Joint 3D Placement and Interference Management for Drone Small Cells

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Abstract—In this paper, considering a heterogeneous set of drone-mounted base stations (DBSs) that operate on the same frequency band, we optimize the 3D position of the DBSs to maximize the aggregate throughput with minimal power consumption. Given a minimum acceptable received power in the downlink, we formulate an optimization problem to guarantee the wireless coverage for all users. However, the interference between the adjacent cells with overlapping coverage areas can degrade the downlink transmission rate for users. In order to decrease the intercell interference, we formulate a novel and computationally-efficient beamforming method to alleviate the inter-cell interference between the overlapping DBSs. We find the closed-form Nash equilibrium for the beamforming vectors which results in maximum downlink transmission rate without requiring any coordination among the DBSs. This is particularly useful for the drone-based wireless network where implementing computationally-intensive protocols is not a viable option due to strict constraints over the battery of DBSs. Simulation results demonstrate the effectiveness of the proposed solution and provide valuable insights into the performance of the heterogeneous drone-based small cell networks.

Index Terms—Drone Base Station (DBS); Power efficiency; Resource optimization; Game Theory.

I. INTRODUCTION

The cellular telecommunications may avail from drone-mounted base stations (DBSs) to satisfy the coverage and rate requirements of wireless users in areas which lack coverage or are heavily congested, such as hotspot areas [1], [2]. However, developing fully fledged drone-based wireless networks brings forward unique technical challenges. In particular, the 3D deployment of DBSs is of paramount importance as it directly impacts the coverage and power consumption of the network [3]. Furthermore, managing the network interference becomes more challenging due to the fact that ground users may receive strong line-of-sight (LoS) signals from multiple DBSs [4], [5]. On the other hand, the limited on-board energy of the DBSs calls for effective and yet, computationally efficient algorithms to address the interference [6].

The deployment of DBSs has been the subject of concerted research in recent years. The authors in [7], [8] derived the optimal flight altitude for a single DBS to maximize the coverage area for a fixed value of transmit power. The work in [9] extended the previous results for the Rician fading channels. The positioning and the effect of intercell interference for two identical DBSs that hover at the same altitude is studies in [10]. The authors in [11] proposed a method for fast deployment of identical BSs with overlap-

ping coverage areas without addressing the severe intercell interference. Most existing studies in the recent of the art, consider the deployment of homogeneous DBSs in which the DBSs have identical characteristics such as the flight altitude and transmit power. Furthermore, the number of DBSs to be deployed in a region is considered as a known parameter, whereas in practical scenarios, this parameter depends on the particular application, the environment, and the availability of the resources.

In this work, our goal is to bridge this gap by considering the deployment of a heterogeneous repository of DBSs in an interference channel where the intercell interference is mitigated by a computationally-efficient algorithm. In particular, the main contributions of this work are as follows:

- Considering a repository of heterogeneous DBSs with varying transmit power and flight altitude, we jointly derive the optimal resource allocation strategy (i.e., selecting a subset of available DBSs) and 3D placement of the DBSs. The goal is to minimize the total transmit power while maintaining the desired downlink received signal strength. In contrast to the existing literature, the type and the number of DBSs that need to be deployed are not known in advance.
- Assuming that the DBSs are operating in the same spectrum, we formulate a novel beamforming method in the context of game theory to alleviate the impact of intercell interference between the DBSs.

The rest of this paper is organized as follows. Section II presents the system model and describes the air-to-ground channel model. The problem formulation and the proposed methodology for optimal horizontal placement of the DBSs are presented in Section III. The interference management is addressed in Section IV where a non-cooperative game is formulated between the neighboring DBSs to optimize their downlink beamforming vectors and the closed form solution for *Nash Equilibrium* is derived. Numerical results are provided in Section V. Finally, Section VI concludes the paper and discusses the future path of this research.

II. SYSTEM MODEL

We consider a repository of DBSs in which the DBSs are classified into C different groups based on their nominal transmit power. The transmit power of a DBS in class $c \in \{1,2,\ldots,C\}$ is modeled as $P_c \pm \sigma_c$ where P_c is the nominal transmit power and σ_c is the standard deviation from

the nominal value. We further assume that there exist N_c DBSs of type c in the repository such that $N = \sum_{c=1}^C N_c$ is the total number of available DBSs. Also, there are K mobile ground users distributed in a 2D geographical area with low to medium mobility. Let $\mathcal U$ denote the set of users.

