Impacts of UC formulation tightening on computation of convex hull prices

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Abstract— In U.S. electricity markets, energy prices are traditionally determined by optimal dual variables corresponding to system demand constraints of the economic dispatch problem. With fixed unit commitment (UC) decisions, however, such prices cannot reflect actual marginal costs of serving load, and market participants might not want to follow commitment and dispatch decisions made by Independent System Operators. To address this, convex hull pricing provides a promising way to accurately calculate prices and reduce uplift payments in electricity markets. One way to find the convex hull prices is to take the dual of the UC problem and solve the resulting problem by Lagrangian relaxation. In this study, impacts of tightening unit formulations on prices from integer relaxation are investigated, where formulation tightening is combined with the surrogate Lagrangian relaxation method to calculate convex hull prices. The performance of the proposed model is compared with the conventional UC formulation and the effect of integer-relaxation is also studied. IEEE 118 bus system with 54 units is used for testing. Results demonstrate that the tightened formulation can produce accurate prices upon integer-relaxation for a smaller 3-hour example and approximate prices for a larger 24-hour example. This demonstrates the potential of formulation tightening on the calculation of convex hull prices.

Keywords—Unit commitment, convex hull pricing, formulation tightening, Surrogate Lagrangian Relaxation.

I. Introduction

In the U.S. electricity markets, Independent System Operators (ISOs) solve unit commitment (UC) and economic dispatch problems in their daily operations. Prices are traditionally determined by optimal dual variables for system demand constraints of the economic dispatch problem, with UC decision variables fixed at their optimal values [1]. With fixed UC, however, unit start-up and no-load costs are not captured, and prices cannot reflect actual marginal costs of serving load. These inaccuracies in pricing compel the ISOs to make out-of-pocket uplift payments to incentivize the generators to follow the dispatch. Such payments compromise the transparency of the market and discourage new competitors. One of the novel methods to overcome these challenges is the convex hull pricing (CHP) method introduced by Gribik et. al. [1] in 2007. The core idea of this approach is to convexify the nonconvex cost function using Fenchel convex conjugate and to form a convex envelope over the convex hull of the constraints. This method can reduce uplift payments and improves market transparency [1].

Several methods have been used to get CHPs, as reviewed in Section II. One type of approach to get CHPs is to develop a convex hull of the feasible set and the convex envelope of the objective function over the convex hull [2]–[8]. However, getting the exact convex hull and envelope is difficult due to the presence of a large number of binary decision variables. Formulation tightening provides a promising way to obtain the convex hull, since tight constraints directly delineate the convex hull of the feasible set [9]. In the literature, most tightened formulations were presented without providing a procedure. The other type of approach to get CHPs is to take the dual of the UC problem and solve the resulting problem using decomposition and coordination approaches like Lagrangian relaxation-based methods [10]–[12]. However, this approach involves the high computational effort, and multipliers may suffer from zigzagging, resulting in slow convergence.

In this paper, the impacts of formulation tightening on the computation of convex hull prices by using decomposition and coordination methods are investigated. In section III, a standard UC formulation is presented. The problem is solved by using our Surrogate Lagrangian Relaxation (SLR) [13] as described in Section IV. This method overcame all the major difficulties of standard Lagrangian relaxation. It does not require all sub-problems to be solved optimally. Therefore, computational efforts are reduced, and the surrogate directions are smoother, thus multiplier zigzagging is much alleviated. In this decomposition and coordination framework, unit-level constraints are tightened by using our novel and systematic approach [9] to further enhance the quality of CHPs and speed up the computational time as well.

In Section V, three examples based on the IEEE 118-bus system are considered with transmission capacity constraints ignored for simplicity. The first small example is to illustrate the concept of convex hull pricing. The second 3-hour example is to demonstrate that with tight unit formulations, the integer relaxation problem leads to the same dual variables of the original problem. The third 24-hour example is to show that with tighter unit formulations, the prices obtained by solving integer relaxation problems are getting closer to convex hull prices.

II. Literature Review

CHP is a new and emerging area of research, and there are limited studies in the literature. In this section, different approaches for calculating CHPs are reviewed.

