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Relating Euclidean correlators and light-cone correlators beyond leading twist

S. Bhattacharya,[a] **K. Cichy,**[b] **M. Constantinou,**[a] **A. Metz,**[a,*] **A. Scapellato**[a] **and F. Steffens**[c]

[a] *Department of Physics, Temple University, Philadelphia, PA 19122 - 1801, USA*

[b] *Faculty of Physics, Adam Mickiewicz University, ul. Uniwersytetu Poznańskiego 2, 61-614 Poznań, Poland*

[c] *Institut für Strahlen- und Kernphysik, Rheinische Friedrich-Wilhelms-Universität Bonn, Nussallee 14-16, 53115 Bonn*

E-mail: metza@temple.edu

A perturbative matching procedure is needed in order to relate the parton quasi-distributions, that are calculable in lattice-QCD, to the corresponding light-cone distributions which enter physical processes. In the past, this type of matching has been studied extensively for twist-2 distributions. Here, we report on our recent calculation of matching coefficients for the twist-3 parton distributions $g_T(x)$, $e(x)$ and $h_L(x)$. A focus of the presentation are zero-mode contributions which pose a challenge for the matching procedure. We also briefly address the latest twist-3 matching results by other authors which include, in particular, quark-gluon-quark correlations.

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*Speaker

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1. Introduction

Parton distribution functions (PDFs) are key quantities that characterize the structure of the nucleon in quantum chromodynamics (QCD) [1]. With the help of QCD factorization theorems the (light-cone) PDFs can be accessed in a variety of high-energy scattering processes [2]. An important property of PDFs is their twist which determines the order in $1/Q$ at which PDFs appear in a cross section formula, with Q denoting the large energy scale of the process. Leading-twist (twist-2) PDFs, such as the unpolarized PDF $f_1(x)$, can be considered probability densities for finding a parton which carries the fraction x of the proton momentum. Twist-3 PDFs can be of equal magnitude as twist-2 PDFs and, therefore, are very important as well. Furthermore, they contain novel information about quark-gluon-quark correlations [3, 4] which cannot be addressed through the (twist-2) densities.

For a spin-$\frac{1}{2}$ hadron, three twist-3 quark PDFs can be identified. They are typically denoted by $g_T(x)$, $e(x)$, $h_L(x)$ [5]. The PDF $g_T(x)$ appears in the cross section for (polarized) inclusive deep-inelastic lepton-nucleon scattering (DIS). While several measurements of this observable have already been performed, the present uncertainties of $g_T(x)$, based on experimental data, are still very large; see, for instance, Ref. [6]. The other two twist-3 PDFs are chiral-odd and therefore decouple from the "simple" inclusive DIS process, but they could be measured through more complicated reactions such as the (polarized) Drell-Yan process [5], or single-inclusive particle production in proton-proton collisions [7].

In this situation, obtaining information on twist-3 PDFs from first principles in lattice QCD (LQCD) is highly desirable. For a long time, LQCD calculations of the parton structure of hadrons were limited to the lowest Mellin moments of PDFs, whereas the full x-dependence of the PDFs remained elusive. A major change of the situation came with the proposal to compute the so-called quasi-PDFs [8], which are spatial correlation functions that can be accessed in LQCD. The essence of this approach is that quasi-PDFs and the corresponding light-cone PDFs share the same infrared (IR) physics. They differ in the ultraviolet (UV) region, but this difference can be calculated, order by order, in perturbative QCD through a procedure referred to as matching [9–11]. Over the last years, the quasi-PDF approach has already been studied and used extensively for twist-2 PDFs; see Refs. [12–15] for reviews. On the other hand, only recently quasi-PDFs were explored for the first time in the context of twist-3 PDFs [16–23]. We note in passing that other Euclidean correlator approaches for the x-dependence of PDFs exist which are being pursued as well [24–26].

The present contribution is largely based on our previous one-loop matching calculations for $g_T(x)$ [17], as well as $e(x)$ and $h_L(x)$ [18], which served as an important ingredient for the pioneering LQCD estimates of twist-3 PDFs in Refs. [16, 21]. Our treatment exploits a quark target and resembles significant similarities with matching calculations for twist-2 PDFs. However, as discussed in more detail below, a key new element at twist-3 is the presence of zero-mode contributions which can give rise to delta-function singularities at $x = 0$. We argue that those contributions, while representing a challenge for the quasi-PDF formalism, fit into a matching formula when treated properly. Our studies did not include mixing with quark-gluon-quark correlators [17, 18], a point that was addressed recently in Refs. [19, 22]. Here, we also discuss the main findings of the latter two references, along with the pertinent implications for LQCD calculations of twist-3 PDFs.

