

Computationally Distributed and Asynchronous Operational Optimization of Droop-Controlled Networked Microgrids

NIMA NIKMEHR¹ (Senior Member, IEEE), MIKHAIL A. BRAGIN² (Senior Member, IEEE),
PENG ZHANG¹ (Senior Member, IEEE), AND PETER B. LUH² (Life Fellow, IEEE)

¹Department of Electrical and Computer Engineering, Stony Brook University, Stony Brook, NY 11794 USA

²Department of Electrical and Computer Engineering, University of Connecticut, Storrs, CT 06269 USA

CORRESPONDING AUTHOR: P. ZHANG (p.zhang@stonybrook.edu)

This work was supported in part by the National Science Foundation under Grant ECCS-2018492, Grant CNS-2006828, Grant OIA-2040599, Grant OIA-2134840, and Grant ECCS-1810108.

ABSTRACT Networked microgrids (MGs) with inverter-based and droop-controlled distributed energy resources (DERs) require operational optimization with guaranteed stability performance to ensure the stable energy supply with minimum cost, yet it remains an open challenge. Additionally, the discrete nature of MGs leads to convergence issues to existing optimization methods thereby leading to difficulties obtaining feasible solutions for large-scale networks. This article develops a paradigm for discrete droop control to improve microgrids' controllability in managing voltage and frequency fluctuations. With the emergence of Internet of Things, the computational tasks are distributed among local resources. The utilized Distributed and Asynchronous Surrogate Lagrangian Relaxation (DA-SLR) method distributes the optimization tasks among the MGs and efficiently coordinates the distributed subsystems. A small-signal model of the operational optimization is then developed to verify the system's stability. Numerous case studies have proven the DA-SLR's efficacy in comparison to various variations of the alternating direction of multipliers method (ADMM).

INDEX TERMS Networked microgrids, droop control, distributed optimization, operational optimization, stability analysis.

I. INTRODUCTION

MODERN networked microgrids (MGs) are interconnected cyber-energy systems featured by deep penetration of distributed energy re-sources (DERs) and the need of privacy preserving between individual microgrids often owned by different stakeholders. Operational optimization is fundamentally important for networked MGs to maintain economical and safe operations under fast varying conditions [1]. It is highly desirable to develop an operational optimization method that is able to cope with privacy-preserving requirements, distributed nature of MGs, and increasing number of DERs.

Energy management of microgrids is carried out by its central controller and controls the output power of dispatchable DERs, charging status of batteries and the level of exchanging

power with the neighboring microgrids. The energy management and optimization-based model of networked microgrids have been extensively studied in the literature. An optimal coordinated energy management among the networked MGs leads toward economical opportunities [2]. To schedule the generation level of DERs and storage systems under fluctuations associated with loads and non-dispatchable generations, a two-stage energy management framework is developed for networked MGs [3]. Recently, droop-controlled energy management of MGs is an increased focus of researchers. In [4], the economic and environmental aspects of droop controlled-based islanded MGs are considered to satisfy the optimal operation of stand-alone microgrids. To recover the nominal frequency of the system, in [5], droop control gains are adjusted in operational optimization problem of islanded

MGs. However, current research on droop controlled-based energy management of microgrids suffers from poor frequency and voltage regulations because of tuning issues associated with droop control parameters [6]. Up to this day, most investigations on droop-controlled energy management of microgrids have used centralized operation mode. With the increasing penetration of distributed energy resources (DERs), the centralized optimization and control architecture of MGs have several remarkable drawbacks, such as computing memory and communication signals overload, risk of privacy-preserving of entities, and cyber-intrusion issue of shared data [7]. Additionally, the energy management of networked microgrids are generally formulated as a mixed-integer-linear-programming (MILP) problem, which solving this type of problem imposes main optimization issues: first, solving the operational optimization problem of MGs is computationally expensive when the number of microgrids are increased. Second, such problems are inherently discrete optimization problems suffering from combinatorial complexity and leading to difficulties of finding feasible solutions efficiently [8].

The centralized framework for the energy management of networked MGs computationally becomes expensive when the number of decision variables and scale of the optimization problem are increased. To resolve this issue, distributed structure of MGs provides a framework for MGs to be responsible for scheduling and optimization of their local resources [9]. Frequently, to support distributed operational optimization and energy management of networked MGs, the alternating direction of multipliers method (ADMM) to coordinate MGs [1], [10], [11]. In [12]; the economic dispatch-based energy management problem of microgrids is solved in a distributed operation mode using ADMM without considering the network constraints. In [13], a transactive-based energy management of networked MGs is introduced and ADMM is employed to coordinate the energy price. Liu *et al.* [14] considered the distributed operation of networked MGs underlying network constraints and coordinated the power exchanging among the MGs using ADMM.

The ADMM method that relaxes coupling constraints in the problem of energy management of networked MGs, however, suffers from convergence issues when solving discrete optimization problems [8]. Moreover, the convergence of ADMM method is slow and finding a solution, which optimality is not guaranteed in lights of discontinuity of integer variables [15]. The convergence of ADMM has not been proved when discrete decision variables exist. To ensure the convergence in the presence of discrete variables, this paper utilizes the Distributed and Asynchronous Surrogate Lagrangian Relaxation (DA-SLR) method [16] to obtain the feasible and near-optimal operational optimization of networked microgrids even in presence of large-scale integration of DERs and discrete variables. In the energy management problem of microgrids, in addition to ADMM, Lagrangian Relaxation (LR) method [17] and Augmented Lagrangian

Relaxation (ALR) method [18] have been already used as decomposition-based methods, which are unable to guarantee the optimality of solution in presence of non-continuous variables.

The operational optimization of droop-controlled networked microgrids necessitates the stability analysis to ensure the stability of the system. In other words, setting and tuning the droop control gains should guarantee the stability of the system. In the research papers about the droop controlled-based energy management of microgrids, only optimal operation and economic aspect of droop-controlled MGs have been considered without discussing the stability concerns of MGs [5], [6], [19]. Other studies on microgrids have focused on small signal stability without considering the operational optimization of MGs. In [20], the stability-constrained economic operation of islanded MGs is studied. However, the islanded MGs are controlled in a centralized framework, and the oversimplified objective function only considers generation costs of DERs without power flow and network constraints. In [21], the network and stability margin constraints are considered in energy management of an autonomous voltage source converter-based microgrid. However, the authors have not extended their study to cover the networked MGs concept, and thus, the impact of energy trading has been overlooked. The economic dispatch problem is studied in a DC microgrid in presence of limited number of droop-controlled sources in [22], and non-dominated sorting genetic algorithm is used as a solution method to optimize a multi-objective function by finding better points of stability of droop-control curves. However, the study lacks a proof of convergence and feasibility in solving the economic dispatch problem when the genetic algorithm is utilized. Moreover, the study is executed in a centralized framework and the stability is tested for limited points of droop control curve. In [23], the energy management of an autonomous MG is implemented underlying the small signal stability analysis. The authors have only considered the impact of frequency-droop in the operational optimization of the microgrid, and frequency-droop gain of DERs are not involved in the optimization process, instead a fixed range of droop control gain is considered and the stability analysis is carried out accordingly. To bridge the aforementioned gaps in underlying literature, the main features of this paper are listed as follows:

- A droop-based networked MGs including discrete droop parameters is developed to ease the controllability of voltage and frequency fluctuations in discrete steps. In the literature about the energy management and operational optimization of networked microgrids, most researches have neglected the impact of droop control on energy management problem. Some other studies have considered the droop control in optimization of microgrids. However, a fixed range has been determined for droop control gains for the tuning purpose. In this paper, the authors have taken both frequency-droop and voltage-droop into account, and the droop control gains

are achieved from optimization procedure by involving droop-based constraints in the optimization problem.

- A decomposition and coordination-based method, namely DA-SLR is utilized to solve the operational optimization of networked MGs in presence of discrete variables. With the emergence of technologies supporting distributed computational capabilities of multiple distributed MGs as well as communications enabled by the Internet of Things, there is a great potential to carry out computational tasks much more efficiently than by using a single computer. Under an assumption of a single coordinator, to efficiently coordinate distributed MGs while avoiding the synchronization overhead, decomposition and coordination DA-SLR methodology is used to efficiently coordinate distributed subsystems in an asynchronous way. The advantages of DA-SLR are: 1) resolving the difficulties of LR without requiring the optimal dual value; 2) obtaining the “surrogate” subgradient directions toward optimal multipliers after solving one or a few subproblems; 3) fast convergence without waiting for slow subproblems to update the multipliers; 4) converge to near-optimal solution in presence of discrete variables.
- A small signal state-space model of distributed operational optimization of MGs is developed to ensure the stability of the system for the optimized droop gains. As discussed in the literature review, there exist a few studies on stability-constrained energy management of microgrids. In most studies, the networking feature of MGs have been neglected and stability analysis of droop gains are confined to fixed operating points of droop curve while in this paper, for any droop gains obtained by optimization, the stability analysis is executed to ensure the stability of the system.

II. OPERATIONAL OPTIMIZATION OF NETWORKED MICROGRIDS

The control center of MGs should collect data from their local controllers to make an accurate decision about the power exchanging possibility with neighbouring MGs. In the presented networked MGs structure, each MG is considered as an autonomous entity. To guarantee the operational optimization of networked MGs, therefore, a distributed algorithm is employed to coordinate the MGs operation.

A. OPTIMIZATION MODEL OF MGs

1) OBJECTIVE FUNCTION

In this subsection, the optimization model consisting of objective function and technical constraints is presented for the networked microgrids.

