2014; Zambrana et al., 2020). For example, Willard et al. (2019) described how caregivers' encouragement to explore versus encouragement to explain had different effects on children's engagement at a gear exhibit—more time investigating the gears and building complex machines in contrast to more time talking about gears, respectively. Some of these results are based on studies in which interventions were in place to support caregivers such as showing the solution to an inquiry activity beforehand (Franse et al., 2020), providing instruction specific to an exhibit or build (e.g., Vandermaas-Peeler et al., 2016; Willard et al., 2019), or providing questions to pose while reading a book together (e.g., Birbili & Karagiorgou, 2009). However, this should not discount the math questions posed by caregivers during everyday family inquiries as family questioning practices are grounded in family routines that capture how children come to understand their world and what they notice and attend to in their day-to-day living (Goodwin, 2007; Keifert, 2015), as well as how caregivers perceive their children's math ability (Duong, 2021; Uscianowski et al., 2020).

Theoretical Grounding

As the above research highlighted, the engagement of children as learners of mathematics is shaped through participation in activities within an out-of-school context and through the support of the family. Hence, in this study, we utilized a socio-cultural perspective, which views learning as active participation and joint engagement in cultural and social activities within a community of practice (Mejia-Arauz et al, 2018; Rogoff et al., 1993). We employed Rogoff et al. (1993) guided participation in which participation is guided, supported, and challenged with an 'other' in a shared endeavor. In this study, this other referred to the caregiver and the shared endeavor was the development of a robot (see below) within a culturally valued activity, engineering. Rogoff (2008) defined participation as an interpersonal process where individuals are actively observing and/or communicating with their words and hands. It builds upon the notion of the Zone of Proximal Development as it involves "not only the face-to-face interaction, which has been the subject of much research, but also the side-by-side, joint participation that is frequent in everyday life" (Rogoff, 2008, p. 60). As argued by Valle (2009), joint and guided participation with caregivers supports the development of children's habitual ways of being, knowing, and reasoning that are valued in their familial and cultural community. Additionally, we acknowledge that guided participation between a caregiver–child is affected by relational and developmental assumptions, a child's position within the family structure, and caregiver's perspective of expectations and behaviors based on their child's age, learning needs, and maturity (Sillars, 1995). Simply, guided participation is grounded in a caregiver's understanding of their child (Umphress, 2016; Uscianowski et al., 2020).

Methods

An intrinsic case study was utilized to understand the particular nature of a situation of interest, the manner in which a caregiver–child dyad (Tanya and Cindy, pseudonyms) engaged in mathematical moments during an engineering design project developed and designed by the dyad (Stake, 1995). This case was selected from a larger research project conducted between February to May

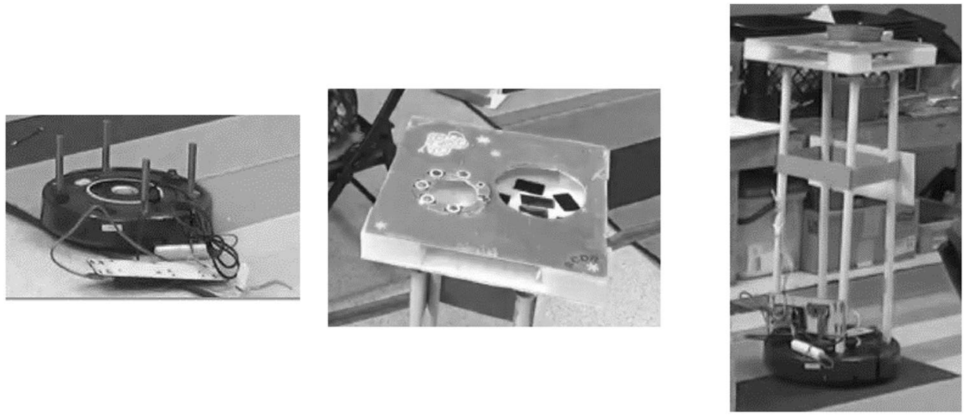

Fig. 1 Cindy's Remote-Controlled Delivery Robot. The image on the left is the Roomba. The image in the middle is the top of the tray with a place for a cup and bowl. The image on the right is the completed version of the delivery robot

