

Oscillator-based Dynamical Computing Platforms to Solve Combinatorial Optimization

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Abstract

Synchronized oscillator networks provide a promising pathway towards realizing application specific analog platforms capable of solving computationally intractable problems in combinatorial optimization. Specifically, electronic oscillators offer a low-power and integrated circuit (IC)-compatible hardware solution for designing such systems. In this work, we will present the opportunities as well as the challenges of realizing oscillator-based Ising machines to solve NP-hard problems like the Maximum Cut (Max-Cut) of a graph. Using experiments and simulations, we will discuss the performance and the scalability trade-offs that need to be considered in designing such platforms. Finally, we will showcase the opportunities in novel materials and beyond-CMOS devices that can be leveraged to augment the hardware implementation and performance of such systems.

(Keywords: oscillators, synchronization, optimization, analog computing)

I. Introduction

Despite the tremendous strides achieved across the entire digital hardware-software ecosystem, certain computational problems are still considered challenging to solve efficiently using digital machines. For instance, combinatorial problems such as computing the Maximum Cut (Max-Cut) of a graph – the process of dividing the nodes of a graph into two sets such that the number of common edges is as large as possible – typically entails an exponential increase in the required computational resources with increasing problem size [1]. Consequently, calculating the optimal solutions to problems of even moderate size can become challenging. Furthermore, while heuristic algorithms can provide some degree of speed up, their performance (i.e., convergence time, solution quality) can be sensitive to the characteristics of the input problem, and hence, is highly application-specific [2]. This has naturally motivated the exploration of alternate computing paradigms for solving such problems. In fact, an important application of a field like quantum

computing is to accelerate such problems with the goal of overcoming the fundamental complexity of the problem [3,4]. While such an approach, if successful, may possibly provide an exponential speed, it is obvious that realizing such quantum computing platforms will require cryogenic temperatures [5]. Consequently, the use of such platforms will likely be specific to large cloud-compatible/server applications that can afford the energy resources required for cryogenic cooling. However, in certain applications where energy is constrained/scarcely, implementing such a quantum platform may not be feasible. Subsequently, this has encouraged the exploration of non-Von Neumann systems and non-Boolean models which may be more suitable for solving such problems in the above scenario. The underlying premise here is that while such classical systems may not overcome the theoretical hardness of the problem, the system could still offer better performance and higher energy efficiency during runtime for a large number of problems. Moreover, such implementations do not require cryogenic cooling and can be implemented using highly scalable CMOS technologies.

One such classical non-Von Neumann approach is based on using dynamical systems – the focus of the present research. Since combinatorial optimization problems entail the minimization (maximization) of an objective function in the discrete domain, the underlying idea here is to design a physical system such that the objective function can be mapped to a physical property such as energy (function) of the system. Consequently, as the system evolves to minimize its energy, it also solves the objective function representing the combinatorial problem [6,7]. Synchronized oscillator-based dynamical systems, in principle, can offer a promising option for designing such a platform since the devices can be made low-power, compact, and can be realized using mature CMOS-compatible technologies [8,9].

II. Oscillator Ising Machines

In this invited paper, we will discuss the opportunities and challenges of realizing such