The objective function is described as:

$$OF = \sum_{t \in T} \sum_{i \in I} [C_g^{MT}(P_{t,i}^{g,MT}) + C_g^{FC}(P_{t,i}^{g,FC}) + C_g^{CHP}(P_{t,i}^{g,CHP})]$$

$$+ \sum_{t \in T} \sum_{b \in B} [C_{dch}(P_{t,b}^{BAT,dch}) - C_{ch}(P_{t,b}^{BAT,ch})] + \sum_{m \in M} \sum_{t \in T} [\zeta_{IL} \cdot P_{t,m}^{IL} + \zeta_{IL} \cdot Q_{t,m}^{IL}] \quad (1)$$

where, $P_{t,m}^{g,MT}$, $P_{t,m}^{g,FC}$ and $P_{t,m}^{g,CHP}$ are power generation level of micro turbine (MT), fuel cell (FC) and combined heat and power (CHP), respectively. Also, the charging and discharging power of batteries are denoted by $P_{t,b}^{BAT,ch}$ and $P_{t,b}^{BAT,dch}$, respectively. Decision variables $P_{t,m}^{IL}$ and $Q_{t,m}^{IL}$ are real and reactive power load shedding in microgrid m at hour t . Also, ζ_{IL} is load shedding price and m is used to describe microgrid index. Sets I , B , M and T are used to denote generation units, batteries, microgrids and time period, respectively. Additionally, generation cost corresponding to each DER (C_g) and battery charging cost (C_{ch}) and discharging cost (C_{dch}) are linear functions with respect to generated power of DERs, charging power and discharging power of batteries, respectively [11].

2) CONSTRAINTS

The objective function (1) is minimized subject to the technical constraints.

• Capacity-based constraints

The capacity-based constraint applies to real/reactive power generation capacity of DERs, charging/discharging power capacity of batteries, real/reactive power flow capacity in the lines between buses, and real/reactive power interruption capacity in each MG [11], [24]. It should be mentioned that real and reactive power outputs and power flows are not aggregated in capacity-based constraint.

$$\underline{P}^{cap} \leq P_t^{cap} \leq \overline{P}^{cap}, \quad \forall t \in T, \quad (2a)$$

$$\underline{Q}^{cap} \leq Q_t^{cap} \leq \overline{Q}^{cap}, \quad \forall t \in T. \quad (2b)$$

where, P_t^{cap} and Q_t^{cap} can be either real/reactive power generation capacity of DERs, charging/discharging power capacity of batteries, real/reactive power flow capacity in the lines between buses, and real/reactive power interruption capacity in each MG and the aforementioned constraints are confined to minimum ($\underline{P}^{cap}/\underline{Q}^{cap}$) and maximum ($\overline{P}^{cap}/\overline{Q}^{cap}$) values at hour t .

• Voltage and frequency restrictions

Voltage and frequency restrictions are described as follows:

$$|V_n| \leq |V_{t,n}| \leq \overline{|V_n|}, \quad \forall t \in T, n \in N, \quad (3)$$

$$\underline{f} \leq f_t \leq \overline{f}, \quad \forall t \in T. \quad (4)$$

where, the node voltage magnitude at node n in set N is confined to minimum ($\underline{|V_n|}$) and maximum ($\overline{|V_n|}$) voltage magnitudes. Similarly, the frequency variations should be between a minimum value ($\underline{f}$) and a maximum value ($\overline{f}$).

• Droop control

The $f - P$ and $V - Q$ characteristics of a droop control can be described as follows [25]:

$$f_t = f_{ref} - m_{p,t,i} \cdot (P_{t,i}^g - P_{ref,t,i}^g), \quad \forall i \in I, t \in T, \quad (5a)$$

$$|V_t| = |V^{ref}| - m_{q,t,i} \cdot (Q_{t,i}^g - Q_{ref,t,i}^g), \quad \forall i \in I, t \in T. \quad (5b)$$

where, f and f_{ref} are fluctuated frequency and reference frequency, which is set to 60 Hz. Similarly, $|V|$ and $|V_{ref}|$ are bus voltage magnitude and nominal voltage magnitude, respectively. Also, P_i^g and Q_i^g are real and reactive power of DGs. Droop control coefficients for frequency and voltage based controller are m_p and m_q , respectively. The constraints associated with m_p and m_q are described as following:

$$\underline{m}_p \leq m_{p,t,i} \leq \bar{m}_p, \quad t \in T, i \in I, \quad (6a)$$

$$\underline{m}_q \leq m_{q,t,i} \leq \bar{m}_q, \quad t \in T, i \in I. \quad (6b)$$

where, $\underline{m}_p$ and $\bar{m}_p$ are minimum and maximum values of frequency-based droop controller respectively. Also, minimum and maximum values of voltage-based droop controller are described by $\underline{m}_q$ and $\bar{m}_q$ respectively. The droop control coefficients $m_{p,t,i}$ and $m_{q,t,i}$ are assigned to each unit i at any given time t . For the sake of better controllability of a droop control and similar to tap-changers in transformers, the droop gains m_p and m_q are discrete variables.

• Power flow

For the power flow study, the linearized DistFlow model is deployed. The model has been utilized and justified in distribution systems and microgrids studies [26], [27]. In recent years, the Distflow model is used in energy management of networked MGs and guarantees the accuracy of power flow model [28], [29]. An approximation of AC power flow is considered in linearized DistFlow model [30]:

$$P_{t,n-k}^{flow} = \sum_{(\kappa:(k,\kappa)) \in N} P_{t,k-\kappa}^{flow} - (P_{t,k}^g - P_{t,k}^L), \quad \forall t \in T, n \in N, \quad (7a)$$

$$Q_{t,n-k}^{flow} = \sum_{(\kappa:(k,\kappa)) \in N} Q_{t,k-\kappa}^{flow} - (Q_{t,k}^g - Q_{t,k}^L), \quad \forall t \in T, n \in N \quad (7b)$$

$$|V_{n,t}| = |V_{k,t}| - \frac{R_{n,k} \cdot P_{t,n-k}^{flow} + \chi_{n,k} \cdot Q_{t,n-k}^{flow}}{|V_0|}, \quad (7c)$$

$$\forall t \in T, n, k \in N. \quad (7d)$$

where, the bus index is described by n and k and the set N denotes a set of nodes. The real and reactive power flow between sending node n and receiving node k at hour t are defined by $P_{t,n-k}^{flow}$ and $Q_{t,n-k}^{flow}$, respectively. Also, real and reactive loads are denoted by $P_{t,k}^L$ and $Q_{t,k}^L$, respectively, and $|V_{n,t}|$ and $|V_0|$ are voltage magnitudes at bus n and the point of common coupling in an MG, respectively. The resistance and reactance between lines n and k are determined by $R_{n,k}$ and $\chi_{n,k}$.

• Real power balance

The real power balance should be satisfied at any node of m^{th} MG in time slot t as follows:

$$\sum_{i \in I} (s_{n,t}^g \cdot P_{t,i}^g) + s_n^{WT} \cdot P_t^{g,WT} + s_n^{PV} \cdot P_t^{g,PV} + \sum_{i \in I} \left(s_{n,t}^g \cdot \frac{1}{m_{p,t,i}} (f_{ref,m} - f_{i,m}) \right)$$

$$+ s_n^{BAT} (P_t^{BAT,dch} - P_t^{BAT,ch}) + P_{n,t}^{IL} + (P_{t,k-n}^{flow} - \sum_{(k:(n,k)) \in N} P_{t,n-k}^{flow}) + (\sum_{m \in M} (s_{n-m}^{Tran} \cdot P_{t,n-m}^{buy} - s_{n-m}^{Tran} \cdot P_{t,n-m}^{sell})) = P_{t,n}^L, \quad \forall t \in T. \quad (8)$$

where, $P_{t,i}^g$, $P_t^{g,WT}$ and $P_t^{g,PV}$ are real power generation by i^{th} dispatchable units, WT and PV panel. Also, s_n^g , s_n^{WT} , s_n^{PV} , s_n^{BAT} and s_{n-m}^{Tran} are connection indicator of a dispatchable generator, WT, PV, battery, and transacted power of node n to a node in m^{th} microgrid, respectively. The purchased and sold real powers by m^{th} MG at node n is described by $P_{t,n-m}^{buy}$ and $P_{t,n-m}^{sell}$, respectively. From (8), the real power balance is applied to each node of the MGs at any given scheduling time slot.

• Reactive power balance

The reactive power balance should be satisfied at any node of m^{th} MG in time slot t as follows:

$$\sum_{i \in I} (s_{n,t}^g \cdot Q_{t,i}^g) + s_n^{WT} \cdot Q_t^{g,WT} + s_n^{PV} \cdot Q_t^{g,PV} + \sum_{i \in I} \left(s_{n,t}^g \cdot \frac{1}{m_{q,t,i}} (|V_{n,t}^{ref}| - |V_{n,t}|) \right) + (Q_{t,k-n}^{flow} - \sum_{(k:(n,k)) \in N} Q_{t,n-k}^{flow}) + (\sum_{m \in M} (s_{n-m}^{Tran} \cdot Q_{t,n-m}^{buy} - s_{n-m}^{Tran} \cdot Q_{t,n-m}^{sell})) + Q_{n,t}^{IL} = Q_{t,n}^L, \quad \forall t \in T. \quad (9)$$

where, $Q_{t,i}^g$, $Q_t^{g,WT}$ and $Q_t^{g,PV}$ are reactive power generation by dispatchable units, WT and PV panel. The purchased and sold reactive powers by m^{th} MG at node n is described by $Q_{t,n-m}^{buy}$ and $Q_{t,n-m}^{sell}$, respectively. From (9), the reactive power balance is applied to each node of the MGs at any given scheduling time slot.

