A Precoding Approach for Dual-Functional Radar-Communication System With One-Bit DACs

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***Abstract*— In this paper, we investigate the precoder design for multiple-input multiple-output (MIMO) dual-functional radar-communication (DFRC) system with one-bit digital-to-analog converters (DACs). In order to form the dual-functional beampattern, we formulate the precoding problem as a weighted optimization problem with the constant modulus constraint, which aims at minimizing the average error power and guaranteeing radar waveform similarity. The problem is divided into three sub-problems corresponding to the multiple variables, i.e., the precoding factor, transmit signal matrix, and radar waveform matrix. Due to the discrete and non-convex properties of the optimization problem, we propose a multi-variable alternating minimization (MVAM) framework to achieve the near-optimal solutions. The precoding factor and radar waveform can be solved in closed-forms. For the transmit signal matrix, we devise a binary particle swarm optimization-simulated annealing (BPSO-SA) algorithm to obtain it under the MVAM framework. Extensive simulations validate the effectiveness of the proposed approach under various scenarios, including the case without perfect channel state information. The simulation results show that, compared with existing non-linear precoders, the proposed approach achieves 7dB SNR gain at the bit error rate of 10^{-4} in the 8-antenna system, and the gain of SNR is 0.2dB in the massive MIMO system with 128 antennas.**

***Index Terms*— Dual-functional radar-communication, massive multiple-input multiple-output, digital-to-analog converter, alternating minimization.**

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I. INTRODUCTION

DUAL-FUNCTIONAL radar-communication (DFRC) system has emerged as a promising paradigm to simultaneously enable radar sensing and wireless communications [1] [2]. Benefit from the efficient spectrum sharing, the DFRC is able to address the explosive need of frequency spectrum resources for a huge number of wireless connected devices. In particular, the DFRC system, employing the massive multiple-input multiple-output (MIMO) as a core technique, is capable of achieving better capacity, higher spectral efficiency and superior target detection performance [3]. The massive MIMO equipped in DFRC can reduce inter-user interference via precoding for downlink. However, it inevitably brings high-cost hardware and high circuit complexity for fully-digital DFRC system. To settle the practical constraints, decreasing the cost of radio frequency (RF) chains is considered with the aid of configured low-resolution digital-to-analog converters (DACs) [4]–[6].

The deployment of DACs in a massive MIMO DFRC structure is confronted with a key issue [7], that is, the circuit power consumption caused by data converters is made exponentially proportional to the quantization level, and more than eight-bit DAC quantizers are generally used in conventional MIMO systems, thereby resulting in prohibitively high power consumption. Besides, the demand for hardware complexity and cost of high-resolution converters is a challenge in full-digital massive MIMO DFRC system [8]. In view of this, it is desirable to study ultra low-resolution DACs, i.e., one-bit DACs [9], which can reduce the requirements of high linearity and low noise in the surrounding RF circuitry, in the meantime mitigating the hardware design bottleneck and power consumption. On the one hand, the constant modulus (CM) output signal enables the RF chain to be equipped with low-cost and efficient power amplifiers, which further reduces the hardware complexity [10]. On the other hand, the one-bit DACs precoding problem involves a discrete and nonconvex constraint. Consequently, it is challenging to implement reliable transmission under coarse quantization.

Previous works have investigated the design of one-bit precoder in the massive MIMO systems for downlink transmission. In [11], the authors provide a simple linear quantized maximal ratio transmission (MRT) scheme with low complexity. The authors in [12] propose the zero-forcing (ZF) precoding. In [13], a minimum mean-square error

(MMSE)-optimal quantized approach is investigated, which exhibits more improved performance in bit error rate (BER) than the ZF precoder. Although the above linear quantized schemes have simple computational complexity, these precoders are unable to support reliable and stable transmission as a result of coarse one-bit quantization. Further, the attempt to design non-linear precoding with one-bit DACs is proposed based on convex relaxation [14], which can relax the non-convex alphabet constraint to a convex problem, and hence achieves superior performance of non-linear algorithms. The authors in [15] consider the semi-definite relaxation, showing a precoder with robust performance has higher computational complexity. To enhance efficiency, the authors in [16] use squared-infinity norm Douglas-Rachford splitting (SQUID) to develop an algorithm with one-bit DACs, where only straightforward operations of matrix are needed during each iteration. In [17], two low complexity methods via biconvex relaxation, called C1PO and C2PO, are presented. In [18], the authors utilize neural-network optimization to tune parameters of C2PO method, which significantly improves precoder's performance. The design of one-bit precoder have attracted extensive attention for MIMO communication, while there is relatively little work to expand one-bit precoder for radar. For example, the radar beam-pattern design has been shown in [19] that uses an approximation approach to obtain solutions for the one-bit DACs case, where the objective function is subject to the weighted squared-error. In [20], the radar waveform is designed based on the integrated sidelobe to mainlobe ratio (ISMR) criterion.

Nevertheless, the existing precoding methods primarily focus on either the wireless communication scenario or the radar case, i.e., the output signal generated in the base station (BS) with one-bit DACs supports multi-user downlink communication or target detection, while ignoring the collaborative design of the dual function. By far, the precoder design for the DFRC system using one-bit DACs is widely unexplored, as this topic is much more complicated and requires to follow performance metrics of communication based on specific radar criterion. By introducing constraints on the similarity of specific radar waveforms, the aforementioned one-bit precoding algorithms are not suitable for the DFRC scenarios. Besides, the transmitted signal based on 1-bit DACs can be regarded as a constant-modulus signal with four phase values. Therefore, the objective of the design problem with one-bit quantification is a non-convex problem [17]. Due to the NP-hardness of such problems, traditional methods based on convex relaxed versions, such as SQUID, C1PO and C2PO, will cause regrettable performance loss.

Motivated by the aforementioned limitations, we strive to design a novel one-bit precoder for the MIMO DFRC system, where the transmit signal carries out target detection and downlink communication simultaneously. Our goal is to obtain a better peak-to-sidelobe ratio (PSLR) of the waveforms at radar side and lower error rate performance at the communication side. To that end, we formulate the dual-functional precoder design as a weighted optimization problem so as to minimize the mean value of the power of the downlink residual error and approach specific radar waveform. The formulated optimization is considered as an NP-hard problem due to the quantization constraints on the real and imaginary parts of the transmitted signal [21]. Inspired by [22], the existing block coordinate descent (BCD) algorithm based on the alternating minimization (AM) can use separable reformulations to solve the problem of separately constrained variables. However, the traditional alternative minimization method faces serious challenges in designing DFRC one-bit precoder. *i)* the generic AM method focuses on a function of two separately constrained variables, while our problem involves triples (i.e., transmitted signal, radar waveform, precoding factor) and the inter-correlation among these three variables is more complicated; *ii)* for the general case, the AM framework requires each subproblem being convex so that ensure that the function value monotonically converges to the lower bound on the constraint set. This is often not the case in our problem since such convex assumption does not hold.

