The London Millennium Footbridge Revisited: Emergent Instability Without Synchronization

By Igor Belykh, Mateusz Bocian, Alan Champneys, Kevin Dally, Russell Jeter, John H.G. Macdonald, and Allan McRobie

The pedestrian-induced instability of London’s Millennium Bridge is widely held up as the canonical example of synchronization in complex networks [9]. The popular explanation maintained that once the number of pedestrians reached a certain threshold, the pedestrians could supposedly synchronize their footstrokes with each other at the bridge’s natural frequency. The result was the onset of dangerous sideways oscillations. Multiple engineering analyses and publications have debated this original interpretation [4, 7, 8]. Nevertheless, the belief that a textbook example of coupled pedestrian synchronization caused the Millennium Bridge instability remains part of numerous presentations in print, film, and radio [6].

We propose an alternative theory by arguing that any synchronization in the timing of pedestrian footsteps is a consequence—not a cause—of the instability [3]; this result is consistent with observations on 30 bridges, including the Brooklyn Bridge and Golden Gate Bridge. We show that unsynchronized pedestrians produce negative damping—a positive feedback effect wherein energy is transferred from pedestrians who are trying not to fall due to perturbations that are caused by bridge motion. Prior studies found negative damping empirically from measurements on London’s Millennium Bridge itself [3], though the researchers believed that synchronization caused this effect. In contrast, we show that negative damping is more fundamental and can occur without synchronization.

To quantify the effective total negative damping, we use a mathematical model that assumes that the coordination between pedestrians transpires solely because of sensory stimuli from the moving bridge. We parsimoniously assume that walking is fundamentally a process in which the stance leg acts as a rigid strut, thus causing the body’s center of mass to behave like an inverted pendulum in the frontal plane during each footstep. The step ends when the other leg meets the ground and—ignoring the brief double-stance phase that occurs in realistic gaits—the pedestrian switches to an inverted pendulum on that other leg, which prevents them from falling over. We consider a single lateral vibration mode for the bridge, which is forced by the motion of N pedestrians who walk perpendicularly to this vibration. We assume that the displacement of the lateral bridge mode $x_i(t)$ is governed by a second-order equation of motion:

$$Mx + Cx + Kx = -H^T(r_i x_i(t)),$$

where $M$, $C$, and $K$ are respectively the mass, damping, and stiffness coefficients of the bridge mode, and $y_i(t)$ is the lateral displacement of the $i$th pedestrian’s center of mass.

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Biological Near-symmetries Explain the Similarity and Diversity of Life

By Matthew R. Francis

Despite the wide variety of shapes that organisms exhibit, their forms are not random. Rather, much of life possesses relatively clear structural patterns; humans and other mammals are bilaterally symmetrical, sea stars have five-fold symmetries, and flowers display a wide variety of symmetrical patterns. However, none of these biological symmetries are exact in a mathematical sense. The near-symmetries in living things are nevertheless real—to the extent that evolutionary, developmental, and environmental pressures must play a part.

Yet when symmetries are not mathematically precise, subtle patterns and symmetries might exhibit variations in the closeness with which they hew to a particular symmetry. For these cases, TI can in principle determine invariants. “I think that TI is a really nice hands-off quantification of symmetries in an unbiased way,” Dawes said. “It requires less time, there’s less opportunity for bias (intentional or not) to be introduced into the measure, and it also gives you a way of then saying, ‘How different is this flower from that flower?’”

In more general terms, TI helps extract symmetry information from biological systems where—for instance—a single plant might have numerous non-identical flowers, or several specimens from one species might exhibit variations in the closeness with which they hew to a particular symmetry. For these cases, TI can in principle identify the underlying symmetry separately from random variation. “Most daisies are going to have rotational symmetry,” Dawes said. “You’ve got your round center and you’ve got your white petals around the outside. We identify a daisy by that rotational symmetry and the color properties” (see Figure 1).

While daisies may deviate from this Platonic ideal, Dawes argued that investigating representative specimens can directly inspire the identification of general symmetries. “If you want to know the common symmetry features among all daisies, it would make sense to do TI on a bunch of different samples and look at the common features in the TI function,” she said.

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**Figure 1.** Outline of the mathematical model of pedestrian-induced lateral instability. (a) Pedestrians are added sequentially at fixed time intervals. The addition of a pedestrian ($n \rightarrow n+1$) causes the overall damping coefficient to become negative, meaning that the amplitude of motion increases rather than diminishes. (b) Inverted pendulum model of bridge mode and pedestrian lateral motion. Figure courtesy of [2].

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**Figure 1.** A daisy is a prime example of a flower with apparent rotational and reflection near-symmetries. The challenge is recognizing the symmetries that are real properties of the species and identifying their levels of robustness between plants. Public domain image.
mass relative to the bridge (see Figure 1, on page 1). The forcing term $B_i(t)$ is the lateral component of the $i$th pedestrian’s foot force on the bridge. According to Newton’s second law of motion, the lateral component of the center of mass for a pedestal of mass $m$ obeys the equation

$$m \ddot{y}_i(t) = -m \ddot{z}_i(t) + B_i(t),$$

where $i = 1, \ldots, N$.

$B_i(t)$ is a piecewise smooth function with abrupt changes at foot transitions that are associated with pedestrian’s gait. We have listed three variants of $B_i(t)$ that correspond to three different gait controls strategies and yield different patterns of pedestrian foot adaptation [2, 3, 8], one of which has a strong propensity for synchronization.