• Interface Power Exchange Limits

The amount of exchanging power by each MG is limited by:

$$0 \leq P_{t,m-w}^{buy} \leq u_{t,m-w}^{buy} \cdot \bar{P}^{buy}, \quad \forall t \in T, \{m, w\} \in M, \quad (10a)$$

$$0 \leq P_{t,m-w}^{sell} \leq (1 - u_{t,m-w}^{buy}) \cdot \bar{P}^{sell}, \quad \forall t \in T, \{m, w\} \in M, \quad (10b)$$

$$0 \leq Q_{t,m-w}^{buy} \leq u_{t,m-w}^{buy} \cdot \bar{Q}^{buy}, \quad \forall t \in T, \{m, w\} \in M, \quad (10c)$$

$$0 \leq Q_{t,m-w}^{sell} \leq (1 - u_{t,m-w}^{buy}) \cdot \bar{Q}^{sell}, \quad \forall t \in T, \{m, w\} \in M. \quad (10d)$$

where, $u_{t,m-w}^{buy}$ is the binary variables denoting the status of purchasing power by m^{th} MG from w^{th} MG. The maximum amount of purchased and sold real and reactive powers are illustrated by $\bar{P}^{buy}$, $\bar{P}^{sell}$, $\bar{Q}^{buy}$ and $\bar{Q}^{sell}$, respectively.

• *Interface power flow constraints*

The following interface power flow constraints are considered to ensure that the power bought by microgrid m from microgrid w equals to the power sold by microgrid w to microgrid m . In other words, the following constraints are coupling with respect to the microgrids:

$$P_{t,m-w}^{buy} = P_{t,w-m}^{sell}, \quad \forall t \in T, \{m, w\} \in M, \quad (11)$$

$$Q_{t,m-w}^{buy} = Q_{t,w-m}^{sell}, \quad \forall t \in T, \{m, w\} \in M. \quad (12)$$

B. PROBLEM LINEARIZATION

In (8), the frequency $f_{t,m}$ is a continuous variable while $\frac{1}{m_p}$ is a discrete variable. Similarly, in (9), voltage magnitude $|V_{n,t}|$ and $\frac{1}{m_q}$ are continuous and discrete variables, respectively.

Therefore, the cross products of decision variables in droop control make the problem constraints nonlinear, which results in a nonlinear optimization problem [31]. Nevertheless, the presented terms are linearized as described below.

1) GENERIC LINEARIZATION PROCEDURE

The linerization procedure will be explained by using an expression $z = A \cdot d$, where A is a continuous variable and d is a binary variable. If A has bounds $[\underline{A}, \bar{A}]$, then the exact form of the linearized inequalities are as following for expression d [31]:

$$\min\{0, \underline{A}\} \leq z \leq \bar{A}, \quad (13a)$$

$$\underline{A} \cdot d \leq z \leq \bar{A} \cdot d, \quad (13b)$$

$$A - (1 - d) \cdot \bar{A} \leq z \leq A - (1 - d) \cdot \underline{A}, \quad (13c)$$

$$z \leq A + (1 - d) \cdot \bar{A}. \quad (13d)$$

Therefore, products of binary variable and continuous variables are linearized by using new introduced continuous variables (z). However, in the designed optimization model, d is a discrete variable (integer non-negative variable). If d_j is bounded by positive integer D , so that $d_j \in \{0, 1, \dots, D\}$, we can introduce binary variables $a_0, a_1, \dots, a_D$ and add the following constraints:

$$\sum_{l=0}^D a_{lj} = 1, \quad \forall j \in J, \quad (14a)$$

$$d_j = \sum_{l=0}^D l \cdot a_{lj}, \quad \forall j \in J. \quad (14b)$$

Therefore:

$$z_j = \sum_{l=0}^D l \cdot a_{lj} \cdot A, \quad \forall j \in J. \quad (14c)$$

Now, we can linearize each of the products of $a_{lj} \cdot A$ based on inequalities (13a)-(13d). In $f - P$ droop control, m_p is a discrete value, while frequency (f) is a continuous variable. Since m_p is discrete, therefore, $k_p = \frac{1}{m_p}$ is also discrete. Therefore,

$$a_{p,j} = k_{p,j} \cdot f_m, \quad \forall j \in J. \quad (15)$$

Following (13a)-(13d), to resolve the non-linearity difficulty, we replace (15) by the following constraints:

$$z_{p,j} = \sum_{l=0}^D l \cdot a_{p,lj} \cdot f_m, \quad \forall j \in J. \quad (16)$$

Similarly, by introducing $k_q = \frac{1}{m_q}$, the nonlinear terms

$$a_{q,j} = k_{q,j} \cdot |V_n|, \quad \forall j \in J. \quad (17)$$

are linearized as follows:

$$z_{q,j} = \sum_{l=0}^D l \cdot a_{q,lj} \cdot |V_n|, \quad \forall j \in J. \quad (18)$$

III. DISTRIBUTED AND ASYNCHRONOUS SURROGATE LAGRANGIAN RELAXATION MODEL FOR NETWORKED MGs

In this section, DA-SLR algorithm is modeled for operational optimization of networked MGs in subsection III-A, and then, the solution steps are explained in III-B.

In this paper, three different architectures including DA-SLR, fully distributed ADMM, and centralized consensus ADMM (C-CADMM) are discussed and the performance of each method is compared. Unlike the fully distributed ADMM algorithm architecture for the operational optimization problem, in which all agents can directly communicate with each other, DA-SLR and C-CADMM algorithms realize the connection between existing microgrids through a coordinator and global fusion center, respectively. Therefore, compared to fully distributed ADMM, the DA-SLR method due to lack of requirement to create a direct connection among the MGs, requires less communication infrastructures to interact signals for coordination purpose. A comprehensive optimization procedure of C-CADMM can be found in [32, P.48], [33]. In fully distributed ADMM, increasing the size of the network requires more communication infrastructures than DA-SLR method, which can be costly for the system.

A. DISTRIBUTED MODEL OF SURROGATE LAGRANGIAN RELAXATION

In the networked microgrids structure, assume there are m MGs connected to several neighbouring MGs. To coordinate the MGs, Lagrangian multipliers are introduced first to relax constraints (11) and (12) that couple MGs. The Lagrangian function then becomes:

$$\begin{aligned} L(P, Q, \lambda) &= \sum_{t \in T} \sum_{i \in I} [C_g(P_{t,i}^{g,MT}) + C_g(P_{t,i}^{g,FC}) + C_g(P_{t,i}^{g,CHP})] \\ &+ \sum_{t \in T} \sum_{b \in B} [C_{dch}(P_{t,b}^{BAT,dch}) - C_{ch}(P_{t,b}^{BAT,ch})] \\ &+ \sum_{m \in M} \sum_{t \in T} [\zeta_{IL} \cdot P_{t,m}^{IL} + \zeta_{IL} \cdot Q_{t,m}^{IL}] \end{aligned}$$

$$\begin{aligned}
& + \sum_{t \in T, \{m, w\} \in M} \lambda_{p, t, m-w} \cdot (P_{t, m-w}^{buy} - P_{t, w-m}^{sell}) \\
& + \sum_{t \in T, \{m, w\} \in M} \lambda_{q, t, m-w} \cdot (Q_{t, m-w}^{buy} - Q_{t, w-m}^{sell}). \quad (19)
\end{aligned}$$

where, $\lambda_{p, t, m-w}$ and $\lambda_{q, t, m-w}$ are Lagrange multipliers associated with real and reactive power levels, respectively.

The relaxed problem is then decomposed into individual subproblems. The formulation for MG m is formulated as:

$$\begin{aligned}
\min_{MG_m} \{ & \sum_{i \in I} [C_g(P_{t, m, i}^{g, MT}) + C_g(P_{t, m, i}^{g, FC}) + C_g(P_{t, m, i}^{g, CHP})] \\
& + \sum_{b \in B} [C_{dch}(P_{t, m, b}^{BAT, dch}) - C_{ch}(P_{t, m, b}^{BAT, ch})] \\
& + [\zeta_{IL} \cdot P_{t, m}^{IL} + \zeta_{IL} \cdot Q_{t, m}^{IL}] \\
& + \sum_{t \in T, \{m, w\} \in M} \lambda_{p, t, m-w} \cdot (P_{t, m-w}^{buy} - P_{t, w-m}^{sell}) \\
& + \sum_{t \in T, \{m, w\} \in M} \lambda_{q, t, m-w} \cdot (Q_{t, m-w}^{buy} - Q_{t, w-m}^{sell}). \quad (20)
\end{aligned}$$

Note that decision variables $P_{t, w-m}^{sell}$ and $Q_{t, w-m}^{sell}$ within (19) belong to subproblem w and are thus not included in (20). While subproblem formulation m includes variables $P_{t, m-w}^{buy}$ and $Q_{t, m-w}^{buy}$ indicating power bought, the microgrid can in fact sell power, in which case the corresponding optimized values will be negative. The formulation for selling MG w is formulated using the same logic with the exception that variables $P_{t, w-m}^{sell}$ and $Q_{t, w-m}^{sell}$ will appear within the objective function with the negative sign.

