Teaching Series and Parallel Connections

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Abstract*—*Contribution: **A new operational definition of series connections is given based on elements belonging to the same two meshes, which is properly dual to the usual definition of parallel elements being connected to the same two nodes. Furthermore, computer-based exercises have been developed and tested to teach students about such connections in gateway linear circuits courses, using color coding of nodes and meshes as a pedagogical device.**

Background: **Series and parallel connections are a crucial but difficult concept. Existing textbooks give them limited attention, resulting in later difficulties learning circuit analysis.**

Research Questions: **RQ1: Can an improved definition of series elements aid student understanding and student satisfaction? RQ2: Can a computer-based "game" lead to effective mastery and student satisfaction at a wide range of institutions, including minority-serving ones?**

Methodology: **Standard and new definitions were elaborated in a multiple-choice tutorial. A game was developed focusing on identifying series and parallel connections, with color coding of both nodes and meshes. Student learning was assessed over eight years using pretest and posttest in 14 varied institutions. Student opinions were assessed using several types of surveys.**

Findings: **Strong learning gains were observed every semester from built-in pretest and posttest, with average scores of 28% and 87%, respectively. Large improvements were observed at every institution including five minority-serving ones. The posttest score is increased by a statistically significant amount after introducing the new definition of series elements. Students preferred the new definition of series and recommended its use, and very strongly endorsed color coding.**

***Index Terms*—Circuit topology, color coding, computer-aided instruction, conceptual learning, linear circuit analysis, parallel connections, series connections.**

I. INTRODUCTION

LINEAR circuit analysis is a key gateway course in electrical engineering and is often required for other engineering majors as well. Success rates are often undesirably low, which can lead to overall failure in an engineering program. The methods to improve instruction in such courses are of great interest [1]. The pervasive difficulties students experience in general with learning the basic properties of electricity and electrical circuits, which have been extensively documented [2]–[5], are likely to be a significant factor. Even students passing such courses often show poor understanding of qualitative circuit concepts [6], which are needed for engineers to develop sound intuition and expertise in this area [7], [8]. The specific focus on conceptual issues can, however, improve those outcomes [6].

A basic concept that underlies a great deal of circuit analysis is that of series and parallel connections. In the first author's experience, students make many errors in advanced topics, such as deriving Thévenin and Norton equivalent circuits due to misunderstanding of these basic ideas. Prior studies and data presented here show that many students entering a first course in circuit analysis lack a sound understanding of these concepts. A computer-based interactive tutorial and randomly generated examples and exercises were therefore developed to promote effective learning of these topics, and several versions were tested in classes from Fall 2012 through Fall 2020 [9]–[13]. The exercises can color code nodes as a pedagogical aid (mainly to help identify parallel elements, especially those that are nonadjacent). An improved operational definition of series elements was developed in Spring 2020 and incorporated into the software and tested in Spring and Fall 2020. That version adds the ability to color code meshes to help identify series elements (especially those that are nonadjacent). Student learning was assessed in controlled experiments and surveys were used to assess student opinions and motivation.

The main research questions addressed here are: RQ1: Can an improved operational definition of series elements that is properly dual to the definition of parallel elements aid student learning and satisfaction? and RQ2: Can a computer-based interactive tutorial and "game" (examples and exercises) lead to student mastery of this topic based on posttests and student satisfaction at a wide range of institutions, including minority-serving ones? In the following, prior work is discussed, a new operational definition of series elements is developed and justified, the software used in this study is described along with the assessment results and conclusions regarding the research questions, and directions for future work are outlined. All reported data are from Circuits I (EEE 202) at Arizona State University (ASU) or from similar courses at other institutions.

II. BACKGROUND

Several studies described difficulties students have understanding series and parallel connections. Undergraduates who studied circuits only in high school did not recognize different graphical representations of the same circuit as being

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This work involved human subjects or animals in its research. The authors confirm that all human/animal subject research procedures and protocols are exempt from review board approval.

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equivalent, and many relied on geometrical interpretations rather than ones based on connections [4], [14]. They misidentified elements as being in series even when another element was connected to their common node, and misidentified resistors as being in parallel even when one of them was in series with a battery. Some students in calculus-based university physics classes did not understand that resistors in series have the same current and that resistors in parallel have the same voltage, and some even believed the reverse [15]. Only 37% of first-year electrical engineering students could correctly identify the elements in series in a simple five-element circuit [5], many ignoring the requirement that no other element be connected to the junctions between series elements.

It is clear from these studies and the data presented below that students have considerable difficulty understanding series and parallel connections. However, many textbooks give only brief and noninteractive discussions of basic series and parallel concepts, often based on definitions that do not include all relevant cases as discussed below. Whereas most books include examples of combining elements in series and parallel, and most do not discuss why elements that students might incorrectly think are in series or parallel are *not* actually so connected. Many books have minimal to no exercises directed at just identifying series and parallel sets [16]–[27].

Color coding of nodes and other items was used very effectively in preliminary work by the present authors [9]–[13] and independently in [28] and later in [29]. Existing textbooks generally only use different colors for different classes of circuit elements and/or for voltage and current labels, but do not color nodes, meshes, or equations at all [16]–[27], [30], [31].

Only a few prior exercises on identifying elements in series and parallel have been developed. An early intelligent tutor generated problems to combine resistors, inductors, and capacitors in series and parallel but was not formally assessed [32]. DePiero *et al.* [33] developed a Web-based software that offers problems identifying nodes and series/parallel and shorted elements in a variety of circuit topologies using true/false questions. The software colors connecting nodes in series sets and the two nodes of a parallel set but does not otherwise color nodes (they are numbered instead). It does not explain why answers are correct or incorrect (like the earliest version developed in this work) and has not been proven to improve learning. Most importantly, it cannot be configured as a required assignment, meaning that student usage will typically be very low in the authors' experience (~5%–10%). It will therefore have limited impact.