The first type of approach relies on the convexified UC problem. However, the convex envelope and convex hull of the entire UC problem are difficult to formulate. To overcome

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that, the convex envelope and convex hull of each unit are used to equivalently construct the convexified UC problem. The difficulty comes down to formulating the convex hull and convex envelope of each unit. The studies [3], [8] used a network flow model with integer vertices, whereas in [2] the authors use a vertex representation polytope to create a convex hull. These models were able to generate exact convex hull prices in the absence of ramp rate constraints. However, they are not practical for UC problems with ramp rate constraints. Since the unit formulation loses its tightness in the presence of ramp rates, some researchers have tried to overcome this problem by enumerating all possible commitment statuses of units and developing a convex combination of its constraints [4-7]. In [4], disjunctive programming is used to enumerate the commitment statuses. The study [5] also employs an enumeration methodology, where decision variables indexed by the unit on and off intervals were introduced. Researchers in [6], [7], used dynamic programming equations of the conventional UC problem and transformed them into a linear programming (LP) problem, where the dual of the LP problem gives the description of convex hull and convex envelope. These methods demonstrate that integer relaxation leads to an exact convex envelope. However, in such enumeration-based approaches, the computational burden increases dramatically with the number of periods. To obtain the convex hull, formulation tightening provides another way since tight constraints directly delineate the convex hull of the feasible set. In the literature, most tightened formulations were presented without providing a procedure [14]–[16]. In our previous work [9], a systematic approach was developed to tighten single unit formulations. Tight constraints were established based on novel integration of "constraint-and-vertex conversion," "vertex elimination," and "parameterization" processes. Testing results on the IEEE 118-bus and Polish 2383-bus systems demonstrated the benefits of improving computational efficiency and solution quality.

In the second type of approach, the Lagrangian dual of the UC problem is used to get CHPs. The multipliers of the dual problem corresponding to the system demand constraints are the CHPs, and there is no need to describe the convex hulls and convex envelopes. In [10], the authors used a subgradient simplex cutting plane method to remove non-optimal solutions using subgradients and dual variables at each iteration. But this method suffers from multiplier zigzagging and slow convergence. An extreme point subdifferential method was used in [11], [12]. Here, the authors minimize the square error between the demand and generation levels to obtain the steepest accent direction. However, this method requires the extreme points of a unit to be calculated in each iteration, requiring a heavy computational effort. The above Lagrangian relaxation-based methods involves the high computational effort, and multipliers may suffer from zigzagging, resulting in slow convergence. These major difficulties have been overcome by our recent SLR method [13]. This method does not require all sub-problems to be solved optimally, thereby alleviating zigzagging and reducing computational requirements. It is a promising method with demonstrable results on different kinds of optimization problems [17].

III. UC Formulation

In this section, a standard UC formulation is presented [9]. Unit-level and system-level constraints are presented in Subsections A and B, respectively. The objective function is discussed in Subsection C.

A. Unit-level constraints

For unit k at node i, at each time t, major decision variables include unit on/off status x (binary), start-up decision u (binary), and generation level p (continuous). At the unit level, constraints include generation capacity, offer price block, start-up, ramp rate, and minimum up/down-time. Node index i and unit index k are omitted below for brevity.

1). Generation capacity: If a unit is online, its generation level p should be within its minimum P^{min} and maximum P^{max}; otherwise, p has to be zero, i.e

$$x(t)P^{min} \leq p(t) \leq x(t)P^{max}, \ \forall t \quad (1)$$

2). Offer price block: Assume offer prices are monotonically non-decreasing, a few offer price blocks with constant prices in each block are considered. For each block, a continuous decision variable p_b is considered, and their sum equals p, i.e.,

$$p_b(t) \leq P_b^{max}, \sum_b p_b(t) = p(t), \ \forall t \quad (2)$$

where $P^{max}{}_b$ (MW) is the maximum generation of block b.

3). Ramp rate: The change of generation levels between two consecutive hours cannot exceed hourly ramp R, and p cannot exceed P^{min} plus 30-min ramp when the unit is online at the first or last hour following standard industrial practice, i.e.,

$$p(t) - p(t-1) \leq (R/2 - P^{min})x(t-1) + (P^{min} + R/2)x(t), \ \forall t$$

$$p(t-1) - p(t) \leq (P^{min} - R/2)x(t-1) + (R/2 - P^{min})x(t), \ \forall t \quad (3)$$

4). Start-up: If a unit is turned on at hour t, binary startup variable $u(t)$ equals 1, i.e.,

$$u(t) \geq x(t) - x(t-1) \quad (4)$$

5). Minimum up/down-time: A unit must stay online or offline for its minimum up or down time [15], respectively, i.e.,

$$\sum_{\tau=1}^{MOn} x(\tau) = T^{MOn}$$

$$\sum_{\tau=1}^{t+T^{MU}-1} x(\tau) \geq T^{MU}\big(x(t) - x(t-1)\big),$$

$$t \in [1 + T^{MOn}, T - T^{MU} + 1],$$

$$\sum_{\tau=t}^{T}(x(\tau) - (x(t) - x(t-1))) \geq 0,$$

$$t \in [T - T^{MU} + 2, T] \quad (5)$$

where T^{MU} is the minimum up time, and T^{MOn} is the number of hours the unit must be on at the beginning of the time horizon. Modeling of minimum down time is similar.