To cope with these challenging issues, we propose a multi-variable alternating minimization (MVAM) framework, via introducing a proximal term and a dynamic inertial parameter to remove the strict convexity assumption, hence providing a feasible solution for the formulated optimization problem. To specify, within the proposed MVAM framework, we conduct convex and non-convex reformulations corresponding to the sub-problems of multiple variables. The sub-problem of precoding factor and radar waveform matrix are convex, each iteration in the related algorithm can be solved in closed-forms. For non-convex subproblems of transmitted signals, the classical precoding algorithms, such as SQUID, C1PO and SDR, will suffer performance loss caused by relaxation of non-convex alphabet constraints. While binary particle swarm optimization (BPSO) algorithm that guide the evolution of optimal solution is easy to fall into local optimal solution, we introduce simulated annealing (SA) to avoid premature convergence. The proposed novel hybrid algorithm, namely, binary particle swarm optimization-simulated annealing (BPSO-SA), does not involve relaxation normalization and enhance the local search ability, thereby obtaining the near-optimal solution defined in the discrete set.

The contributions of this paper are highlighted as follows.

- We deploy one-bit DACs to massive MIMO DFRC downlink system, and formulate the precoder design as weighted optimization problem to realize radar sensing and downlink communication simultaneously. Particularly, the formulated optimization problem is dedicated to minimize the mean value of the power of the residual error at downlink and guarantee specific radar waveform similarity. In this way, the DFRC system can achieve a better PSLR of the waveforms for radar sensing and superior error rate performance of communication.
- We design the iterative framework called MVAM, which introduce a proximal term and a dynamic inertial parameter, to relax the convergence conditions of original AM algorithm. The proposed MVAM improves the convergence speed, and solve the formulated optimization problem of the precoder design. Specifically, we derive the closed-form solutions for the sub-problems of precoding factor and radar waveform matrix. For the non-convex

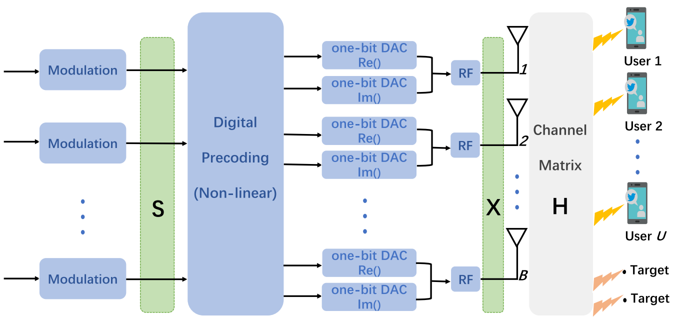


Fig. 1. Massive MIMO DFRC downlink system with one-bit DACs.

sub-problem of transmitted signal, which is caused by the one-bit quantization, we design the BPSO-SA algorithm to optimize the transmitted signal under the MVAM framework.

- We conduct extensive simulations and the results validate that the proposed method can outperform the conventional precoders, in the meantime, the performance of radar sensing is guaranteed. Besides, the proposed method obtains reliable performance at the communication side in the case of imperfect channel state information (CSI).

The remainder of this paper is organized as follows. In Section II, we outline the system model. In Section III, we propose the problem formulation for DFRC one-bit precoder design and analyze the multi-bit quantization. After that, we solve the problem via the BPSO-SA algorithm under the MVAM framework in Section IV. In Section V, we present the simulation results. Finally, we conclude the paper in Section VI.

Notation: Without special note, bold lowercase letters, uppercase boldface letters and normal font designate column vectors, matrices and scalar, respectively. For a matrix $\mathbf{A}$, we denote its transpose, Hermitian transpose and complex conjugate by $\mathbf{A}^T$, $\mathbf{A}^\dagger$ and $\mathbf{A}^*$, respectively. $a_{i,j}$ indicates the entry on the i-th row and on the j-th column of $\mathbf{A}$, $\mathbf{a}_n$ represents the n-th vector. For a vector $\mathbf{b}$, we use b_k to denote the k-th element in $\mathbf{b}$. $tr(\cdot)$ stands for the trace of a matrix, and $vec(\cdot)$ is the vectorization operations. $\mathbf{I}_N$ is an identity matrix with dimension $N \times N$. $\mathbb{E}[\cdot]$ represents the expectation operator, and j denotes the imaginary unit. $\Re(\cdot)$ and $\Im(\cdot)$ extract the real and imaginary part of the argument. $\mathcal{N}\left(\mu, \epsilon^2\right)$ is a complex Gaussian random variable with mean μ and variance ϵ^2. $\otimes$ represents the Kronecker product. We use $\|\cdot\|_2$ and $\|\cdot\|_F$ to denote the l_2 norm and the Frobenius norm, respectively. Table I presents the notations of the main variables.

II. System Model

The massive MIMO DFRC model includes B antennas and serves U single-antenna user equipments (UEs). As illustrated in Fig. 1, a pair of one-bit DACs is equipped with each antenna. We assume that the system supports target sensing and downlink communication with the same time-frequency resource, which is fitted with a uniform linear array (ULA).

TABLE I
List of Key Notations

B	Number of transmitting antennas
U	Number of all the users
P_T	The total transmit power
$\otimes$	The Kronecker product
ρ	The precoding factor
$\mathbf{P}$	The precoding diagonal matrix
δ	The weighting factor for dual functions
$\mathcal{P}$	The precoding strategy of DAC quantization
$\mathcal{Q}$	The quantization block
$\mathcal{X}_{\text{DAC}}$	The complex valued quantization alphabet
θ_q	Azimuth angle of the q-th target
$\mathbf{a}(\theta)$	The transmit steering vector
$\mathbf{v}_i$	The velocity of i-th particle
$\mathbf{d}_i$	The position of i-th particle
$\mathbf{X}$	The transmit signal matrix
$\mathbf{H}$	The channel matrix
$\mathbf{Y}$	The signal matrix received by downlink users
$\mathbf{Z}$	The additive Gaussian noise matrix
$\mathbf{S}$	The desired constellation symbol matrix at UEs
$\mathbf{C}$	The covariance matrix of ideal radar signal
$\mathbf{X_R}$	The matrix of ideal radar waveforms
$\bar{\mathbf{X}}$	The unquantized transmit signal matrix

A time-division duplex (TDD) system is considered, and we focus on the radar transmitting beamforming and downlink communication.

A. Transmit Model With One-Bit DACs

In the one-bit MU-MIMO DFRC system, the transmitted signal matrix, i.e., the quantized matrix, is denoted by $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_L]^T \in \mathbb{C}^{B\times L}$, where L is the number of snapshots in a radar pulse. The number of symbol frames, i.e., $\mathbf{x}_l \in \mathbb{C}^{B\times 1}$ is employed for detecting target in the l-th snapshot and transmitting the l-th symbol frame to the downlink UEs. For the UEs, $\mathbf{S} \in \mathbb{C}^{U\times L}$ is the ideal symbol matrix, whose entry can be drawn from the same constellation. We consider a block-fading scenario. The channel matrix $\mathbf{H} \in \mathbb{C}^{U\times B}$ depends on the diagonal matrix of channel gains for each user and matrix $\bar{\mathbf{H}}$. Hence we can define $\mathbf{H} = diag\{C_{g1}, C_{g2}, \ldots, C_{gU}\}\bar{\mathbf{H}}$, where $\{C_{g1}, C_{g2}, \ldots, C_{gU}\} \in \mathbb{C}^{U\times U}$ denotes the diagonal matrix composed of individual channel gains, C_{gu} is the channel gains of u-th UE, and each entry of $\bar{\mathbf{H}}$ follows a standard complex Gaussian distribution with zero-mean and unit-variance. We assume that the CSI is perfectly known to the BS by pilot symbols. In practice, as the perfect CSI assumption is questionable, the CSI of the receiver can be obtained through training and then shared with the transmitter through limited feedback. The channel reciprocity in TDD system can be utilized to use channel training on the reverse link and obtain the channel estimation at the base station, i.e., the channel estimation of the downlink can be obtained from its uplink counterpart [23].