III. Defining Series and Parallel Connections

Clear definitions can promote learning. Here, the focus is on series and parallel *connections*, as distinct from series and parallel *circuits*. In the latter, which are often covered in high school physics and rather easy to identify, all elements are connected in a single loop or to the same pair of nodes.

The definitions given for series and parallel connections in many common introductory linear circuits and introductory electrical engineering textbooks [16]–[27], [30], [31] are surveyed in the following. Many define series connections by connectivity, usually saying that two elements must be exclusively connected to a single common node (i.e., no other element that can carry current is connected to that node) [16], [20], [21], [23], [24], [26], [27], [31]. They go on to state or deduce from Kirchhoff's current law (KCL) that such elements must have the same current. Generally, they further state or imply the necessary additional property of transitivity (never expressed using that term) in order to allow more than two elements to be in series; namely, that if A is in series with B and B is in series with C, then A is in series with C. Other sources start with the definition that elements are in series if they have the same current [17]–[19], [22] and usually go on to specify that this condition implies the connectivity requirement given above [17], [19], [22] (which is actually not true, as shown in the next paragraph).

The connectivity requirement is not inclusive as a definition and is not a consequence of having the same current. A simple counterexample is shown in Fig. 1(a). Applying KCL to a closed surface surrounding the two elements on either end of this circuit (shown as a dotted line for those on the left) shows that the 7 Ω and 3 Ω resistors must have the same current, because the charge entering that surface through the 7 Ω resistor must leave through the 3 Ω resistor. They are, therefore, be in series according to [17]–[19] and [22], but not according to the connectivity definitions in [16], [20], [21], [23]–[27], [30], and [31]. They have *no* nodes in common (nor are they both in series with another individual element). It is clearly possible to combine them into a single 10 Ω resistor (replacing either original element) without affecting any other branch current, voltage, or power. If a nonbranch voltage is to be found (i.e., a voltage that is not across any single circuit element or branch), such as that between the nodes at lower left and upper right, the resistors could not be combined without changing that value. However, such nonbranch sought voltages similarly prohibit combining resistors in a simple chain of elements, so it should not preclude their being in series.

The circuit in Fig. 1 has two subcircuits (one-ports) consisting of the 1 Ω and 2 A elements, and the 5 Ω and 4 A elements, respectively. These subcircuits can be said to be in series with both the 7 Ω and 3 Ω resistors, though series relationships are usually only defined for individual circuit elements. Much more complicated examples are easily constructed, with two or more elements separated by two intervening nontrivial subcircuits (one on each side of one element). The subcircuits can each contain an arbitrary number of elements in an arbitrarily complicated network, as long as they have two terminals.

Given that nearly all textbooks give definitions or statements that would deny that these resistors are in series (either purposefully or just because they never considered cases like Fig. 1), it seems important to clarify the purpose of the series concept to confirm if they should indeed be considered to be in series. The series idea is useful in the following applications.

1) Resistors, inductors, and capacitors (or any impedances in ac circuits or in circuits in the Laplace or Fourier domains) and independent voltage sources can be combined if they are in series (provided their individual

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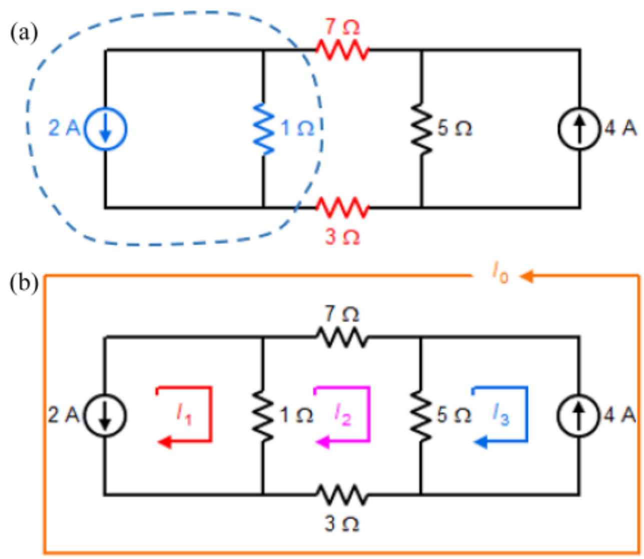

Fig. 1. (a) Circuit where the 7 Ω- and 3 Ω resistors highlighted in red must have the same current and are therefore in series, despite not satisfying conventional definitions of series in terms of connections to nodes. A closed surface is drawn around a subcircuit (colored blue) consisting of the two leftmost elements, to which KCL can be applied. (b) Same circuit, showing color-coded mesh currents for all four of its meshes, including the outer mesh. The 7 Ω and 3 Ω resistors carry the same two mesh currents (I_0 and I_2), confirming that they must have the same current $|I_0 - I_2|$.

voltages and powers are not sought, and provided no nonbranch sought voltages are defined involving intermediate nodes). Such simplification usefully reduces the number of nodes in nodal analysis, for example.

2) The current of any element in a series set can be determined from that of any other element in the set, which may be more readily found. For example, the current through a voltage source cannot be found directly but can be found from a resistance or impedance in series with it using Ohm's law.

3) Any circuit element or subcircuit in series with an ideal current source is redundant and can be replaced by a short circuit without affecting the remainder of the circuit, as long as there are no sought variables on the element or within the subcircuit and the voltage and power of the current source are not sought (desired) [6].