B. System-level constraints

At the system level, constraints include system demand (power balance), and transmission capacity.

1). System demand: Total generation equals total demand, i.e.,

$$\sum_{i,k} p_{i,k}(t) = \sum_i P_i^D(t), \forall t \quad (6)$$

where $P_i^D(t)$ is the demand of node i at time t.

2). Transmission capacity: DC power flow $f_l(t)$ of line l, linear combinations of nodal injections $P_i(t)$ from all nodes weighted by generation shift factors $a_{i,l}$, cannot exceed capacity f_l^{max}, i.e.,

$$-f_l^{max} \leq f_l(t) \leq f_l^{max}, f_l(t) = \sum_i a_{i,l} P_i(t),$$

$$P_i(t) = \sum_k p_{i,k}(t) - P_i^D(t), \forall t \quad (7)$$

C. Objective function

The total cost to be minimized is the commitment cost plus the dispatch cost, i.e.,

$$\sum_t \sum_i \sum_k (u_{i,k}(t) S_{i,k} + x_{i,k}(t) S_{i,k}^{NL}) + \sum_t \sum_i \sum_k \sum_b C_{i,k,b} p_{i,k,b}(t) \quad (8)$$

where $S_{i,k}$, $S_{i,k}^{NL}$, and $C_{i,k,b}$ are start-up, no-load, and generation costs, respectively.

IV. Methodology

The above problem is solved by using Surrogate Lagrangian Relaxation (SLR) [13] as decried in Subsection A. Then unit-level constraints are tightened by using our novel and systematic approach [9] as introduced in Section B.

A. Surrogate Lagrangian Relaxation (SLR) method

The SLR method is a novel decomposition and coordination algorithm that overcame all the major difficulties of standard Lagrangian relaxation [13]. The process begins with the relaxation of system coupling constraints (system demand and transmission capacity) Lagrangian multipliers. The relaxed problem is given as:

$$\min L(\lambda(t), p(t)) = \min \Big\{ \sum_t \sum_i \sum_k (u_{i,k}(t) S_{i,k} + x_{i,k}(t) S_{i,k}^{NL}) + \sum_t \sum_i \sum_k \sum_b C_{i,k,b} p_{i,k,b}(t) \Big\} + \sum_t \lambda(t) \big(\sum_i P_i^D(t) - \sum_{i,k} p_{i,k}(t) \big) + \sum_l \sum_t \big(\mu_l^+(t).(f_l(t) - f_l^{max}) \big) + \big(\mu_l^-(t).(-f_l(t) - f_l^{max}) \big) \Big\} \quad (9)$$

With system-wide constraints relaxed, the relaxed problem can be decomposed into individual unit subproblems. In the method, not all subproblems are solved at a time and only the surrogate optimality condition [13] is needed to be satisfied. Therefore surrogate directions do not change drastically from one iteration to the next and zigzagging difficulties are thus alleviated. Multipliers λ are updated by the following equation (time index t is omitted):

$$\lambda^{n+1} = \lambda^n + c^n \tilde{g}(p^n),$$
$$\mu_l^{+,n+1} = \mu_l^{+,n} + c^n \tilde{g}^+(p^n),$$
$$\mu_l^{-,n+1} = \mu_l^{-,n} + c^n \tilde{g}^-(p^n). \quad (10)$$

Where, c^n is the positive scalar step size at iteration n and $\tilde{g}$ is the surrogate subgradient vector given as:

$$\tilde{g}(p^n) = \sum_i P_i^D(t) - \sum_{i,k} p_{i,k}^n,$$
$$\tilde{g}^+(p^n) = a_{i,l} P_i^n - f_l^{max},$$
$$\tilde{g}^-(p^n) = -a_{i,l} P_i^n - f_l^{max}. \quad (11)$$