When one-bit DACs are employed, the elements in matrix $\mathbf{X}$ will be quantized on both real and imaginary parts, which leads to

$$\mathbf{X} = \mathcal{Q}(\mathcal{P}(\mathbf{S}, \mathbf{H})). \tag{1}$$

In (1), $\mathcal{P}$ repesents the nonlinear or linear precoding strategy to map the symbol matrix $\mathbf{S}$ into the unquantized matrix [24]. The quantizer-mapping function $\mathcal{Q}$, which is used to quantize the real and imaginary parts of per element in unquantized matrix, describes the joint operation of a pair of DACs at the BS. Hence, the entries in $\mathbf{X}$ can be given as

$$x_{i,j} \in \mathcal{X}_{\text{DAC}}, \quad \forall i \in \mathcal{I}, \quad \forall j \in \mathcal{J}, \tag{2}$$

where $\mathcal{I} = \{1, 2, \ldots, B\}$, $\mathcal{J} = \{1, 2, \ldots, L\}$, and $\mathcal{X}_{\text{DAC}}$ denote the quaternary symbol transmited by each antenna in the MIMO DFRC system.

B. Communication Model

The matrix $\mathbf{Z} = [\mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_L] \in \mathbb{C}^{U \times L}$ represents the additive Gaussian noise matrix at the receiver side with zero mean and covariance N_0. $\mathbf{z}_j \sim \mathcal{CN}(0, N_0 \mathbf{I}_B)$, for $j = 1, 2, \ldots, L$. The input-output relation can be expressed by

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{Z}. \tag{3}$$

According to [15], the desired symbol signal of the u-th user in l-frame $s_{u,l}$ can be recovered as $\hat{s}_{u,l}$ from its received signal $y_{u,l}$, that is $\hat{s}_{u,l} = \rho_u y_{u,l}$, wherein $\rho_u \in \mathbb{R}$ is a precoding factor. We consider the block-fading [25] in the communications model, hence the matrix $\mathbf{H}$ maintains quasi-static within a fading block (over all L time slots). In addition, the continuous symbols in this period of time suffer the same fading. Accordingly, for the same user, the precoding factor is regarded as constant within L symbol intervals [25].

Recall that the target of our work is to minimize the mean value of the power of the error at the user side via designing a dual-functional waveform matrix $\mathbf{X}$. Here the total mean value of downlink multi-user interference (MUI) and quantization error can be measured as

$$\mathbb{P}_{com} = \sum_{l=1}^{L} \mathbb{E}_{\mathbf{Z}} \left[\|\mathbf{s}_l - \widehat{\mathbf{s}}_l\|_2^2 \right] = \|\mathbf{S} - \rho\mathbf{H}\mathbf{X}\|_F^2 + \rho^2 U L N_0. \tag{4}$$

Inspired by [26], the improved performance of communication system can be obtained by designing appropriate precoding factor ρ and transmitted signal matrix $\mathbf{X}$. Accordingly, we formulate the optimization problem of communication side, which is given as follows.

$$\begin{aligned} &\min_{\mathbf{X},\rho} \mathbb{P}_{com} = \min_{\mathbf{X},\rho} \ \|\mathbf{S} - \rho\mathbf{H}\mathbf{X}\|_F^2 + \rho^2 U L N_0. \\ &\text{s.t. } x_{i,j} \in \mathcal{X}_{\text{DAC}}, \quad \forall i \in \mathcal{I}, \ \forall j \in \mathcal{J}. \end{aligned} \tag{5}$$

C. Radar Model

MIMO radar can transmit linearly independent probing signals allowing more Degrees of Freedom (DoFs), hence it is capable of detecting maximum multiple targets simultaneously [27]. In the case of our work, we investigate the directional beams design of MIMO radar so that formulate the beam-pattern for probing targets angles. It is noteworthy that the optimization of omnidirectional beam-pattern is a potential problem considered in recent studies such as [28].

Given the q-th detection angle, the transmit steering vector is obtained by

$$\mathbf{a}(\theta_q) = [1, e^{j2\pi \frac{k}{\lambda} sin\theta_q}, \ldots, e^{j2\pi(B-1)\frac{k}{\lambda} sin\theta_q}]^{\mathrm{T}}, \tag{6}$$

where k and λ denote the spacing of antennas and the signal wavelength, respectively. We define $k = \frac{\lambda}{2}$. Then, the designed radar beam-pattern at angle θ_q can be described as

$$\hat{b}_R(\theta_q) = \mathbf{a}^{\dagger}(\theta_q)\mathbf{C}\mathbf{a}(\theta_q), \tag{7}$$

where $\mathbf{C} \in \mathbb{C}^{B \times B}$ is the covariance matrix of radar transmitted signal designed to match the ideal radar transmit beam-pattern.

Based on the critical contribution from [29], the probing beam-pattern problem can be addressed by designing the covariance matrix of transmitted signal from MIMO radar. Motivated by this, we obtain the covariance matrix $\mathbf{C}$ via matching the ideal radar beam-pattern b_R and the designed probing beam-pattern $\hat{b}_R$. Following that, we recapture the constrained least-squares problem to obtain the approximately desired covariance matrix of radar transmit signal, which can be written as follows.

$$\begin{aligned} &\min_{\alpha,\mathbf{C}} \ \frac{1}{Q} \sum_{q=1}^{Q} \left| \alpha b_R(\theta_q) - \mathbf{a}^{\dagger}(\theta_q)\mathbf{C}\mathbf{a}(\theta_q) \right|^2 \\ &\text{s.t. } c_{bb} = \frac{P_T}{B}, \quad b = 1, 2, \ldots, B, \\ &\quad\ \mathbf{C} \succeq 0, \end{aligned} \tag{8}$$

where Q is the number of points of discretization in spatial grid covering the direction of arrival (DOA) of targets, α is the proportion parameter satisfying $\alpha \geq 0$, and $\mathbf{C}$ is the covariance matrix of radar transmit signal. To specify, the radar waveforms designed $\mathbf{X_R} \in \mathbb{C}^{B \times L}$ can be obtained by factorizing the covariance matrix $\mathbf{C}$, i.e., $\mathbf{C} = \frac{1}{L}\mathbf{X_R}\mathbf{X_R}^{\dagger}$.

Regarding problem (8), we note that the covariance matrix can be obtained efficiently via using the numerical tools as it is convex. Once $\mathbf{C}$ has been determined, the dual-functional transmitted matrix $\mathbf{X}$ can detect the target of interest via minimizing the power of the residual error between $\mathbf{X}$ and the ideal radar waveforms $\mathbf{X_R}$ [30]:

$$\min_{\mathbf{X}} \mathbb{P}_{rad} = \min_{\mathbf{X}} \ \|\mathbf{X} - \mathbf{X_R}\|_F^2. \tag{9}$$

III. Problem Formulation

The main objective of this work is to propose a novel precoding approach so that the waveform can be applied to the communication between the BS and UEs, as well as the radar sensing. By introducing a weighting factor δ $(0 \leq \delta \leq 1)$

in the DFRC system, which makes a balance between communication and radar sensing functions, we formulate the optimization problem that can be expressed by

$$\begin{aligned}\min_{\mathbf{X},\mathbf{X_R},\rho}\ & (1-\delta)\left(\|\mathbf{S}-\rho\mathbf{H}\mathbf{X}\|_F^2+\rho^2ULN_0\right)+\delta\|\mathbf{X}-\mathbf{X_R}\|_F^2\\ \text{s.t. }& x_{i,j}\in\mathcal{X}_{\text{DAC}},\quad \forall i\in\mathcal{I},\ \forall j\in\mathcal{J},\\ & \frac{1}{L}\mathbf{X_R}\mathbf{X_R}^\dagger=\mathbf{C}.\end{aligned} \tag{10}$$

Problem (10) describes that the DFRC system realizes dual function transmission under the constraint of finite resolution DACs. Here, the complex-valued quantization set $\mathcal{X}_{\text{DAC}}$, i.e., the complex-valued transmit alphabet, has finite cardinality and depends on the quantization accuracy per RF chain.