Using a multiple-scale asymptotic analysis, we derived a general expression that quantifies the average contribution to the bridge damping from the interaction force of a single pedestrian over one gait cycle. This quantity $\sigma$ has three components: (i) A coefficient of lateral bridge damping; (ii) the lateral dependent component of force due to the adjustment of pedestrian lateral gait timing; and (iii) a coefficient of lateral velocity dependent component of force due to adjustment of the forward gait of $\sigma_i$. One should numerically calculate the expressions $(\sigma_{p})$ and $(\sigma_{s})$ as time averages (integrales) of partial derivatives of $B_i(t)$, which are associated with the pedestrian’s lateral and forward gaits. We observe that these expressions are evaluated individually for each pedestrian and depend on the pedestrian’s stride frequency $\omega_{p(i)}$, the bridge vibration frequency $\omega_{s}$.

The terms $\sigma_{p}$ and $\sigma_{s}$ depend on the pedestrian’s stepping behavior in response to bridge motion. Yet in all of our simulations, we found that $\sigma_{p}$ plays the most important role in triggering large-amplitude vibrations. This effect is cumulative because in the absence of phase synchrony between the bridge and pedestrian, one must account for the fact that the lateral force on the bridge will average to zero. However, this is not the case. Figure 2 provides an explanation.

We calculate the total effective damping coefficient $c_{ef}$ as

$$c_{ef} = c_{p} + N \sum_{i=1}^{N} \sigma_{p}(\omega_{p(i)}) + \sigma_{s}(\omega_{s}),$$

where $c_{p}$ is the coefficient of the bridge’s inherent damping and $\sigma$ represents the mean pedestrian stride frequency. We found that $\sigma$ is negative over a large range of bridge and pedestrian frequency ratios. The overall modal damping $c_{ef}$ hence becomes zero when the number of pedestrians exceeds a critical value of $N$.

As a result, negative damping causes bridge vibrations to grow. In all of our simulations, the occurrence of negative damping and onset of bridge instability always precede the occurrence of increased coherence among pedestrian step-timestep timings — even for the model that is highly prone to synchronization.

In a key scientific conclusion of our work, we argue that negative damping due to pedestrian attempts to maintain balance is the essential mechanism of cases of lateral bridge instability. Indeed, our simulations revealed that increased coherence in the pattern of pedestrian footstep timing is a partial nonlinear adjustment to the amplitude of vibration after initiation of the instability. This secondary effect typically produces saturation of vibration amplitude, which can further exacerbate instability in extreme cases.

We achieve these findings via asymptotic analysis, which is applicable to a wide class of foot force models. Moreover, we conducted a comprehensive review of the literature on real bridges that have experienced large-amplitude lateral pedestrian-induced vibrations. Based on our review, we conclude that any clear evidence of synchronization is scant at best. In contrast, our theory is consistent with many observed observations.

Our findings indicate that large amplitude oscillations can occur for a wide range of bridge frequencies. Therefore, bridge engineering techniques that try to resolve the problem by ensuring that bridge frequency is not close to typical pedestrian stride frequencies (often referred to as frequency tuning) are potentially dangerous. For a wide range of systems in nature and society, our work argues more generally that this macro-scale instability (in this case, the bridge motion) may emerge from macro-scale behavior (in this case, of many individual pedestrians) without obvious casual synchrony. It also points to other examples in economic cycles and the timing of remarkably sensitive hearing organs in mammals and insects.

This article is based on [2] and Igor Belykh’s minisymposium presentation at the 2021 SIAM Conference on Applications of Dynamical Systems, which took place virtually last year. It is dedicated to the memory of John Macdonald, who passed away unexpectedly as the proofs were being prepared. His memory and legacy will be sorely missed.

Acknowledgments: This work was supported by the U.S. National Science Foundation under DMS-1909924 (to Igor Belykh). Kevin Daley and Russel McRobie, and the Edinburgh University Department of Mathematics. SIAM, in all its forms and manifestations, is what it is because of the support of SIAM’s members. The editors reserve the right to select and modify all material presented in SIAM News.

References

How Can I Help Ukraine? And Why Should Applied Mathematicians Care?

By Jeff Sachs

As a SIAM member with deep connections to Ukraine, I am heartbroken, horrified, and furious at the atrocities being committed there. News media have largely focused on the military conflict and its casualties and refugees, along with the geopolitics and economic consequences. But we should also remember the cost to the scientific and mathematical communities. The immediate consequences are clear and include casualties, refugees, and impairment of communication, nutrition, shelter, and medical infrastructure; the additional impact of delaying research—in fields like energy production, computer science, and medicine, for example—are also very real, even if difficult to quantify.

Academics and researchers have been calling for development of novel drugs but was underway in Ukraine. These include more than 100 trials for therapies against cancer. Even if they do not become direct military casualties, these cancer patients no longer have the hope that those trials provided. The resulting suffering and mortality are multiplied by the impediment to development and approval of the therapies. Given the fragility of thousands of people who suffer from the diseases for which trials are delayed, thousands of additional deaths—“excess mortality”—in epidemiological terms—are likely.

Other direct impacts on our scientific and mathematical communities arise through the rift that conflict creates and the dissolution of collaborations. The Massachusetts Institute of Technology has announced that it is ending a decade-long collaboration with the Skolkovo Institute of Science and Technology (Skoltech), one of Russia’s leading institutes. CERN (the European Council for Nuclear Research) has cancelled a number of recent trips to Russia. The many examples of this type of disunion will isolate some of our dear and talented colleagues at all stages of their careers, regardless of their position on the conflict. It also clearly impacts both Ukrainians and Russians who study abroad — whether through loss of funding or access to research materials, the stress of concern for loved ones, or directly through the conflict’s impact.

It is not lost on any of us that our work relationships can humanize those on the other side of geopolitical boundaries. Perhaps the resulting empathy and communication can help prevent catastrophes like the one that is currently unfolding. To that end, it is probably best to avoid pre-judging colleagues based on their nationality or the language that they speak. A poignant reminder of this is an open letter1 that clearly rejects the invasion, a flagship data science course across the state. Participants will investigate the data science cycle via a project-based learning method that allows student choice in learning experiences (see Figure 2, on page 6). The proposed course focuses on five units: data and society, data and ethics, data and communication, data modeling, and data and computing. Students will utilize open-source technology tools to try and explore problems that involve relational database concepts and data-intensive computing in order to ultimately find solutions. They will also engage in problem-solving structures to interact with large data sets and formulate problems; collect and sort big data, create data-based models; and effectively communicate data-sampled solutions.