The value of c^n is calculated with the following equation

$$c^n = \alpha_n \frac{c^{n-1}\left(\|\tilde{g}(p^{n-1})\|_2 + \|\tilde{g}^+(p^{n-1})\|_2 + \|\tilde{g}^-(p^{n-1})\|_2 \right)}{\|\tilde{g}(p^n)\|_2 + \|\tilde{g}^+(p^n)\|_2 + \|\tilde{g}^-(p^n)\|_2},$$
$$0 < \alpha_n < 1 \quad (12)$$

The values of α_n depends upon the given variables:

$$\alpha_n = 1 - \frac{1}{Mn^p}, p = 1 - \frac{1}{n^r}, M \geq 1, 0 < r < 1 \quad (13)$$

The values of Lagrangian multipliers corresponding to the system demand constraints (λ) of the relaxed problem gives the required CHPs.

B. Formulation tightening

Formulation tightening is a promising but relatively overlooked research area. It involves the transformation of the constraints in the pre-processing stage to directly delineate the convex hull of the unit. Our approach first generates vertices of the integer relaxation problem from constraints by using algebraic manipulation of unit parameters with algorithms well established in existing software Porta. Then the vertices of the original convex hull are obtained by the elimination of vertices with fractional values for binary variables [9]. Then these vertices are converted back to tight constraints by using the software Porta as a reverse process of the first step. These constraints are reusable as numerical coefficients are converted to unit parameters and stored in look-up tables for future use. The biggest advantage of this approach is that it enables the use of linear programming methods to solve the UC problem. Since it is hard to develop tight formulations of units for 24 hours, near-tight formulations can be used as good approximations [2], [9].

To illustrate the idea, consider a simple problem with two binary variables x_1 and x_2, and $x_1 + x_2 \geq 0.5$. In Figure 1(a), constraints are represented by blue lines, and the convex hull is described by red lines with vertices represented by solid red dots. Those red lines are hard to obtain directly. For the integer relaxation problem in Figure 1(b), constraints are still blue lines. After constraint-to-vertex conversion, vertices are represented by blue dots. There are two sets of vertices in Figure 1(b). One set consists of binary vertices represented by solid blue dots, and the other set consists of fractional vertices represented by open blue dots. By dropping the open blue dots, the remaining binary vertices are the same as the vertices in Figure 1(a). After vertex-to-constraint conversion, tight constraints, i.e., red lines, are obtained.

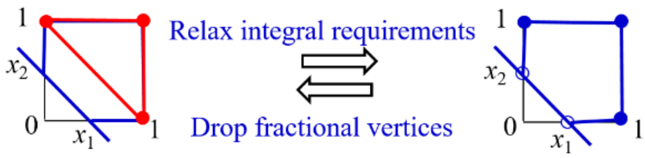

Figure 1(a): Convex hull of a problem with binary variables x_1, x_2

Figure 1(b): Convex hull of its integer relaxation problem

V. Testing and Results

In this section, three examples are considered. The first small example is to illustrate the concept of convex hull pricing. The IEEE 118 bus system is used in examples 2 and 3. The second 3-hour example is to demonstrate that with tight unit formulations, the integer relaxation problem leads to the same dual variables of the original problem. The third 24-hour example is to show that with tighter unit formulations, the prices obtained by solving integer relaxation problems are getting closer to convex hull prices. The system consists of 54 units, 186 branches, and 118 buses. The experiments are performed using IBM ILOG CPLEX Optimization Studio V 12.10.0.0 on a PC with 2.30 GHz Intel(R) Core (TM) i7- 10510U CPU and 16 GB RAM.

Example 1

For the UC problem under consideration, the convex hull cost function is an alternative well behaved convex approximation of the total cost function. It produces a supply function that has marginal cost, i.e., convex hull price, increasing in load, and provides a lower bound to the total cost function. To illustrate the idea, consider the two-unit problem in [1] as an example. The unit parameters are described in Table I below. For simplicity, P^{min} is assumed as 0, and R is assumed as infinitely large so that ramp rate constraints are no longer needed. Also, minimum up/down-time constraints are not considered. With units A and B, the total commitment and dispatch cost, and the marginal cost with respect to system load are shown in Figures 2 and 3, respectively.

As shown in Figure 2, the total cost curve follows the total cost curve of unit A until the load level is high enough (approximately 178 MW) to commit unit B, and then it is switched to the total cost curve of the combination of units A and B. Note that in this case, the increase in the rate of the total cost drops. The marginal cost first increases and then decreases, and then increases again with the load as shown in Figure 3. Technically, it is not monotonically increasing.