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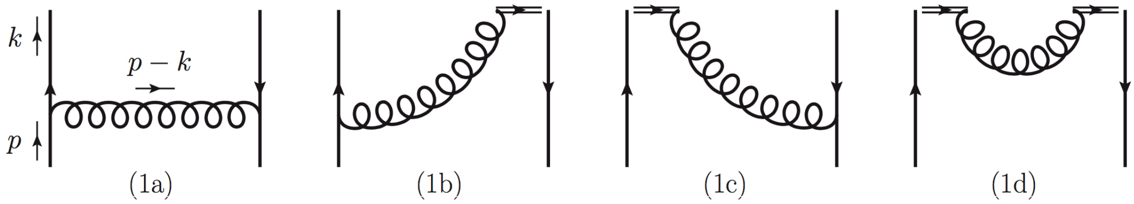

Figure 1: One-loop real diagrams for quark target contributing to both light-cone PDFs and quasi-PDFs.

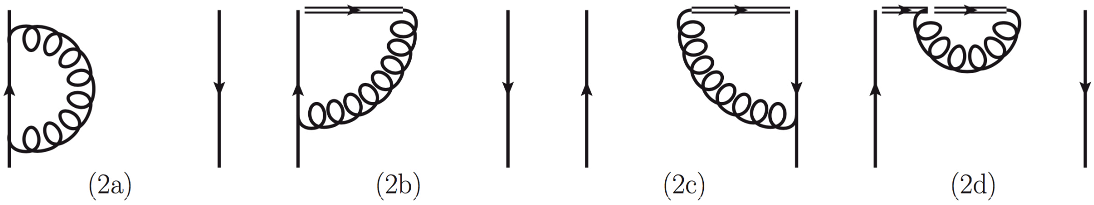

Figure 2: One-loop virtual diagrams for quark target contributing to both light-cone PDFs and quasi-PDFs. The Hermitean conjugate diagrams of (2a) and (2d) are not displayed.

2. Elements of matching for twist-2 PDFs

Before addressing the twist-3 case, we would like to briefly outline the matching procedure using a twist-2 example. In order to obtain the matching formula one can compute both the light-cone and quasi-PDFs for a quark target, with the relevant one-loop real and virtual Feynman diagrams shown in Fig. 1 and Fig. 2, respectively. Both types of PDFs exhibit IR singularities which must be regulated. In Refs. [17, 18], we presented results for three different IR regulators: nonzero gluon mass m_g, nonzero quark mass m_q, and dimensional regularization (DR).

To illustrate the essence of the quasi-PDF approach, let us consider the unpolarized twist-2 PDF f_1 and concentrate on the contribution from the diagram (1a). Using a nonzero gluon mass as an IR regulator, the result for the light-cone PDF reads

$$f_1^{(1a)}(x) = \frac{\alpha_s C_F}{2\pi} (1-x) \left(\ln \frac{\mu^2}{x\, m_g^2} - 2 \right), \tag{1}$$

where $\alpha_s = g^2/(4\pi)$, with the strong coupling g. The color factor of the diagram is $C_F = 4/3$, while the scale μ appears when exploiting DR for the UV divergence of the diagram. For the corresponding quasi-PDF, one finds

$$f_{1,\mathrm{Q}}^{(1a)}(x, p_3) = \frac{\alpha_s C_F}{2\pi} \begin{cases} (1-x) \ln \dfrac{x}{x-1} + 1 & x > 1 \\ (1-x) \ln \dfrac{4(1-x) p_3^2}{m_g^2} + x & 0 < x < 1 \\ (1-x) \ln \dfrac{x-1}{x} - 1 & x < 0 , \end{cases} \tag{2}$$

where p_3 is the momentum of the quark target. The light-cone PDF is nonzero only in the region $0 \le x \le 1$, but the quasi-PDF has the support $(-\infty, +\infty)$. The most important feature of the above results is that the term proportional to $\ln m_g^2$ is exactly the same for both PDFs. This explicitly illustrates, for the present example, that quasi-PDFs and light-cone PDFs share the same IR physics. Note also that the one-loop real contributions to the quasi-PDF do not exhibit a UV divergence. (The one-loop virtual diagrams in Fig. 2 do lead to a UV divergence for the quasi-PDFs as well.) The full UV-divergent contribution present in the light-cone PDF is recovered when computing the lowest x-moment of the quasi-PDF.