In addition, the course intends to build data literacy through “data talks” — low-threshold, high-ceiling activities that engage and motivate students in data-related topics like correlation versus causation, bias and appropriate visualizations; a similar technique known as “number talks” builds computational fluency. Short, targeted lessons called “mini bytes” can either offer timely instruction to the whole class or serve as differentiation or scaffolding approaches for individuals on an as-needed basis. Community and business connections permit students to investigate local and real-world problems, engaging in the research and exploration of data science professions.

Development of the proposed data science standards involved three phases of collaboration among a diverse group of K-12 educators, industry representatives, and higher education professionals in Virginia.

The phase 1 team created the course standards based on the data science cycle (see Figure 2, on page 6). The phase 2 team then used feedback from the first phase as well as universal backwards design principles to identify ideas and concepts for the Data Science SOL, along with the resulting knowledge and skills. Finally, the phase 3 team generated draft unit guides, rubrics, and project templates that outlined the way in which students demonstrate evidence of learning; these materials include a self-assessment and a built-in feedback cycle from teachers, mentors, and peers.

Preparing Virginia’s Students for New Post-secondary Pathways in Data Science

By Tina Mazzacane, Deborah Crawford, Lisaussian, Aamand Vasudevan, and Padmanabhan Seshayari

Inspired by a groundbreaking 2018 study entitled Catalyzing Change in High School Mathematics from the National Council of Teachers of Mathematics [1], the Commonwealth of Virginia is considering a redesign of its Mathematics Standards of Learning (SOL) to promote deeper learning. A diverse group of mathematicians, mathematics educators from higher education, and K-12 leaders across Virginia convened to share ideas that inspired critical conversations about the way in which K-12 mathematics education prepares all students for an evolving post-secondary landscape.

From 2019 to 2021, business and industry stakeholders discussed possible ways to modernize mathematics education in Virginia public schools. Conversations addressed (i) What essential content should be included in the Mathematics SOL, (ii) where content across grade levels could be better focused on college and career readiness, and (iii) the introduction of additional course options for students might create distinct mathematics pathways. A common emerging thread was the need for mathematics content that focused on data science and would better equip students for post-secondary studies. The proposed Data Science SOL—meant to support a locally designed, high-school-level data science course—were presented to the Virginia Board of Education in November 2021 and are currently under review. These prospective standards mark the start of a modernization of the Virginia Mathematics SOL, which are scheduled for revision in 2023.

While higher education institutions around the world have been steadily incorporating some version of data science into a wide range of undergraduate curricula (from business to healthcare and even the social sciences), K-12 education tends to teach three continuous years of integrated math wherein concepts and computational fluency. Short, targeted lessons called “mini bytes” can either offer timely instruction to the whole class or serve as differentiation or scaffolding approaches for individuals on an as-needed basis. Community and business connections permit students to investigate local and real-world problems, engaging in the research and exploration of data science professions.

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Preparatory work among a diverse group of K-12 educators, industry representatives, and higher education professionals in Virginia. The phase 1 team created the course standards based on the data science cycle (see Figure 2, on page 6). The phase 2 team then used feedback from the first phase as well as universal backwards design principles to identify ideas and concepts for the Data Science SOL, along with the resulting knowledge and skills. Finally, the phase 3 team generated draft unit guides, rubrics, and project templates that outlined the way in which students demonstrate evidence of learning; these materials include a self-assessment and a built-in feedback cycle from teachers, mentors, and peers.
Near-symmetries

Continued from page 1

Perfect and Imperfect Symmetries

G.V. Vstovsky first introduced TI in 1997 to apply basic concepts of information theory to broken symmetries in physics-based applications [4]. Approximate symmetries play important roles across scientific fields, from frustrated lattices (solid states that do not form perfect crystal structures) to computer vision problems.

To quantify asymmetry, one must first define a function $\mu : D \to \mathbb{R}^+$ that maps points on the object $D$ to positive real values. For a two-dimensional (2D) object,

$$T_I = \int_D \int_D \frac{\mu(x) \mu(y)}{P(x,y)} \, dD$$

If a symmetry transformation is mathematically exact, its TI is zero because the transformation will map the domain onto a perfect copy. For real biological systems, near-symmetries correspond to minimum (albeit nonzero) TI values. The group also examined developmental problems. "Projection issues are definitely something we worried about," Gandhi said. "But it’s all occurring within the constraint of a spherical cell embryo and understanding why it’s all happening is a huge diversity of form evolved from simpler ancestors in the distant past [1]. Near-symmetries may help explain why—despite the huge diversity of life—organisms have symmetries in the first place.

Endless Forms Most Wonderful

Because the group also examined development within single cells and embryos, Cocanell’s AAA talk addressed the challenge of symmetry that breaks as cells divide and develop. "We were interested in using a spherical cell embryo and understanding how the symmetry of that system would break through rules that are somewhat conserved across model organisms," she said. Similar to how physicists look for broken symmetries to explain different particle properties, mathematical biologists can seek out meaningful underlying principles beneath near-symmetries. Such asymmetries might arise from external influences—like light, gravity, environmental chemistry, and so forth—or from developmental or genetic factors. "One of the things that we looked into with these models of cell polarity was symmetry breaking, driven by different initial conditions: different protein localizations around the boundary of the cell," Cocanell said. "We found that the measure can capture those things really well but also shows robustness.""Dawes presented a TI-driven principal component analysis for C. elegans—a type of nematode worm—that grows within an egg. "When you watch the worm, you can see that it’s undergoing these tremendous changes in body shape," she said. "But it’s all occurring within the constraint of the egg shell. What happens is that the TI really picks up on that ellipsoidal constraint from the egg shell.