The air-to-ground (A2G) channel is typically modeled by considering two different propagation groups, namely the line-of-sight (LOS) and the non-LoS (NLOS) links. The NLOS links incur additional loss due to shadowing and scattering. The total path loss for A2G channel is given by [7]:

$$\Gamma|_{\rm dB} = 20 \log\left(\frac{4\pi f_c d}{c}\right) + \eta_{\xi},$$
 (1)

where Γ is the total path loss, f_c is the carrier frequency, c is the speed of light, d is the distance between the DBS and ground receiver, and η_{ξ} is additional loss which depends on the propagation group $\xi \in \{\text{LOS}, \text{NLOS}\}$. The corresponding probability of a LOS link is given by the seminal work [7]:

$$\Psi_{\text{LOS}} = \left[1 + Ae^{-B(\theta - A)}\right]^{-1},\tag{2}$$

in which, A and B are constant parameters determined by the environmental factors and $\theta = \tan^{-1}(\frac{d}{r})$ is the elevation angle between a DBS located at (x,y,z) and a generic user located as (x',y',z) where $d=\sqrt{(x-x')^2+(y-y')^2+h^2}$ and $r=\sqrt{(x-x')^2+(y-y')^2}$. The probability of a NLOS link is $\Psi_{\rm NLOS}=1-\Psi_{\rm LOS}$.

Our goal is find the optimal deployment of DBSs to guarantee the wireless coverage for all users. A ground user k is covered if its signal-to-noise ratio is (SNR) γ is greater than a threshold ϵ , i.e., $\gamma_k > \epsilon$ It is shown in [12] that the flight altitude of DBSs can be optimized independently from their horizontal location. Thus, we assume that all the deployed DBSs hover at their optimal altitude, which results in maximum coverage disk for a given transmit power [12].

III. PROBLEM FORMULATION AND PROPOSED METHODOLOGY

Considering a repository of DBSs, we investigate the joint problem of selection and the 3D placement for a heterogeneous set of of DBSs to provide wireless coverage for the ground users. After the locations of all DBSs are determined, each ground user is associated with the DBS that has the highest SINR, which is the common practice in cellular wireless networks. We formulate the following optimization problem:

$$\underset{I(c), M_c, (x,y)}{\text{minimize}} \quad \sum_{c=1}^{C} \sum_{i=1}^{M_c} I(c) P_c(x,y), \tag{3}$$

s.t.

$$I(c) \in \{0, 1\},$$
 (4)

$$M_c \in \{0, 1, \dots, N_c\},$$
 (5)

$$P_c - \sigma_c \le P_c \le P_c + \sigma_c, \tag{6}$$

$$h \le h^{\text{opt}},$$
 (7)

where I(c) is an indicator function that is set to one if class c is selected for deployment and otherwise, it is set to zero.

Constraint (5) controls the number of DBSs from each selected class c to be less than the total available DBSs in that class, i.e., N_c . Constraint (6) guarantees that the DBSs transmit at their nominal power, and constraint (7) requires the flight altitude to be less than the optimal value, as increasing the altitude beyond $h^{\rm opt}$ results in smaller coverage area [12]. Note that $h^{\rm opt}$ is computed to have $\gamma_k > \epsilon$ for all users $k = 1, \ldots, K$. Finally, the goal of this optimization problem is to find the optimal horizontal location of DBSs as well as the optimal resource allocation strategy in order to minimize the aggregate transmit power.

As there is no constraint to prevent potential overlaps between the adjacent DBSs' coverage disks, inter-cell interference is unavoidable. Next, we propose a novel algorithm to solve the optimization problem (3) and then, we formulate a Nash Bargaining Solution (NBS) to alleviate the interference.

A. Methodology

Due to its non-convexity, non-linear constraints, and the large number of unknowns, the optimization problem stated in (3) is very challenging to solve. Note that restricting the DBS flight altitude to $h \in (0, h_i^{\text{max}}]$ in (7), causes the coverage radius to be an increasing function of transmit power P_i^t [12]. Therefore, we can minimize the DBSs' coverage radii instead of their transmit power. We will do so by minimizing the coverage radii of DBSs one at a time, starting from the largest coverage radius. Indeed, for a given resource allocation strategy, i.e., for a fixed indicator vector $[I(1), \dots, [I(C)]]$, the problem reduces to the one of covering all ground users with a set of disks with given radii. Let us consider a simpler problem in which there is only one coverage disk and its location has to be optimized in such a way that it covers the maximum number of ground users. This problem is know as *on-center* problem (OCP) in 2D space. A linear time $\mathcal{O}(n)$ algorithm for solving OCP is proposed in [13]. Here, we propose a recursive algorithm to tackle the optimization problem (3) for a fixed indicator vector $[I(1), \ldots, [I(C)].$