Each application above works for series elements separated by subcircuits as in Fig. 1. Application #3 shows that it can even be useful to know what subcircuits are in series with a single element. Thus, elements separated by subcircuits should indeed be considered to be in series. Another argument follows from the important unifying principle of duality for planar circuits [18], [23], [34]. The exact dual of the circuit in Fig. 1 is shown in Fig. 2, constructed as described in [18]. No one would deny that the 7 and 3 S conductances are in parallel because they are physically separated by a subcircuit, given that they have the same voltage and are connected to the same two nodes. As they are the duals of the 7 Ω and 3 Ω resistors in Fig. 1, it would be illogical to say that the latter are not in series, given that the duals of elements in series must be in parallel [18], [23].

To properly include cases such as Fig. 1, the following fundamental definition is proposed.

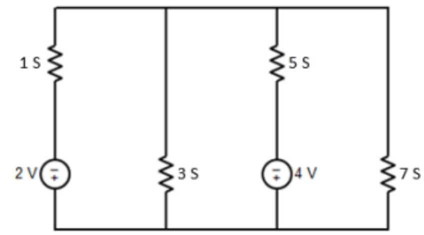

Fig. 2. Exact dual of the circuit in Fig. 1, whose node equations are identical to the mesh equations of the original circuit if mesh currents in Fig. 1 are changed to node voltages in this circuit.

A. Elements and Subcircuits Are in Series if They Must Have the Same Physical Current

Physical current means that the same electrons must pass successively through each series element or subcircuit (neglecting random motion). The current of a subcircuit is meant here to be current through either of its two terminals, not internal currents. The above definition is clear and simple and includes all relevant cases. It is not very useful as an operational definition (or "production rule"), however, because it does not prescribe how to identify series elements in a simple algorithmic way. Instead, the user must know how to apply KCL to determine if it is true. The node connection rule above is not satisfactory as it is not fully inclusive. Before giving an operational definition, prior definitions of parallel connections are reviewed.

Existing textbooks generally parallel elements as those having the same voltage [17]–[19], [22], or equivalently as those that are connected to the same pair of nodes [16], [20], [21], [23]–[25], [27], [30]. Curiously, however, most books defining elements in series as those that are connected to a common node (with no other conducting elements connected to that node) do *not* give the dual of that definition for parallel elements. The latter would be that parallel elements belong to the same mesh, and no other element belongs to that mesh, as stated in only one surveyed text [31]. This definition implies the 3 and 7 S elements are not in parallel in Fig. 2 because they have no common mesh, even though they obviously are. Making it inclusive requires including subcircuits and transitivity, exactly as is necessary to make the dual connectivity definition of series be inclusive.

A simpler definition [26] would be: "Elements are said to be connected in parallel when they form a loop containing no other elements." This approach obviates the need for subcircuits, but still needs transitivity for more than two elements. A valid dual statement would be that elements are connected in series when they form a cutset containing no other elements, given that cutsets are the duals of loops [35]. Transitivity is still needed. However, no source was found that states this definition, and it is more complicated than the one given as follows.

The fundamental definition of parallel elements is proposed to be that as follows.

B. Elements and Subcircuits Are in Parallel if They Must Have the Same Physical Voltage

This definition is suitably dual to the definition given above for series elements. It is useful to include subcircuits because they can be replaced by open circuits if they are parallel to a voltage source (redundant) without affecting the remainder of the circuit, as long as there are no sought variables on the element or within the subcircuit and the current and power of the voltage source are not sought [6]. (This application of the parallel concept is dual to #3 above for the series concept.) As before, the voltage of a subcircuit is that across its terminals, not an interior voltage.

This definition is however not an operational one, in that it does not prescribe how to easily identify elements in parallel. An operational version can be stated as follows.

C. Elements Are in Parallel if and Only if They Are Connected to the Same Pair of Nodes

Just as is done in nearly all the cited textbooks. It follows from the fundamental (same voltage) definition because each branch voltage is the difference of the voltages of the nodes to which the element is connected. This definition is very simple, fully inclusive and exclusive, and very easy to apply either by inspection of a circuit diagram or algorithmically by the use of a netlist (or nodelist, as it might be termed). The latter, as used in SPICE and other circuit analysis programs, lists each element by type, value, and polarity (where appropriate) along with the two nodes to which it is connected. Such a list is easily scanned to identify all elements in parallel. There is no need to invoke transitivity or to identify subcircuits.

Given the beauty (and wide usage) of the above operational definition, it seems remarkable that the dual of this definition has never (to the knowledge of the authors) been stated for series elements. A new operational definition is therefore proposed as follows.

D. Elements Are in Series if and Only if They Belong to the Same Pair of Meshes

(For nonplanar circuits, this definition is easily generalized to elements that are part of the same set of fundamental loops.) Its validity follows directly because the current of each branch in this set is given by the difference of the same two mesh currents. The visual application of this definition is easy using labeled mesh currents as in Fig. 1(b). There, the 7 Ω and 3 Ω resistors both have mesh currents I_0 and I_2. The algorithmic application is easy by scanning a *meshlist* (analogous to the nodelist defined above but listing the meshes containing each element rather than the connected nodes). Again, there is no need for transitivity or finding subcircuits. Finding the "correct" subcircuits to connect individual elements in series requires very complex procedures, as the first author can attest after writing code to do so in Circuit Tutor. The complexity arises because subcircuits can contain smaller nontrivial subcircuits and can also overlap. The code to find all series elements using a meshlist is very much shorter and simpler than code using subcircuits and the node connection rule to find series elements.

