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Mechanism Design for Peak Demand Management in Energy Communities

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Abstract— **We consider a demand management problem of an energy community, in which several users obtain energy from an external organization such as an energy company, and pay for the energy according to pre-specified prices that consist of a time-dependent price per unit of energy, as well as a separate price for peak demand. Since users' utilities are private information which they may not be willing to share, a mediator, known as the planner, is introduced to help optimize the overall satisfaction of the community (total utility minus total payments) by mechanism design. A mechanism consists of message spaces, and a set of tax and allocation functions for each user. Once we implement the mechanism, each user reports a message chosen from her own message space, and then receives some amount of energy determined by the allocation function and pays the tax specified by the tax function. A desirable mechanism induces a game, the Nash equilibria (NE) of which, result in an allocation that coincides with the optimal allocation for the community.**

As a starting point, we design a standard, centralized mechanism for the energy community with desirable properties such as full implementation, strong budget balance and individual rationality for both users and the planner. Then we extend this mechanism to the case of communities where message exchanges only happen among neighborhoods, and consequently, the tax and allocation functions of each user are only determined by the messages from her neighbors. All the properties designed for the centralized mechanism are preserved in the distributed mechanism.

I. Introduction

Resource allocation is an essential task in networked systems such as communication networks, energy/power networks, etc. In such systems, there is usually one or multiple kinds of limited and divisible resources allocated among several agents. When full information regarding agents interests is available, solving the optimal resource allocation problem reduces to a standard optimization problem. However, in many interesting scenarios, strategic agents may choose to conceal or misreport their interests in order to get more resources. In such cases, it is possible that appropriate incentives are designed so that selfish agents are incentivized to report truly their private information, thus enabling optimal resource allocation.

In existing work related to resource allocation problems, *mechanism design* is frequently used for resolving the obstacles mentioned above. In the framework of mechanism design, the participants reach an agreement regarding how they exchange messages, how they share the resources, and how much they should pay. Such agreements are designed to incentivize the agents to provide the information needed to solve the optimization problem.

In this paper, we develop mechanisms to solve a demand management problem in energy communities. In an energy community, users obtain energy from an energy company and pay for it. Users' demand is subject to constraints relating to equipment capacity and minimum comfort level. Each user possesses a utility as a function of her/his own demand. Utilities are private information for users. The welfare of the community is the sum of utilities minus the energy cost. If users were willing to report truthfully their utilities, one could easily optimize energy allocation to maximize social welfare. However, since users are strategic and might not be willing to report utilities directly, to maximize the welfare, we need to find an appropriate mechanism that incentivizes them to reveal some information about their utilities, so that optimal allocation is reached even in the presence of strategic behaviors. These mechanisms are usually required to possess several interesting properties, among which, full implementation in Nash equilibria (NE), individual rationality and budget balance [1]–[3].

A. Contributions

The main contribution of this work is to design a mechanism for implementing the optimal allocation of the demand management problem in an energy community with strategic users and a pre-specified message exchange network. Emphasizing more on non-VCG mechanisms, we look into indirect mechanisms similar to the ones in [4], [5]. Unlike [4], [5] where a radial allocation scheme is used, the allocation scheme used in this work is much simpler and can be used in more general settings (e.g., environments with non-monotonic utilities). We start from a centralized mechanism without communication constraints. We design a mechanism that satisfies the properties of full implementation, budget balance, and individual rationality. We then generalize the mechanism into a distributed version.

B. Related Literature

The starting point of this work is network utility maximization (NUM), which is one typical category of resource allocation. The interested reader might refer to Chapter 2 of [6] for a detailed approach to models and algorithms for solving NUM problems. For the demand management problem in energy communities, the model formulation follows in a similar way. As mentioned previously, when strategic users are involved, mechanism design is a powerful approach. One well-known framework proposed by [7]–[9]

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is the VCG mechanism. In this mechanism, users have to communicate utilities (i.e., entire functions), which leads to a high cost of information transmission. To ease the burden of communication, Kelly mechanism in [10] uses logarithmic functions as surrogates of utilities. The agents only need to report one parameter for the logarithms. This saves a lot of communication costs but only works for price-taking agents. There is a line of work based on these two mechanisms. The mechanism proposed by [11] is an online mechanism modified from VCG, with a δ-strategy-proof. The work in [12] extends Kelly's mechanism to an environment with multiple divisible resources, but still needs the assumption of price-taking. By introducing additional message components of prices, [13] develops a mechanism for strategic agents based on Kelly's. Moreover, under some certain assumptions on the utilities, [13] provides a surrogate optimization-based algorithm for learning the Nash equilibrium.

There are a number of mechanisms reported in the literature that are not based on VCG or Kelly mechanism. The mechanisms proposed by [14] fully implement the Walrasian equilibrium (for divisible private goods) and Lindahl equilibrium (for divisible public goods) with feasibility on and off equilibria, individual rationality, budget balance, and convergence guarantees. The works in [15], [16] adopt an idea of penalty functions to incentivize feasibility on equilibrium. The mechanism in [5] presents a technique called radial allocation, which ensures the feasibility on and off equilibria in a centralized mechanism. With a loss of feasibility off equilibria, the work in [4] extends the mechanisms with the help of demand proxies to distributed environments where message exchanges are only allowed between neighbors.

The paper is structured as follows. In Section II the demand management problem in energy communities is formulated. Some necessary concepts of mechanism design are also presented. Section IV presents a centralized mechanism for demand management without communication constraints. The mechanism is characterized by the message spaces, allocation functions and tax functions. We show that the centralized mechanism possesses several desirable properties. In Section V, we incorporate communication constraints by introducing the concept of message exchange network, and propose a distributed mechanism. We conclude the paper in Section VI. The proofs of lemmas and theorems can be found in the appendix.