Without loss of generality, we arrange the DBSs in a decreasing order of their transmit power. Recall that since the flight altitude is restricted to $h \leq h^{\text{opt}}$, the coverage area is a monotonically increasing function of the transmit power. Thus, in essence, the DBSs are arranged in decreasing order of their coverage area. Starting from the DBS class with the largest coverage disk (i.e., c = 1), the location of DBS (i.e., the center of disk) is optimized using linear OCP algorithm to cover the maximum number of users. Since one DBS of class 1 is deployed, we update the M_1 and subtract M_1 from N_1 to update the number of available DBSs of class 1. In the next step, from the remaining DBS classes, we select one that covers the maximum number of "uncovered" users, and the algorithm continues until either all the users are covered or all the resources are exhausted. The pseudocode for horizontal placement of DBS is shown in Table 1.

The pseudocode in Table 1 finds the number of required DBSs of each class along with their optimal horizontal location for a given vector of indicator functions. There are 2^C such

Algorithm 1: Horizontal Placement of DBSs

Data: $[I(1), \ldots, [I(C)], P_c \text{ and } \overline{N_c \ \forall c, \text{ location of }}$

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users: \Psi = \{(x_k', y_k') | \forall k\}. Result: \{M_i, (x_i, y_i)\}_{i=1}^N
    Initialization: M_i \leftarrow 0 for i = 1, 2, \dots, C
                           \mathcal{U}' \leftarrow \emptyset
 1 repeat
          for c = 1, 2, ..., C do
 2
               if I(c) = 1 then
 3
                     Solve OCP to find the center (x_c, y_c) that
 4
                       covers largest user subset K
                     \mathcal{U}' \leftarrow \mathcal{K}
 5
                     \mathcal{U} \leftarrow \mathcal{U} - \mathcal{K}
                     M_c \leftarrow M_c + 1
 7
                     if M_c = N_c then
                          I(c) \leftarrow 0
                     end
10
                end
11
          end
12
13 until U' = U or I(c) = 0 \quad \forall c;
14 return \{M_i, (x_i, y_i)\}_{i=1}^N
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vectors and an exhaustive search to find the best resource allocation policy can get out of hand for large values of C. However, in practice, the number of DBSs classes in a repository that are suitable for a specific task are limited to small numbers. As the algorithm in Table 1 employs linear OCP, it is possible to run an exhaustive search over all indicator vectors to find optimal resource allocation strategy. For large values of C, however, one can benefit from smart search algorithms such as genetic algorithm [14] to find a near optimal solution.

IV. NASH EQUILIBRIUM FOR DOWNLINK BEAMFORMING VECTORS

Enabling airborne adhoc systems to efficiently operate in the same spectral band is a key challenge for the drone small cells. Similar to the terrestrial wireless cells, the intercell interference caused by communication in an interference channel degrades the quality of received signals on the ground stations. There are many algorithms and solutions to alleviate the impact of destructive co-channel interference for the conventional terrestrial networks [15]. However, the difficulty in the airborne small cell networks stems from the fact that the DBSs are very battery-limited and thus, implementing the conventional interference management methods for DBSs is not a viable option due to overwhelming computational complexity. In this section, we adopt the framework of *bargaining game theory* [16] to introduce a simple and low-complexity beamforming method to address the inter-cell interference.

We consider the scenario whereby M interfering DBSs are trying to transmit their information in the downlink to M ground users located in the overlapping region of their corresponding coverage disks. Assuming that each DBS per-

forms single-stream transmission, and given that all channels are frequency flat, we have the following complex baseband symbols y_m received by the ground users T_m :

$$y_m = h_{mm}^T w_m s_m + \sum_{l=1, l \neq m}^M h_{lm}^T w_l s_l + e_m,$$
 (8)

where $s_m, 1 < m < M$ is the transmitted symbols from DBS D_m , h_{lm} represents the $K \times 1$ channel vector between DBS D_l and user T_m , w_m is the $k \times 1$ beamforming vector used by DBS D_m , and n_m is the zero mean additive Gaussian noise with variance σ^2 . The maximum transmit power per DBS is normalized to 1 which yields the following power constraint on each DBS D_m : $\|w_m\|^2 \le 1, \forall m \in \{1, 2, ..., M\}$.