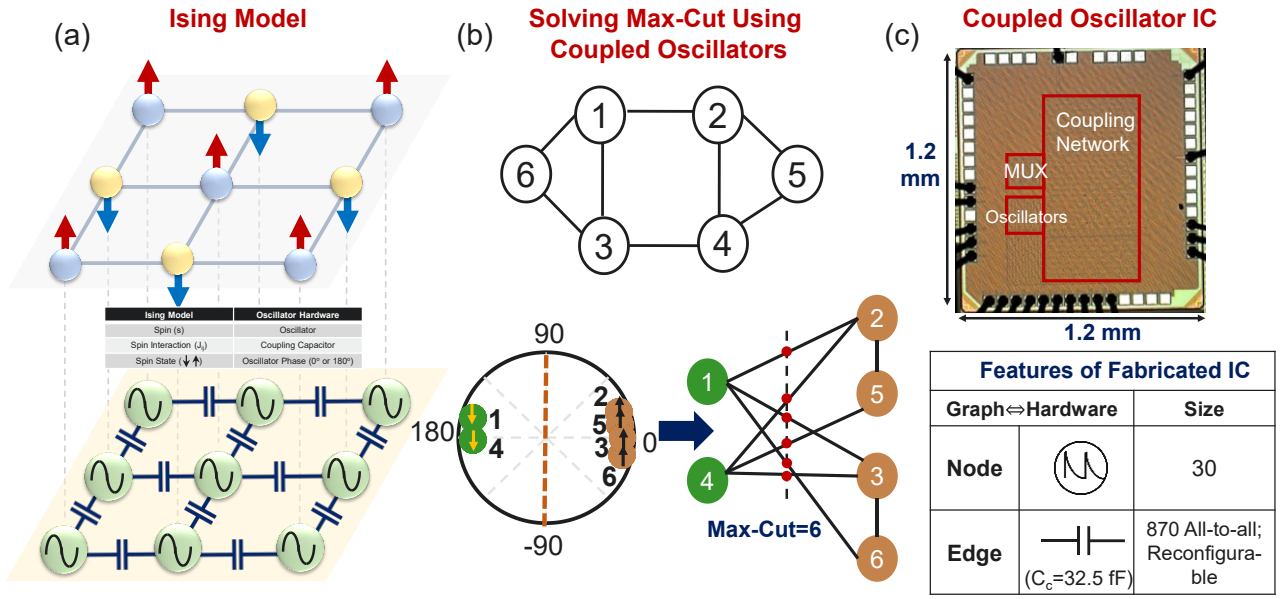


Fig. 1. (a) Ising Hamiltonian can be minimized using a topologically equivalent network of coupled oscillators under second harmonic injection. (b) Illustration of coupled oscillators as Ising machines. The oscillators, under second harmonic injection, converge to a phase of 0 and π , which represent the two sets created by the Max-Cut; the Max-Cut (for an unweighted graph) can be calculated by counting the number of common edges among the two sets. (c) Die photograph of a coupled oscillator fabricated by Mallick et. al [10]. The IC consists of 30 oscillators wherein each oscillator can be coupled to each and every other oscillator in a programmable manner.

oscillator-based dynamical compute platforms to solve computationally intractable problems in combinatorial optimization. Specifically, we will focus on the ability of a synchronized oscillator network to minimize the Ising Hamiltonian which is given by, $H = -\sum_{i,j}^N J_{ij} s_i s_j$ where N represents the number of nodes in a particular problem set, J_{ij} represents the weight values interconnecting the nodes, $s = [s_1 \dots s_n]$ represents the solution space where s_i can take the value of either +1 (spin $\uparrow$) or -1 (spin $\downarrow$). It can be shown that the oscillators in a topologically equivalent (oscillator $\equiv$ spin (s); coupling element $\equiv$ spin interaction (J_{ij})) coupled network, under the influence of a second harmonic injection, partition into two set of phases (0, π) in a way that minimizes the H (Fig. 1a-b). Subsequently, this capability can be directly applied to solving problems such as the Max-Cut.

While oscillator-based analog platforms are being investigated for their promise of delivering a significant speedup, important questions about the scalability of the hardware and the performance, the performance vs. accuracy trade-offs as well as the role of variability and noise still remain to be answered. One well known challenge of realizing such platforms is the square law dependence of the

coupling network (applicable when realizing a fully programmable all-to-all coupling network) on the system size which significantly impacts the feasibility of larger systems. One way to overcome this challenge is to design sparser connectivity among the oscillators. However, this in turn can increase the pre-processing required in order to map the input graph on the network. Additionally, the need for external injection of engineered signals to realize the desired system dynamics (e.g., second harmonic signal injection required in oscillator Ising machines) can also create an energy and area overhead. There is a strong performance trade-off between the quality of the solution measured and the computation time; annealing the system for a longer time can improve the solution quality but entails a penalty in the time-to-compute, and subsequently, the throughput of the platform.

We will present our group's efforts in addressing these questions through the development of CMOS oscillator-based integrated circuits (ICs) at different scales. Figure 1-c shows an illustrative example of a 30 oscillator-based integrated circuit (fabricated using 65nm bulk CMOS technology) with a programmable all-to-all coupling that was used to demonstrate an oscillator-based Ising machine to

solve the Max-Cut problem [10]. We will also present recent results on scaled oscillator systems [11].

Beyond CMOS-based implementations, emerging materials and devices, with novel functionalities can also help augment the implementation and performance of such platforms. For instance, the natural instabilities in materials such as insulator-metal transition oxides (VO_2 , NbO_2) [12-13], highly scalable ferroelectrics ($\text{Hf}_{1-x}\text{Zr}_x\text{O}_2$), spin transfer torque MRAM (STTM-RAM) can facilitate extremely compact and energy efficient oscillator architectures.

III. Conclusion

In summary, while coupled electronic oscillator-based dynamical systems provide a promising option to accelerate combinatorial optimization problems, the eventual feasibility and success of this paradigm will depend on the ability to implement critical scalability challenges associated with the implementation and engineer crucial design and performance trade-offs.

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