II. Model

Consider an energy community consisting of N users and a given time horizon T, where T can be viewed as the number of days during one billing period. Each user i in the user set $\mathcal{N}$ has her own prediction on her usage over one billing period denoted by $\boldsymbol{x}^i = (x_1^i, \dots, x_T^i)^\mathsf{T}$, where x_t^i is the predicted usage of user i on the t-th time slot of the billing period. Note that x_t^i can be a negative number due to the potential possibility that users in the electrical grid can generate power through renewable technologies (e.g., photovoltaic) and return the surplus back to the grid. The users are characterized by their utility functions as

$$v^i(\boldsymbol{x}^i) = \sum_{t=1}^{T} v_t^i(x_t^i), \forall i \in \mathcal{N}.$$

The energy community, as a whole, pays for the energy. The unit prices are given separately for every time slot t denoted by p_t. These prices are considered given and fixed (e.g., by the local utility company). In addition, the local utility company imposes a unit peak price p_0 in order to incentivize load balancing and lessen the burden of peaks in demand. To summarize, the cost of the energy community is as follows:

$$J(\boldsymbol{x}) = \sum_{t=1}^{T} p_t \left(\sum_{i=1}^{N} x_t^i \right) + p_0 \cdot \max_{1 \le t \le T} \sum_{i=1}^{N} x_t^i, \tag{1}$$

where $\boldsymbol{x}$ is a concatenation of demand vectors $\boldsymbol{x}^1, \dots, \boldsymbol{x}^N$.

The centralized demand management problem for the energy community can be formulated as

$$\underset{\boldsymbol{x} \in \mathcal{X}}{\text{maximize}} \quad \sum_{i=1}^{N} v^i(\boldsymbol{x}^i) - J(\boldsymbol{x}). \tag{2}$$

The meaning of the feasible set $\mathcal{X}$ is to incorporate possible lower bounds on each user's demand (e.g., minimal indoor heating or AC) and/or upper bounds due to the capacities of the facilities.

In order to solve the optimization problem (2) using convex optimization methods, the following assumptions are made.

Assumption 1: All the utility functions $v_t^i(\cdot)$'s are twice differentiable and strictly concave.

Assumption 2: The feasible set $\mathcal{X}$ is a polytope formed by several linear inequality constraints, and $\mathbf{0} \in \mathcal{X}$.

By Assumption 2, $\mathcal{X}$ can be written as $\{\boldsymbol{x} | A\boldsymbol{x} \le \boldsymbol{b}\}$ for some $A \in \mathbb{R}^{L \times NT}$ and $\boldsymbol{b} \in \mathbb{R}_+^L$, where L is the number of linear constraints in $\mathcal{X}$, and

$$\begin{aligned} A &= \begin{bmatrix} \boldsymbol{a}^1 & \dots & \boldsymbol{a}^L \end{bmatrix}^\mathsf{T}, \\ \boldsymbol{a}^l &= \begin{bmatrix} a_1^{1,l} & \dots & a_T^{1,l} & \dots & a_1^{N,l} & \dots & a_T^{N,l} \end{bmatrix}^\mathsf{T}, \ l = 1, \dots, L, \\ \boldsymbol{b} &= \begin{bmatrix} b^1, \dots, b^L \end{bmatrix}^\mathsf{T}. \end{aligned}$$

With Assumptions 1, 2, the energy community faces an optimization problem with a strictly concave objective function over a nonempty compact convex feasible set. Therefore, from convex optimization theory, the optimal solution for this problem always exists and is unique.

Substituting the max function in (1) with a new variable w, the optimization problem in (2) can be equivalently restated as

$$\underset{\boldsymbol{x}, w}{\text{maximize}} \quad \sum_{i=1}^{N} v^i(\boldsymbol{x}^i) - \sum_{t=1}^{T} p_t \left(\sum_{i=1}^{N} x_t^i \right) - p_0 w \tag{3a}$$

$$\text{subject to} \quad A\boldsymbol{x} \le \boldsymbol{b}, \tag{3b}$$

$$\sum_{i=1}^{N} x_t^i \le w, \ \forall t \in \{1, \dots, T\}. \tag{3c}$$

The new optimization problem has a differentiable concave objective function with a convex feasible set, which means it is still a convex optimization problem, and therefore, KKT conditions are the sufficient and necessary conditions for a solution $(\boldsymbol{x}, \boldsymbol{\lambda}, \boldsymbol{\mu})$ to be an optimal solution, where $\boldsymbol{\lambda} = [\lambda^1, \ldots, \lambda^L]^\top$ are the Lagrange multipliers for each linear constraint $\boldsymbol{a}^l \boldsymbol{x} \leq b^l$ in constraint $x \in \mathcal{X}$, and $\boldsymbol{\mu} = [\mu_1, \ldots, \mu_T]^\top$ are the Lagrange multipliers for (3c). The KKT conditions are listed as follows:

1) Primal Feasibility:

$$\boldsymbol{x} \in \mathcal{X}, \tag{4a}$$

$$\sum_{i=1}^{N} x_t^i \leq w. \tag{4b}$$

2) Dual Feasibility:

$$\lambda^l \geq 0,\ l = 1, \ldots, L; \mu_t \geq 0,\ t = 1, \ldots, T. \tag{4c}$$

3) Complementary Slackness:

$$\lambda^l (\boldsymbol{a}^{l\top} \boldsymbol{x} - b^l) = 0,\ l = 1, \ldots, L, \tag{4d}$$

$$\mu_t (\sum_{i=1}^{N} x_t^i - w) = 0,\ t = 1, \ldots, T. \tag{4e}$$

4) Stationarity:

$$p_0 = \sum_t \mu_t, \tag{4f}$$

$$\dot{v}_t^i(x_t^i) = p_t + \sum_l \lambda^l a_t^{i,l} + \mu_t,\ t = 1, \ldots, T,\ i \in \mathcal{N}. \tag{4g}$$

where $\dot{v}_t^i(\cdot)$ is the first order derivative of $v_t^i(\cdot)$.

III. Mechanism Design Preliminaries

In an energy community, utilities are users' private information. Due to privacy and strategic concerns, users might not be willing to report their utilities. As a result, (3) cannot be solved directly. In order to solve (3) under the settings stated above, we introduce a planner as an intermediary between the community and the energy company. To incentivize users to provide necessary information for optimization, the planner signs a contract with users, which prespecifies the messages needed from users and rules for determining the allocation and taxes/subsidies from/to the users. The planner commits to the contract. Informally speaking, the design of such a contract is referred to as *mechanism design*.