A. The MVAM Framework Based Precoding Approach

Problem (10) is considered as NP-hard [21] due to the constraint of constant modulus in real part and imaginary part. Such problem can be solved by 4^{BL} exhaustive searches, while inevitably resulting in high complexity. It entails a prohibitive complexity due to the coupled variables, i.e., precoding factor, ideal radar matrix and transmitted signal matrix. This leads to the classical convex relaxation algorithms, such as SQUID, C1PO and C2PO, being not suitable for the non-convex problem in (10).

To tame this challenge, we adopt the MVAM framework based on the decoupling of $\mathbf{X}$, $\mathbf{X_R}$ and ρ. The framework involves a three-step iterative procedure, where derive one variable while keep the other variables fixed at each internal iteration. For the precoding design of DFRC, each iteration of the framework consists of two convex problems and a discrete nonconvex problem. Specifically, the proposed method can be summarized as follows:

- We derive the precoding factor ρ by solving a problem without constraint, in which $\mathbf{X}$ and $\mathbf{X_R}$ are fixed.
- We obtain the approximately desired transmitted matrix of radar $\mathbf{X_R}$ by solving an orthogonal procrustes problem [31], given fixed $\mathbf{X}$ and ρ.
- We obtain the dual-functional transmitted matrix $\mathbf{X}$ by solving an optimization problem with the constraint of constant modulus in real part and imaginary part, in which ρ and $\mathbf{X_R}$ are fixed.

To clarify, we summarize the MVAM framework into three uncoupled steps at $(q+1)$-th iteration as follows:

$$\begin{aligned}\rho^{(q+1)}=\arg\min_{\rho\in\mathbb{R}}\ &(1-\delta)\left(\mathbb{P}_{com}(\mathbf{X}^{(q)},\mathbf{X_R}^{(q)},\rho)\right)\\ &+\delta\left(\mathbb{P}_{rad}(\mathbf{X}^{(q)},\mathbf{X_R}^{(q)},\rho)\right),\end{aligned} \tag{11a}$$

$$\begin{aligned}\mathbf{X_R}^{(q+1)}=\arg\min_{\mathbf{X_R}}\ &(1-\delta)\left(\mathbb{P}_{com}(\mathbf{X}^{(q)},\mathbf{X_R},\rho^{(q+1)})\right)\\ &+\delta\left(\mathbb{P}_{rad}(\mathbf{X}^{(q)},\mathbf{X_R},\rho^{(q+1)})\right),\end{aligned} \tag{11b}$$

$$\begin{aligned}\mathbf{X}^{(q+1)}=\arg\min_{x_{i,j}\in\mathcal{X}_{DAC}}\ &(1-\delta)\left(\mathbb{P}_{com}(\mathbf{X},\mathbf{X_R}^{(q+1)},\rho^{(q+1)})\right)\\ &+\delta\left(\mathbb{P}_{rad}(\mathbf{X},\mathbf{X_R}^{(q+1)},\rho^{(q+1)})\right)\\ &+\frac{1}{2}\|\mathbf{X}-\overline{\mathbf{X}}^{(q)}\|_F^2\end{aligned} \tag{11c}$$

$$\overline{\mathbf{X}}^{(q+1)}=\mathbf{X}^{(q)}+\beta^{(q)}\left(\mathbf{X}^{(q)}-\mathbf{X}^{(q-1)}\right), \tag{11d}$$

where $\beta^{(q)}$ is the introduced dynamic inertial parameter meets $\beta^{(q)}\in[0,1]$. In this paper, we setting $\beta^{(q)}=(q-1)/(q+2)$ and achieved good performance in the simulation.

For the MVAM framework, we first give an random initial solution of transmitted signal meeting discrete one-bit constraint to start the iterative algorithm, i.e., $\mathbf{X}^{(0)}=\left\{x_{i,j}^{(0)}\in\mathcal{X}_{\text{DAC}}\right\}$ for $i=1,2,\ldots,B$ and $j=1,2,\ldots,L$. Then, based on the mathematical derivation, we can get ρ and $\mathbf{X_R}$ from a given value of $\mathbf{X}$.

IV. The Proposed Algorithm

In order to present the aforementioned precoding approach, we decompose the formulated problem (10) into three sub-problems and solve them iteratively.

A. Sub-Problem 1 for ρ

We first consider the precoding factor ρ of the DFRC system. Given the fixed $\mathbf{X}$ and $\mathbf{X_R}$, problem (10) of optimizing ρ is defined as

$$\min_{\rho}\ \|\mathbf{S}-\rho\mathbf{H}\mathbf{X}\|_F^2+\rho^2ULN_0, \tag{12}$$

which is a convex problem without constraint, resulting in optimal ρ. The closed-form expression of ρ is given by

$$\rho=\frac{\operatorname{tr}\left(\mathbf{S}^\dagger\mathbf{H}\mathbf{X}\right)}{\|\mathbf{H}\mathbf{X}\|_F^2+ULN_0}. \tag{13}$$

B. Sub-Problem 2 for $\mathbf{X_R}$

Given the fixed ρ and $\mathbf{X}$, problem (10) of optimizing $\mathbf{X_R}$ can be recast as

$$\begin{aligned}\min_{\mathbf{X_R}}\ &\|\mathbf{X}-\mathbf{X_R}\|_F^2\\ \text{s.t. }&\frac{1}{L}\mathbf{X_R}\mathbf{X_R}^\dagger=\mathbf{C}.\end{aligned} \tag{14}$$

Without loss of generality, we assume $\mathbf{C}$ is positive-definite, so that ensure $\mathbf{C}$ is invertible, which is given by

$$\mathbf{C}=\mathbf{G}\mathbf{G}^\dagger. \tag{15}$$

$\mathbf{G}\in\mathbb{C}^{B\times B}$ denotes the determined lower triangular matrix. Relying on the decomposition of covariance matrix, the constraint in (14) with respect to $\mathbf{G}$ is expressed in the form

$$\frac{1}{L}\mathbf{G}^{-1}\mathbf{X_R}\mathbf{X_R}^\dagger\mathbf{G}^{-\dagger}=\mathbf{I}_B. \tag{16}$$

Since $\bar{\mathbf{X}}_\mathbf{R}=\sqrt{\frac{1}{L}}\mathbf{G}^{-1}\mathbf{X_R}$, problem (14) can be considered as an orthogonal procrustes problem, which is defined as

$$\begin{aligned}\min_{\bar{\mathbf{X}}_\mathbf{R}}\ &\|\mathbf{X}-\sqrt{L}\mathbf{G}\bar{\mathbf{X}}_\mathbf{R}\|_F^2\\ \text{s.t. }&\bar{\mathbf{X}}_\mathbf{R}\bar{\mathbf{X}}_\mathbf{R}^\dagger=\mathbf{I}_B.\end{aligned} \tag{17}$$