2019. In that project, several dyads were asked to engineer a solution "that would either fix a problem or make life easier or better for you, your family or your community." They were then guided through parts of the engineering design cycle—define the problem, brainstorm solutions, prototype, iterate and communicate problem and solution. The uniqueness of this larger study was that each dyad had the opportunity to define a problem salient to them as opposed to every dyad addressing the same engineering problem. The case of Cindy and Tanya was selected after data was collected, but prior to data analysis. They were chosen for this study for three reasons—(a) convenience (Stake, 1995); (b) their relationship in this program was not different from their relationship at home (i.e., "Yeah, I think it is pretty much the way it is normally."), indicating that any spontaneous mathematical moments in this study would be a normative way of interacting with each other; and (c) the possibility of their self-defined problem to elicit spontaneous mathematical moments.

As such, Cindy and Tanya's project were summarized by Cindy as, "My project is a remote-controlled delivery robot to help people who can't get out of bed or are sick…I was thinking about someone in a nursing home." Cindy's prototype was a Roomba (i.e., robot) that would deliver food to various rooms in her home environment, which consisted of constructing a tray to carry and deliver items, as well as programming an Arduino microcontroller to map the robot's movement through the home (see Fig. 1). Prior research regarding educational robots points to the potential to support Cindy as a mathematical learner (e.g., Lopez-Caudana et al., 2020; Sisman et al., 2020), while also highlighting the integral role that educators and caregivers play in developing this mathematical understanding (e.g., Forsström, 2019; Zhong & Xia, 2020).

Data Collection

The main source of data was video recordings of each monthly session with Tanya and Cindy working alongside an engineer and/or member of the research team, as well as home video recordings. During monthly sessions the research team positioned cameras to capture the interactions between Tanya and Cindy, as well as with the research team and engineers. Families were provided with mobile tablets that they could use to record video at home and a method for uploading videos to cloud storage for access by the research team. Video data for Tanya and Cindy amounted to 11 h from the monthly sessions and approximately 3.5 h of video from home visits.

Data Analysis

The analysis was conducted in two phases. During the first phase, the first two authors watched all the videos in chronological order, individually looking for mathematical moments. Each researcher noted the time range and provided a brief overview of the relevant interaction in terms of engagement with mathematical ideas or concepts within the engineering project. For instance, one researcher noted the following moment in the construction of the tray, specifically in computing the average height of three beds to determine the height of the tray from the top of the Roomba: "Cindy did the computation of the average by hand (some thinking aloud); took a partitive division approach by drawing three circles (for three beds) and equally distributing the total height of the beds; mom probed into Cindy's strategy for dividing." Our analytical goal was not to establish interrater reliability, but to capture identifiable mathematical moments, or events, for further analysis (Rogoff, 2008). Our understandings of and experiences with mathematics were from two different lenses—as a mathematics teacher educator and STEM education researcher, and science teacher and science education doctoral student, respectively, which Denzin (1984) identified as investigator triangulation. Therefore, we met five times to discuss our observations and interpretations, which allowed the first two authors to articulate, explore, and challenge one another's interpretations and supporting evidence through an evolving process (Charmaz, 2006). We further discussed how our observations and interpretations were grounded in our reading of literature regarding mathematics between caregivers and child(ren). The final meeting focused on identifying specific moments or chunks that addressed the research question, which were transcribed verbatim and included non-verbal acts of communication. In total, 12 events, ranging from 35 s to 15 min in length, were identified for further analysis.

In the second phase, we utilized an inductive approach to the transcripts, in which insights and themes emerged from the raw data without the constraints of structured approaches (Thomas, 2006). We contend this approach is appropriate for an intrinsic case study where the intent is to understand the case itself (Stake, 1995). We individually read and re-read through the transcripts and noted the ways Tanya initiated and continually engaged Cindy in spontaneous mathematical moments. We provide one example to illustrate the process of moving from the specific, or the raw data, to the general. As such, we noted specific instances in which Tanya introduced or utilized mathematical vocabulary (e.g., average, degrees, scale factor), symbolic notation (e.g., how to represent a repeating decimal), practices (e.g., precision), or concepts (e.g., dividing by multiples of ten) throughout the 12 identified events. These specific instances were also situated in both formal or informal ways of knowing and doing mathematics. At times, Tanya would refer to mathematical strategies and concepts that Cindy was learning in school (e.g., division) or grounded in a similar experience of creation and innovation (e.g., construction of a dollhouse). When we met to discuss our notes, we were similar in our understanding of these spontaneous moments, which moved the specifics to a more general way in which Tanya engaged Cindy in mathematical moments across the video data (see #1 below). Through our analysis and ongoing discussions, we identified three ways Tanya routinely initiated and engaged Cindy in spontaneous mathematical moments during the engineering design process: (a) Discussing mathematical concepts and engaging in mathematical practices that were based on an awareness of formal and informal ways of thinking; (b) Encouraging use of physical objects, tools and materials for mathematical application and visualization; and (c) Questioning that invited exploration, contribution of ideas, and reasonableness of responses.