More formally, a *mechanism* is a collection of message sets and an outcome function [2]. Specifically, in resource allocation problems, a mechanism can be defined as a tuple $(\mathcal{M}, \hat{x}(\cdot), \hat{t}(\cdot))$, where $\mathcal{M} = \mathcal{M}^1 \times \ldots \times \mathcal{M}^N$ is a space of message profile, $\hat{x} : \mathcal{M} \mapsto \mathcal{X}$ is an allocation function determining the allocation $\boldsymbol{x}$ according to the received messages m, $\hat{t} : \mathcal{M} \mapsto \mathbb{R}^N$ is a tax function which defines the payments (or subsidies) of users based on m. Once defined, the mechanism induces a game $(\mathcal{N}, \mathcal{M}, \{u^i\}_{i \in \mathcal{N}})$. In the game, each user i chooses her message m^i from message space $\mathcal{M}^i$, with the objective to maximize her payoff $u^i(m) = v^i(\hat{x}^i(m)) - \hat{t}^i(m)$. The planner charges taxes and pays for the energy cost to the company, so the planner's payoff turns out to be $\sum_i \hat{t}^i(m) - J(\hat{x}(m))$ (the net income of the planner).

For the mechanism-induced game $\mathcal{G}$, Nash equilibrium (NE) is an appropriate solution concept. At the equilibrium point m^*, if $\hat{x}(m^*)$ coincides with the optimal allocation, we say that the mechanism *implements* the optimal allocation at m^*. A mechanism has the property of *full implementation* if all the NE m^*'s implement the optimal allocation.

There are other desirable properties in a mechanism. *Individual rationality* is the property that everyone volunteers to participate in the mechanism-induced game instead of quitting. For the planner, this means that the sum of taxes $\sum_i \hat{t}^i(m^*)$ collected at NE is larger than the cost paid to the energy company $J(\hat{x}(m^*))$. In the context of this paper, *strong budget balance* is the property that the sum of taxes is exactly the same as the cost paid to the energy company, so no additional funds are required by the planner or the community to run the mechanism other than the true energy cost paid to the energy company.

IV. Centralized Mechanism

In this section, we temporarily assume there are no communication constraints, i.e., all the message components are accessible for the calculations of the allocation and taxation. The mechanism designed under this assumption is called a centralized mechanism. In the next section, we will extend this mechanism to an environment with communication constraints.

In the proposed centralized mechanism we define the user i's message m^i as

$$m^i = \left(\{y_t^i\}_{t=1}^T, \{q^{i,l}\}_{l \in \mathcal{L}}, \{s_t^i\}_{t=1}^T, \{\beta_t^i\}_{t=1}^T \right). \tag{5}$$

Each message component has an intuitive meaning. Message $y_t^i \in \mathbb{R}$ can be regarded as the demand for time slot t announced by user i. Message $q^{i,l} \in \mathbb{R}_+$ is the additional price that user i expects to pay for the constraint l, which corresponds to the Lagrange multiplier λ^l. Message $s_t^i \in \mathbb{R}_+$ is proportional to the peak price that user i expects to pay at time t. Intuitively, setting one s_t^i greater than $s_{t'}^i$ means user i thinks day t is more likely to be the day with the peak demand rather than t'. This component corresponds to the Lagrange multiplier μ_t. Message $\beta_t^i \in \mathbb{R}$ is the prediction of user $(i+1)$'s usage at time t by user i. This message is included for technical reasons that will become clear in the following (for a user index $i \in \mathcal{N}$, let $i-1$ and $i+1$ denote modulo N operations).

Denote the message space of user i by $\mathcal{M}^i$, and the space of the message profile is represented as $\mathcal{M} = \mathcal{M}^1 \times \ldots \times \mathcal{M}^N$. The allocation functions and the tax functions are functions defined on $\mathcal{M}$. The allocation functions follow the simple definition:

$$\hat{x}_t^i(m) = y_t^i,\ t = 1, \ldots, T,\ \forall i \in \mathcal{N}. \tag{6}$$

i.e., users get exactly what they request.

Prior to the definition of the tax functions, we want to find some variable that acts like μ_t at NE. Although s_t^i is designed to be proportional to μ_t, it does not guarantee $\sum_t s_t^i = p_0$, which is one of the essential conditions (4f) in KKT conditions. To solve this problem, we utilize a technique similar to the radial allocation in [4], [5] to shape the suggested peak price vector $\boldsymbol{s}$ into the form which satisfies (4f). Define a radial pricing operator $\mathcal{RP}^i : \mathbb{R}_+^T \times \mathbb{R}^T \mapsto \mathbb{R}_+^T$, such that

$$\mathcal{RP}^i(\boldsymbol{s}, \boldsymbol{\zeta}^{-i}) = \left(\mathcal{RP}_1^i(\boldsymbol{s}, \boldsymbol{\zeta}^{-i}), \ldots, \mathcal{RP}_T^i(\boldsymbol{s}, \boldsymbol{\zeta}^{-i})\right),$$

where

$$\mathcal{RP}_t^i(\boldsymbol{s}, \boldsymbol{\zeta}^{-i}) = \begin{cases} \frac{s_t}{\sum_{t'} s_{t'}}, & \text{if } \exists t', s_{t'} > 0, \\ \frac{p_0 \cdot \mathbf{1}\{t \in \arg\max_{\tilde{t}} \zeta_{\tilde{t}}^{-i}\}}{\#(\arg\max_{\tilde{t}} \zeta_{\tilde{t}}^{-i})}, & \text{if } \forall t', s_{t'} = 0, \end{cases}$$

and

$$\zeta_t^{-i} = \sum_{j \neq i} y_t^j + \beta_t^{i-1}, \quad \forall i \forall t,$$

where $\#(\cdot)$ represents the number of elements in a finite set, ζ_t^{-i} is the total demand on day t predicted by users other than i. The output of the radial pricing $\mathcal{RP}(\cdot,\cdot)$ will be taken as the peak price in the subsequent tax functions. When the given suggested prices are not all zeros, the unit peak price will be separated to each day proportional to s_t. If the suggested price vector $\boldsymbol{s} = \mathbf{0}$, then divide p_0 to the days with peak demand with equal proportion.