These biological examples pertain to the question of why many organisms have near symmetry at all, as opposed to perfect forms or complete amorphism (in fact, exact symmetry can trigger the “uncanny valley” response because we are psychologically primed to expect imperfections), Gandhi and his colleagues suspect that there are reasons for variations in basic symmetry; otherwise, these patterns would be undesir- able from an evolutionary standpoint. The data back up this supposition. “There’s evidence of some advantage to having slight asymmetries in terms of resilience,” he said.

In the concluding paragraph of On the Origin of Species, Charles Darwin waxed eloquent about “endless forms most beautiful and most wonderful” and indicated that diversity of form evolved from simpler ancestors in the distant past [1]. Near-symmetries may help explain why—despite the huge diversity of life—organisms have symmetries in the first place.

References


Matthew R. Francis is a physicist, science writer, public speaker, educator, and frequent wearer of jaunty hats. His website is BowtieHatsScience.org

Panelists then examined bay leaves and desmids (a type of green algae) for bilateral symmetry, along with fern leaves and urchin spines. "We were interested in taking near-symmetries to explain different localizations around the boundary of the cell," Cocanell said. "We found that the measure can capture those things really well but also shows robustness."

Dawes elaborated on this concept. “This is where a little bit of user discrimination comes into play,” she said. “Karl [Niklas] and I went through pictures and decided which ones were appropriate for the analysis because of differences in viewpoints that weren’t capturing the symmetry characteristics we were interested in.” She pointed out that one could use TI in three dimensions in principle, though doing so raises its own set of challenges. “How do you do a meaningful comparison of TI with 3D data compared to 2D data?” Dawes asked. Of course, there is the question of how to obtain usable 3D images in the first place.

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Looking Ahead to the 2022 SIAM Annual Meeting

By Edmund Chow and Sanjica Canić

The 2022 SIAM Annual Meeting1 (AN22) will take place in a hybrid format from July 11-15 in Pittsburgh, Pa. On a broad scale, SIAM’s Annual Meeting aims to bring together researchers who are working in many different areas of industrial, applied, and computational mathematics. Yet the meeting has a focused aspect as well—the minisymposia that are organized in various application areas can make the Annual Meeting feel like a smaller version of your favorite SIAM conference.

Along with the dozens of minisymposia that participants put together at every Annual Meeting, AN22 will also feature tracks of minisymposia that are sponsored by the SIAM Activity Groups on Linear Algebra2 and Mathematical Aspects of Materials Science.3 Furthermore, AN22 will be held jointly with three other SIAM conferences: the SIAM Conference on Applied Mathematics Education,4 the SIAM Conference on the Life Sciences,5 and the SIAM Conference on Mathematics of Planet Earth.6 Each of these meetings will present a joint invited lecture at AN22.

Every year, SIAM bestows many awards and hosts several prize lectures during the Annual Meeting, including the John von Neumann Prize, W.T. and Idalia Reid Prize, AWM-SIAM Sonia Kovalevsky Lecture, SIAM Prize for Distinguished Service in the Profession, and various student and paper prizes. Anne Greenbaum (University of Washington) will deliver the 2022 Sonia Kovalevsky Lecture7 and speak about two of her favorite problems, while Leah Edelstein-Keshet (University of British Columbia) will deliver the John von Neumann Prize Lecture8—SIAM’s highest honor and flagship lecture. Edelstein-Keshet is recognized for her far-reaching contributions to mathematical biology, specifically her work on cellular biophysics and her influential textbook. Mathematical Models in Biology: AN22 likewise boasts the I.E. Block Community Lecture, which is free and open to the public. This iteration of G2S3 to take place in Africa, also experimented with multiple deep learning methods; machine learning theory and applications in science and engineering; methods for partial differential equations; optimization, control, and uncertainty; high-performance and large-scale computing; and scientific computing algorithms, mostly in the area of fluid dynamics. The Organizing Committee has selected invited presentations to cover these and other topics. A few highlights of information aboutAN22 include a great place for students to network and build connections. The “Student Days” program includes special sessions that appeal directly to students, such as tutorials that offer accessible introductions to active research areas in applied mathematics.

Themes for AN22 include probabilistic, statistical, and stochastic algorithms; computational physics and physics-inspired methods; machine learning theory and applications in science and engineering; methods for partial differential equations; optimization, control, and uncertainty; high-performance and large-scale computing; and scientific computing algorithms, mostly in the area of fluid dynamics. The Organizing Committee has selected invited presentations to cover these and other topics. A few highlights of information about AN22 include a great place for students to network and build connections. The “Student Days” program includes special sessions that appeal directly to students, such as tutorials that offer accessible introductions to active research areas in applied mathematics.

SIAM meetings since March 2020 have to have an in-person component. All participants, whether in-person or remote, will have full access to every session. Recordings of the sessions will also be available to conference attendees. The virtual platform for the hybrid meeting is powered by Pathable and is different from the platform that SIAM used for strictly virtual conferences in 2021. In addition to providing easy access to interac-

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1. https://www.siam.org/conferences/cm/conference/an22
2. https://www.siam.org/membership/activity-groups/detail/linear-algebra
5. https://www.siam.org/conferences/cm/conference/life22
6. https://www.siam.org/conferences/cm/conference/mpe22
10. See page 12 of this issue of SIAM News for an article by Kristin Lauter on this topic.
11. https://www.siam.org/conferences/cm/program/program-and-abstracts/an22-program-abstracts

Reflecting on the 2021 Gene Golub SIAM Summer School in South Africa

By Babacar Bah

The African Institute for Mathematical Sciences (AIMS) South Africa1 hosted the 11th Gene Golub SIAM Summer School2 (G2S3) from July 19-30, 2021. The event—which was originally scheduled for 2020 but got postponed due to the ongoing COVID-19 pandemic—took place mostly virtually, which allowed for the inclusion of additional attendees from African institutions. The theme of the school was “Theory and Practice of Deep Learning,” with a focus on the theory, implementation, and application of neural networks. Course material, which was complemented by notes and video lectures, covered concepts from functional and harmonic analysis and optimization theory, which they used to explore the mathematical underpinnings of deep learning.