According to the singular value decomposition (SVD) [31], the globally optimal solution of $\bar{\mathbf{X}}_\mathbf{R}$ can be obtained as

$$\bar{\mathbf{X}}_\mathbf{R}=\bar{\mathbf{U}}\mathbf{I}_{B\times L}\bar{\mathbf{V}}^\dagger, \tag{18}$$

where $\bar{\mathbf{U}} \in \mathbb{C}^{B\times B}$ and $\bar{\mathbf{V}} \in \mathbb{C}^{L\times L}$ are the unitary matrices given by $\bar{\mathbf{U}}\boldsymbol{\Sigma}\bar{\mathbf{V}}^\dagger = \mathbf{G}^\dagger\mathbf{X}$, $\mathbf{I}_{B\times L} \in \mathbb{C}^{B\times L}$ stands for the matrix consisting of $\mathbf{I}_B$ and zero matrix of $B \times (L-B)$ dimensions. Consequently, the variable $\mathbf{X_R}$ can be computed as

$$\mathbf{X_R} = \sqrt{L}\mathbf{G}\bar{\mathbf{U}}\mathbf{I}_{B\times L}\bar{\mathbf{V}}^\dagger. \tag{19}$$

C. Sub-Problem 3 for X

Given the fixed ρ and $\mathbf{X_R}$, problem (10) of optimizing $\mathbf{X}$ is equivalent to

$$\begin{aligned}&\min_{\mathbf{X}} \ (1-\delta)\|\mathbf{S}-\rho\mathbf{H}\mathbf{X}\|_F^2 + \delta\|\mathbf{X}-\mathbf{X_R}\|_F^2 \\ &\text{s.t.} \ \ x_{i,j} \in \mathcal{X}_{\text{DAC}}, \quad \forall i \in \mathcal{I}, \ \forall j \in \mathcal{J}.\end{aligned} \tag{20}$$

Upon letting $\mathbf{A} = \left[\sqrt{1-\delta}\rho\mathbf{H}^T, \sqrt{\delta}\mathbf{I}_B\right]^T \in \mathbb{C}^{(U+B)\times B}$ and $\mathbf{W} = \left[\sqrt{1-\delta}\mathbf{S}^T, \sqrt{\delta}\mathbf{X_R}^T\right]^T \in \mathbb{C}^{(U+B)\times L}$, the problem (20) can be equivalently expressed as

$$\begin{aligned}&\min_{\mathbf{X}} \ \|\mathbf{A}\mathbf{X}-\mathbf{W}\|_F^2 \\ &\text{s.t.} \ x_{i,j} \in \mathcal{X}_{\text{DAC}}, \quad \forall i \in \mathcal{I}, \ \forall j \in \mathcal{J}.\end{aligned} \tag{21}$$

Defining $\mathbf{x} = vec(\mathbf{X}) \in \mathbb{C}^{LB\times 1}$, $\mathbf{w} = vec(\mathbf{W}) \in \mathbb{C}^{L(U+B)\times 1}$, we obtain $\mathbf{x}$ by solving the problem as

$$\begin{aligned}&\min_{\mathbf{x}} \ \|\dot{\mathbf{A}}\mathbf{x}-\mathbf{w}\|_2^2 \\ &\text{s.t.} \ x_n \in \{\pm 1 \pm j1\}, \quad \forall n,\end{aligned} \tag{22}$$

where $\dot{\mathbf{A}} = \sqrt{P_T/2B}\mathbf{I}_L \otimes \mathbf{A} \in \mathbb{C}^{L(U+B)\times LB}$ is based on the relationship between the Kronecker product and vectorization product, i.e., $vec(\mathbf{AXB}) = (\mathbf{B}^T \otimes \mathbf{A})vec(\mathbf{X})$, $\otimes$ dentoes the Kronecker product. In the next step, we define the following function $\phi(\cdot)$ to map the each element of a vector from a complex value to two real numbers. The function $\zeta(\cdot)$ maps each element of a matrix from a complex value to four real numbers. Thus, for each complex value m in vector and matrix, we have

$$\begin{aligned}\phi(m) &= \begin{bmatrix}\Re(m) & -\Im(\mathrm{m})\end{bmatrix}^T, \\ \zeta(m) &= \begin{bmatrix}\Re(m) & -\Im(m) \\ \Im(m) & \Re(m)\end{bmatrix},\end{aligned} \tag{23}$$

in which they operate element-wise, for instance, $\phi([m_1, m_2]^T) = [\Re(m_1), \Im(m_1), \Re(m_2), \Im(m_2)]^T$. Hence, the equivalent real-valued expression of problem (19) can be cast as

$$\begin{aligned}&\min_{\tilde{\mathbf{x}}} \ \|\tilde{\dot{\mathbf{A}}}\tilde{\mathbf{x}}-\tilde{\mathbf{w}}\|_2^2 \\ &\text{s.t.} \ \tilde{x}_n = \pm 1, \quad \forall n,\end{aligned} \tag{24}$$

with $\tilde{\dot{\mathbf{A}}} = \zeta(\dot{\mathbf{A}}) \in \mathbb{R}^{2L(U+B)\times 2LB}$, $\tilde{\mathbf{x}} = \phi(\mathbf{x}) \in \mathbb{R}^{2LB\times 1}$, and $\tilde{\mathbf{w}} = \phi(\mathbf{w}) \in \mathbb{R}^{2L(U+B)\times 1}$. 4^{BL} times of searching is required to obtain the transmitted signal via exhaustive search, which brings considerable complexity. To address this problem, we design a heuristic global optimization algorithm, namely, the binary particle swarm optimization-simulated annealing (BPSO-SA) algorithm to solve this sub-problem.

The typical particle swarm optimization (PSO) algorithm, which is based on the behavior of birds flocking and fish schooling [32], is to employ a particle representing a feasible solution of the problem. For each iteration, the speed and position of particles are updated according to the trajectory of itself and other group members. In the case of our study, when the variable of sub-problem (24) is setting in a discrete space, an algorithm called binary particle swarm optimization algorithm (BPSO) is required [32], which poses an efficient solution for the discrete problem.