The tax functions are defined as

$$\hat{t}^i(m) = \text{cost}^i(m) + \sum_{t=1}^{T} \text{pr}\boldsymbol{\beta}_t^i(m) + \sum_{l \in \mathcal{L}} \text{con}^{i,l}(m) + \sum_{t=1}^{T} \text{con}_t^i(m), \tag{7}$$

where

$$\begin{aligned} \text{cost}^i(m) =& \sum_{t=1}^{T} (p_t + \mathcal{RP}_t^i(\boldsymbol{s}^{-i}, \boldsymbol{\zeta}^{-i})) \hat{x}_t^i(m) \\ &+ \sum_{l \in \mathcal{L}_i} q^{-i,l} \sum_{t=1}^{T} \boldsymbol{a}^{i,l} \hat{\boldsymbol{x}}^i(m), \\ \text{con}^{i,l}(m) =& (q^{i,l} - q^{-i,l})^2 + q^{i,l}(b^l - \sum_{j \neq i} \boldsymbol{a}^{j,l} \boldsymbol{y}^j - \boldsymbol{a}^{i,l} \boldsymbol{\beta}^{i-1}), \\ \text{con}_t^i(m) =& (s_t^i - s_t^{-i})^2 + s_t^i(z^{-i} - \zeta_t^{-i}), \\ \text{pr}\boldsymbol{\beta}_t^i(m) =& (\beta_t^i - y_t^{i+1})^2, \end{aligned}$$

and

$$\begin{aligned} s_t^{-i} &= \frac{1}{N-1} \sum_{j \neq i} s_t^j \quad \forall i \ \forall t, \\ q^{-i,l} &= \frac{1}{N-1} \sum_{j \neq i} q^{j,l} \quad \forall i \ \forall l, \\ z^{-i} &= \max_t \left\{\zeta_t^{-i}\right\} \quad \forall i. \end{aligned}$$

The tax function for user i consists of three parts. The first part $\text{cost}^i(m)$ is the cost for the demand. According to this part, user i pays the fixed price and the peak price for her demand. The second part $\text{pr}\boldsymbol{\beta}_t^i(m)$ is a penalty term for the imprecision of prediction $\boldsymbol{\beta}^i$, which incentivizes $\boldsymbol{\beta}^i$ to align with $\boldsymbol{y}^{i+1}$ at NE. The third part consists of two penalty terms $\text{con}^{i,l}(m)$ and $\text{con}_t^i(m)$ for constraints l and peak demand inequalities t. Both of them have a quadratic term and possess a form that looks similar to the complementary slackness conditions (4d), (4e). This special design facilitates the suggested price to come to an agreement, and ensures the primal feasibility and complementary slackness hold at NE, which will be shown in Lemma 2.

The main property we want from this mechanism is full implementation. We expect the allocation scheme under the Nash equilibrium of the mechanism-induced game to coincide with that of the original optimization problem. Full implementation can be shown in two steps. First, we can show that if there is a (pure strategy) NE, it must induce the optimal allocation. Then we prove the existence of such (pure strategy) Nash equilibrium.

From the form of the tax functions, we can immediately get the following lemma.

Lemma 1: At any NEs, for each user i, the demand proxy β_t^i's are the same as the demand of her next neighbor, i.e., $\beta_t^i = y_t^{i+1}$ for all t.

Proof: Suppose m is a NE where there exists at least one usre i, whose message $\boldsymbol{\beta}^i$ does not agree with next user's demand, i.e., $\boldsymbol{\beta}^i \neq \boldsymbol{y}^{i+1}$. Say, $\beta_t^i \neq y_t^{i+1}$. Then we can find a profitable deviation $\tilde{m}$, which keeps everything other than $\boldsymbol{\beta}^i$ the same as m, but modifies β_t^i with $\tilde{\beta}_t^i = y_t^{i+1}$. Compare the payoff value u^i before and after the deviation:

$$\begin{aligned} u^i(\tilde{m}) - u^i(m) =& -(\tilde{\beta}_t^i - y_t^{i+1})^2 + (\beta_t^i - y_t^{i+1})^2 \\ =& (\beta_t^i - y_t^{i+1})^2 > 0. \end{aligned}$$

Thus, if there is some $\boldsymbol{\beta}^i \neq \boldsymbol{y}^{i+1}$, user i can always construct another announcement m^i, such that user i get a better payoff. ■

Lemma 1 plays an important role in the mechanism. Recall that $\text{con}^{i,l}(m), \text{con}_t^i(m)$ facilitate complementary slackness by giving feedback to the prices with the slackness of constraints. While if we directly use the announced demand of user i here, this term would somehow impede the full implementation of the optimal demand because quoting the self-announced demand in the tax function raises the possibility of unexpected strategic moves for user i to obtain extra profit. Instead, using proxy eliminates the control on the slackness factor of user i, which excludes the disturbance to the feedback for the prices.

With the introduction of these proxies, we show in the following lemmas, that at NE, all KKT conditions required for the optimal solution are satisfied.

Lemma 2: At any NEs, user suggested prices are equal:

$$q^{i,l} = q^l, \ \forall l \in \mathcal{L}_i \ \forall i \in \mathcal{N}, \tag{8}$$

$$s_t^i = s_t, \ t = 1, \ldots, T, \ \forall i \in \mathcal{N}. \tag{9}$$

Furthermore, users' announced demand profile satisfies $\boldsymbol{y} \in \mathcal{X}$, and the equal prices, together with the demand profile, have the property of complementary slackness:

$$\boldsymbol{q}(A\boldsymbol{x}-\boldsymbol{b})=\mathbf{0}, \tag{10}$$

$$s_t(z-\sum_i y_t^i)=0,\ \forall t=1,\ldots,T, \tag{11}$$

and (11) implies

$$\mathcal{RP}_t^i(\boldsymbol{s},\boldsymbol{\zeta}^{-i})\left(z-\sum_i y_t^i\right)=0,\ \forall t=1,\ldots,T, \tag{12}$$

where z is the peak demand during the billing period.