Fig. 2 Image of Cindy and Tanya's Comb-bot

Dyad Profile and Context

Case study design includes a rich description of the physical situation as to provide the reader with a sense of being present (Stake, 1995). Utilizing Parker (2016) as an example, the case will be provided from the perspective of the first author, beginning when I first met Cindy and Tanya on a Saturday morning in January 2018. Cindy, a third-grade student, and her mom, Tanya, entered the vibrantly painted room at a local community organization, walking straight to the long table filled with materials and tools for the day's activities. The room itself was stocked with books, seven desktop computers, and four square tables, each with four seats. Due to the number of dyads in attendance, Cindy and Tanya sat with another dyad, Walt and Mac, a father and son. As an initial insight into the overarching program, all families that attended in January were asked to "think about something you can build that might improve the quality of life of someone you know or someone you know about." Following the "rules" provided by the third author, Cindy and Tanya silently jotted their individual ideas on paper. Through this shared, yet parallel experience, Cindy and Tanya had a similar idea that became the focus of their time together. As stated by Tanya to the larger group, "She wrote she could help by picking up trash so it doesn't go in the ocean, recycling cardboard and plastic."

As this example illustrated, Tanya often positioned Cindy as lead engineer of her ideas. I continued to observe such behavior in the creation of the prototype (see Fig. 2). As I walked over to a table where hot glue guns are located, I observed Tanya cutting straws for Cindy to glue on her "comb-bot," a rotating device that would gather waste from the ocean. When finished cutting the straws, Tanya held the prototype in place while Cindy hot glued the straws on a PVC pipe. This was done without little verbal interaction, which I perceived as a mutual understanding of what needed to be done to create the prototype. In an interview, Tanya described her pedagogical style, or role, as an observer through "giving Cindy space." She further articulated her role as ensuring that "I wasn't directing what was being done and being a support for what Cindy was directing to be done.... So that was hard for me to not interfere, but also realize it's important to not interfere."

Results

Throughout this section we present specific instances where Tanya initiated or continually engaged and guided Cindy in spontaneous and authentic mathematical moments situated within the engineering design process. Tanya's actions and behaviors were not isolated events, but occurred throughout the engineering design process, from defining the problem to testing and making necessary changes to the prototype. The spontaneous mathematical moments included in the results are representative of the data set as a whole. Nonverbal actions relevant to these moments are italicized in the transcripts.

Mathematical Concepts and Practices

In the examples below, we illustrate how Tanya's knowledge of mathematical concepts and elicitation of mathematical practices engaged Cindy as a mathematics learner within the context of Cindy's project. The first two transcripts occurred during the April workshop when Tanya and Cindy were brainstorming how to securely attach the tray on top of the Roomba (see Fig. 1). Cindy's initial idea of creating a scissor lift that would adjust to various bed heights was not feasible within the time constraints of the project. The first transcript begins as they are discussing the appropriate height for the tray once mounted on top of the Roomba.

1.1 Tanya Okay. And so you were talking about the height of your stands and what you, you had said that- oh, well maybe you'll do it a certain way.
1.2 Cindy Yeah, in the middle of the three beds.
1.3 Tanya: Okay. So what would that measurement be here? How would you figure out that measurement?
1.4 Cindy That would, wait…it would be all the beds to get all of it? No, it'd be the biggest height and then split that in half. So 32 in half is…
1.5 Tanya Are you trying to find the average?
1.6 Cindy Yeah.
1.7 Tanya So if you are going to take an average, you would take the three numbers. You would add them together and then you would divide them by three, if you're trying to get the average. Is that what you want? Or are you trying to do it one particular height to get to the person that…it's kind of your choice here.
1.8 Cindy No. I want it to be the average. So then it could get to anything. And it would either be a little too tall or a little too short. They [people in bed] would have to reach down a little bit or reach up, or like sit up.
1.9 Tanya Okay. So you think we should do the measurement or do you want to figure out the actual height?
1.10 Cindy I want to figure out the average.