Participants also experimented with multiple deep-learning applications such as computer vision and forecasting, and listened to additional lectures by industry practitioners who work on real-life problems. Because the 2021 school was the first iteration of G2S3 to take place in Africa, it garnered a lot of interest. A total of 46 attendees represented 36 nations from 17 different countries around the world. About 50 percent of participants were African, and only a few of these students were located in Africa; this was thus the first school of its kind to have such significant participation from Africa itself. Most participants were graduate students in either Ph.D. or M.S. programs, and roughly 31 percent were female. Figure 1 provides a more detailed breakdown of attendance.

The school was held in a hybrid format due to COVID-19. Although most attendees participated virtually, several in-person cohorts did work together at the AIMS centres in Rwanda and Cameroon. The lectures and tutorials took place on Zoom, with the social sessions transpired on Wonder. Daniel Nickelsen (AIMS South Africa) served as the moderator.

Barry Green, the director of AIMS South Africa, delivered the opening remarks that marked the beginning of G2S3 activities. As with any virtual event, time zone differences added a logistical complication. All lectures were scheduled for the evening in Africa and Europe, which corresponded to the morning and early afternoon in North America. Lecturers included Babacar Bah (AIMS South Africa), Coralia Cartis (University of Oxford), Gitta Kutyniok (Ludwig Maximilian University of Munich), Kasia克斯托夫 (Tufts University), and Jared Tanner (University of Oxford).

The Organizing Committee—which consisted of Bah, Cartis, Kutyniok, and Oksanen—arranged tutorials in the afternoon for cohorts in Africa and Europe that were then repeated for the North American cohort at a later time. A social event for all cohorts in the form of a coffee/tea break took place between the two main lectures every day. Students, tutors, and lecturers were invited to bring their coffee, tea, or beverage of choice and spend some time interacting with each other on Wonder. Despite modest participation (likely due to fatigue from excessive screen time), the virtual gatherings nonetheless generated a sense of international interaction between participants. In contrast, the high-quality lectures, tutorials, demonstrations, practicals, and exercises were extremely interactive. The students greatly appreciated and benefited from these learning opportunities, which offered them a great place to connect with fellow peers around the world.

At the school’s conclusion, we solicited feedback from attendees to gauge their levels of satisfaction with both the program and implementation. The response was overwhelmingly positive. “It was very useful to have a wide variety of subjects covered,” said one. Another praised the format of the tutorials, adding that there was “enough information” to represent the breadth of the course material. “Students would get lost in the material.” A third student reflected on the structure of the school as a whole. “The thought put

1. https://aims.ac.za
2. https://sites.google.com/aims.ac.za/g2s3

Figure 1. Breakdown of attendance at the 11th Gene Golub SIAM Summer School by gender and current research status. Figure courtesy of SIAM.
Alumni Panel Series Offers a Glimpse into Life as an Early-Career Mathematician

University of North Carolina at Chapel Hill SIAM
Student Chapter Organizes Inaugural Career Event

By Katherine Slyman

On the fourth floor of Phillips Hall at the University of North Carolina (UNC) at Chapel Hill, mathematics graduate students have various iterations of the same conversation within their respective cohorts. Where will my degree take me in life? What am I going to do with my doctorate in mathematics? What kind of career should I pursue? These types of questions are constantly present in the minds of all Ph.D. students, regardless of year or subject. Most early-stage mathematicians have broad ideas about their career paths when they start their programs, but they tend to develop more interests—as well as some doubts—as the programs proceed.

In short, students generally crave insight into a prospective career before actually committing to that path. For example, they might wonder how their academic degrees will earn them jobs in business, industry, or government. Suddenly, they find that they are finishing their dissertations and still do not know what comes next. We sought to break this cycle and provide mathematics graduate students with firsthand accounts of different career possibilities.

To do so, the UNC Chapel Hill SIAM Student Chapter began hosting the UNC Math Alumni Career Panel Series. Katherine Slyman and Katherine Daftari—the chapter’s co-presidents—organized the first-ever panel, which was inspired by the aforementioned questions and general graduate student interest in utilizing the knowledge and stories of previous students who had recently entered postdoctoral life (much more recently than some of the faculty with whom they work). The three-day event, which took place in November 2021, offered all mathematics graduate students the opportunity to learn from the experiences of a diverse group of successful graduates in teaching positions, postdoctoral appointments, and industry jobs.

It also afforded students and mentors opportunities to see how data is collected, processed, analyzed, interpreted, visualized, and shared with a variety of stakeholders, and that information about the student experience is compiled. The experience is project-based that students collaborate with each other and their community to investigate questions of interest through data visualization, modeling, prediction, analysis, and communication. Student teams are structurally similar to teams in the workforce, as every individual contributes their own assets to each project. These efforts are essential to the creation of a strong and diverse undergraduate and graduate workforce. A data science skillset is valuable, easily transferable across disciplines, and fills a critical need in most workplace settings.

Students who study and practice data science are more likely to see them- selves in different data science roles based on their unique strengths in areas like statistics, program- ming, and visualization. Taliha Washington, director of the Data Science initiative, describes the value of diverse data science positions, which can employ an asset-based approach and build student agency. This effort will also help data science education move beyond a deficit model that is driven by accountability and problems in practice, and towards an asset model that is driven by a pos- itive approach that examines student strengths, interests, and cultures.