It is unclear why this definition is not already in general use, but it might be due to the almost universally used definition of a mesh in modern texts as a "loop that does not enclose any smaller loops." It was argued that the definition of a mesh should include the outer mesh on the same basis as interior meshes for logical consistency [36], given that any planar circuit can equivalently be drawn on the surface of a sphere [37], where there is no distinction between the two. Also, any planar circuit can be redrawn on a plane with any mesh as the outer mesh [36], [37]. Furthermore, constructing the geometric dual of a circuit requires treating the outside of a circuit as a mesh where a node of the new circuit must be placed, as is done for the interior meshes [18]. Thus, a mesh can be defined as *a loop that does not enclose any smaller loops, or that is not enclosed by or a portion of any larger loop in a planar circuit* [36].

It might appear that the new definition of series elements does not include the case of a single-loop circuit. In fact it does, because even in such a circuit the "interior" and "exterior" of the one loop are properly regarded as meshes, following Guillemin's suggestion that a mesh should really be regarded as a region of the plane surrounded by a loop of circuit elements rather than as the loop itself [38]. Thus, the simplest complete circuit is a single mesh-pair circuit, which is the dual of a single node-pair circuit. The very method used to construct the geometric dual of a circuit requires that both the inner and outer mesh be mapped to different nodes [18].

The new definition of series does not require abandoning the common existing statement about elements in a chain being in series. It is a sufficient but not necessary condition, which can be generalized to include cases like Fig. 1 using subcircuits. However, this more complicated approach is not likely to be nearly as useful in practice as the simple statement about common meshes. Once students learn the mesh definition, it may become natural to use that idea for all series connections.

IV. Series-Parallel Recognition Exercises

As part of a step-based tutoring system known as Circuit Tutor, a specific tutorial and "game" have been developed to teach students about series and parallel connections [9]–[13]. It presents randomly generated, fully connected circuit diagrams to students and asks them to identify elements that are in series or parallel sets, as illustrated in Fig. 3. Each student gets unique circuit topologies each time so that copying others' answers is not possible. Problems are presented at each of four progressive levels of difficulty and complexity, starting from four elements laid out on a grid with 2 × 2 squares with one series and one parallel set on the easy level, up to 15 elements on a 4 × 4 grid with four to five sets to identify on the mastery level. One problem always involves exactly two elements that are both in series and in parallel (the only nonhinged case where this is possible); this topic is covered explicitly in the tutorial. Exactly isomorphic, fully solved examples are available in unlimited quantity on each level, where each set is highlighted red in turn to help visualize them easily. The level of difficulty is carefully graded by providing hints like coloring of nodes (and now meshes) and information on the number of sets of each type on the lower levels, which are gradually phased out on the harder levels.

In the exercises, students are informed immediately via textual and auditory feedback (a beep) if they enter an invalid or

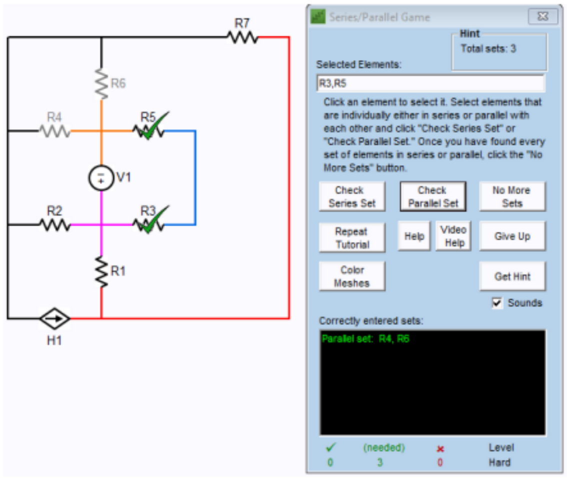

Fig. 3. Screen shot of a "hard" problem in the series parallel game, where the student has correctly identified one parallel set (now greyed out) and has checked a series set in preparation to check it. Node coloring was turned on.

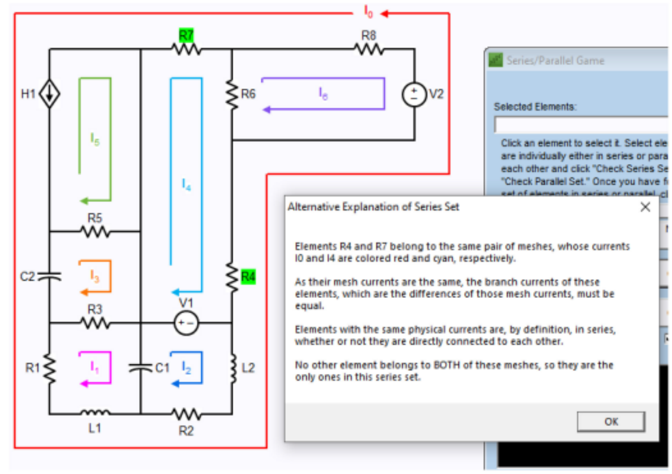

Fig. 4. Screen shot of a "mastery" problem in the series parallel game after giving up, where an explanation of the series set consisting of R4 and R7 (highlighted in green) is being explained with the aid of colored mesh currents.

incomplete set, and receive an immediate, detailed explanation of any error when in the learning mode. Students can "give up" on any problem at any time for no penalty, whereupon they are shown the answers using color coding to help understand them. Grading is based solely on completion of the required number of problems (three at each of four levels), though students can start on any level and still earn full credit.