The MG subproblems are coordinated by Lagrangian multipliers, which are updated based on violation of relaxed constraints as:

$$\lambda_{p, t, m-w}^{r+1} = \lambda_{p, t, m-w}^r + e_r \cdot (P_{t, m-w}^{buy, r} - P_{t, w-m}^{sell, r}), \quad (21a)$$

$$\lambda_{q, t, m-w}^{r+1} = \lambda_{q, t, m-w}^r + e_r \cdot (Q_{t, m-w}^{buy, r} - Q_{t, w-m}^{sell, r}). \quad (21b)$$

where, r is the coordinator iteration number, $P_{t, m-w}^{buy, r}$ is the most recent value available of active power at iteration r that MG m has the intention to buy and e_r is the stepsize. In [34], the contraction mapping concept is employed to derive e_r in the following way:

$$e_r = \gamma^r \frac{e_{r-1} \|g(P^{r-1}, Q^{r-1})\|}{\|g(P^r, Q^r)\|}, 0 < \gamma^r < 1, r = 1, 2, \dots \quad (22)$$

where, $g(P^r, Q^r)$ is the vector of constraint violations, γ^r is stepsize-setting parameter which can be updated in each iteration. An exact formula to determine γ^r can be found in [34], [35].

There are three main tenets, upon which fast convergence is established as summarized below:

- 1) No need for synchronization by waiting to the “slowest subproblem” solution to arrive at the coordinator or to satisfy the “long-term” frequency requirement [36]; only a “freshness” assumption [34] is required for

subproblem solution to arrive at the coordinator at least once within a finite number of iterations;

- 2) Solution of one subproblem is sufficient to update multipliers as originally proved in [35] and then generalized in [34].
- 3) Because of the presence of discrete variables, the coordination of microgrids is equivalent to the optimization of a non-smooth dual function, the optimization of which requires stepsizes to be set by using the so-called the “optimal dual value” (see e.g., the classical work of Polyak [37]). The stepsize formula (22), originally developed [35] and then proved to lead to convergence in [34], results in fast convergence because (22) does not require the knowledge of the estimations of the optimal dual value.

As mentioned in Section II and as shown in (20), MG subproblems do not involve decision variables from other subproblems, thus, unlike ADMM, the DA-SLR method does not require data exchange among MGs thereby preserving privacy.

B. DA-SLR SOLUTION STEPS

1) FEASIBLE COST SEARCH

Feasible solutions are searched by using heuristics. Heuristics are operationalized by solving the original problem (1)-(12) with decision variables fixed at the most recent values obtained by solving MG subproblems.

If the feasible solution to the original problem is not found, the multipliers are updated for several iterations before feasible solutions are searched again.

2) DUAL VALUE OF DA-SLR

Dual values provide the lower bound for the feasible cost to quantify its quality, and the dual values are obtained by minimizing each MG subproblem by using the most recent available values of multipliers.

3) STOPPING CRITERIA

The stopping criteria is the difference between the feasible cost and the dual value:

$$SC = \frac{cost^{feasible} - cost^{dual}}{cost^{feasible}} \quad (23)$$

IV. STABILITY ANALYSIS OF OPERATIONAL OPTIMIZATION OF NETWORKED MICROGRID

The stability considerations are one of the main determining factors in setting droop controller parameters [38]. In this section, the small signal model of the studied system is obtained. In this paper, the stability analysis associated with real and reactive powers are considered and since the other dynamics such as voltage control loop and the current control loop are faster, their effects therefore omitted [39].

The filtered version of real and reactive powers are often taken into consider in a droop control:

$$P_i^g = \frac{\omega_f}{s + \omega_f} P_{ins, i}^g \rightarrow \tau_{P, i} \cdot \dot{P}_i^g = -P_i^g + P_{ins, i}^g \quad (24a)$$

$$Q_i^s = \frac{\omega_f}{s + \omega_f} Q_{ins,i}^s \rightarrow \tau_{Q,i} \cdot \dot{Q}_i^s = -Q_i^s + Q_{ins,i}^s. \quad (24b)$$

where, $P_{ins,i}^s$ and $Q_{ins,i}^s$ are instantaneous real and reactive powers of i^{th} DER, respectively. Also, ω_f , $\tau_{P,i}$ and $\tau_{Q,i}$ describe cutoff frequency, real and reactive powers' time constants of low-pass filter, respectively.

To obtain the frequency dynamic, the filtered real power in (24a) should be plugged into (5a). Similarly, by combining the filtered reactive power (24b) and voltage droop control (5b), the voltage dynamic is derived [39], [40]:

$$\dot{\omega}_i^{ang} = \frac{1}{\tau_{i,P}}(\omega_{ref}^{ang} - \omega_i^{ang}) - \frac{m_{p,i}}{\tau_{P,i}}(P_{ins,i}^s - P_{ref,i}^s), \quad (25a)$$

$$|\dot{V}|_i = \frac{1}{\tau_{Q,i}}(|V_{ref,i}| - |V_i|) - \frac{m_{q,i}}{\tau_{Q,i}}(Q_{ins,i}^s - Q_{ref,i}^s). \quad (25b)$$

where, ω_i^{ang} and ω_{ref}^{ang} are angular frequency and reference angular frequency of source i respectively.

The real and reactive power injections into the node n are given as follows:

$$P_n^{flow} = -g_{nn}|V_n|^2 + \sum_{n \neq k} [g_{nk}|V_n||V_k|\cos(\delta_{nk}) + b_{nk}|V_n||V_k|\sin(\delta_{nk})], \quad (26a)$$

$$Q_n^{flow} = b_{nn}|V_n|^2 + \sum_{n \neq k} [g_{nk}|V_n||V_k|\sin(\delta_{nk}) - b_{nk}|V_n||V_k|\cos(\delta_{nk})], \quad (26b)$$

where, g_{nk} and b_{nk} are conductance and susceptance between nodes n and k respectively. Moreover, $g_{nn} = \sum_{n \neq k} g_{nk}$ and $b_{nn} = \sum_{n \neq k} b_{nk}$ without shunt conductance and susceptance respectively. The phase angle difference between n^{th} and k^{th} nodes is described by $\delta_{nk} = \delta_n - \delta_k$. The linearized form of (26a) and (26b) using the first-order Taylor expansion can be written as follows considering small deviations from the equilibrium [40]:

$$\Delta P_n^{flow} \approx \sum_{n \neq k} [b_{nk} \cdot \Delta \delta_{nk} - g_{nk}(\Delta |V_n| - \Delta |V_k|)], \quad (27a)$$

$$\Delta Q_n^{flow} \approx \sum_{n \neq k} [g_{nk} \cdot \Delta \delta_{nk} + b_{nk}(\Delta |V_n| - \Delta |V_k|)]. \quad (27b)$$

A. CLOSED-LOOP DYNAMICS OF DER_{*i*}

The integral of the angular frequency results in the phase angle as follows:

$$\dot{\delta}_i = \omega_i^{ang} \quad (28)$$

The Dynamic model of frequency can be derived for each DER using (25a), real power injection in the node including that DER (27a) and real power balance (8). The general form of real power balance can be expressed as $P_{ins,i}^s = P_i^{flow} + \frac{1}{m_{p,i}}\omega_i^{ang} + P_i^L - P_i^s$. Combining the real power balance

and (25a) results in:

$$\begin{aligned} \dot{\omega}_i^{ang} = & \frac{1}{\tau_{i,P}}(\omega_{ref}^{ang} - \omega_i^{ang}) \\ & - \frac{m_{p,i}}{\tau_{P,i}}(P_n^{flow} + \frac{1}{m_{p,i}}\omega_i^{ang} + P_i^L - P_i^s - P_{ref,i}^s). \end{aligned} \quad (29)$$

Similarly, to obtain the dynamic model of voltage for each DER, the reactive power balance in the form of $Q_{ins,i}^s = Q_i^{flow} + \frac{1}{m_{q,i}}|V_i| + Q_i^L - Q_i^s$ should be combined with (25b), which results in:

$$\begin{aligned} |\dot{V}|_i = & \frac{1}{\tau_{Q,i}}(|V_{ref,i}| - |V_i|) \\ & - \frac{m_{q,i}}{\tau_{Q,i}}(Q_i^{flow} + \frac{1}{m_{q,i}}|V_i| + Q_i^L - Q_i^s - Q_{ref,i}^s). \end{aligned} \quad (30)$$

The state-space model of the system can be obtained in the form of $\dot{x} = Ax + W\omega$ by combining the (28), (29), (30), (27a) and (27b), where $x = [\delta \ \omega^{ang} \ |V|]^T$, and ω is the disturbance input. The A matrix is derived as follows:

$$A = \begin{bmatrix} 0 & I & 0 \\ -\frac{1}{T_P} \cdot M_P \cdot B & -\frac{1}{T_P} & \frac{1}{T_P} \cdot M_P \cdot G \\ -\frac{1}{T_Q} \cdot M_Q \cdot G & 0 & -\frac{1}{T_Q} \cdot (2I + M_Q \cdot B) \end{bmatrix}, \quad (31)$$

where, M_P and M_Q are diagonal matrices consisting of frequency and voltage droop gains of all DERs, respectively. Also, T_P and T_Q are diagonal matrices, where the diagonal elements are time constants $\tau_{P,i}$ and $\tau_{Q,i}$ of low-pass filter in i^{th} DER, respectively. Matrices G and B demonstrate the conductance and susceptance of the network, respectively.

To satisfy the stability of the network, all obtained eigenvalues for the optimized droop gains should be negative.

To indicate the optimization process of the droop-based operational optimization of networked microgrids, Algorithm 1 is provided.