The transcript highlights several things. First, Tanya reminded Cindy how to find an average (Line 1.7); thus, encouraging Cindy to apply a mathematical concept in an everyday context. Second, Tanya provided Cindy with an opportunity to decide whether the average of the height of the three beds or the height of one bed was preferred (e.g., "It's kind of your choice here."; Line 1.7). While it is more than likely that Tanya knew the most appropriate approach within this context, she allowed Cindy to make her own decision (i.e., agency; Norén, 2015). Cindy revealed her reasoning in Line 1.8 as to why the average was appropriate in that the person in bed would have to reach down or up to gain access to food on the tray. Third, Tanya provided Cindy with the definition and language to describe the approach, which Cindy adopted as part of her language (Lines 1.8 & 1.10). Fourth, this example illustrates how Tanya was "with" Cindy in these moments as she gathered evidence of Cindy's thinking and made in-the-moment decisions regarding the project and Cindy's process and progress. This was often done through questioning (e.g., Line 1.3).

In the next transcript, Cindy just completed cutting four PVC pipes to the appropriate average height of the beds. She was interested in converting 27 ½ inches to centimeters.

1.11 Cindy (*Speaking into a tablet.*) Centimeters to inches.
1.12 Tanya (*Reaches across the table to grab a tape measure which has two sides, one side shows centimeters, the other side shows inches.*) Instead of using that, there's a way that you can figure it out using this [tape measure]. What do you think it is?
1.13 Cindy (*Grabs tape measure and pulls the tape from the housing. Smiles.*)
1.14 Tanya Yeah, you don't always need that [tablet]. You can figure it out without just trying to get the quick answer.
1.15 Cindy Eight and a half. (*Let's go of the end of the tape measure and it retracts.*) I mean, no. (*Pulls the tape measure out again and seems to examine.*)
1.16 Tanya Yeah, that doesn't… Does that make sense to you? [Asking—How can 27 ½ inches equal 8 cm?]
1.17 Cindy It said eight. (*Continues looking at the tape.*) Oh no, I get it. I get it. Sixty…sixty…sixty-eight and a half.
1.18 Tanya (*Takes the tape measure.*) These are decimals, so it actually would be 68 and six-tenths. When you're doing measurements, sometimes that tenth of a centimeter is going to make a big difference.

This mathematical moment was sparked through Tanya's question that pushed Cindy to think of another conversion strategy, namely, reading the tape measure as opposed to relying on a technological device for the conversion centimeters to inches (Line 1.12). In other words, Tanya encouraged Cindy to consider an informal approach of conversion through utilizing the tape measure as a mathematical tool as opposed to converting measurement units by multiplying or dividing quantities, an approach more common in school mathematics. In Line 1.16, we also observe Tanya questioning the reasonableness of Cindy's first response of 8, indicating that 27.5 inches was the same as 8 cm. This question was intentional; it served a purpose as Cindy was encouraged to reflect upon her response. Lastly, in Line 1.18, Tanya explained to Cindy the importance of accuracy and precision appropriate to this particular context.

Use of Physical Objects, Tools, and Materials

The example presented here illuminates how Tanya spontaneously utilized available tools to assist Cindy in thinking spatially about her design and floor plan of her house. In the example above, the tape measure was utilized as an alternative mathematical tool to computing a conversion as opposed to a digital or semiotic mathematical tool. As another example, in February, Tanya, Cindy, and a volunteer engineer were discussing the size of the tray; specifically, how long and wide the tray should be designed to securely hold a drink, a bowl, and silverware. This transcript begins as Cindy is using a tape measure and thinking out loud that the tray needed to be at least 9 inches wide to hold a drink and silverware.