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During the University of North Carolina (UNC) Math Alumni Career Panel Series—hosted by the UNC Chapel Hill SIAM Student Chapter—a diverse group of successful graduates in teaching, postdoctoral appointments, and industry jobs described their experiences and career-related decisions during graduate school. Figure courtesy of Katherine Daftari.

Pathways in Data Science

Continued from page 3

The data science standards seek to ensure that data is collected, processed, analyzed, interpreted, visualized, and shared with a variety of stakeholders, and that information about the student experience is compiled. The experience is project-based that students collaborate with each other and their community to investigate questions of interest through data visualization, modeling, prediction, analysis, and communication. Student teams are structurally similar to teams in the workforce, as every individual contributes their own assets to each project. These efforts are essential to the creation of a strong and diverse undergraduate and graduate workforce. A data science skillset is valuable, easily transferable across disciplines, and fills a critical need in most workplace settings.

Students who study and practice data science are more likely to see themselves in different data science roles based on their unique strengths in areas like statistics, programming, and visualization. Taliha Washington, director of the Data Science initiative, describes the value of diverse data science positions, which can employ an asset-based approach and build student agency. This effort will also help data science education move beyond a deficit model that is driven by accountability and problems in practice, and towards an asset model that is driven by a positive approach that examines students’ strengths, interests, and cultures.

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Pathways in Data Science

Continued from page 3

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Sofia Kovalevskaya: Mathematician and Writer

In 1874, Kovalevskaya completed her thesis, which consisted of three papers: her aforementioned work on PDEs, a paper on the dynamics of Saturn’s rings, and a paper on elliptic integrals. Weierstrass successfully persuaded the University of Berlin to issue Kovalevskaya a doctorate even though she never attended classes or defended her thesis. For many years afterward, however, she was unable to obtain any kind of university position—no institute in Germany or Russia would consider such a thing. Finally, in 1888, thanks to the efforts of Gösta Mittag-Leffler, Kovalevskaya received an appointment as privy-docent at Stockholm University. In 1883, she was later promoted to ordinary (full) professor in 1889. Kovalevskaya led an altogether extraordinary life in her 41 years. She was a socialist, a feminist, and a radical. One illustrative occurrence was an encounter in early 1871 during the aftermath of the Franco-Prussian War. At the time, Paris was ruled by the Commune and under attack from the national army. Kovalevskaya and her husband traveled from Berlin and somehow smuggled themselves past the nearby German lines and into the besieged city to help her beloved elder sister, Anna, who was deeply involved with the Commune.

Kovalevskaya was also a gifted writer. Her best-known work, A Russian Childhood, is a personal memoir that is evocative, perceptive, moving, and extremely entertaining. Her account of novelist Fyodor Dostoevsky’s unsuccessful courtship of her sister Anna is a masterpiece of narrative and vivid characterization. While I have not read her semi-autobiographical novel, Nihilist Girl, it is said to be remarkable. Translator Sandra DeLozier Coleman’s engagement with Kovalevskaya is itself a fascinating story that Coleman recounts in great detail. In addition to her 30-year career as a mathematics professor, Coleman is a poet, writer, and artist. In 1996, she served as a book reviews editor for AMATIC Review (a publication of the American Mathematical Association of Two-Year Colleges) and penned a review of three then-recent biographies of Kovalevskaya. Coleman became obsessed—the word is not too strong—with Kovalevskaya’s work and avowed to translate her writings, despite the fact that she did not even know the Cyrillic alphabet at the time. In the 25 years since—and assisted by several Russian-speaking colleagues and friends—Coleman has pursued this goal with immense energy and dedication.

Her new book, Mathematician with the Soul of a Poet, is the fruit of these labors. The text includes translations of nine of Kovalevskaya’s poems and a pair of plays that were not previously translated into English, along with extensive commentary and biographical analysis. It also contains a translation from German of a short poem by Weierstrass—a toast that he gave his own 70th birthday.

Kovalevskaya’s poems are a mixed bag. They are mostly written in rhyme and meter—all but one is in free verse and another is a prose poem. Thematically, they include semi-humorous descriptions of her married life (one is even written from her husband’s point of view!) personal, introspective musings; and a strange fantasy of a young girl who daydreams about becoming a martyr. Coleman’s translations with extreme care to convey not only the meaning but also the meter, rhyme scheme, and music of the original works. Doing so is often very challenging. For instance, Kovalevskaya’s 32-line poem called “If You in Life” alternates between 12-syllable and 10-syllable lines and sticks to two rhymes in an AABAB scheme for its entire duration. The translation faithfully follows the poetry’s complex rhymes.

Kovalevskaya’s two plays—How It Was and How It Might Have Been—are “parallel to that form a pair of didactic dramas” that form a pair called The Struggle for Happiness. They were written collaboratively by Kovalevskaya and her friend Anne Charlotte Edgren-Leffler, who was an actress, writer, and sister of Mittag-Leffler. The two plays feature the same characters and opening situation.


cos(βα + ε) = cos β cos α + sin β sin α

Here is a simple physical interpretation of the familiar identity

This equation is the identity (1) in disguise.

Each of the three terms in (1) acquires a special meaning, just like in (2).

Depending on one’s standards of rigor, the aforementioned expression might be considered a proof. I did not strive for full rigor, trying to keep this piece short.

© Sofia Kovalevskaya 1890

A partnership between SIAM and COMAP, Guidelines for Assessment and Instruction in Mathematical Modeling Education (GA&ME) enables the modeling process to be understood as part of a STEAMS education curriculum and, taught as a basic tool for problem solving and logical thinking. GA&ME helps define core competencies to include in student experiences, and provides direction to enhance math modeling education at all levels.

The second edition includes changes primarily to the Early and Middle Grades (K–8) chapter.

The GA&ME report is freely downloadable from both the COMAP and SIAM websites (summaries/articles/reports) in English and Spanish.

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July 2022 SIAM NEWS • 7
The computing field is currently experiencing a management crisis. In 2017, Wall Street Journal columnist Peggy Noonan described Silicon Valley executives as “moralMartians who operate on some new world now postmodern ethical wavelength.”