Each DBS D_m seeks to optimize its weight vector w_m in order to maximize the quality of service received by its corresponding ground user. However, there exists an interplay between the strategies (i.e., optimizing the weight vectors) of the DBSs as any selected value of w_m impacts the choice of $w_l, l \neq m$, and vice versa. Thus, the key question is how to optimize the beamforming vectors so as to maximize the downlink rate for each individual DBS. To answer this question, we assess the non-cooperative scenario in which the DBSs act selfishly with no exchange of information. We formulate a non-cooperative zero-sum game between the interfering DBSs for which the Nash equilibrium is the accepted outcome [17]. It is worth noting however, that allowing cooperation between the DBSs can improve the performance. However, it adds noticeable overhead to the system which puts additional constraint on the battery of DBS. Moreover, the dynamic nature of networking with DBSs requires frequent channel estimation and information exchange across the network of DBSs. Nonetheless, a cooperative beamforming solution with low complexity is an interesting subject for future research.

In this work, we consider the non-cooperative downlink beamforming game G as the triplet $G = \{\mathcal{M}, (S_m)|_{m \in \mathcal{M}}, (u_m)|_{m \in \mathcal{M}}\}$ where:

- \mathcal{M} is the set of players, i.e., the interfering DBSs;
- S_m is the strategy of DBS D_m which is its choice of weight vector w_m such that $||w_m||^2 \le 1$;
- S_{-m} is the vector of strategies of all DBSs except D_m ;

$$S_{-m} = [S_1, \dots, S_{m-1}, S_{m+1}, \dots, S_M];$$

• $u_m: [S_m, S_{-m}] \to \mathbb{R}$ is the utility of each DBS D_m which is the rate it achieves at its correponding ground

For a given tuple of beamforming vectors (w_1, w_2, \dots, w_M) , the received rate at the ground users is given by:

$$R_m = \log_2 \left(1 + \frac{|w_m^T h_{mm}|^2}{\sigma^2 + \sum_{l=1}^M \frac{|w_l^T h_{lm}|^2}{|w_l^T h_{lm}|^2}} \right).$$
(9)

We define the utilities of the DBSs as

$$u_m(S_m, S_{-m}) = R_m(w_1, w_2, \dots, w_M).$$
 (10)

As the utilities depend on the strategies of the competing players, we have a noncooperative game among the DBSs. In the absence of coordination among the DBSs, the outcome of the game will generally be the Nash equilibrium. A vector of strategies $(S_1^{\rm NE}, S_2^{\rm NE}, \dots, S_M^{\rm NE})$ is the Nash equilibrium if it satisfies the following condition:

$$u_m(S_m^{\text{NE}}, S_{-m}^{\text{NE}}) \ge u_m(S_m, S_{-m}^{\text{NE}}), \quad 1 \le m \le M,$$
 (11)

which means that no DBS can unilaterally deviate from its optimal Nash equilibrium strategy without decreasing its own utility. By substituting (9) and (10) in (11) and by performing some algebraic manipulations, we can find the unique equilibrium strategies as,

$$w_m^{\text{NE}} = \frac{h_{mm}^*}{||h_{mm}||}, \quad 1 \le m \le M,$$
 (12)

where h_{ij}^* is the complex conjugate of h_{ij} . The equilibrium strategies in (12) correspond to the maximum-ratio transmission beamforming. This conclusion is resulted from the fact that when DBS D_m uses the beamforming vector $w_m^{\rm NE}$ at the Nash equilibrium, there exists no other vector that can yield a larger rate while satisfying the power constraint $\|w_m\|^2 \le 1$.

V. SIMULATION RESULTS

For simulations, we consider the drone-based communications over 2 GHz carrier frequency, i.e., $f_c=2$ GHz, in an urban environment with parameters $a=9.61,\ b=0.16$ [7]. We assume that the minimum allowable received signal power for a successful transmission is $\epsilon=-60$ dBm. We also consider a repository of 12 DBSs in which there are three different types of DBSs with maximum transmit power of 35 dBm, 39 dBm, and 43 dBm, and there are four identical DBSs of each kind. The goal is to provide wireless coverage for the ground users that are distributed in a 10 Km \times 10 Km area. We consider the uniform and truncated Gaussian distributions for users location. Assuming that the x-coordinate and the y-coordinate are independent random variables, these distributions for a rectangular area with size of $L_x \times L_y$ are respectively given by [18]:

$$f^{\text{tG}}(x,y) = \frac{1}{G} \exp\left(\frac{L_x - \mu_x}{\sqrt{2\sigma_x}}\right)^2 \exp\left(\frac{L_y - \mu_y}{\sqrt{2\sigma_y}}\right)^2,$$
(21)

in which G is the normalization constant and μ_x , σ_x , μ_y , σ_y are the mean value and standard deviation in the x and y directions.