2.1 Tanya (*T takes hold of the ends of a piece of white standard A4 paper while C marks 9 inches on the tape measure.*) This is a good reference. Do you know what size, this is very standard. Do you know what size this is?
2.2 Cindy Eighteen by eleven.
2.3 Tanya Eight and half by eleven. Sometimes it's nice for me at least to have a visual because depending on how your brain works with the spatial stuff. I know this is 8.5 by 11. (*Tanya removes her hands from the sheet of paper and sits back in her chair.*)
2.4 Cindy Yeah, and then the food area (*C takes hold of tape measure and considers a length.*) would have to be pretty big. . . .I'd say 18 inches. (*C extends tape measure to indicate 18 inches.*)
2.5 Tanya I'm thinking about the hallway… (*C repositions tape measure across the width (8.5") of the paper.*) and the size, and if the tray is bigger than the hallway. (*C repositions tape measure across the length of the paper.*)
2.6 Cindy (*C measures along one side of the paper—11 inches—and moved left hand from one side of the paper to the other—indicating 8.5 inches.*) So it would be like this big, (*C taps the middle of the sheet of paper.*) but a little bigger.

In this excerpt, the mathematical moment was initiated as Tanya provided Cindy with a two-dimensional tool to think about the size of the tray, an 8.5×11-inch sheet of paper. Tanya created a mathematical tool for visualization from the available materials in the environment. We did not observe Cindy use the sheet of paper as a tool until Tanya challenged Cindy to think about the size of the hallway in relation to the size of the tray (Line 2.5); asking Cindy to think spatially about her design. Cindy noted that the tray should be a little bit bigger than the size of the sheet of paper (Line 2.6).

Questioning

Throughout the excerpts above, we highlighted Tanya's questions, questions that invited explorations and reasonableness of response (Line 1.16). Tanya's questions are sensitive and build upon Cindy's ways of thinking (Line 1.5), and are posed in a way that Cindy can enter the intellectual space as a collaborator and mathematical thinker (Line 1.7). The transcript that follows is another example of Tanya's questioning and was captured in Cindy and Tanya's home environment. The use of a tablet served as a remote control for Cindy to gain an understanding of the robot's path in her home environment before programming the path in *Python*. Cindy was at one end of the hallway and pushing "buttons" on the tablet to rotate the robot to return to the kitchen. Cindy determined that it takes "13 times to go 90 degrees." The excerpt begins as Tanya observed Cindy from the other end of the hallway and posed a question that encouraged Cindy to continue exploring her claim or conjecture, a statement based on an observation specific to this project.

3.1 Tanya You want to try it again to test your hypothesis?
3.2 Cindy Yes. (*Cindy begins pushing the tablet and counting to herself.*) No, 14 is for 180 [degrees].
3.3 Tanya Okay, so if 14 ends up 180 [degrees], how many would be 90 [degrees]?
3.4 Cindy Seven.
3.5 Tanya Okay. Then you can test it again to see if you are right.

As noted in Line 3.5, Tanya was not satisfied with Cindy's response of 7 as "right" and encouraged her to continue testing. In other words, Tanya attended to more than the answer and asked Cindy to support her response through evidence. Cindy again tested this claim that pushing the tablet seven times would rotate the robot 90 degrees from any position. Cindy determined that her claim was incorrect—"8 is 90 degrees"—which made her question her initial claim that pushing the tablet 14 times would be a 180-degree rotation. Cindy once again tested this claim, but this was done without a prompt or question from Tanya. In other words, Cindy was primed and empowered to continue this exploration. Through iterations of testing her claims, Cindy determined that "16 is 180 [degrees] and 8 is 90 [degrees]."

As we documented, this spontaneous mathematical moment was initiated by a question from Tanya that was perceived and taken up by Cindy as a question that sparked an exploration to test a claim (Line 3.1). This moment also highlighted the power of not evaluating an answer as "right" (e.g., 7 is 90 degrees) or "wrong" as Cindy determined that her initial claim was incorrect through evidence based on additional testing. We observed a similar situation in May as Cindy was tasked with demonstrating how the robot delivered food through using her home as a prototype blueprint. This included determining the appropriate scale for reducing the length of her hallway to the room setting (Fig. 1A). Tanya had stated, "Remember ratios, or putting things to scale. What if you want to do a half scale, or a third scale, or a quarter scale?" Cindy immediately stated, without any mathematical justification, "I'd probably do a half scale." Tanya suggested to Cindy that she test her initial claim that a half scaled-model would be an appropriate-size for the room, in which the longest path would have to be at least 4.5 yards long.