Proof: The proof can be found in Appendix A. ■

We now prove stationarity under Nash equilibrium.

Lemma 3: At NE, stationarity holds, i.e.,

$$\dot{v}_t^i(\hat{x}_t^i(m))=p_t+\mathcal{RP}_t^i(\boldsymbol{s},\boldsymbol{\zeta}^{-i})+\sum_{l\in\mathcal{L}_i}q^l a_t^{i,l}, \tag{13}$$

$$p_0=\sum_{t=1}^{T}\mathcal{RP}_t^i(\boldsymbol{s},\boldsymbol{\zeta}^{-i}). \tag{14}$$

Proof: (13) can be shown by evaluating the derivative of $u^i(m)$ w.r.t. y_t^i at NE. (14) comes from the definition of $\mathcal{RP}$. The details can be found in Appendix C of [17]. ■

With Lemmas 1, 2 and 3, it is straightforward to derive the first part of our result, i.e., efficiency of the allocation at any NE.

Theorem 1: For the mechanism-induced game $\mathcal{G}$, if there exist Nash equilibria, then these NEs result in the same allocation as the optimal solution of the centralized problem.

Proof: For a NE message profile m^*, one can verify that the induced allocation $\boldsymbol{x}$ together with the prices $\boldsymbol{q}^*$ and $\boldsymbol{s}^*$ after radial pricing satisfy all the KKT conditions. The details can be found in the proof of Theorem 1 in [17]. ■

By construction, one can easily prove the existence of Nash equilibrium.

Theorem 2: For the mechanism-induced game $\mathcal{G}$, there has to be at least one NE.

Proof: From the theory of convex optimization, we know that the optimal solution of (3) exists. Based on this solution, one can construct a message profile which satisfies all the properties we present in Lemmas 1,2,3 and prove there is no unilateral deviation for all users. The details can be found in Appendix D of [17]. ■

Full implementation indicates that if all users are willing to participate in the mechanism, the equilibrium outcome is nothing but the optimal allocation. For each user i, the payoff at NE will be

$$u^i(m^*)=v^i(\hat{\boldsymbol{x}}^i(m^*)) -\sum_{t=1}^{T}\underbrace{\left(p_t+\mathcal{RP}_t^i(\boldsymbol{s},\boldsymbol{\zeta}^{-i})+\sum_{l\in\mathcal{L}_i}q^{-i,l}a_t^{i,l}\right)}_{\text{Aggregated unit price for }\hat{x}_t^i}\hat{x}_t^i(m^*). \tag{15}$$

In other words, the users pay for their own demands by the aggregated unit prices given by the consensus at NE. By counting the planner as a participant of the mechanism with utility $\sum_{i\in\mathcal{N}}\hat{t}^i(m^*)-J(\boldsymbol{x}^*)$, a strong budget balance is automatically achieved. However, there are still two questions remaining. Are the users willing to follow this mechanism or would they rather not participate? Will the planner have to pay extra money for implementing such a mechanism? The two theorems below answer these questions.

Theorem 3 (Individual Rationality for Users): Assume agent i gets $\boldsymbol{x}^i=\mathbf{0}$ and pays nothing if she chooses to not participate in the mechanism. Then, at NE, participating in the mechanism is weakly better than not participating, i.e.,

$$u^i(m^*)\geq v^i(\mathbf{0}).$$

Proof: The main idea for the proof of Theorem 3 is to find a message profile with m^{-i*}, in which user i's payoff is $v^i(\mathbf{0})$, and then we can argue that following NE won't be worse since m^* is a best response to m^{-i*}. The details are presented in Appendix B. ■

Theorem 4 (Individual Rationality for the Planner): At NE, the planner does not need to pay extra money for the mechanism:

$$\sum_{i\in\mathcal{N}}\hat{t}^i(m^*)-J(\hat{\boldsymbol{x}}(m^*))\geq 0. \tag{16}$$

Moreover, by a slight modification of the tax functions defined in (7), the total payment of users and the energy cost achieve a balance at NE:

$$\sum_{i\in\mathcal{N}}\tilde{t}^i(m^*)-J(\hat{\boldsymbol{x}}(m^*))=0. \tag{17}$$

Proof: The verification of individual rationality of the planner can be done by substituting m^* in (16) directly. By giving the income of the planner back to the users in a certain way, the total payment of users is exactly $J(\hat{\boldsymbol{x}}(m^*))$ and consequently no money is left after paying the energy company. The details are presented in Appendix C. ■

V. Distributed Mechanism

In the previous mechanism, allocation functions and tax functions of users depend on the global message profile. If one wants to compute t^i for certain user i, m^j for all $j\in\mathcal{N}$ are needed. Such mechanisms won't work under environments with communication constraints, where such global message exchanges are restricted. To tackle this problem, we provide a distributed mechanism, in which the calculation of the allocation and tax of a certain user depends only on the messages from the "available" users, and therefore satisfies the communication constraints. In this section, we will first introduce communication constraints using a message exchange network model. We then develop a distributed mechanism, which accommodates the communication constraints and preserves the desirable properties of the centralized mechanism.

A. Message Exchange Network

In an environment with communication constraints, all the users are organized in an undirected graph $\mathcal{GR}=(\mathcal{N},\mathcal{E})$, where the set of nodes $\mathcal{N}$ is the set of users, and the set of edges $\mathcal{E}$ indicates the accessibility to the message for each user. If $(i,j)\in\mathcal{E}$, user i can access the message of user j, i.e., the message of j is available for user i when computing the allocation and tax of user i, and vice versa. Here we state an assumption for the message exchange network:

Assumption 3: The graph $\mathcal{GR}$ is a connected graph.