The problem of present-day computing lies not with AI technology per se, but with its current use in the computing industry. AI is the fundamental technology behind “surveillance capitalism,” which Shoshana Zuboff defines as an economic system that centers on the commodification of personal data with the core purpose of profit-making. Under the mantra of “information wants to be free,” several tech companies have become advertising companies and perfected the technology behind micro-targeted advertising, using us to construct a “surveillance society.” By 2021, more than 1,000,000 people worldwide year. Nevertheless, the fatality rate has been decreasing over the past 100 years with improved road and automobile safety, licensed drivers, drunk driving laws, and the like. The solution to automobile crashes is not ethics training for drivers, but it is a public policy, which makes transportation safety a public priority. I share this ethics skepticism with Dutch philosopher Ben Wagner, who wrote that “...[m]uch of the debate about ethics seems to provide an easy alternative to government regulation.”

At the same time, democracy, or we can have a surveillance society,” she wrote in 2021 article in The New York Times, “but we cannot have both.” [10]. Internet companies are using AI technology to support a business model that is arguably unethical.

The biggest problem that computing faces today is not that AI technology is unethical—but that large and powerful corporations use AI technology to support a business model that is arguably unethical.
Plan to attend the 2022 AWM Research Symposium showcasing research from women in the mathematical sciences from across the career spectrum in academia, government and industry.

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The Rational Krylov Toolbox

By Stefan Güttel

Rational Krylov methods have become an indispensable tool in the field of scientific computing. Originally invented for the solution of large, sparse eigenvalue problems, these methods have seen an increasing number of other uses over the last two decades. Example applications include model order reduction, matrix function approximation, matrix equations, non-linear eigenvalue problems, and nonlinear rational least squares fitting.

Here I aim to provide a short introduction to rational Krylov methods, using the MATLAB Rational Krylov Toolbox (RKToolbox). The RKToolbox is freely available online and features an extensive collection of examples that users can explore and modify. The following text contains pointers to select RKToolbox examples, which are printed in typewriter font. Readers can also utilize these examples for teaching purposes, as they might serve as good starting points for computational coursework projects.

Rational Krylov: A Brief Introduction

A rational Krylov space is a natural generalization of the more widely known standard Krylov space. The latter is defined as

$$K_m \equiv \text{span}\{ b, Ab, …, A^{m-1}b \}$$

where $A$ is a square matrix and $b$ is a nonzero vector of computable size. We can represent every element in $K_m$ as a polynomial $r(A)b$. On the other hand, a rational Krylov space is a linear space that is spanned by rational functions $r(A)$.

For illustrative purposes, assume that we have three distinct complex numbers $\zeta_1$, $\zeta_2$, and $\zeta_3$, all of which are different from $A$'s eigenvalues. We can then define the fourth-dimensional rational Krylov space as

$$Q_m \equiv \text{span}\{ b, (A - \zeta_1 I)^{-1}b, (A - \zeta_2 I)^{-1}b, (A - \zeta_3 I)^{-1}b \}$$

(in artificial cases, $Q_m$ can have a dimension that is less than four, but we assume that this does not happen here). In order to work with $Q_m$, numerically, we compute an orthonormal basis with a method that is very similar to the standard Arnoldi algorithm. This method computes a matrix decomposition of the form

$$A[v_1, v_2, …, v_m] = [r_1, r_2, …, r_m]$$

where the columns of $V$ form an orthonormal basis of $Q_4$ and the rectangular four-by-three matrices $H$ and $K$ are of upper block Hessenberg form (i.e., all entries below the first subdiagonal are zero).

The computation of (2) requires the solution of three shifted linear systems with $A$. This requirement is typically the computational bottleneck of rational Krylov methods, but there is potential for coarse-grain parallelism if we solve these systems simultaneously; more information is available in the RKToolbox example parallel.html.

Eigenvale Computations

Decompositions of the form (2) play a central role in eigenvalue computations [4]. It turns out that Ritz values of appropriate submatrices of $H$ and $K$—so-called Ritz values—may actually approximate some of $A$’s eigenvalues quite well. This capability is completely analogous to the way in which we use the standard Arnoldi and Lanczos algorithms to approximate eigenvalues of large matrices. If $A$ is a symmetric matrix, we even have a good theoretical understanding of which eigenvalues of $A$ are well approximated by Ritz values [2] (Figure 1). Figure 1 illustrates a neat electrostatic interpretation of this concept. The RKToolbox provides examples for the numerical solution of linear and nonlinear eigenvalue problems at example_example_eigenproblems.html and example_example_nlep.html.

RKFUN Calculus

A useful feature of the decomposition in (2) is that matrices $H$ and $K$ encode rational functions, or so-called RKFUNs. The RKToolbox uses this matrix representation to enable an array of numerical operations with rational functions. The object-oriented MATLAB implementation of RKFUNs is provided by the Chebfun package [3]. In addition to root and pole finding, it is possible to add, multiply, and concatenate RKFUNs—as well as convert them into partial fraction, quotient, and continued fraction forms. We can implement all of these operations with established numerical linear algebra routines. For example, the conversion to continued fraction form utilizes the nonsymmetric Lanczos tridiagonalization process.

RKToolbox demonstrations of RKFUN calculus include example_rkfun.html, example_example_feast.html, and example_example_filter.html. Figure 2 is an example of rational filter construction. As Figure 3 illustrates, we can easily plot the filters with a command such as

```matlab
plot(butterworth([0,0.2]), elliptic([0,0.2]), chebyshev([0,0.2]))
```

The elliptic filter—also known as the Cauer filter—is based on Zolotarev’s equioscillating rational functions, which are implemented in the RKToolbox examples, which are printed in typewriter font. Readers can also utilize these examples for teaching purposes, as they might serve as good starting points for computational coursework projects.