3.6 Tanya I want you to see if it's even big enough. You can test it and see.
3.7 Cindy Okay. I'm going to start back here. (*as she repositions her body*) Will you hold it right here mom?
3.8 Tanya If you really need me too. But you're visualizing it right now, right? I'm just getting a general idea to see if it [half-scaled model] will work.
3.9 Cindy I was actually going to… (*C has marked a spot on the floor at 36 inches or 1 yard and now moving the tape measure so one end matches this marked spot.*)

In this short except, we observed Cindy enact upon Tanya's suggestion of testing the notion that a half-scaled model would work within the confines of the room (Line 3.7). This initiated a spontaneous mathematical moment as Cindy used the tape measure (Line 3.9) to iterate a length of 36 inches or 1 yard four times (i.e., 120 inches) and knew that she had enough room for an additional half yard to complete the path.

Discussion

The scenarios presented in this paper illustrated how one caregiver engaged her child in mathematical ideas and concepts that spontaneously arose throughout a shared endeavor, designing a remote-controlled delivery robot. These spontaneous mathematical moments extend current research that situate out-of-school family mathematical experiences in everyday activities such as cooking, budgeting, and home improvement projects (e.g., Esmonde et al., 2012; Pea & Martin, 2010) to mathematical experiences that are spontaneous and embedded within an ill-defined authentic engineering design problem (e.g., de Abreu, 2000). Such mathematical moments were initiated and/or continually expressed by the caregiver in three ways: (1) discussing mathematical concepts and engaging in mathematical practices that were based on an awareness of formal and informal ways of thinking; (2) encouraging use of physical objects, tools and materials for mathematical application and visualization; and (3) questioning that invited exploration, contribution of ideas, and reasonableness of responses. Based on these findings, we assert that caregivers are capable of engaging, supporting, challenging, and enhancing their child(ren) through mathematical moments that arise and unfold spontaneously in out-of-school contexts not designed to elicit mathematical moments.

We observed instances when Tanya made in-the-moment or spontaneous decisions that supported Cindy's mathematical practices and concepts. First, formal and informal mathematics was not framed as distinct ways of thinking about and doing mathematics (Moschkovich, 2013), but a way of thinking about mathematics more generally in everyday and authentic contexts. This was exemplified through finding an average height of three beds using paper and pencil or in converting between inches and centimeters using a tape measure. Second, the use of physical tools, materials, and objects was a way of engaging in mathematics. Similar to prior research, Tanya encouraged the utilization of an appropriate tool as needed within the engineering design process (e.g., Goldman & Booker, 2009), which may be more common to a workplace environment than a formal school setting (e.g., use of tape measure to convert inches to centimeters). Further, Tanya encouraged the use of physical tools such as paper-and-pencil than use of a technological tool when Cindy had some knowledge of the concept from school—division of whole numbers versus division of decimals. Third, a mathematical practice encouraged by Tanya was attention to precision and accuracy; for example, how to read a tape measure to the nearest tenth of a centimeter.

Additionally, Tanya posed a variety of questions that encouraged exploration (e.g., "How would you figure it out?"), agency (e.g., "What would you do?"), explanation (e.g., "Why do you want to make sure it's straight?"), and reasonableness (e.g., "Does that make sense?"). While these questions may seem similar to rhetorical pedagogical questions in that Tanya may have known the response and sought to elicit learning (Yu et al., 2019), we argue that Tanya's questions were purposeful within the context of the engineering design process and led to spontaneous mathematical moments that were not due to some intervention (e.g., Franse et al., 2020) or prior instructions (e.g., Willard et al., 2019). We agree with prior research that such questioning is likely grounded in family routines and practices (e.g., Keifert, 2015) and substantiated in what caregivers know about their child(ren) (e.g., Umphress, 2016).

We noted previously that there have been recent debates around what counts as mathematics (e.g., Greiffenhagen & Sharrock, 2008; Nemirovsky et al., 2017), which is typically grounded in school-specific ways of doing mathematics that are usually detached from real-world applications (e.g., Goldman & Booker, 2016). We argue that one significant contribution of this study is the provision of a counter-narrative to deficit views and assumptions of caregivers as mathematics educators (Langer-Osuna et al., 2016). This study highlights how young children may be engaged as mathematics learners within a larger learning ecosystem that extends to anywhere and everywhere within their communities (Rogoff, 2008). In addition, while we cannot make claims as to Cindy's development as a mathematics learner, prior research on spontaneous and serendipitous science moments would suggest that her engagement in spontaneous mathematical moments with a caregiver had a positive influence on her disposition towards, interest in, and understanding of mathematics (e.g., Goodwin, 2007; Vedder-Weiss, 2018). We recommend that future research should examine how engaging in spontaneous mathematical moments with caregivers, as well as other family members, may shape different child outcomes, including knowledge and identity development.