We denote the learning factors or acceleration factors as c_1 and c_2, and the random numbers r_1 and r_2 with uniform distribution in the range of $[0,1]$. Then, following the BPSO rules [32], for the i-th ($i = 1, 2, \ldots, N$) particle in the $(t+1)$-th iteration, the velocity adjustment $\mathbf{v}_i^{(t+1)} \in \mathbb{R}^{2LB\times 1}$ can be expressed based on previous position $\mathbf{d}_i^{(t)} \in \mathbb{R}^{2LB\times 1}$:

$$\begin{aligned}\mathbf{v}_i^{(t+1)} = \omega\mathbf{v}_i^{(t)} + c_1 r_1\left(\mathbf{p}_{i,\text{ best}}^{(t)} - \mathbf{d}_i^{(t)}\right) \\ + c_2 r_2\left(\mathbf{p}_{gbest}^{(t)} - \mathbf{d}_i^{(t)}\right),\end{aligned} \tag{25}$$

where ω is the inertia factor, $\mathbf{p}_{i,\text{ best}}^{(t)} \in \mathbb{R}^{2LB\times 1}$ is the individual's best solution, and $\mathbf{p}_{gbest}^{(t)} \in \mathbb{R}^{2LB\times 1}$ is the best-so-far solution in the whole population. By mapping velocity to probability constrained to the interval $[0,1]$ as $\mathbf{f}_i \in \mathbb{R}^{2LB\times 1}$, we obtain

$$f_{i,j}^{(t+1)} = \frac{1}{1+o_{i,j}^{(t+1)}}, \tag{26}$$

where $\mathbf{o}_i^{(t+1)} \in \mathbb{R}^{2LB\times 1}$ is defined as $o_{i,j}^{(t+1)} = e^{-v_{i,j}^{(t+1)}}$ with $j = 1, 2, \ldots, 2LB$. Here, we introduce update scheme of particle position as

$$d_{i,j}^{(t+1)} = \begin{cases}1, & \text{if } r_3 < f_{i,j}^{(t+1)}, \\ -1, & \text{otherwise},\end{cases} \tag{27}$$

where $r_3 \in [0,1]$ is a random parameter.

Despite the advantage of fast convergence speed, the BPSO has the tendency of moving to the local optimal area, resulting in all particles gathering together. As such, it is difficult to obtain global optimal solution [33]. Inspired by [34], we propose a hybrid method combining the simulated annealing (SA) algorithm and BPSO to avoid premature convergence. If there is no improvement of $\mathbf{p}_{gbest}$ during the every μ iterations, the probability of accepting new solution in the SA algorithm can be written as

$$P = \begin{cases}1, & \text{if } obj\left(\mathbf{s_n}\right) - obj\left(\mathbf{s_c}\right) < 0, \\ \exp\left(\dfrac{-\left|obj\left(\mathbf{s_n}\right) - obj\left(\mathbf{s_c}\right)\right|}{T}\right), & \text{otherwise},\end{cases} \tag{28}$$

where T is the temperature that follows the descend criterion as $T = \gamma T$, $\gamma = 0.99$ is a factor. $obj()$ denotes the objective function of sub-problem (24), $\mathbf{s_n} \in \mathbb{R}^{2LB\times 1}$ and $\mathbf{s_c} \in \mathbb{R}^{2LB\times 1}$ are the new solution and current solution, respectively.

For the sake of clarity, we outline the principle of hybrid algorithm in Algorithm 1, which solves problem (24) iteratively.

Algorithm 1 BPSO-SA Algorithm for Solving (24)

Input: input $\tilde{\mathbf{w}}$, $\tilde{\tilde{\mathbf{A}}}$, t_{max}, μ, k_{max}, T_{min}, population of particles N, inertia factor ω.
Output: $\mathbf{x}$

Set $t=1$ and $T=1000$. Initialize randomly $\mathbf{d}_i^{(0)}$ with $d_{i,j}^{(0)}=\pm 1$, $i=1,2,\ldots,N$, $j=1,2,\ldots,2LB$
while $t \le t_{max}$ **do**
  for $i=1:N$ **do**
    if $obj(\mathbf{d}_i^{(t)}) < obj(\mathbf{p}_{i,best}^{(t)})$ **then**
      $\mathbf{p}_{i,best}^{(t)} = \mathbf{d}_i^{(t)}$.
    if $obj(\mathbf{p}_{i,best}^{(t)}) < obj(\mathbf{p}_{gbest}^{(t)})$ **then**
      $\mathbf{p}_{gbest}^{(t)} = \mathbf{p}_{i,best}^{(t)}$.
  for $i=1:N$ **do**
    Compute the velocity of the i-th particle based on (25).
    Compute the position of the i-th particle based on (27).
  if p_{gbest} is not changed during last μ iterations **then**
    Set $k=1$, $\mathbf{s_c}=\mathbf{p}_{gbest}^{(t_{max})}$.
    while $T > T_{min}$ **do**
      while $k < k_{max}$ **do**
        Generate $\mathbf{s_n}$ randomly with $\mathbf{s_{n}}_i = \pm 1$.
        Compute the probability based on (28).
        if $p > rand(0,1)$ **then**
          $\mathbf{p}_{gbest}^{(t_{max})} = \mathbf{s_n}$, $\mathbf{s_c}=\mathbf{s_n}$.
        $k=k+1$.
      Update T as $T=\gamma T$.
  $t=t+1$.
return $\tilde{\mathbf{x}} = \mathbf{p}_{gbest}^{(t_{max})}$, $\mathbf{x}=\phi^{-1}(\tilde{\mathbf{x}})$.

D. Complexity Analysis

The proposed MVAM method for solving the three aforementioned sub-problems is summarized in Algorithm 2. In this section, we analyze the overall computation cost of MVAM. As presented in Algorithm 2, we can obtain three variables via multiple iterations of the MVAM framework, where the computation cost of the proposed method is linear with the number of iterations. To this end, we conclude that the total complexity in each iteration as follows.

We calculate the computation cost of precoding factor ρ updated by (13), which needs the complexity of $\mathcal{O}(UL + BUL + BL^2)$. The update of radar waveform $\mathbf{X_R}$ includes one Cholesky decomposition, four matrix multiplications and one SVD with a complexity of $\mathcal{O}(BL^2 + B^2L + B^3)$. The design of transmitted matrix $\mathbf{X}$ is solved by using the BPSO-SA algorithm. Each BPSO-SA iteration involves the calculation of particles' velocity and position, doing so takes $\mathcal{O}(BLN)$ complex floating-point-operations. It is worth noting that if the optimal solution does not change in the last μ iteration, the increased complexity of algorithm is the computation of probability, i.e., $\mathcal{O}(BL^2(U+B))$. Accordingly, the maximum computation cost of the proposed algorithm in one iteration can be computed as $\mathcal{O}(UL + BUL + BL^2 + B^2L + B^3 + t_{max}(BLN) + kBL^2(U+B))$.

Algorithm 2 MVAM Framework for Solving (10)

Input: input $\mathbf{H}$, $\mathbf{S}$, P_T, n_{max}, weighting factor $0 \le \delta \le 1$, tolerance threshold β.
Output: $\mathbf{X}$

Set $n=1$. Initialize randomly $\mathbf{X}^{(0)}$ with $x_{i,j} \in \mathcal{X}_{\text{DAC}}$.
Compute the value of objective function of (10), defined as $\mathbb{F}^{(0)} = \mathbb{F}(\rho^{(0)}, \mathbf{X_R}^{(0)}, \mathbf{X}^{(0)})$.
while $n \le n_{max}$ and $|\mathbb{F}^{(n)} - \mathbb{F}^{(n-1)}| \ge \beta$ **do**
  Update $\rho^{(n)}$ based on (13).
  Update $\mathbf{X_R}^{(n)}$ based on (19).
  Update $\mathbf{X}^{(n)}$ based on Algorithm 1.
  Update the value of objective function $\mathbb{F}^{(n)}$.
  $n=n+1$.
return $\mathbf{X}^{(n)}$.

E. Convergence Analysis

Since convex optimization techniques can be used for the precoding factor and radar waveform, the transmitted signal can be designed via BPSO-SA, three variables are solved iteratively under the MVAM framework until the termination condition is reached.

Many works have focused on the convergence analysis of traditional alternate minimization approach, which provides a useful framework for iterative optimization with computationally cheap updates. In order to ensure the convergence of alternating optimization, in the q-th iteration, the subproblem is required to derive the unique minimum point, i.e., the subproblem corresponding to each variable is strictly convex. Following up on this idea, Powell proved that if this assumption is not satisfied, the algorithm may cycle indefinitely [35].