Notice that in the previous mechanism, user i is expected to announce a β_t^i equal to the demand of the next user $(i+1)$, but here it is possible that $(i, i+1) \notin \mathcal{E}$, and owing to the communication constraint, user i is not able to compare β_t^i with y_t^{i+1}. Instead, β_t^i should be a proxy of the demand of user i's direct neighbor. This motivates us to define the function $\phi(i)$, where $\phi(i) \in \mathcal{N}(i)$, $\mathcal{N}(i)$ is the set of user i's neighbors, and $\phi(i) = j$ denotes that in user i's tax function, the proxy variable $\boldsymbol{\beta}$ is provided by user j.

In the next part we are going to use the summaries of the demands to deal with the distributed issue. For the sake of convenience, we define $n(i,k)$ as the nearest user among the neighbors of user i and user i itself to user k. $n(i,k)$ is well-defined because one can show that $n(i,k) = j$ provides a partition for all the users. The proof is omitted here. The details can be found in [18, Ch. 4, Sec. 7.1].

B. The Message Space

In the distributed mechanism, the message m^i in user i's message space $\mathcal{M}^i$ is defined as

$$m^i = \Bigg(\left\{y_t^i\right\}_{t=1}^T, \left\{q^{i,l}\right\}_{l\in\mathcal{L}}, \left\{s_t^i\right\}_{t=1}^T, \left\{\beta_t^{i,j} : \phi(j) = i\right\}_{t=1}^T, \left\{n^{i,j,l} : j \in \mathcal{N}(i)\right\}_{l\in\mathcal{L}}, \left\{\nu_t^{i,j} : j \in \mathcal{N}(i)\right\}_{t=1}^T \Bigg).$$

Here $n^{i,j,l}$ is a summary for demands of users related to constraint l and connected to user i via j. Message $\nu_t^{i,j}$ serves a similar role for the peak demand.

C. Allocation and Tax Functions

The allocation functions $\hat{x}_t^i(m) = y_t^i$ are still straightforward. There are some modifications on tax functions, including adjustments on prices, the consensus of new variables, and terms for complementary slackness.

$$\begin{aligned} \hat{t}^i(m) = \text{cost}^i(m) + \sum\nolimits_l (\text{pr}\boldsymbol{n}^{i,l}(m) + \text{con}^{i,l}(m)) \\ + \sum\nolimits_t (\text{pr}\boldsymbol{\beta}_t^i(m) + \text{pr}\boldsymbol{\nu}_t^i(m) + \text{con}_t^i(m)), \end{aligned} \tag{18}$$

where

$$\begin{aligned} \text{cost}^i(m) &= \sum_{t=1}^T (p_t + \mathcal{RP}_t^{-i}(\boldsymbol{s}^{-i}, \boldsymbol{\zeta}^{-i}))\hat{x}_t^i(m) \\ &\quad + \sum_{l\in\mathcal{L}_i} q^{-i,l}\boldsymbol{a}^{i,l}\hat{\boldsymbol{x}}^i(m), \\ \text{con}^{i,l}(m) &= (q^{i,l} - q^{-i,l})^2 \\ &\quad + q^{i,l}(b^l - \boldsymbol{a}^{i,l}\boldsymbol{\beta}^{\phi(i),i} - \sum_{j\in\mathcal{N}(i)} f^{i,j,l}), \\ \text{con}_t^i(m) &= (s_t^i - s_t^{-i})^2 + s_t^i(z^{-i} - \zeta_t^{-i}), \\ \text{pr}\boldsymbol{n}^{i,l}(m) &= \sum_{j\in\mathcal{N}(i)} \left(n^{i,j,l} - f^{i,j,l}\right)^2, \\ \text{pr}\boldsymbol{\beta}_t^i(m) &= \sum_{j:\phi(j)=i} (\beta_t^{i,j} - y_t^j)^2, \\ \text{pr}\boldsymbol{\nu}_t^i(m) &= \sum_{j\in\mathcal{N}(i)} \left(\nu_t^{i,j} - f_t^{i,j}\right)^2 \\ f^{i,j,l} &= \boldsymbol{a}^{j,l}\boldsymbol{y}^j + \sum_{h\in\mathcal{N}(j)\setminus\{i\}} n^{j,h,l}, \\ f_t^{i,j} &= y_t^j + \sum_{h\in\mathcal{N}(j)\setminus\{i\}} \nu_t^{j,h}. \end{aligned}$$

and

$$\begin{aligned} s_t^{-i} &= \frac{1}{|\mathcal{N}(i)|}\sum_{j\in\mathcal{N}(i)} s_t^j \quad \forall i\ \forall t, \\ q^{-i,l} &= \frac{1}{|\mathcal{N}(i)|}\sum_{j\in\mathcal{N}(i)} q^{j,l} \quad \forall i\ \forall l, \\ \zeta_t^{-i} &= \sum_{j\in\mathcal{N}(i)} f_t^{i,j} + \beta_t^{\phi(i),i}, \quad \forall i\ \forall t, \\ z^{-i} &= \max_t \left\{\zeta_t^{-i}\right\} \quad \forall i. \end{aligned}$$

D. Properties

This mechanism is distributed because all the messsages needed for the allocation and tax functions of user i come from her neighborhood $\mathcal{N}(i)$ and herself. The mechanism satisfies properties similar to those in Lemmas 2, 3, and consequently Theorems 1, 2. The reason is that the components n and ν behave the same as the absent y^h, $h \notin \mathcal{N}(i)$ in the user i's functions at NE, which renders the proofs of the lemmas applicable here as well. These are summarized in Lemmas 4, 5.