In the initial version (used in 2012–2013), a student could still get credit for a problem after any number of incorrect answers, and right and wrong answers were not explained beyond giving the correct ones. The analysis of log entries showed that the percentage of correct identifications of series and parallel sets (averaged over student scores) declined monotonically from difficulty level 1 to level 3, and did not rise above the initial values on level 4 [11]. Only 51% of sets identified (all levels combined) were valid. The game was, therefore, revised to limit students to two incorrect answers per problem to receive credit (discouraging guessing), and to provide detailed explanations of the correct solutions and of any wrong answers they had given at the end of a problem and when viewing examples. The result was a substantial increase in accuracy and improved trends as the difficulty level increased [11]. From Fall 2014 to Fall 2020 (after these changes), 75% of identified sets were valid.

The students click on elements to identify them as series or parallel (automatically placing a check mark on selected elements). Elements used in a prior set are greyed out to minimize extraneous cognitive load. Students may complete a problem for no credit when they exceed the allowable errors. A pretest and posttest consisting of fixed problems at levels 1 (easy) and 3 (hard) (with no aids or immediate feedback available) is administered automatically. Congratulatory sounds are played when students get right answers or complete a level to build confidence. A help video is available on YouTube [39] to both demonstrate the operation of the interface and to work example problems; it has been viewed 470 times since Fall 2019. It is available from a button in the software, in addition to written help. The system now automatically generates and stores transcripts of all student work on the exercises as PDF files, showing both correct and incorrect answers they entered, for use when reviewing or studying for exams.

Administrative features include an instructor dashboard that graphs student progress, pretest and posttest scores, accuracy, time on task, and usage of hints and examples. A downloadable gradebook with detailed per-student views is available to instructors at www.circuittutor.com with an instructor manual and other features. The software runs on Windows but is also available in virtualized form in a Web browser, using Citrix Workspace software on university servers [40].

The sequence now followed by students is to take a pretest with no preparation, view and complete a multiple choice, interactive tutorial that introduces the concepts (which can be used in place of a traditional textbook), optionally view examples at each level, complete the required number of problems at each level, take a post-test like the pretest, complete a brief survey, and receive a certificate of completion. The introductory tutorial and written or video help can be accessed or repeated at any time without losing progress, and students can spread their work over multiple sessions.

The introductory tutorial was revised in late Spring 2020 to incorporate the new operational definition of series elements in terms of common meshes. The ability to show colored mesh current arrows for all meshes [Figs. 1(b) and 4] was added, as was the automatic coloring of a subcircuit connecting individual series elements when node connection-based explanations are being given. It now explains series connections using both the new and traditional approaches, using subcircuits when needed in the latter. The prior tutorial used the conventional (incomplete) definition of series elements based on connections to nodes.

V. Effect on Student Learning in Various Institutions

In the previously reported work, the effectiveness of Circuit Tutor compared to paper homework was evaluated in a laboratory-based experiment at ASU involving 33 paid student volunteers who had already covered series-parallel concepts in Circuits I within the prior year [9], [10], and in a classroom-based experiment at the University of Notre Dame involving sophomore students enrolled in Introduction to Electrical Engineering (EE 20224) [13]. In the former controlled, randomized experiment in December 2012, average scores on the relevant portion of the pretest increased from 71% to 91% on the posttest for students assigned to use the software for 25 min, but decreased from 71% to 68% for students assigned to work textbook [16] problems involving combining series and parallel elements (since no qualitative exercises were available in the book involving series and parallel elements). The difference was statistically significant with ($p = 0.0075$) using a two-tailed t-test with an effect size (Cohen d-value) of 0.92 σ comparing the two groups [9], [10]. Similarly, improved scores were found on a motivational survey [41].

In the Notre Dame study, one class section was assigned to complete the Series-Parallel game in Circuit Tutor along with the series-parallel with Terminals game described below. The other section was assigned to read a textbook [23] discussion of the topic and do a textbook assessment problem, and then do a paper exercise to identify series and parallel elements in 20 circuits taken from the book (as the book had no specific exercises to do that). Both groups completed pretest and posttest to assess learning. The software group had an adjusted mean posttest score of 36.7 (adjusted for the pretest scores as a covariate) compared to 30.5 for the textbook users, for a statistically significant ($p < 0.001$) effect size of $d = 0.97$ σ [13].

These prior studies implied an advantage for the software over paper exercises but had two limitations. The lab-based study at ASU was not conducted in an authentic classroom-based setting and its participants had received significant prior instruction on the topic, so were not typical of students encountering it for the first time in a circuits course. The Notre Dame study showed that the software worked well for students at a highly selective private university but did not establish its effectiveness at a wide range of institutions, including large public universities, minority-serving institutions, etc. Here, RQ2 (effectiveness at a wide range of institutions) is addressed by examining data from classroom settings from Fall 2014 to Fall 2020 on ~6700 distinct student users in 195 different class sections taught by ~60 distinct instructors in 14 different colleges and universities in the U.S. and Canada. The institutions included three large public universities, a large selective private university and two small private universities, five minority-serving institutions, and three community colleges. These students were usually sophomores (33%), juniors (45%), and seniors (20%); about 20% were electrical engineering majors and the remainder were mostly other engineering majors (mechanical, aerospace, biomedical, etc.)