V. TESTING RESULTS

In this section, the efficiency of the DA-SLR based operational optimization method is tested and validated using two case studies: 1) a modified IEEE 33-bus distribution network [41] including four MGs, 2) a nine-MG networked microgrids system based on a modified IEEE 123-bus system [42].

In both studies, the range of frequency-based droop coefficients m_p is assumed to fall in $[0.02, 0.2]$ with a discrete step of m_p considered as 0.0018. Similarly, the range of voltage-based droop coefficients m_q is set as $[0.05, 0.5]$ with a discrete step of 0.0045. Additionally, the efficiency of proposed DA-SLR algorithm is compared to three different versions of ADMM: 1) vanilla-ADMM [43], which is a fully distributed method and independent from the results of past iterations, 2) and to improve the convergence of vanilla-ADMM, a new variant, so called relaxed-ADMM [44], which is a fully distributed method, is employed where the results of subproblems recorded in the past iterations are taken into

Algorithm 1 Execution of DA-SLR on Operational Optimization of Networked MGs

Result: Exchanging powers ($P_{m-w}^{buy}/P_{m-w}^{sell}, Q_{m-w}^{buy}/Q_{m-w}^{sell}$) between microgrids

1 Initialize microgrids' exchanged powers $P_{m-w}^{0,buy}/P_{m-w}^{0,sell}, Q_{m-w}^{0,buy}/Q_{m-w}^{0,sell}$, Lagrange multipliers λ^0 , updating stepsize e^0

2 iteration $r \leftarrow 0$

3 Coordinator receives MGs subproblem solutions and updates multipliers without waiting for all solutions to arrive. Coordinator then broadcasts multipliers to all MGs

4 Each idle MG starts solving its subproblem by using the latest available multipliers and sends its solution to the coordinator

5 **if** criteria to search to feasible solution are satisfied **then**

search for a feasible solution

6 **if** the feasible solution is obtained **then**

7 obtain the dual value and calculate the gap (23)

else

go to step 3

end

else

go to step 3

end

8 **if** the stopping criteria is satisfied **then**

the stability of the system should be checked

9 **if** the eigenvalues of small signal model are negative **then**

10 the optimization process is finished

else

change the range of droop coefficients and start the process from step 1

end

else

go to step 3

end

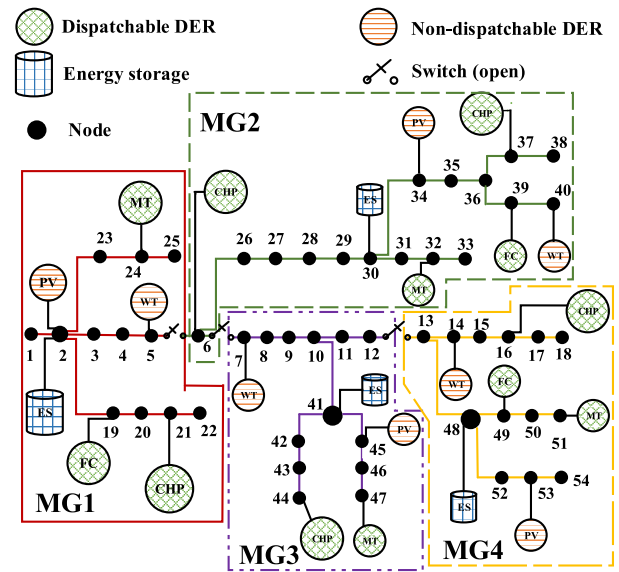


FIGURE 1. Modified IEEE 33-bus distribution system with four MGs.

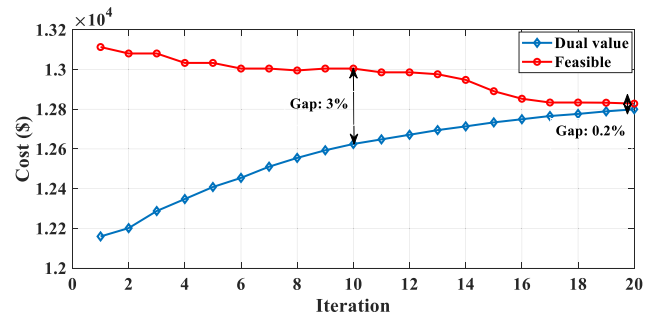


FIGURE 2. The DA-SLR method in tracking the feasible solution in modified IEEE 33-bus system.

TABLE 1. Comparison of feasible costs using different methods.

Method	Objective function (\$)				
	MG1	MG2	MG3	MG4	Total
Vanilla	4621	5759	16680	4742	31802
ADMM	3286	5755	7650	3818	20509
Consensus	2953	5755	6027	3589	18324
ADMM	2002	5114	2731	2953	12800
DA-SLR					

account in the calculation of current iteration. 3) The centralized consensus ADMM, in which a fusion center is serving as a single coordinator. The operational optimization problem is solved as a 24-hour scheduling problem for each case study and the time interval is one hour. The MATLAB software is used and the optimization problem is solved by Gurobi solver.

A. TEST RESULTS ON A 33-BUS NETWORKED MICROGRIDS SYSTEM

In Fig. 1, the single line diagram of the 33-bus networked microgrids system with four MGs is shown.

The power base of the system is set to be 10 MVA. The line resistance and reactance of the rest of the network for added buses are set to be 0.06 and 0.1 p.u., respectively. The maximum real power generation level is 200kW, 150kW, 400kW, 300kW and 400kW for PV panels, WTs, MTs, FCs

and CHPs, respectively. The maximum reactive power generation level is 80kW, 60kW, 200kW, and 300kW for PV panels, WTs, MTs, and CHPs, respectively.

As demonstrated in Fig. 2, the operation cost of the system obtained by the DA-SLR method decreases fast thereby reaching the gap of less than 0.2% within 20 iterations.

In Table 1, the optimization results obtained for a day by DA-SLR are compared with variants of ADMM.

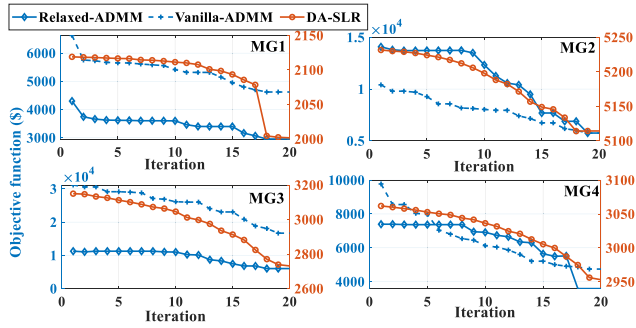


FIGURE 3. Update of objective function using different methods in IEEE 33-bus system.

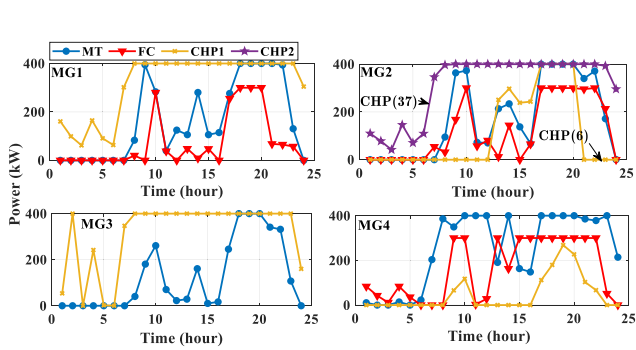


FIGURE 4. Generated real power by DERs in each microgrid obtained by DA-SLR method.

According to Table 1, the DA-SLR method can lead the networked MGs' operation toward the minimum cost of the system better than ADMM. While, the relaxed version of ADMM can improve the convergence of vanilla-ADMM, the performance of DA-SLR is much more efficient. As discussed, the represented results in Table 1 shows that the ADMM method suffers from slow convergence in solving MILP problems.

To show the optimized operation cost of existing microgrids in more details, Fig. 3 shows the update of objective function for every microgrids in each iteration.

In Fig. 3, DA-SLR method outperforms different versions of ADMM in reaching the minimum value of objective function after 20 iterations. It should be mentioned that the left y-axis of each subplot represents the objective function values for two versions of ADMM while the right side y-axis is for DA-SLR.

From computing time point of view, DA-SLR is the fastest method in reaching the minimum value of objective function after 20 iterations. In this regard, DA-SLR, vanilla-ADMM, relaxed-ADMM and consensus-ADMM take 4, 12.2, 15.7 and 18.6 seconds, respectively.

The contribution of active and reactive power levels of each DER obtained by DA-SLR are shown in Fig. 4 and Fig. 5, respectively.

According to Fig. 4 and Fig. 5, CHP and MT units have a significant contribution in providing the real and reactive

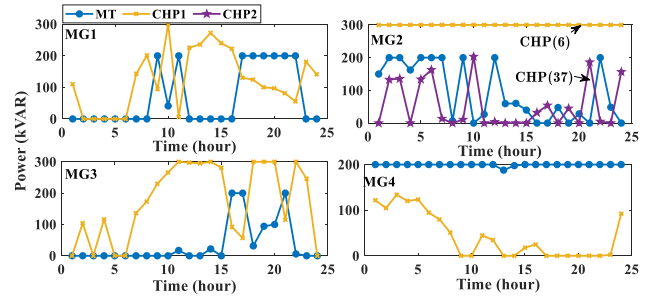


FIGURE 5. Generated reactive power by DERs in each microgrid obtained by DA-SLR method.