Lemma 4: At any NE, we have the following results regarding the proxies:

$$\beta_t^{i,j} = y_t^j, \quad \forall j : \phi(j) = i, \tag{19}$$

$$n^{i,j,l} = \boldsymbol{a}^{j,l}\boldsymbol{y}^j + \sum_{h\in\mathcal{N}(j)\setminus\{i\}} n^{j,h,l}, \forall i, j \in \mathcal{N}(i), \forall l \in \mathcal{L}, \tag{20}$$

$$\nu_t^{i,j} = y_t^j + \sum_{h\in\mathcal{N}(j)\setminus\{i\}} \nu_t^{j,h}, \quad \forall t, \forall i, \forall j \in \mathcal{N}(i). \tag{21}$$

Proof: $\beta^{i,j}, n^{i,j,l}$ and $\nu_t^{i,j}$ only appear in the quadratic penalty terms of user i's tax function. Therefore, for any user i, the only choice to minimize the tax is to bid $\beta^{i,j}, n^{i,j,l}$ and $\nu_t^{i,j}$ by (19)-(21). ■

Then, with the help of the structure of the message exchange network, we have

Lemma 5: At any NE, $n^{i,j,l}$ and $\nu_t^{i,j}$ satisfies

$$n^{i,j,l} = \sum_{t=1}^T \sum_{h:n(i,h)=j} a_t^{h,l} y_t^h, \quad \forall i \in \mathcal{N}, \forall j \in \mathcal{N}(i), \forall l \in \mathcal{L}, \tag{22}$$

$$\nu_t^{i,j} = \sum_{h:n(i,h)=j} y_t^h, \forall i \in \mathcal{N}, \forall j \in \mathcal{N}(i), \forall t. \tag{23}$$

Proof: The proof can be found in Appendix G in [17]. ∎

With this lemma, we immediately get the following.

Corollary 1: At any NE, for all user i, we have

$$\sum_{j\in\mathcal{N}(i)} f^{i,j,l} = \sum_{j\in\mathcal{N}\setminus\{i\}} \boldsymbol{a}^{j,l}\boldsymbol{y}^j, \quad \forall l \in \mathcal{L}_i, \tag{24}$$

$$\sum_{j\in\mathcal{N}(i)} f_t^{i,j} = \sum_{j\in\mathcal{N}\setminus\{i\}} y_t^j, \quad \forall t. \tag{25}$$

This corollary plays a similar role as Lemma 1. With this corollary, the properties in Lemmas 2 and 3 can be reproduced in the distributed mechanism. We then obtain the following theorem.

Theorem 5: For the mechanism-induced game $\mathcal{G}$, NE exists. Furthermore, any NE of game $\mathcal{G}$ induces the optimal allocation.

Proof: By substituting (24), (25) in (18), we obtain exactly the same form of the tax function in centralized mechanism on equilibrium, which yields the desirable results as we show in Lemmas 2, 3. Then we can conclude that any NE induces the optimal allocation. The existence of NE can be proved by a construction similar to that of Theorem 2. ∎

In the distributed case, the planner may also have concerns about whether the users have an incentive to participate, and whether the mechanism requires an external source of money to maintain the balance. The good news is, Theorems 3 and 4 still hold here. As a result, users would rather join the mechanism-induced game, and the mechanism results in a balanced budget. The proofs and the construction of the subsidies follow those for the centralized case and are therefore omitted.

VI. Conclusions

In this paper, we propose two indirect mechanisms to address the demand management problem in energy communities with private utilities. Such mechanisms can be applied to any market with a similar structure. The proposed mechanisms possess desirable properties including full implementation, individual rationality, and budget balance.

It is worth noting that although NEs induced by the mechanisms lead the allocation to the optimal solution, how users get to *learn* an equilibrium still remains a problem. Without the information of others' utilities, one user is not able to evaluate the equilibrium point offline. Therefore, designing an online learning algorithm is one of the future research directions.

Furthermore, some questions relating to generalizations of the proposed mechanism are of interest. For example, whether it is possible to extend the mechanism to more general environments, such as an environment with a general convex feasible set or utilities with fewer restrictions, and whether there exists a strategy-proof modification of the mechanism, are open problems for future research.

Appendix

A. Proof of Lemma 2

Proof: Here we only present the proof for (8) and (13). The details can be found in Appendix B in [17].

At NE m^*, for $l \in \mathcal{L}$, consider the message components $q^{i,l}$ for each user i. In user i's tax function, denote the part relative to $q^{i,l}$ by $\hat{t}_q^{i,l}$. We have

$$\hat{t}_q^{i,l}(m^i, m^{-i*}) = (q^{i,l} - q^{-i,l*})^2 + q^{i,l} \underbrace{\left(b^l - \sum_j \boldsymbol{a}^{j,l}\boldsymbol{y}^{j*}\right)}_{\text{denoted by } e^l(\boldsymbol{y}^*)}$$

$$(\boldsymbol{\beta}^{i-1} = \boldsymbol{y}^i \text{ by Lemma 1})$$

For any user i, if we fix m^{-i*} and all the message components of m^{i*} except $q^{i,l}$, a necessary condition for NE is that user i cannot find a better response than $q^{i,l*}$.

Consider the best response of $q^{i,l}$ for different $e^l(\boldsymbol{y}^*)$.

Case 1. $e^l(\boldsymbol{y}^*) > 0$, i.e., the constraint l is inactive at NE. Note that $\hat{t}_q^{i,l}$ is a quadratic function of $q^{i,l}$:

$$\hat{t}_q^{i,l} = (q^{i,l})^2 - (2q^{-i,l*} - e^l(\boldsymbol{y}^*))q^{i,l} + (q^{-i,l*})^2.$$

Therefore, the best response is

$$q^{i,l*} = (q^{-i,l*} - e^l(\boldsymbol{y}^*)/2)^+.$$

Observe that $(q^{-i,l*} - e^l(\boldsymbol{y}^*)/2)^+ \leq (q^{-i,l*})^+ = q^{-i,l*}$. Equality holds only if $q^{-i,l*} \leq e^l(\boldsymbol{y}^*)/2$ and $q^{-i,l*} = 0$. Thus, for all i, $q^{i,l*} \leq q^{-i,l*}$, equality holds only if $q^{i,l*} = 0$ and $q^{-i,l*} = 0$. In other words, if for one user i we have $q^{i,l*} = q^{-i,l*}$, then all the $q^{i,l*} = 0$.

Notice that $q^{i,l*} < q^{-i,l*}$ implies $q^{i,l}$ is smaller than one of the $q^{j,l}$ among user $j \neq i$, which means $q^{i,l}$ is not the largest. Assume that $q^{i,l*} < q^{-i,l*}$ for all i, then no $q^{i,l}$ can be the largest among $\{q^{i,l}\}_{i\in\mathcal{N}}$, but $\{q^{i,l}\}_{i\in\mathcal{N}}$ is a finite set and therefore it must have a maximum. Here comes the contradiction. As a result, there must exist at least one i, such that $q^{i,l*} = q^{-i,l*}$, which implies that all the $q^{i,l*} = 0$.