To do so, a wide variety of data was collected via log entries stored on a central server while students used the system, including scores on the built-in pretest and posttest; time on task, accuracy, and completion answering the multiple-choice questions in the introductory tutorial; and use of examples, hints, node or mesh coloring (when not provided by default), instructions, and video help. Other recorded items

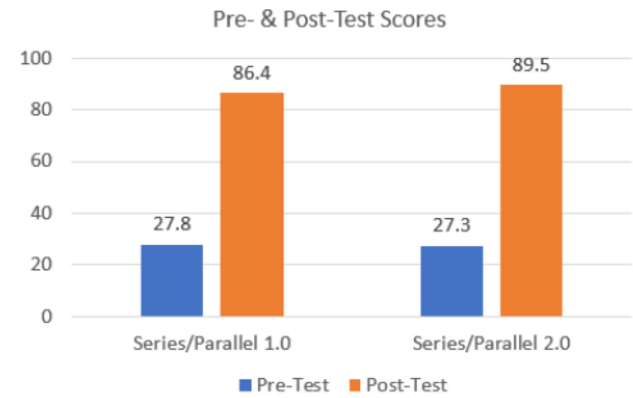

Fig. 5. Average pretest and posttest scores for series-parallel games using the conventional (v.1.0) and new (v.2.0) definitions of series elements.

included a count of exercise problems at various levels that are successfully completed or given up, and corresponding time on task; correct and incorrect answers entering (what students believe to be) series and parallel sets; a classification of errors entering those sets; and student responses to a brief two-question survey at the end of the game.

The first question is whether students can complete the exercises successfully. Students attempted a total of ~105 000 distinct problems, successfully completing (with two or fewer errors) ~83 000 (79%), giving up (voluntarily or involuntarily because of too many errors on a problem) ~18 000 times (17%), and aborting the rest. The percentages of proposed series and parallel sets that were correct were 77% and 73%, respectively, implying that series sets are usually a bit easier to identify than parallel ones. The most common error by far identifying parallel sets was proposing ones that were directly connected only on one end, not on both ends. The most common error by far for series sets was proposing ones that were properly connected in a chain or loop, but another element not in that set was connected to one of the internal nodes. Less common errors mainly included proposing series sets where two or more of those elements were actually in parallel, or where three or more of those elements formed a star (had a common node) and were not a valid chain. These data show high rates of completion.

Students generally completed the exercise as a whole, finishing an average of 3.83 of the four levels of difficulty. (This value was fairly consistent among institutions; averages of the institutional averages for those having at least 30 participants were 3.81/4 levels and the minimum value among those ten institutions was 3.63/4.) The students were allowed to start on any level, but 89% chose to complete all four levels. The average time to complete all four levels of exercises was about 32 min (median 26 min), and 90% completed them within 1 h, so most students learned the relevant concepts and skills quickly.

To measure overall learning, the built-in (untimed) pretest and posttest (where two similar forms of the test were given randomly either as pretest or as posttest) were used to assess student learning across multiple institutions. The overall average pretest score was 27.8%, rising to 86.4% on the posttest for the original version of the game (see Fig. 5). This large 59 pt. gain suggests that most students initially learned the material well (although longer term retention has not yet been studied). It also shows that students generally have a very poor prior understanding of the topic. Pretest scores varied by institution from 17% to 57% (presumably due to student characteristics and amount of prior instruction), and posttest scores varied from 72%–89% (considering only institutions with at least 30 scores). Nine of the ten institutions with sufficient scores had posttest averages of at least 78%.

To assess student satisfaction at the end of the game, users are asked to rate it in one of four categories, and for comments and suggestions. Ratings have varied little year to year and average 72.4% "very useful," 23.3% "somewhat useful," 2.7% "not very useful," and 1.7% "a waste of time." In the open-ended comments, color coding of nodes or meshes was mentioned (nearly always positively) about 199 times (in 1332 comments) and has been one of the most well-received features of Circuit Tutor. The word "fun" was used in 17 comments, and the congratulatory sounds were mentioned (usually favorably) 14 times.

VI. Impact of the New Mesh-Based Definition of Series Connections

In the original version (1.0) of the Series-Parallel game, used through Spring 2020, only the conventional connectivity-based definition of series elements was given in the introductory tutorial and implemented in the software. The randomly generated problems sometimes, however, unintentionally included problems having series elements that do not satisfy the conventional definition. The generation of many circuits using the algorithm in use at that time has shown that such problems occurred about 0% of the time on level 1, 1.5% of the time on level 2, 2.5% of the time on level 3, and 7.3% of the time on the larger, more complex circuits on level 4. Thus, at least 29% of the students completing the minimum three problems on each level (which was typical) would encounter such a problem. If students entered such sets as being in series, however, they would be erroneously informed that they were incorrect. It is very unlikely that students did so, as no one ever complained that the game was grading them incorrectly (which they often do when there are bugs) and they were never taught from any source that such sets of elements are in fact in series. As a result, students never learned that elements physically distant from one another and separated by subcircuits can be in series. Yet, they would regularly be taught and required to learn that elements far apart from each other and separated by intervening subcircuits can still be in parallel (as shown in a simple case for the 3 and 7 S resistors in Fig. 2, for example).

In version 2.0 of the series-parallel game, the improved common-mesh definition of series elements was introduced both in the introductory tutorial and in the game. The problem generation algorithm was changed to control the generation of problems having series elements separated by subcircuits, which is now done only on level 4 for every other problem (given that identifying distant series sets is more difficult than identifying those in a simple chain). The students completing level 4, therefore, now must complete at least one such

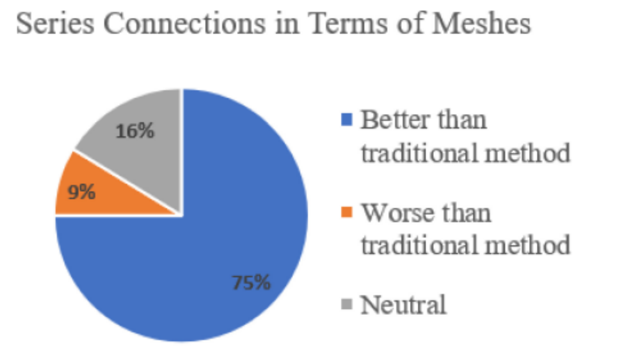

Fig. 6. Student opinions in Fall 2020 about how the new definition of series elements compares to the traditional one (students had experienced both).

problem successfully. Since most students can do so, the new version is significantly more effective in teaching the identification of all series elements than the old one. Furthermore, scores on the posttest are significantly higher with the new version than with the old one ($p < 0.00001$ in a two-tailed students' t-test), as shown in Fig. 5, with an effect size of Cohen $d = 0.14\ \sigma$, where σ is the standard deviation. This improvement is in addition to the improved ability to identify distant series sets, as that more advanced skill is not tested on the posttest.