The results of this study have implications for early childhood practitioners across a variety of learning environments. First, the results from this study illustrate how practitioners may engage young children in mathematics through a shared endeavor, namely within an engineering design task. These engineering tasks do not have to be developed and integrated with a mathematical goal in mind, but instead attend to when one might engage students in spontaneous mathematical moments. As noted by McMullen et al. (2019), supporting such mathematical moments in any setting will likely improve children's development of mathematical thinking and learning of mathematical skills and concepts. Second, this research may provide practitioners ways to empower caregivers who may lack confidence in supporting their child(ren) as mathematics learners. This can be done through understanding caregiver's cultural and familial ways of engaging in mathematics in their home and other community institutions such as the playground or grocery store. Thus, practitioners can highlight for caregivers, and other family members, how they utilize mathematical practices and concepts through their own understanding and everyday interactions. Realistically, this may occur through family interviews, video or photographic journaling, and family STEM nights or playground activities in which practitioners observe and/or play alongside families. Third, practitioners should encourage caregivers to have frequent conversations around mathematical concepts and ideas through their everyday interactions with their children. Initially, caregivers might need support in ways to do this such as take-home math bags (Linder & Emerson, 2019), prompts posted in a playground or a grocery store (Hassinger-Das et al., 2018), or through sending mathematical ideas for interactions through a text-based app. These supports can be faded out as caregivers begin to understand how to engage their child(ren) in mathematics in any environment.

Limitations

The main data source for this study was video data from a single caregiver–child dyad—Cindy and Tanya—engaged in the engineering design process over a 4-month timeframe. Some may argue that focusing on a single case limits our ability to triangulate the results. However, we employed investigator triangulation in that two researchers with different backgrounds and experiences examined the same phenomenon (Denzin, 1984). Further, the goal of this case study was to provide a description of spontaneous mathematical moments that occurred within a particular social setting, not to examine or present a case that would be generalizable. We acknowledge that some may view this as a limitation. While we agree that we are limited in our ability to generalize the results, we contend that the results of this study attempted to generate a "local" or substantive theory that future research can build upon (Fouche, 2002; Rule & John, 2015). Lastly, some might argue that Tanya possessed some level of mathematical expertise and social resources that other caregivers may not possess. While this may be true, prior research has illustrated how caregivers are able to support their child without requiring an expertise in technology (Barron et al., 2009), engineering (Simpson et al., 2021a), or making activities (Sadka & Zuckerman, 2017).

Conclusions

Engagement in mathematics can occur anywhere and everywhere—homes, playgrounds and parks, museums, sidewalks, forests, school, and so forth. This implies that opportunities to engage in spontaneous mathematical moments are infinite. Yet, we need to expand our views of what constitutes mathematics beyond the school curriculum as this limited perspective may have negative implication on young children's development and identity as a mathematics learner. In this study, we highlighted the nature of one caregiver engaging their child in spontaneous mathematical moments in the development and construction of a remote-controlled robot—(a) Discussing mathematical concepts and engaging in mathematical practices that were based on an awareness of formal and informal ways of thinking; (b) Encouraging use of physical objects, tools and materials for mathematical application and visualization; and (c) Questioning that invited exploration, contribution of ideas, and reasonableness of responses. The authenticity of these moments is more aligned with a humanistic approach than one might expect in designed-based environments where a focus on mathematics is intentional.

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Data Availability Not available as the video data include identifiable information. Additional examples can be provided upon request.

Code Availability Not applicable.

Declarations

Conflict of interest There are no competing interest to declare.

Ethical Approval The study was approved by the appropriate institutional research ethics committee (Indiana University Institutional Review Board). We certify that the study was performed in accordance with the ethical standards as laid down in the 1964 Declaration of Helsinki and its later amendments or comparable ethical standards.

Consent to Participate Informed consent was obtained from all individual participants included in the study.

Consent for Publication The participant has consented to the submission of the case report to the journal.

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