Case 2. $e^l(\boldsymbol{y}^*) = 0$, i.e., the constraint l is active at NE. In this case, $\hat{t}_q^{i,l} = (q^{i,l} - q^{-i,l*})^2$, so every user's best response is to make her own price align with the average of the others. Consequently, $q^{i,l*} = q^{j,l*}$ for all $i, j \in \mathcal{N}$.

Case 3. $e^l(\boldsymbol{y}^*) < 0$, i.e., the constraint l is violated at NE. In this case,

$$\hat{t}_q^{i,l} = (q^{i,l})^2 - (2q^{-i,l*} - e^l(\boldsymbol{y}^*))q^{i,l} + (q^{-i,l*})^2,$$

which leads to a condition for all users i as

$$q^{i,l*} = q^{-i,l*} + (-e^l(\boldsymbol{y}^*)/2) > q^{-i,l*}.$$

However, if this condition is true for all users, it means there is no smallest $q^{i,l}$ among $(q^{i,l})_{i=1}^N$, which is impossible. Therefore, Case 3 won't happen at NE.

In summary, at NE, we always have $e^l(\boldsymbol{y}^*) \geq 0$, and $q^{i,l}$'s are equal. Moreover, $q^{i,l*}e^l(\boldsymbol{y}^*) = 0$. These prove the primal feasibility, equal prices and complementary slackness on prices q in the Lemma 2. ∎

B. Proof of Theorem 3

Proof: For any user i, if she chooses to participate with other users when everyone anticipates the NE, user i's payoff is of the form (15) if she only considers modifying $\boldsymbol{y}^i$ and keeps other components unchanged. Thus, user i is facing the following optimization problem

$$\boldsymbol{y}^i = \arg\max_{\boldsymbol{y}^i \in \mathbb{R}^T} \quad \Bigg\{ v^i(\boldsymbol{y}^i) - \sum_{t=1}^{T} \left(p_t + \mathcal{RP}_t^i(\boldsymbol{s}, \boldsymbol{\zeta}^{-i})\right) y_t^i - \sum_{l \in \mathcal{L}_i} q^{-i,l} \sum_{t=1}^{T} a_t^{i,l} y_t^i \Bigg\}.$$

By the definition of NE, $\boldsymbol{y}^{i*}$ is one of the best solutions, which yields a payoff $u^i(m^*)$. User i can also choose $\tilde{\boldsymbol{y}}^i = \boldsymbol{0}$. Denote the corresponding message by $\tilde{m}^i$. Then, the payoff value becomes $u^i(\tilde{m}^i, m^{-i*}) = v^i(\boldsymbol{0})$, which coincides with the payoff for not to participate. Since m^{i*} is the best response to m^{-i*}, we have $u^i(m^*) \geq u^i(\tilde{m}^i, m^{-i*}) = v^i(\boldsymbol{0})$. In other words, if every one anticipates the NE as the outcome, to participate is at least no worse than not to participate. ■

C. Proof of Theorem 4

Proof: Suppose the optimal solution for the original problem given by NE is $(\boldsymbol{x}^*, \boldsymbol{\lambda}^*, \boldsymbol{\mu}^*)$, then

$$\hat{t}^i(m^*) - J(\hat{\boldsymbol{x}}_t^i(m^*)) = \sum_{t=1}^{T} (p_t + \mu_t^*) x_t^{i*} + \sum_{l \in \mathcal{L}_i} \lambda^{l*} \sum_{t=1}^{T} a_t^{i,l} x_t^{i*} - J(\boldsymbol{x}^{i*}).$$

The total amount of tax is

$$\begin{aligned} &\sum_{i \in \mathcal{N}} \hat{t}^i(m^*) - J(\boldsymbol{x}^{i*}) \\ &= \sum_{i \in \mathcal{N}} \sum_{t=1}^{T} (p_t + \mu_t^*) x_t^{i*} + \sum_{i \in \mathcal{N}} \sum_{l \in \mathcal{L}_i} \lambda^{l*} \sum_{t=1}^{T} a_t^{i,l} x_t^{i*} - J(\boldsymbol{x}^{i*}) \\ &= \sum_{l \in \mathcal{L}} \lambda^{l*} \sum_{t=1}^{T} \sum_{i \in \mathcal{N}} a_t^{i,l} x_t^{i*}. \end{aligned}$$

For each constraint l, by the complementary slackness, we have

$$\lambda^{l*} \left(b^l - \sum_{t=1}^{T} \sum_{i \in \mathcal{N}} a_t^{i,l} x_t^{i*} \right) = 0.$$

Therefore,

$$\sum_{i \in \mathcal{N}} \hat{t}^i(m^*) - J(\boldsymbol{x}^{i*}) = \sum_{l \in \mathcal{L}} \lambda^{l*} b^l \geq 0,$$

which shows that at NE, the planner's payoff is nonnegative.

Furthermore, to save unnecessary expenses on the planner, the energy community can adopt the mechanism with the following tax function $\tilde{t}^i(m)$ instead

$$\tilde{t}^i(m) = \hat{t}^i(m) - \sum_{l \in \mathcal{L}} q^{-i,l} b^l / N.$$

Note that user i has no control on the additional term because no components of m^i are in that term, and thus the additional term won't change NE. Since the prices are equal at NE, so the planner gives $\sum_{l \in \mathcal{L}} \lambda^{l*} b^l$ back to the users. Hence,

$$\sum_{i \in \mathcal{N}} \tilde{t}^i(m^*) - J(\boldsymbol{x}^{i*}) = 0,$$

As a side comment, the choice of $\tilde{t}^i(m)$ is not unique. Any adjustment works here as long as it does not depend on m^i for each $t^i(\cdot)$, and sums up to $\sum_{l \in \mathcal{L}} \lambda^{l*} b^l$ at NE. ■

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