To assess student satisfaction with the new version, three sections of students in Spring 2020 at ASU who had initially completed version 1.0 of the series-parallel game were allowed to complete the new version 2.0 for extra credit at the end of the semester. They were then surveyed on their comparative opinions of the two approaches, having experienced both. Out of 88 students, 72% strongly or somewhat agreed that the new mesh-based definition of series is better than the traditional one they learned originally, and 72% strongly or somewhat agreed that future students should use the new version of the series-parallel game incorporating that definition. (These percentages rose to 80% and 87%, respectively, for students in a section whose instructor discussed the new definition in class as well as in the game.)

In Fall 2020, all students used version 2.0 of the game as a required assignment and were asked to complete a survey at the end of the term. Of the 80 respondents, 75% (mainly from four class sections at two institutions) felt that the new approach to series connections in terms of meshes was much better than or somewhat better than the traditional definitions in their textbooks (Fig. 6). Only 9% felt it was much or somewhat worse (the remainder were neutral). A total of 29% felt that new students studying this idea should learn it using only the mesh-based definition; 51% felt they should use a combination of the two approaches [as is now done in Circuit Tutor (see Fig. 7)]. Only 10% felt they should use the traditional definition exclusively (the remainder felt it did not matter). There is, therefore, strong support among students to adopt the new definition while maintaining the chain-based definition as an alternative.

Open-ended student comments in Spring 2020 about these approaches were classified into themes (a given comment could be scored in more than one category). The most common

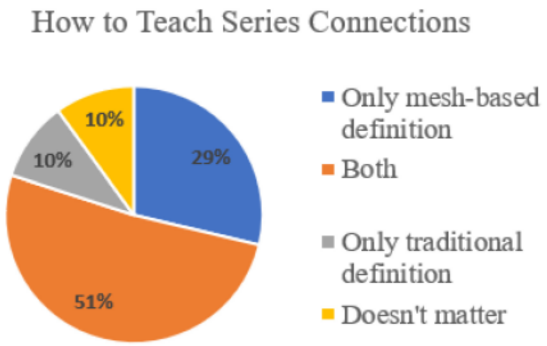

Fig. 7. Student opinions in Fall 2020 about how series connections should be taught in the future.

was that the new method leads to a more complete understanding and/or identifies series elements that the traditional method would miss (29 cases). A typical comment is "I liked the new version because it was a bit more challenging and I feel it would be better for really mastering the topic. The new explanation with meshes was really informative and made finding the more complex sets easier." Similarly, 29 students perceived the new method to be better than the old/traditional method for an array of other reasons, including the helpfulness of the early introduction to meshes (six cases), its more visual nature (six cases), being more user friendly in general (11 cases), and favoring its duality aspect (four cases). In total, 19 of the students perceived the old method to be better, some indicating that the early introduction to meshes was overwhelming (four cases), the new approach is too complicated (nine cases), or for miscellaneous reasons (six cases). A typical negative comment was "I prefer the old version because it seems to be complex to understand the concept with the several rules included." Seven students simultaneously found the new method to be more difficult and complex, yet also preferred it to the old method. The smaller number of comments in Fall 2020 was generally similar and mostly favorable. It seems that the new approach may generally be liked by average and stronger (most) students, but that students having more difficulty preferred a simpler approach.

VII. Analysis and Discussion

The first research question here (RQ1) is if an improved definition of series connections that is dual to the usual definition of parallel elements can improve student learning and satisfaction. All prior instructional approaches, including those used by the first two authors, failed to even recognize that possibly distant series elements separated by subcircuits are in series at all, even though similarly separated parallel elements have always been considered to be in parallel. It would not make sense to carry out a controlled study of student learning of that concept both with and without the instruction now provided in version 2.0 of the game, as students would almost certainly not understand a concept that has never been taught or explained to them. Instead, the very fact that nearly all students are now able to complete exercises where they are required to identify nonchain-connected elements demonstrates that learning of the subject has improved. The introductory tutorial is evidently very effective in conveying the related concepts, as several instructors using Circuit Tutor did not cover the novel approach in lecture. Furthermore, the pretest and posttest data show that students' overall ability to identify both series and parallel elements when using the new definition has improved by a statistically significant, even if modest amount. A large improvement is not really possible given that the posttest scores were already quite high with the original version (86.4%). Given the overall increase in posttest proficiency and the successful learning and application of a more complete and logical definition of series connections, it is concluded that student learning did improve significantly.

Regarding student satisfaction with the new definition, the high percentage of students who stated that the new definition is superior to the standard one (72% in Spring 2020 and 75% in Fall 2020) provides strong evidence of higher satisfaction. Furthermore, a full 72% of students in spring and 80% in fall favored using either only the new definition or a combination of the two, as now done in the software, providing further support. Qualitative open-ended comments also favored the new approach by a margin of 36 to 19. It is therefore concluded that the new approach aids both learning and satisfaction, answering the first research question in the affirmative.