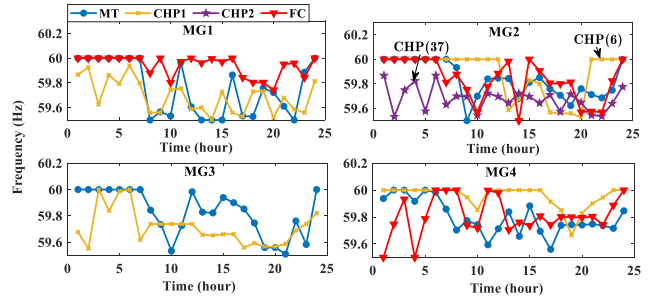


FIGURE 6. Frequency fluctuation due to droop control performance in each DER.

loads in each MG. Since dispatchable DERs are equipped with droop controllers, MT, FC, and CHP generators contribute to frequency-based droop control, in which frequency variations can cause changes in generation level of DERs, while MT and CHP generators are used for voltage-based droop control to adjust the reactive power in a predetermined range.

The frequency fluctuation of each DER caused by frequency-based droop control is shown in Fig. 6.

According to Fig. 6, a DER decreases its frequency to enhance its droop control contribution in real power generation and vice versa. In general, if a DER contributes the maximum power generation level using frequency-based droop control, the lowest frequency level will be achieved.

Also, as the frequency and voltage droop coefficients are considered discrete variables, the optimized values for droop coefficients is shown in Fig. 7 and Fig. 8.

A droop controller prefers to adjust the frequency and voltage droop coefficients in either higher or lower step to keep the power balance guaranteed. According to Fig. 7, when the real power generation of a DER increases the frequency-based droop control coefficient increases compared to the hours that the generated real power is lower. Similarly, the reactive power generation level of a DER in Fig. 5 has a direct relationship with the contribution of voltage-based droop control thereby the droop control coefficient dealing with reactive power will be changed to guarantee the reactive power balance.

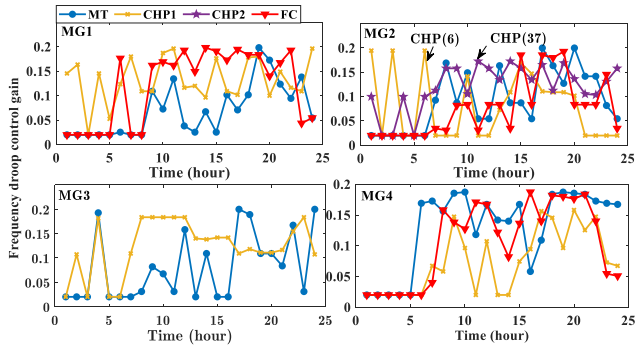


FIGURE 7. Optimized values for real power droop control coefficients obtained by DA-SLR method.

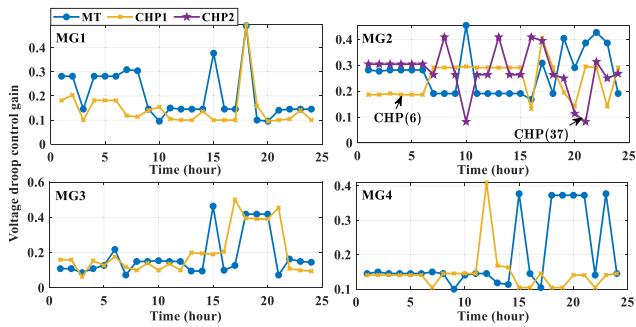


FIGURE 8. Optimized values for reactive power droop control coefficients obtained by DA-SLR method.

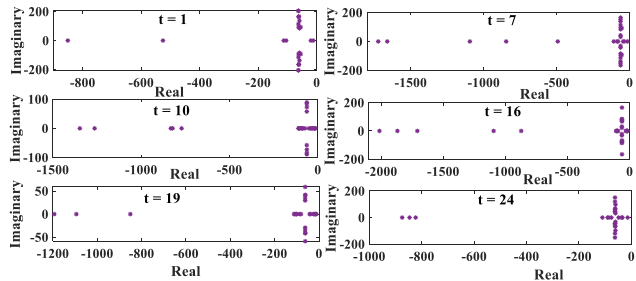


FIGURE 9. Eigenvalues of modified 33-bus network based upon the optimized values of droop gains.

1) STABILITY VERIFICATION

As shown in Fig. 7 and Fig. 8, the optimized values for both frequency and voltage droop gains were obtained so that the operation cost of microgrids are minimized. However, the eigenvalue analysis should be considered to check the stability of networked microgrids after optimization. Therefore, the eigenvalue analysis based upon optimized droop gains of each DER is shown in Fig. 9 in different scheduling hours.

According to Fig. 9, the optimized frequency and voltage droop gains satisfy the negative eigenvalues of the system in different scheduling hours. To obtain the eigenvalues of the system, the optimized droop gains (m_p and m_q) of DERs are plugged into the A matrix (31). The plotted eigenvalues of

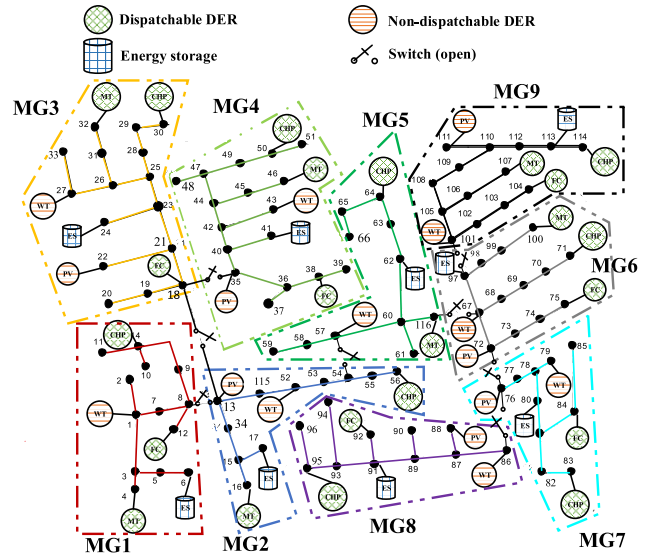


FIGURE 10. Modified IEEE 123-bus distribution system with nine MGs.

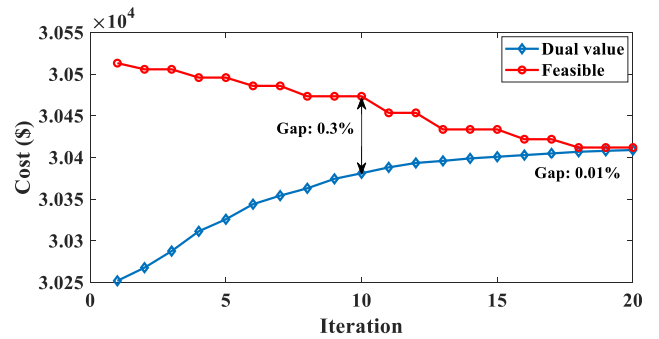


FIGURE 11. Tracking the feasible solution using SLR and DA-SLR in modified IEEE 123-bus system.

IEEE 33-bus distribution network guarantees the stability of the system.

B. TEST RESULTS ON A 123-BUS NETWORKED MICROGRIDS SYSTEM

To demonstrate the scalability of DA-SLR, a larger network is considered. Fig. 10 shows the single line diagram of a 123-bus networked microgrids system with nine interconnected MGs.

The results obtained by the designed DA-SLR method are analyzed upon approaching the feasible cost (see Fig. 11). The gap reaches 0.3% and 0.01% after 10 and 20 iterations respectively.

In Table 2, the operation cost of MGs is analyzed using three methods.

According to the results of Table 2, the DA-SLR has a better performance of optimization compared to different versions of ADMM. It is seen that the vanilla-ADMM method suffers from convergence issues in coordinating the

TABLE 2. Comparison of feasible costs using DA-SLR and ADMM in 123-bus distribution system.

Method	Objective function (\$)				
	MG1	MG2	MG3	MG4	MG5
Vanilla-ADMM	7289	7624	5537	5799	18281
Consensus ADMM	5212	3267	5411	2548	8956
Relaxed-ADMM	5212	3267	5411	1346	8956
DA-SLR	4104	943	4915	665	3618
	Objective function (\$)				
	MG6	MG7	MG8	MG9	Total
Vanilla-ADMM	25586	8013	7001	9555	94685
Consensus ADMM	11838	4760	6815	5030	53837
Relaxed-ADMM	11838	4760	6800	5029	52979
DA-SLR	4406	2664	6358	2736	30409

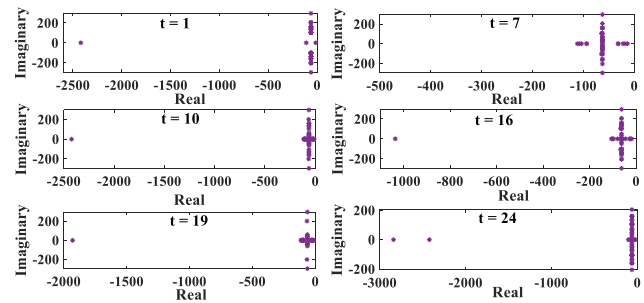


FIGURE 12. Eigenvalues of modified 123-bus network based upon the optimized values of droop gains.

subproblems. While, the relaxed-ADMM and consensus-ADMM can improve the convergence of basic version of ADMM, the optimization results are yet to be as efficient as DA-SLR.