The book’s title, Mathematics with the Soul of a Poet, is derived from a passage in a letter by Kovalevskaya.

Eliot’s passing. Yet whether poet or prosist, Kovalevskaya’s mind and soul were certainly extraordinary, and Coleman’s labor of love sheds a new and interesting light on the esteemed mathematician.

Many researchers suppose that the mathematician she quotes was Weierstrass, and that Weierstrass was speaking not just of mathematics, but in general but of Kovalevskaya specifically.

Judging by her literary works, I would say that Kovalevskaya had more the soul of a gifted writer of prose than the soul of a poet. I do not know why the former

would be considered of lesser prestige. Moreover, she seems to have had an affinity for other great writers of prose; she remained a close friend of Dostoevsky until his death and visited George Eliot at her house a number of times—she even wrote an insightful reminiscence in tribute after Eliot’s passing. Yet whether poet or prosist, Kovalevskaya’s mind and soul were certainly extraordinary, and Coleman’s labor of love sheds a new and interesting light on the esteemed mathematician.

Ernest Davis is a professor of computer science at New York University’s Courant Institute of Mathematical Sciences.

See Rational Krylov on page 11

1 http://skeetlet.com/kikobox/examples/html/example_rkfun.html
2 http://skeetlet.com/kikobox/examples/html/example_example_eigenproblems.html
3 http://skeetlet.com/kikobox/examples/html/example_example_nlep.html

| Ritz values | Butterworth | Chbyshev
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Figure 1. Electrostatic interpretation of rational Ritz values. Atoms in a grey disk are placed on a wire (black line). The with-order Ritz values (blue disk) will then tend to distribute like electrons on the wire, satisfying the following properties: (i) Every two Ritz values are separated by at least one eigenvalue, (ii) the Ritz values repel each other, and (iii) they are attracted by an external field that is produced by the poles (red disk on the left); one can think of these poles as positive charges. Ritz values hence tend to find eigenvalues that are near the poles and in regions with few other eigenvalues.

1. Butterworth, Chebyshev and Elliptic filter forms.
2. Example of rational filter construction.
Public Policy
Continued from page 9

The computing community must address its relationship with surveillance capital-
ism corporations. For example, the ACM’s ACM Turing Award 2021—the highest
award in computing—is now accompanied by a prize of $1 million that is supported by
Google. Yet relationships with tech com-
panies are not the only quandary. We must
also consider the way in which society
views officers and technical leaders with-
in these companies. Holding community
members accountable for the decisions of
the institutions that they lead raises serious
questions. The time has come for mature and nuanced conversations about respon-
sible computing, ethics, corporate behavior,
and professional responsibility.

That being said, it is unreasonable to expect for-profit corporations to avoid prof-
able and legal business models. Ethics cannot be the remedy for surveillance capi-
talism. If society finds the surveillance busi-
ness model offensive, the remedy should be public policy—in the form of laws and
regulations—rather than an ethics outrage.

Of course, we cannot divest public policy from the companies and the human organ
trading because we find it ethically repugnant, but the ban is enforced via public
policy rather than ethical debate.

HIPAA informs the IT industry (ITI) has successfully lobbied for decades against
its relationship with surveillance capital-
isation corporations. For example, the ACM’s
Institute for Ethics in Artificial Intelligence
has successfully lobbied for decades against
its relationship with surveillance capital-
isation corporations. For example, the ACM’s
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Rational Krylov
Continued from page 10

in the RKToolbox’s gallery (a collection of special rational functions).

Model Order Reduction

Model order reduction is one of the key drivers of research on rational Krylov
methods. Interpolatory Methods for Model Reduction 11) particularly chapters three
and five therein—describes the way in which rational Krylov spaces serve as natu-
ral projection spaces for large-scale, linear time-invariant (LTI) dynamical systems.
The iterated rational Krylov algorithm (IRKA) is of particular interest for the opti-
mal interpolatory reduction of LTI systems in the H₂ norm. The RKToolbox contains
several model order reduction problems, such as example_frequency.html.

example_iss.html
10 and example_cdplayer.html
11

Here I have provided a brief introduction to rational Krylov methods and referenced a number of applications, like eigenvalue
problems and model order reduction.

Krylov-based model order reduction is very closely linked with rational approxi-
mation. In a follow-up article to appear in the next issue of SIAM News, the author will discuss the RKFIT method for non-
linear rational approximation—a core algorithm in the RKToolbox.

All figures are courtesy of the author.

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Moshe Y. Vardi is a university professor and the Karen Ostrum George Distinguished
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Private Artificial Intelligence: Machine Learning on Encrypted Data

By Kristin Lauter

Artificial intelligence (AI) refers to the science of utilizing data to formulate mathematical models that predict outcomes with high assurance. Such predictions can be used to make decisions automatically or to re-encrypt models. However, traditional encryption schemes do not allow for any computation on encrypted data. We therefore need a new kind of encryption that maintains the data’s structure so that meaningful computation is possible. Homomorphic encryption allows us to encrypt the data, perform computation on it, and then decrypt the result. This is achieved through a homomorphic encryption scheme that can process any circuit was first proposed in 2009 [3]. Since then, cryptographic libraries and frameworks have been developed to make such solutions practical in a wide range of applications.

A homomorphic encryption scheme can be used in various applications, including: 1) privacy-preserving machine learning, 2) secure cloud computing, 3) secure data sharing, and 4) secure data analytics. These applications allow users to perform computations on encrypted data without revealing the underlying plaintext.

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Figure 1. Diagram of homomorphic encryption. If one begins with two pieces of data a and b, the output is ab, which can be decrypted to recover a and b. The arrows in either direction across and then down (compute, then encrypt or down and then across (encrypt, then compute) Figure courtesy of SIAM.

Figure 2. Private cloud service to predict health outcomes on encrypted medical data. Figure courtesy of SIAM.