The second research question (RQ2) asked if a step-based tutoring system on series and parallel connections could achieve both high mastery based on posttests and student satisfaction at a broad range of institutions including minority-serving ones. The results in Section V show that large gains in performance from the pretest (∼27%) to the posttest (∼86% for version 1.0 and ∼89% for version 2.0) can be achieved for a large number (∼6700) of students in authentic classroom settings at 14 different institutions of various types with many different instructors. Furthermore, these gains were broadly consistent across many institutions. Of the ten schools with at least 30 posttest scores (for a reasonable sample), 90% achieved posttest scores of 78% or better, averaging 85% across those schools (disregarding the number of students at each school). The number of completed levels in the game was also high, averaging 3.81/4 levels across the same institutions. Only 32 min of playing the game was required to achieve this performance, implying that learning with this tool is efficient.

Regarding student satisfaction with the game, almost 96% overall considered it somewhat or very useful for learning series and parallel connections. Analyzing by institution showed fairly consistent ratings, the lowest for any school with at least 30 ratings being 88% and the next lowest 94% somewhat or very useful. The color coding of nodes and mesh currents was especially well received in the open-ended comments. Overall, it is concluded that RQ2 involving the achievement of high levels of mastery and student satisfaction is answered positively.

One of the important outcomes of this work is a revision of standard definitions of series and parallel connections to be properly dual to each other. Combining this change with a novel approach to mesh analysis whereby the latter is made fully dual to nodal analysis for the first time [36] should allow teaching of circuit analysis in a way that emphasizes duality as a central organizing principle. Doing so can help provide a framework to help students organize their knowledge on

TABLE I
AVERAGE SCORES AND RATINGS AS VERY OR SOMEWHAT USEFUL

Game	Pre-Test	Post-Test	Useful	*N*
Series/Parallel 1.0	27.8	86.4	95.7%	3949
Series/ Parallel 2.0	26.7	89.5	95.1%	818
Series/Parallel with Terminals	59.7	80.5	90.7%	3957
Resistor Simplification	53.8	83.2	91.1%	3948
L/C Simplification	49.7	80.8	88.8%	3561
Impedance Simplification	42.7	84.4	87.7%	2677

this subject, much as is conventionally done using duality in Boolean algebra, for example. Further work is planned to develop and test instructional materials emphasizing this approach.

VIII. RELATED GAMES

In addition to the series-parallel game discussed above, Circuit Tutor includes several other related games. All have the same basic features. These include a series parallel with terminals game, in a similar format, but where terminals are present. The latter are shown connected either pictorially or schematically to an ideal voltmeter, an ohmmeter, or an arbitrary subcircuit (represented as a "blob" with terminals). The latter two allow current through the terminals and thereby, destroy any series relationship between elements connected via a node that has a terminal, which is the point of the exercise.

Three simplification games are also included. These involve simplifying networks of resistors, inductors or capacitors, or various types of ac impedances connected in complicated series-parallel networks. The circuits have a set of input terminals from which an equivalent quantity is viewed. Students click on elements to combine as in the series-parallel game, and then enter the combined value. The simplification step is then carried out automatically by the program, and the students continue until only a single equivalent element remains. The errors are trapped at each step and excessive errors result in forfeiting credit for a problem, though students can always work another of the same type instead. In the future, the authors would like to modify the interface to have students explicitly combine the elements themselves in an interactive circuit editor [42], [43], to be even more involved in the process. In the impedance simplification game, students must also compute impedance values in the phasor domain corresponding to element values in the time domain and carry out complex arithmetic operations to combine elements. Average pretest and posttest scores (for *all* students) and percentages rating the games very or somewhat useful are shown in Table I. The change in scores from pretest to posttest scores is statistically significant with $p = 0$ (to the accuracy of a floating point number) in all cases. A further extension to simplify voltage and current sources in series or in parallel (respectively) is planned.

IX. CONCLUSION

A computer-based "game" to identify series and parallel connections in circuits (and four related games including simplification exercises) was developed and tested with over 6700 students studying linear circuit analysis at 14 institutions of many different types including five minority-serving institutions. The game features an unlimited supply of randomly generated circuit topologies as problems and fully explained examples at four progressive levels of difficulty and provides detailed feedback on mistakes. A research question (RQ2) asking if such a system can produce high levels of mastery across a broad range of institutions is answered affirmatively, based on pretest and posttest scores averaging 27% and 90% (for the latest version), respectively, implying effective learning in an average total time of ~42 min. High student satisfaction is also achieved, with about 96% of users rating the software as very or somewhat useful for learning the topic.

The software now uses a novel, simple, and accurate operational definition of series elements as those belonging to the same pair of meshes in a planar circuit (including the outer mesh), which successfully identifies series elements not belonging to a conventional chain of individual elements. This definition is properly dual to the common definition of parallel elements as those connected to the same pair of nodes, unlike the conventional connection-based definition. A research question asking if the new definition leads to improved mastery and satisfaction (RQ1) is answered affirmatively, based on a statistically significant increase in posttest scores for the new version and successful completion of problems requiring students to identify series elements that are not directly connected in the new version. Students favor this new approach by a considerable margin in both quantitative and qualitative data. Based on student comments, it is further concluded that color coding of nodes and meshes can be a valuable pedagogical tool when teaching about series and parallel connections.

In future work, it would be desirable to add elements that are shorted or "dangling" (connected on only one end) to the problems to clarify those ideas. Longitudinal studies to measure long-term retention would be useful, and it is planned to introduce "desirable learning difficulties" such as spacing into the game to improve retention [44]. For example, students could be required to complete the game over an extended period of time, with each level being available at different times, with appropriate explanations of why that approach is being used. Circuit Tutor is free to students and available to any instructor for use in their courses (e-mail the first author for access).

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