Fig. 1 illustrates the optimal resource allocation and the horizontal placement of the DBSs as well as the user-DBS association for snapshot of the ground users' topology. In particular, Fig. 1 shows the 2D projection of the DBSs and their corresponding coverage disks. It can be seen that for each coverage disk, there exists at least two ground users on its boundary. Consequently, one cannot shrink any of these coverage disks without leaving some ground user out of the coverage area. In other words, the DBSs' coverage radii are minimized while providing the required service to the ground

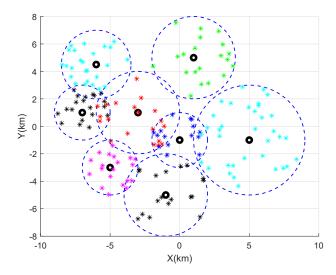


Fig. 1: An illustrative snapshot of the optimal placement of the DBSs and the user-DBS association. Three classes of DBSs with coverage radii of approximately 4km, 3km, and 2km are deployed to cover the ground users.

users. It is seen that the total number of deployed DBSs from each class are $M_1 = 1$, $M_2 = 3$, and $M_3 = 4$, respectively.

Fig. 2 shows the optimal number of DBSs in order to satisfy the coverage requirement of the ground users with minimum average transmit power. In this figure, we can see that number of DBSs is a monotonically increasing function of the number of ground users. However, the number of DBSs does not solely depend on the number of users, it also depends on how the users are distributed in the area. According to Fig. 2, for a large number of users, the required number of DBSs in a congested hotspot scenario is less than the scenario in which the users are evenly distributed in a larger area.

Figure 3 shows the average number of iterations of the proposed algorithm for three different network sizes as the number of users varies. In this figure, we can see that the number of algorithm iterations is an increasing function of the number of users and the network size. It shows that the average number of iterations varies from 1.09 and 1.1 at K=3 to 7.8 and 10.2 at K=180, for the cases of 15 DBSs and 25 DSCs, respectively. Given that the OCP problem in the proposed algorithm is solvable with time complexity of order $\mathcal{O}(1)$, figure 3 demonstrates that the proposed algorithm converges within a reasonable number of iterations and scales well with the network size.

VI. CONCLUSIONS

This paper discusses the practical scenario where a subset of available DBSs needs to be deployed in a geographical region to provide wireless connectivity for the ground users. In particular, considering a heterogeneous repository of DBSs with varying nominal transmit powers, we propose a novel technique for optimal resource allocation (i.e., DBS selection) and deployment of DBSs to satisfy the downlink received

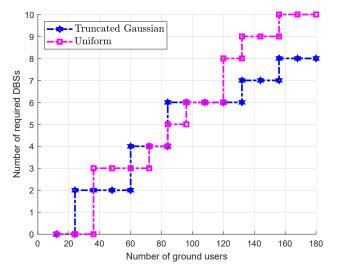


Fig. 2: The number of required DSBs vs. the number of users for two random models of user distribution. The truncated Gaussian distribution models the hotspot area where the density of users is maximum around a particular location and it gradually fades away as the distance from the center increases.

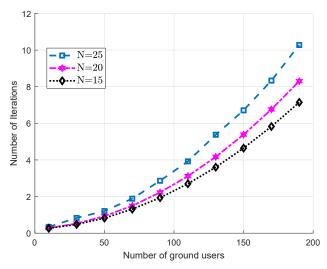


Fig. 3: Average number of algorithm iterations for reaching the optimal horizontal location of DBSs, for different number of users with N=25, N=20, and N=25 DBSs.

signal strength indicator (RSSI) for one snapshot of the users' distribution. The selection and deployment of the DBSs are dictated by the user distribution in the geographical area. The goal is to cover *all* the ground users, if possible. Thus, the coverage disks of the DBSs will unavoidably overlap, which results in inter-cell interference. We propose a simple and effective beamforming method for the interfering DBSs in the downlink to alleviate the effect of interference on the RSSI. In the proposed technique, a non-cooperative game is formulated among the interfering DBSs and the optimal strategy which

corresponds to the Nash equilibrium is derived. As for the future directions of research, it is interesting to investigate the effect of users' mobility and dynamic changes in the A2G channel on the system performance. Moreover, it is promising to study a cooperative scenario where the DBSs can exchange local information with adjacent cells to dynamically adjust their beamforming for improved overall performance.

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