The computation time of DA-SLR, vanilla-ADMM, relaxed-ADMM and consensus-ADMM per iteration is 9, 35, 49 and 57 seconds, respectively. The computation time of different methods shows the fast coordination of DA-SLR in large-size networks compared to conventional distributed methods. It is observable that to have a better convergence using relaxed-ADMM, much time should be spent compared to the vanilla-ADMM.

1) STABILITY ANALYSIS

To ensure the stability of the system based upon the obtained droop gains, the eigenvalues of modified 123-bus system are depicted as Fig. 12 in different scheduling time slots:

In Fig. 12, the negative eigenvalues satisfy the stability of the system for optimized droop control gains.

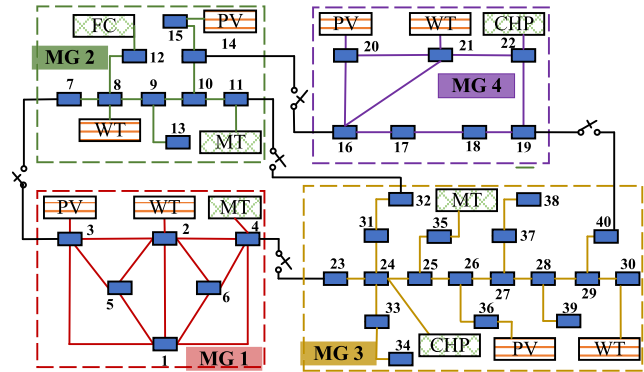


FIGURE 13. A realistic networked microgrid benchmark including four MGs [45].

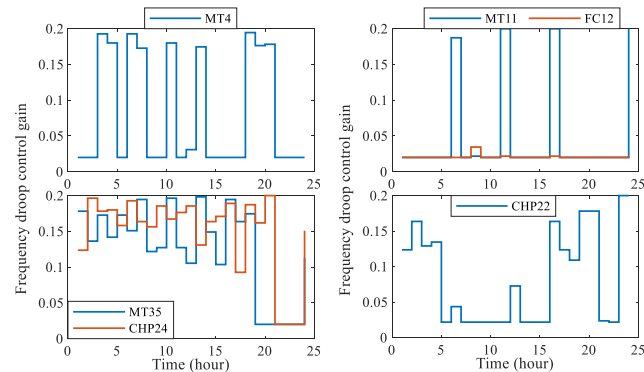


FIGURE 14. Optimized frequency droop control coefficients using DA-SLR in the 40-bus networked MGs system.

C. TEST RESULTS ON A 40-BUS NETWORKED MICROGRIDS SYSTEM

To show the performance of DA-SLR on a realistic networked microgrid benchmark, a 40-bus networked microgrids system is used [45]. In Fig. 13, a single line structure of 40-bus system including four MGs is depicted.

The benchmark test system includes four MGs with 6, 9, 18 and 7 buses for MG1, MG2, MG3 and MG4, respectively.

After solving the operational optimization of networked MGs problem using DA-SLR method, the operation cost of MG1, MG2, MG3, and MG4 are obtained as \$2930, \$3615, \$3336 and \$5824, respectively.

In Fig. 14, the optimized value of frequency droop control gains are shown for different DERs.

VI. CONCLUSION

In this paper, the discrete droop control was modeled and the distributed and asynchronous surrogate Lagrangian relaxation method was used to coordinate droop-controlled networked microgrids to schedule the MGs in a distributed manner in presence of discrete decision variables. It was shown that the DA-SLR method outperforms ADMM from convergence and computation time points of view. Also,

different versions of classical method ADMM, in contrast, could not efficiently manage a distributed problem when the number of microgrids or subproblems are growing. The computational time of DA-SLR described that the proposed method is at least three times faster than ADMM. Additionally, the small signal model of networked MGs was derived and the eigenvalue analysis of the system ensured the stability of the system for the optimized droop control gains. As a future research direction, a fully distributed method -a further improvement of DA-SLR- capable of executing the energy management of networked MGs in a distributed and asynchronous manner without a coordinator will be conducted.

REFERENCES

- [1] W. Shi, X. Xie, C.-C. Chu, and R. Gadh, "Distributed optimal energy management in microgrids," *IEEE Trans. Smart Grid*, vol. 6, no. 3, pp. 1137–1146, Dec. 2014.
- [2] N. Nikmehr, S. Najafi-Ravadanegh, and A. Khodaei, "Probabilistic optimal scheduling of networked microgrids considering time-based demand response programs under uncertainty," *Appl. Energy*, vol. 198, pp. 267–279, Jul. 2017.
- [3] D. Wang, J. Qiu, L. Reedman, K. Meng, and L. L. Lai, "Two-stage energy management for networked microgrids with high renewable penetration," *Appl. Energy*, vol. 226, pp. 39–48, 2018.
- [4] A. Maulik and D. Das, "Optimal operation of droop-controlled islanded microgrids," *IEEE Trans. Sustain. Energy*, vol. 9, no. 3, pp. 1337–1348, Jul. 2017.
- [5] B. Alghamdi and C. A. Canizares, "Frequency regulation in isolated microgrids through optimal droop gain and voltage control," *IEEE Trans. Smart Grid*, vol. 12, no. 2, pp. 988–998, Mar. 2020.
- [6] R. A. Jabr, "Mixed-integer convex optimization for DC microgrid droop control," *IEEE Trans. Power Syst.*, vol. 36, no. 6, pp. 5901–5908, Nov. 2021.
- [7] W. Shi, X. Xie, C. Chu, and R. Gadh, "Distributed optimal energy management in microgrids," *IEEE Trans. Smart Grid*, vol. 6, no. 3, pp. 1137–1146, Dec. 2015.
- [8] M. R. Sandgani and S. Siropour, "Coordinated optimal dispatch of energy storage in a network of grid-connected microgrids," *IEEE Trans. Sustain. Energy*, vol. 8, no. 3, pp. 1166–1176, Jul. 2017.
- [9] A. Rajaei, S. Fattaheian-Dehkordi, M. Fotuhi-Firuzabad, M. Moeini-Aghaie, and M. Lehtonen, "Developing a distributed robust energy management framework for active distribution systems," *IEEE Trans. Sustain. Energy*, vol. 12, no. 4, pp. 1891–1902, Oct. 2021.
- [10] Y. Liu, Y. Li, H. B. Gooi, Y. Jian, H. Xin, X. Jiang, and J. Pan, "Distributed robust energy management of a multimicrogrid system in the real-time energy market," *IEEE Trans. Sustain. Energy*, vol. 10, no. 1, pp. 396–406, Jan. 2017.
- [11] N. Nikmehr, "Distributed robust operational optimization of networked microgrids embedded interconnected energy hubs," *Energy*, vol. 199, May 2020, Art. no. 117440.
- [12] Z. Yang, R. Wu, J. Yang, K. Long, and P. You, "Economical operation of microgrid with various devices via distributed optimization," *IEEE Trans. Smart Grid*, vol. 7, no. 2, pp. 857–867, Mar. 2015.
- [13] S. Fattaheian-Dehkordi, A. Rajaei, A. Abbaspour, M. Fotuhi-Firuzabad, and M. Lehtonen, "Distributed transactive framework for congestion management of multiple-microgrid distribution systems," *IEEE Trans. Smart Grid*, vol. 13, no. 2, pp. 1335–1346, Mar. 2021.
- [14] T. Liu, X. Tan, B. Sun, Y. Wu, and D. H. Tsang, "Energy management of cooperative microgrids: A distributed optimization approach," *Int. J. Elect. Power Energy Syst.*, vol. 96, pp. 335–346, Mar. 2018.
- [15] Z. Li and M. Shahidehpour, "Privacy-preserving collaborative operation of networked microgrids with the local utility grid based on enhanced benders decomposition," *IEEE Trans. Smart Grid*, vol. 11, no. 3, pp. 2638–2651, May 2019.
- [16] M. A. Bragin, B. Yan, and P. B. Luh, "Distributed and asynchronous coordination of a mixed-integer linear system via surrogate Lagrangian relaxation," *IEEE Trans. Autom. Sci. Eng.*, vol. 18, no. 3, pp. 1191–1205, Jul. 2020.
- [17] D. Papadaskalopoulos, D. Pudjianto, and G. Strbac, "Decentralized coordination of microgrids with flexible demand and energy storage," *IEEE Trans. Sustain. Energy*, vol. 5, no. 4, pp. 1406–1414, Oct. 2014.
- [18] R. Aboli, M. Ramezani, and H. Falaghi, "A hybrid robust distributed model for short-term operation of multi-microgrid distribution networks," *Electric Power Syst. Res.*, vol. 177, Dec. 2019, Art. no. 106011.
- [19] W. T. El-Sayed, H. E. Z. Farag, H. H. Zeineldin, and E. F. El-Saadany, "Formation of islanded droop-based microgrids with optimum loadability," *IEEE Trans. Power Syst.*, vol. 37, no. 2, pp. 1564–1576, Mar. 2021.
- [20] A. Maulik and D. Das, "Stability constrained economic operation of islanded droop-controlled DC microgrids," *IEEE Trans. Sustain. Energy*, vol. 10, no. 2, pp. 569–578, Apr. 2018.
- [21] P. H. Divshali, S. H. Hosseini, and M. Abedi, "A novel multi-stage fuel cost minimization in a VSC-based microgrid considering stability, frequency, and voltage constraints," *IEEE Trans. Power Syst.*, vol. 28, no. 2, pp. 931–939, May 2012.
- [22] H. Liu *et al.*, "Small-signal analysis of DC microgrid and multi-objective optimization segmented droop control suitable for economic dispatch," *J. Modern Power Syst. Clean Energy*, vol. 8, no. 3, pp. 564–572, 2020.
- [23] E. Barklund, N. Pogaku, M. Prodanovic, C. Hernandez-Aramburo, and T. C. Green, "Energy management in autonomous microgrid using stability-constrained droop control of inverters," *IEEE Trans. Power Electron.*, vol. 23, no. 5, pp. 2346–2352, Sep. 2008.
- [24] N. Nikmehr, P. Zhang, and M. Bragin, "Quantum distributed unit commitment: An application in microgrids," *IEEE Trans. Power Syst.*, early access, Jan. 11, 2022, doi: [10.1109/TPWRS.2022.3141794](https://doi.org/10.1109/TPWRS.2022.3141794).
- [25] N. Pogaku, M. Prodanovic, and T. C. Green, "Modeling, analysis and testing of autonomous operation of an inverter-based microgrid," *IEEE Trans. Power Electron.*, vol. 22, no. 2, pp. 613–625, Mar. 2007.
- [26] H.-G. Yeh, D. F. Gayme, and S. H. Low, "Adaptive VAR control for distribution circuits with photovoltaic generators," *IEEE Trans. Power Syst.*, vol. 27, no. 3, pp. 1656–1663, Aug. 2012.
- [27] S. Tan, J.-X. Xu, and S. K. Panda, "Optimization of distribution network incorporating distributed generators: An integrated approach," *IEEE Trans. Power Syst.*, vol. 28, no. 3, pp. 2421–2432, Aug. 2013.
- [28] M. Farivar and S. H. Low, "Branch flow model: Relaxations and convexification—Part I," *IEEE Trans. Power Syst.*, vol. 28, no. 3, pp. 2554–2564, Apr. 2013.
- [29] G. Liu, T. Jiang, T. B. Ollis, X. Zhang, and K. Tomsovic, "Distributed energy management for community microgrids considering network operational constraints and building thermal dynamics," *Appl. Energy*, vol. 239, pp. 83–95, Apr. 2019.
- [30] Z. Wang, H. Chen, J. Wang, and M. Begovic, "Inverter-less hybrid voltage/VaR control for distribution circuits with photovoltaic generators," *IEEE Trans. Smart Grid*, vol. 5, no. 6, pp. 2718–2728, Nov. 2014.
- [31] H. D. Sherali and W. P. Adams, *Reformulation—Linearization Techniques for Discrete Optimization Problems*. New York, NY, USA: Springer, 2013, pp. 2849–2896.
- [32] S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein, "Distributed optimization and statistical learning via the alternating direction method of multipliers," *Found. Trends Mach. Learn.*, vol. 3, no. 1, pp. 1–122, 2011.
- [33] M. Ma, A. N. Nikolakopoulos, and G. B. Giannakis, "Hybrid ADMM: A unifying and fast approach to decentralized optimization," *EURASIP J. Adv. Signal Process.*, vol. 2018, no. 1, pp. 1–17, Dec. 2018.
- [34] M. A. Bragin, B. Yan, and P. B. Luh, "Distributed and asynchronous coordination of a mixed-integer linear system via surrogate Lagrangian relaxation," *IEEE Trans. Autom. Sci. Eng.*, vol. 18, no. 3, pp. 1–15, Jun. 2020.
- [35] M. A. Bragin, P. B. Luh, J. H. Yan, N. Yu, and G. A. Stern, "Convergence of the surrogate Lagrangian relaxation method," *J. Optim. Theory Appl.*, vol. 164, no. 1, pp. 173–201, 2015.
- [36] A. Nedić, D. P. Bertsekas, and V. S. Borkar, "Distributed asynchronous incremental subgradient methods," *Stud. Comput. Math.*, vol. 8, no. C, pp. 381–407, 2001.
- [37] B. T. Polyak, "Minimization of unsmooth functionals," *USSR Comput. Math. Math. Phys.*, vol. 9, no. 3, pp. 14–29, Jan. 1969.
- [38] X. Guo, Z. Lu, B. Wang, X. Sun, L. Wang, and J. M. Guerrero, "Dynamic phasors-based modeling and stability analysis of droop-controlled inverters for microgrid applications," *IEEE Trans. Smart Grid*, vol. 5, no. 6, pp. 2980–2987, Nov. 2014.
- [39] Y. Xu, Z. Dong, Z. Li, Y. Liu, and Z. Ding, "Distributed optimization for integrated frequency regulation and economic dispatch in microgrids," *IEEE Trans. Smart Grid*, vol. 12, no. 6, pp. 4595–4606, Nov. 2021.