For a nucleon (of mass M), the general matching formula in the case of f_1 takes the form [9–11]

$$f_1(x,\mu) = \int_{-\infty}^{\infty} \frac{d\xi}{|\xi|}\, C\left(\xi, \frac{\mu^2}{p_3^2}\right) f_{1,\mathrm{Q}}\left(\frac{x}{\xi}, \mu, P_3\right) + O\left(\frac{M^2}{P_3^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{P_3^2}\right), \tag{3}$$

where C is the perturbatively calculable matching coefficient, and $p_3 = (x/\xi)P_3$ is the relation between the nucleon momentum P_3 and the quark momentum. The matching coefficient is essentially the difference between the quark-target results for f_1 and $f_{1,\mathrm{Q}}$, which is IR finite. A practical challenge of the quasi-PDF approach is caused by the higher-twist corrections in Eq. (3), that is, terms which are proportional to $1/P_3^2$. Those contributions depend on x and can be large; see, for instance, Refs. [27–30]. To minimize the higher-twist contamination, lattice calculations should be performed for target momenta P_3 as large as possible [8].

3. Matching beyond leading twist

3.1 Zero-mode contributions to twist-3 light-cone PDFs

An interesting and sometimes controversially-discussed feature of twist-3 light-cone PDFs is the possible existence of singular zero-mode contributions, that is, delta-function singularities. Such terms have been found in model calculations and model-independent treatments [20, 31–36]. If a zero-mode term is present, we can decompose the PDF into a canonical and a singular term, which for the PDF $e(x)$ reads

$$e(x) = e_{\mathrm{can}}(x) + e_{\mathrm{sin}}(x), \quad \text{with} \quad e_{\mathrm{sin}}(x) \sim \delta(x)\,. \tag{4}$$

In model calculations of twist-3 PDFs, both the canonical and the singular terms are needed to satisfy relations such as the Burkhardt-Cottingham sum rule [20, 32, 36].

Delta-function singularities also arise in the matching calculation when using a quark target. Specifically, diagram (1a) in Fig. 1 leads to a term proportional to $\delta(x)$ for all three twist-3 light-cone PDFs which arises from the integral

$$p^+ \int dk^- \, \frac{1}{(k^2 - m_q^2 + i\varepsilon)^2} = \frac{i\pi}{\vec{k}_\perp^{\,2} + m_q^2}\, \delta(x)\,, \tag{5}$$

where $x = k^+/p^+$. The integral in Eq. (5) does not appear for twist-2 PDFs, a result which holds to all orders in perturbation theory. For the discussion of the matching, it is most interesting (and challenging) to focus on the chiral-odd PDFs $e(x)$ and $h_L(x)$, where the $\delta(x)$ term is accompanied by an IR singularity [18]. Using a nonzero quark mass as the IR regulator, one finds for $e(x)$:

$$e_{\mathrm{sin}}^{(1a)}(x) = \frac{\alpha_s C_F}{2\pi} \, \ln \frac{\mu^2}{m_q^2}\, \delta(x)\,. \tag{6}$$

For the matching to work, we must be able to identify a corresponding term in the quasi-PDF $e_{\mathrm{Q}}(x, p_3)$ which basically agrees with the expression in Eq. (6).

3.2 Matching formulas

For the one-loop calculation of $e_{\mathrm{Q}}(x, p^3)$, the counterpart of the expression in Eq. (6), which also arises from the diagram (1a), is given by [18]

$$e_{\mathrm{Q,sin}}(x, p_3) = \frac{\alpha_s C_F}{2\pi} \frac{1}{\sqrt{x^2 + \eta^2}} \overset{\text{naïve}}{\longrightarrow} \frac{\alpha_s C_F}{2\pi} \frac{1}{|x|}, \text{ for } p_3 \to \infty, \tag{7}$$

where $\eta = m_q / p_3$. At first, the calculation provides the term in Eq. (7) which is proportional to $1/\sqrt{x^2 + \eta^2}$. Generally, for the matching calculation one performs a twist-expansion of the results for the quasi-PDFs, that is, one just keeps the dominant term of an expansion in powers of $1/p_3$. This is done to arrive at the expression on the r.h.s. of Eq. (7). While $1/|x|$ is indeed singular at $x = 0$, we (seemingly) do not find an IR singularity like the one in Eq. (6). One therefore could, naïvely, conclude that matching does not work for this contribution.