TABLE I: A TWO-UNIT PROBLEM

Unit	Fixed Cost ($)	P^{max}(MW) of block 1	P^{max}(MW) of block 2	Energy Cost of block 1 ($/MWh)	Energy Cost of block 2 ($/MWh)
A	0	100	100	65	110
B	6000	100	100	40	90

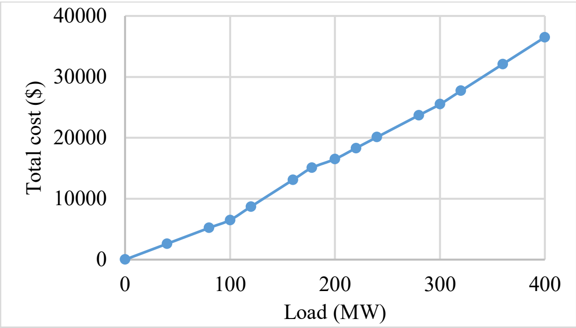

Figure 2. Two-unit problem: Total cost

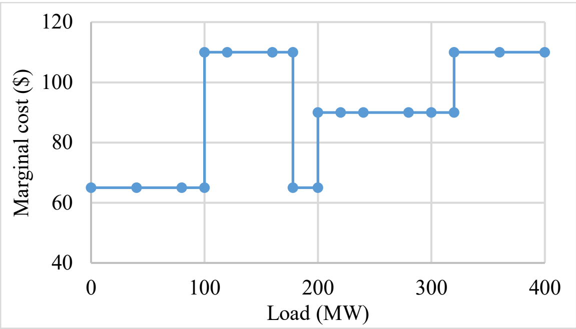

Figure 3. Two-unit problem: Marginal cost

For the above two-unit problem, the convex hull cost yields the total commitment and dispatch cost as shown in Figure 4, and the associated convex hull price curve is shown in Figure 5. The original total cost and marginal cost (price) presented earlier are also shown for comparative purposes.

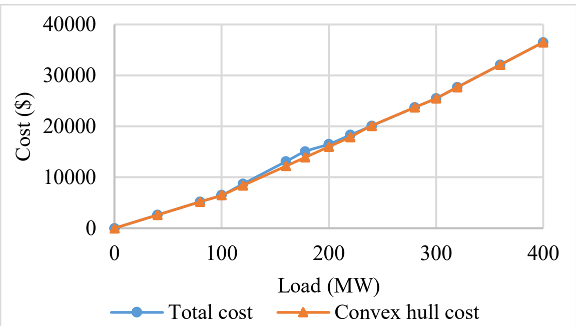

Figure 4. Two-unit problem: Total cost and convex hull cost

As shown in Figure 4, the convex hull cost is as close as possible to the total cost. This convex hull approximation will not reproduce the economic dispatch, but it provides increasing uniform energy prices. In this case, the marginal cost, i.e., the convex hull price, is increasing with the load as shown in Figure 5. It is monotonically increasing.

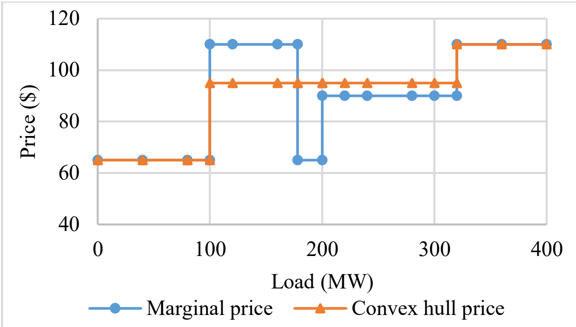

Figure 5. Two-unit problem: Marginal cost and convex hull cost

Example 2

In this example, all 54 units of the IEEE 118 bus system are run for 3 hours only. The motivation behind running it only for 3 hours is that it is possible to get tight formulation and exact CHPs for all hours as the computational load is less due to the smaller time horizon. All units have one offer price block. Transmission capacity constraints are ignored for simplicity. The testing consists of four cases. The first one is a conventional mixed-integer linear programming (MILP) formulation commonly used in the industry, while its integer-relaxation forms the second case. The third and fourth cases are based on our tight formulation presented in [9], case three is a tight MILP formulation while case four is a tight LP formulation (through integer relaxation). Three load profiles are considered, and the results are presented in Tables II, III, and IV.