We emphasize that the traditional alternate minimization does not apply to our problem (10), since the objective function is not strictly convex for all coupled variables. As the closed-form solution of subproblem (12) and (14) can be derived, each iteration of solving precoding factor ρ and radar covariance matrix $\mathbf{X_R}$ is considered to be monotonically decreasing. However, the sub-problem (20) of solving transmitted signal matrix $\mathbf{X}$ is an NP-hard problem, resulting in the inability to obtain the unique global optimal solution. In order to ensure the convergence of the proposed framework, we couple the alternating minimization with a proximal term $\frac{1}{2}\|\mathbf{X} - \overline{\mathbf{X}}^{(q)}\|_F^2$ in (11c) to removing the strict convexity assumption [36]. Further, we introduce the dynamic inertial parameter β^q at $(q+1)-$th iteration. In this way, it enables the proposed MVAM to add part of the old direction to the new direction of the algorithm, thereby accelerating the convergence.

V. Performance Evaluation

In this section, we evaluate the performance of the proposed method for radar and communication under one-bit DACs constraints in the downlink system. Unless otherwise specified, we set $P_T = 1$ and each entry of flat-fading Rayleigh channel matrix $\mathbf{H}$ follows a standard Complex Gaussian distribution $\mathcal{CN}(0,1)$. As for the transmit SNR, we define

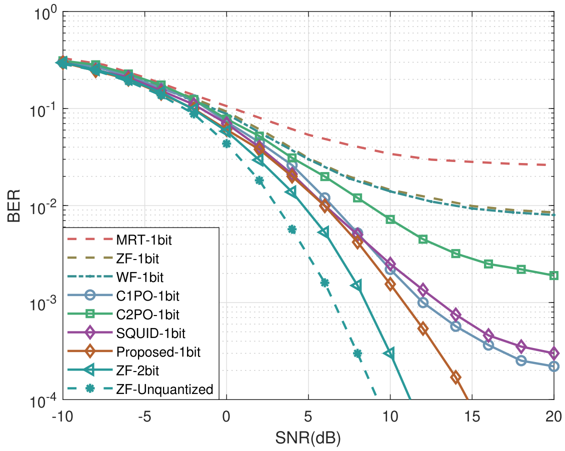


Fig. 2. BER of different precoders vs. SNR with $B = 8$.

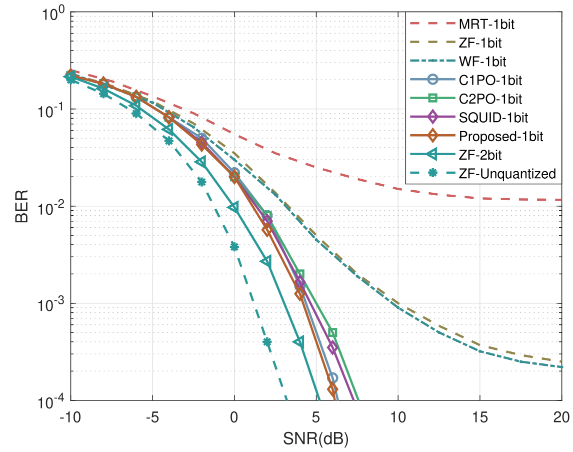


Fig. 3. BER of different precoders vs. SNR with $B = 128$.

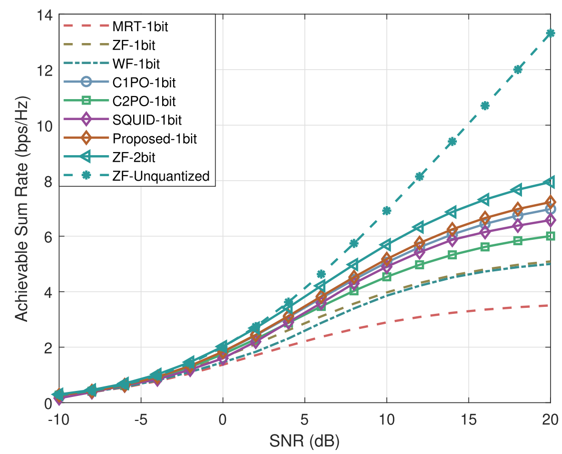


Fig. 4. Sum-rate of different precoders vs. SNR with B=8, U=2.

$\mathrm{SNR} = P_T/N_0$. The QPSK alphabet is considered in the evaluation. In particular, the performance of the proposed algorithm for 16QAM modulation is analyzed in Section V-E. We compare the MVAM framework with various precoding algorithms. Each simulation result is an average over 1000 Monte–Carlo trials. From Fig. 6 and Fig. 11, 'Radar-Only' denotes the one-bit radar beam-pattern obtained by proposed methods when $\delta = 1$.

A. Communication Performance

We first study the communication performance in a small-scale MIMO DFRC downlink system based on Monte Carlo trials, wherein the number of transmit antennas is set as $B = 8$ and the number of users is $U = 2$. In Fig. 2, the ZF-Unquantized precoder is a benchmark, where the output signal is generated by the zero-forcing (ZF) approach with infinite-precision DACs. Through comparing various one-bit precoders, the results illustrate that the proposed approach ($\delta = 0$) achieves the best BER performance as expected. For the quantized linear precoding approaches, i.e., ZF, water filling (WF) and maximal ratio transmission (MRT), they have lower complexity than the proposed non-linear precoder at the cost of poor BER. For the non-linear mapping algorithms, i.e., squared-infinity norm Douglas-Rachford splitting (SQUID), C1PO and C2PO, they obtain transmitted signal via the relaxation-normalization process which results in a performance loss.

Compared to the SQUID and C1PO, the SNR gain of the proposed method is about 7 dB for a target BER of 10^{-4}. For analyze the performance of multi-bit quantizer, ZF-2bit is used to represent ZF precoder with a pair of two-bit DACs per chain. The transmitted signal generated by two-bit DACs quantization is equivalent to 16-PSK symbols, which has shorter quantization step as $\Delta = \pi/2$, and hence derive the smaller quantization error than one-bit quantization. We observe that the performance trend of ZF-2bit approach to the inifinite precision quantization curve.

Next, we evaluate the communication performance as setting $B = 128$ and $U = 16$ in Fig. 3. We observe that non-linear precoders are obviously favorable in the 128×16 system over the linear mapping methods as the SNR increases. When the SNR is above 5 dB, all the schemes of non-linear precoder with one-bit can approach similar and outstanding property, which indicate that massive MIMO system is one of the ways to reduce the error of one-bit DAC system. Specifically, the simulation results show that, compared with ZF infinite resolution precoder, the proposed method incurs only 4dB penalty at the bit error rate of 10^{-3} in the system with 8 antennas, besides, the loss of signal-to-noise ratio is less than 2dB in the massive MIMO system with 128 antennas. A similar conclusion is also given in [37], that is, the antenna extra distortionand and RF hardware impairments caused by low-precision quantization can be compensated by using large-scale antenna arrays. In short, Figs. 2 and 3 present that our proposed method has superiority in terms of BER in both small scale and massive MIMO system.