However, there is a flaw in the argument, since the twist expansion in Eq. (7), while being valid for all nonzero x, becomes incorrect at $x = 0$ where one cannot use $\eta \ll |x|$. We therefore indicated the twist expansion in Eq. (7) as "naïve." Keeping in mind that the results for the quark target determine the matching coefficient C, and that C is always part of an integrand (see Eq. (3)), one can consider the result for $e_{\mathrm{Q,sin}}(x, p^3)$ as a distribution, which allows one to arrive at [18]

$$e_{\mathrm{Q,sin}}(x, p_3) = \frac{\alpha_s C_F}{2\pi} \frac{1}{\sqrt{x^2 + \eta^2}} \overset{\text{correct}}{=} \frac{\alpha_s C_F}{2\pi} \ln \frac{p_3^2}{m_q^2} \, \delta(x) + \ldots . \tag{8}$$

The IR-divergent expression of the r.h.s. of Eq. (8) agrees exactly with the one in Eq. (6), and therefore the singularity proportional to $\ln m_q^2$ drops out in the matching coefficient. Note that on the r.h.s. of Eq. (8) we omitted terms that do not diverge in the limit $m_q \to 0$.

Overall, though the zero-mode contributions appearing at twist-3 pose an extra challenge which is not present at leading twist, they do fit into the structure of the matching formula in Eq. (3) when treated properly, leading to an IR-finite matching coefficient. We have demonstrated this for all three twist-3 PDFs in Refs. [17, 18].

The $O(\alpha_s)$ contribution to the matching coefficient for the PDF $e(x)$, coming from the zero-mode contribution, reads [18]

$$C_{\overline{\mathrm{MS}}}^{e(\mathrm{sin})}\left(\xi, \frac{\mu^2}{p_3^2}\right) = \frac{\alpha_s C_F}{2\pi} \begin{cases} \dfrac{1}{\xi} & \xi > 1 \\[2ex] \delta(\xi)\left(\ln \dfrac{4p_3^2}{\mu^2} + 1\right) + \mathrm{R}_0(|\xi|) & -1 < \xi < 1 \\[2ex] -\dfrac{1}{\xi} & \xi < -1 , \end{cases} \tag{9}$$

where R_0 is a plus-function distribution at $x = 0$ defined through [18]

$$\mathrm{R}_0(|x|) \equiv \left[\frac{1}{|x|}\right]_{+[0]} = \theta(|x|)\,\theta(1 - |x|) \lim_{\beta \to 0} \left[\frac{\theta(|x| - \beta)}{|x|} + \delta(|x| - \beta) \ln \beta\right]. \tag{10}$$

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The result in Eq. (9) is in the modified minimal subtraction ($\overline{\text{MS}}$) scheme. We repeat that only the diagram (1a) exhibits a non-trivial term due to zero-modes. The canonical terms from that diagram, together with the contributions from all the other diagrams in Fig. 1 and Fig 2, provide the following matching coefficient [18]:

$$
\begin{aligned}
C^{e(\text{can})}_{\overline{\text{MS}}}\left(\xi, \frac{\mu^2}{p_3^2}\right) &= \frac{\alpha_s C_F}{2\pi}\begin{cases}\left[\dfrac{2}{1-\xi}\ln\dfrac{\xi}{\xi-1}+\dfrac{1}{1-\xi}+\dfrac{1}{\xi}\right]_+ - \dfrac{1}{\xi} & \xi > 1 \\ \left[\dfrac{2}{1-\xi}\ln\dfrac{4\xi(1-\xi)p_3^2}{\mu^2}-\dfrac{1}{1-\xi}\right]_+ & 0<\xi<1 \\ \left[\dfrac{2}{1-\xi}\ln\dfrac{\xi-1}{\xi}-\dfrac{1}{1-\xi}+\dfrac{1}{1-\xi}\right]_+ - \dfrac{1}{1-\xi} & \xi<0\end{cases} \\
&+ \frac{\alpha_s C_F}{2\pi}\delta(1-\xi)\left(\ln\frac{\mu^2}{4p_3^2}\right),
\end{aligned}
\tag{11}
$$

where $[]_+$ indicates a plus-function at $x = 1$. The general structure of the canonical contribution to the matching coefficient looks like in the case of twist-2 PDFs. The matching coefficients for $g_T(x)$ and $h_L(x)$ can be found in Refs. [17, 18]. Let us finally mention that for the numerical calculations of the twist-3 PDFs in LQCD [16, 21] we transformed the matching coefficients to the so-called modified $\overline{\text{MS}}$-scheme [37], in order to obtain a better convergence of the convolution integral in Eq. (3) for large values of $|\xi|$.