TABLE II: RESULTS OF TESTING EXAMPLE 2 (2 UNITS, 3 HOURS) WITH SYSTEM DEMAND 1

Case #	**Model**	**CHPs ($/MWh)**		
		Hour 1	**Hour 2**	**Hour 3**
1	MILP (Conventional)	15.025	12.532	13.608
2	LP (Conventional)	15.009	12.539	13.571
3	MILP (Tight)	15.025	12.532	13.608
4	LP (Tight)	15.025	12.532	13.608

TABLE III: RESULTS OF TESTING EXAMPLE 2 (2 UNITS, 3 HOURS) WITH SYSTEM DEMAND 2

Case #	**Model**	**CHPs ($/MWh)**		
		Hour 1	**Hour 2**	**Hour 3**
1	MILP (Conventional)	14.776	12.857	13.585
2	LP (Conventional)	14.968	12.532	13.650
3	MILP (Tight)	14.776	12.857	13.585
4	LP (Tight)	14.776	12.857	13.585

TABLE IV: RESULTS OF TESTING EXAMPLE 2 (2 UNITS, 3 HOURS) WITH SYSTEM DEMAND 3

Case #	**Model**	**CHPs ($/MWh)**		
		Hour 1	**Hour 2**	**Hour 3**
1	MILP (Conventional)	15.06	12.382	12.532
2	LP (Conventional)	15.031	12.428	12.644
3	MILP (Tight)	15.06	12.382	12.532
4	LP (Tight)	15.06	12.382	12.532

As shown in the above three tables, the values of the multipliers for all three hours are exactly the same for cases 1, 3, and 4, and they are CHPs. Only case 2 deviates away from the prices. With tight unit formulations, the values of the dual variables corresponding to system demand constraints for the LP problem are exactly the same as the dual variables, i.e., CHPs, for the original problem. This demonstrates that with tight unit formulations, CHPs can be obtained by solving the LP problem. Thereby, reducing the computational burden of solving MILP problems.

Example 3

In this example, the UC problem in Example 2 is extended to 24 hours. The inspiration for this example is to test the performance of our model against the industrial standard for UC problems, which are typically calculated one day ahead for the next day. Figure 6 shows the comparative performance of the near-tight model presented in this paper with conventional model and the effect of integer-relaxation on prices.

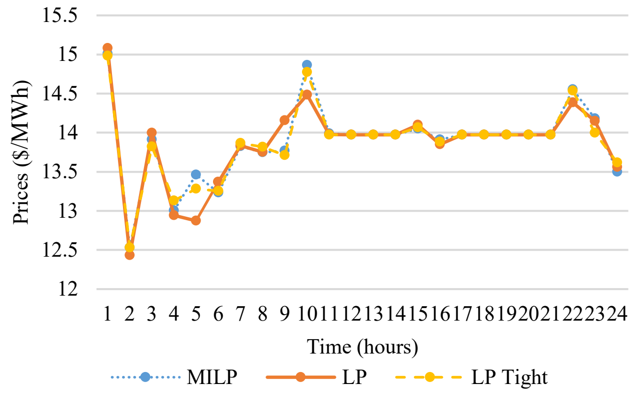

Figure 6. Results of testing example 3 (24 hours)

It is observed that the values of the multipliers produced by near-tight formulations with integer realization are very close to CHPs obtained by the conventional model. However, the prices obtained by the conventional model with integer relaxation deviate considerably, especially for hours 5, 9, and 10. This shows that formulation tightening helps in creating near-tight formulations which can provide a good approximation to the exact CHPs. Since it is easier to solve LP problems than solving MILP programs, this approach reduces the computational requirements for solving the UC problem in electrical markets.

VI. Conclusion

This paper is to investigate the impacts of tightening unit formulations on prices obtained from integer relaxation. The approach of formulation tightening is applied to the UC problem to calculate prices, and the resulting problem is solved by using the SLR method. If the tight constraints can directly delineate the convex hull of the unit, an LP can be used to solve the UC problem for CHPs. Thereby, reducing the computational burden in a major way. The performance of the tightened UC formulation is compared with the conventional UC formulation used in the industry. The results of testing demonstrate that the tightened formulation can produce accurate CHPs upon integer-relaxation for a smaller problem (3 hours) and can provide approximate CHPs upon integer-relaxation for a larger problem (24 hours), for the IEEE 118 bus system with 54 units. This demonstrates the potential of formulation tightening on calculating CHPs.

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