- [40] H. Yuan, F. Li, Y. Wei, and J. Zhu, "Novel linearized power flow and linearized OPF models for active distribution networks with application in distribution LMP," *IEEE Trans. Smart Grid*, vol. 9, no. 1, pp. 438–448, Jan. 2016.
- [41] M. E. Baran and F. F. Wu, "Network reconfiguration in distribution systems for loss reduction and load balancing," *IEEE Trans. Power Del.*, vol. 4, no. 2, pp. 1401–1407, Apr. 1989.
- [42] W. H. Kersting, "Radial distribution test feeders," *IEEE Trans. Power Syst.*, vol. 6, no. 3, pp. 975–985, Aug. 1991.
- [43] S. Mhanna, G. Verbic, and A. C. Chapman, "Adaptive ADMM for distributed AC optimal power flow," *IEEE Trans. Power Syst.*, vol. 34, no. 3, pp. 2025–2035, May 2019.
- [44] E. Ghadimi, A. Teixeira, I. Shames, and M. Johansson, "Optimal parameter selection for the alternating direction method of multipliers (ADMM): Quadratic problems," *IEEE Trans. Autom. Control*, vol. 60, no. 3, pp. 644–658, Mar. 2014.
- [45] M. N. Alam, S. Chakrabarti, and X. Liang, "A benchmark test system for networked microgrids," *IEEE Trans. Ind. Informat.*, vol. 16, no. 10, pp. 6217–6230, Oct. 2020.



NIMA NIKMEHR (Senior Member, IEEE) is currently pursuing the Ph.D. degree in electrical engineering with the Department of Electrical and Computer Engineering, Stony Brook University, Stony Brook, NY, USA.

He is an Academic Summer Intern with Commonwealth Edison (ComEd) in 2022. His research interests include power systems operation and optimization, networked microgrids, and quantum computing. He received the Best Paper Award at the 2022 IEEE Power and Energy Society General Meeting for his research on quantum distributed unit commitment. He was also a recipient of the Outstanding Reviewer Award for IEEE TRANSACTIONS ON SUSTAINABLE ENERGY in 2021.



MIKHAIL A. BRAGIN (Senior Member, IEEE) is an Assistant Research Professor with the Department of Electrical and Computer Engineering, University of Connecticut. His research work has been supported by the U.S. National Science Foundation, BNL, MISO, ISO-NE, ABB, CESMII, and AFRL. His research is geared toward solving complex technical and societal challenges within smart grids, supply chains, and artificial intelligence. Accordingly, his research interests include

operations research, mathematical optimization, including power systems optimization, grid integration of renewables, energy-based operation optimization of distributed energy systems, stochastic scheduling within manufacturing systems, and machine learning through deep neural networks. His research has appeared in top journals such as *Journal of Optimization Theory and Applications*, *IEEE TRANSACTIONS ON POWER SYSTEMS*, *IEEE TRANSACTIONS ON AUTOMATION SCIENCE AND ENGINEERING*, *Decision Support Systems*, and *IEEE ROBOTICS AND AUTOMATION LETTERS* as well as in top conferences such as the PES General Meeting, the INFORMS Annual Meeting, and the International Joint Conference on Artificial Intelligence.



PENG ZHANG (Senior Member, IEEE) received the Ph.D. degree in electrical engineering from The University of British Columbia, Vancouver, BC, Canada, in 2009.

He is a Full Professor of electrical and computer engineering and a SUNY Empire Innovation Professor with Stony Brook University, New York. He has a joint appointment with the Brookhaven National Laboratory as a Staff Scientist. Previously, he was a Centennial Associate Professor and a Francis L. Castleman Associate Professor with the University of Connecticut, Storrs, CT, USA. He was a Systems Planning Engineer at the BC Hydro and Power Authority, Canada, from 2006 to 2010. His research interests include AI-enabled smart grids, quantum-engineered power grids, networked microgrids, power systems stability and control, cybersecurity, and formal methods and reachability analysis. He is an individual member of CIGRÉ. He is an Editor of the *IEEE TRANSACTIONS ON POWER SYSTEMS*, the *IEEE POWER AND ENERGY SOCIETY LETTERS*, and the *IEEE JOURNAL OF OCEANIC ENGINEERING*.



PETER B. LUH (Life Fellow, IEEE) received the B.S. degree from the National Taiwan University, the M.S. degree from M.I.T., and the Ph.D. degree from Harvard University. He was with the University of Connecticut, Storrs, CT, USA, from 1980 to 2020, and Board of Trustees Distinguished Professor and the SNET Professor of communications and information technologies upon retirement in early 2021. He is currently a Distinguished Chair

Professor at the Department of Electrical Engineering, National Taiwan University. His research interests include smart grid, intelligent manufacturing, energy-smart and safe buildings, and mathematical optimization of complex mixed-integer problems with combinatorial complexity. He is a member of the IEEE Publication Services and Products Board. He received the IEEE Robotics and Automation Society 2013 Pioneer Award, the 2017 George Saridis Leadership Award, and the 2018 IEEE T-ASE Best Paper Award.

...