3.3 Further developments

Our treatment in Refs. [17, 18] was performed in close analogy with the corresponding matching calculations for twist-2 PDFs [9–11]. However, for twist-3 parton correlation functions one must also consider mixing with the 3-parton quark-gluon-quark correlators upon UV renormalization. Such a calculation was carried out in Ref. [19] for g_T, and in Ref. [22] for $e(x)$ and $h_L(x)$. Here, we briefly discuss the PDF $g_T(x)$, which can be decomposed into a twist-2 and a (genuine) twist-3 contribution according to

$$
g_T(x) = g_T^{\text{tw2}}(x) + g_T^{\text{tw3}}(x) = \int_x^1 \frac{dy}{y}\, g_1(y) + g_T^{\text{tw3}}(x), \tag{12}
$$

where $g_T^{\text{tw2}}(x)$ is determined through the twist-2 helicity distribution $g_1(x)$, while $g_T^{\text{tw3}}(x)$ can be related to a (twist-3) quark-gluon-quark correlator [38]. The generic form of the matching formula obtained in Ref. [19] reads

$$
g_{T,\text{Q}} = C^{\text{tw2}} \otimes g_T^{\text{tw2}} + C^{\text{tw3}}_{\text{2pt}} \otimes g_T^{\text{tw3}} + C^{\text{tw3}}_{\text{3pt}} \otimes S_{\text{3pt}} + \dots \,. \tag{13}
$$

In Eq. (13), we have omitted the arguments of all functions, $\otimes$ denotes a convolution integral, and the dots on the r.h.s. indicate higher-twist contributions. The quantities $C^{\text{tw3}}_{\text{2pt}}$ and $C^{\text{tw3}}_{\text{3pt}}$ represent matching coefficients for twist-3 contributions with 2-parton and 3-parton correlators, respectively. Generally, the mixing with quark-gluon-quark correlators at twist-3 differs in its structure from the mixing that appears at twist-2 between the quark-singlet and gluon contributions. A quantitative comparison of the analytical expressions for the matching coefficients between our work [17, 18] and the one in Refs. [19, 22] is difficult if not impossible.

While the expressions for the matching coefficients provided in [19, 22] can be considered the complete QCD results through $O(\alpha_s)$, their usage for numerical calculations in LQCD is presently far from trivial. This is mainly due to the fact that there are too many unknowns. For example, in LQCD the (2-parton) quasi-PDFs $g_{1,Q}$ and $g_{T,Q}$ can be computed. But this is not sufficient to determine the three light-cone correlators that appear on the r.h.s. of Eq. (13). As a result, in order to fully solve the problem also quark-gluon-quark quasi-correlators must be calculated in LQCD, which requires an entirely new and dedicated effort. Moreover, it should be noted that neglecting, in a first approximation, the term proportional to $S_{3\text{pt}}$ in Eq. (13), is delicate, since the separation between the last two terms on the r.h.s. of that equation is not unique [19]. Because of these complications, the approach we took in Refs. [16–18, 20] seems justified at the present stage. However, future efforts to find twist-3 PDFs in LQCD must aim at going beyond the methodology we used previously.

4. Summary

Quasi-PDFs offer new opportunities for LQCD calculations of the x-dependence of PDFs and related quantities. Recently, this approach was considered for the first time in the context of twist-3 PDFs. Quasi-PDFs and the corresponding light-cone PDFs have a different behavior in the UV region, but the difference can be accounted for through a matching procedure in perturbative QCD. The matching for twist-3 PDFs has been the focus of this contribution, which is largely based on the results presented in Refs. [17, 18]. Those results were critical for obtaining the first LQCD results for twist-3 PDFs [16, 21]. At twist-3, zero-mode contributions, giving rise to delta-function singularities, pose a challenge for the matching, yet such terms are compatible with a matching formula if treated properly. Our matching calculations did not take into account the mixing with quark-gluon-quark correlations, a point that was addressed more recently in Refs. [19, 22]. At present, however, it is difficult to use the results of those papers for LQCD calculations. In particular, a fully consistent calculation of twist-3 PDFs also requires lattice results for spatial 3-parton correlators, along with (new) matching results for such objects. Progress in this area therefore calls for a dedicated future research program. Such an effort seems warranted, given the importance of twist-3 PDFs and the difficulty to extract them from experiment.

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