Fig. 4 and Fig. 5 present the sum-rate of precoders under the varying of SNR. Particularly, these results intend to prove that the proposed precoder for the DFRC downlink system under one-bit constraint realize a reliable achievable sum-rate performance. In Fig. 4 and Fig. 5, the transmit SNR is employed as metric, the achievable sum-rate is defined as $AS = \sum_{i=1}^{U} \log_2\left(1 + \frac{\mathbb{E}\left(|s_{i,j}|^2\right)}{\mathbb{E}\left(\left|\mathbf{h}_i^T\mathbf{x}_j - \rho^{-1}s_{i,j}\right|^2\right) + N_0}\right)$. By comparing the

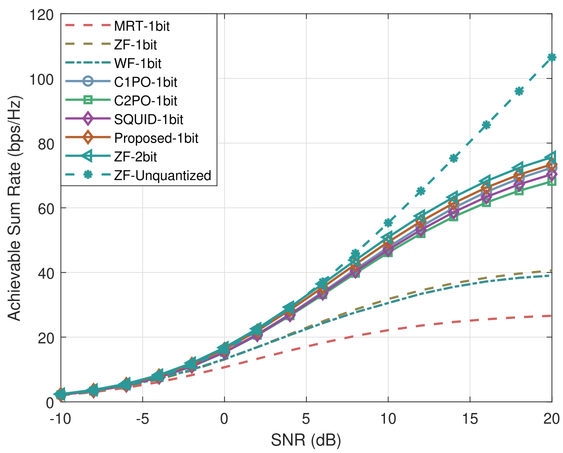


Fig. 5. Sum-rate of different precoders vs. SNR with B=128, U=16.

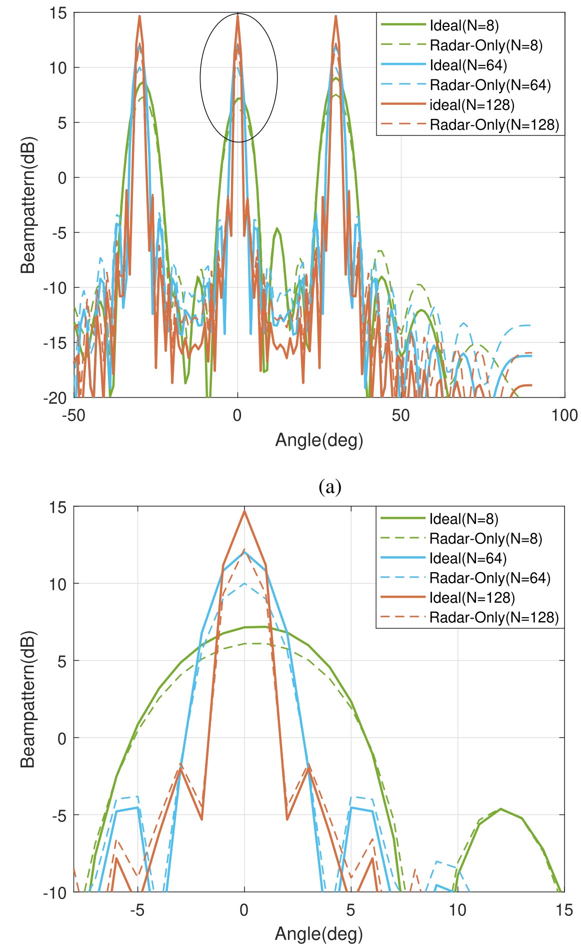
proposed method with others, we find out that the achievable sum-rate can improve by increasing the SNR. Besides, the curves in Fig. 4 and Fig. 5 illustrate that the proposed method is suitable for the moderate scale MIMO as well as massive MIMO case.

B. Radar Performance

Further, we investigate the influence of various antennas on the radar beam-patterns in Fig. 6, where Fig. 6(b) is a locally enlarged graph of Fig. 6(a). Without loss of generality, the angles of the three targets tracked by beams are fixed as $[\theta_1 = -30°, \theta_2 = 0°, \theta_3 = 30°]$. For beam-patterns design, 'Ideal' represents the ideal radar waveform. The results show that the designed beam-patterns under one-bit constraints forms the main lobe in three interested target directions, which meets the detection and sensing function of radar. And it's noteworthy that our method provides the better PSLR of the radar waveforms at targets' directions as the number of antennas increases, and only experiences a slight performance loss when $B = 128$. By further analyzing the results of above figures, we note that the DFRC system can experience tolerable performance loss in massive MIMO systems.

C. Trade-Off Between the Radar and Communication

In order to compare the proposed method with different weighting factors, we present the communication performance in Fig. 7, in which $B = 128$ and $U = 16$. The radar beam-patterns is explicitly shown in Fig. 8, the angles of the three targets tracked by beams are fixed as $[\theta_1 = -30°, \theta_2 = 0°, \theta_3 = 30°]$. δ is a weight factor indicating the degree of bias to radar or communication design, which implies a design trend. In Fig. 7, taking the linear one-bit precoder MRT as the benchmark, the result shows that the '$\delta = 0$' curve (all the weights are allocated to the communication side) provides significant advancement in terms of BER. As for the proposed precoder with $\delta = 0.5$, our method achieves comparable performance compared to MRT at SNR=5, and the performance advantage becomes more obvious with the increase of SNR. In Fig. 8, as the weight factor increases, more power



(b)

Fig. 6. Transmit beam-pattern vs. antennas numbers.

resources are allocated to the radar function and hence better PSLR can be achieved. When the weight is evenly assigned to radar and communications, the beam-pattern suffers acceptable performance loss at the angle ($\delta = 0.5$). Note that We do not regard the tradeoff factor δ as a variable that can be optimized in this paper, instead, it is regarded as a parameter that can be adjusted according to the actual situation in the DFRC design. All the above results validate that the proposed method for the DFRC downlink system under the one-bit constraint can realize a reliable balance performance between the radar and communications.

D. BER Performance Under Imperfect CSI

We evaluate the robustness of the proposed method at the presence of channel-estimation errors. Fig. 9 illustrates the BER performance under imperfect CSI. The results in Fig. 9 show the BER performance of the existing algorithms and the proposed method by varying channel estimation factor with $B = 128$ and $U = 16$ at SNR $= 5$dB. Specifically, we model the channel matrix as

$$\tilde{\mathbf{H}} = \sqrt{1-\varepsilon}\mathbf{H} + \sqrt{\varepsilon}\Delta\mathbf{H}, \tag{29}$$

where $\varepsilon \in [0, 1]$ is the factor of channel-estimation error, $\tilde{\mathbf{H}}$ is the estimated channel matrix, i.e., $\varepsilon = 1$ with the

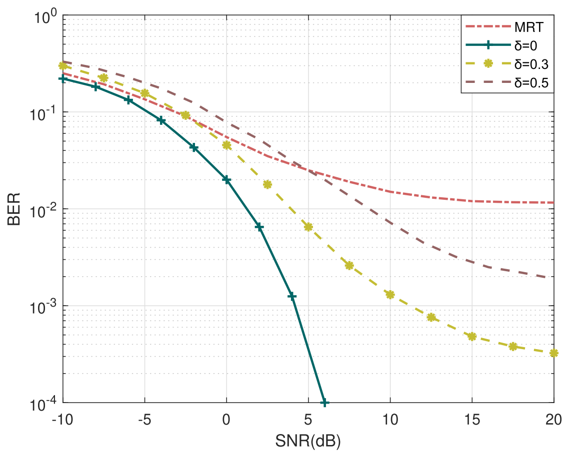


Fig. 7. BER with different weighting factor.

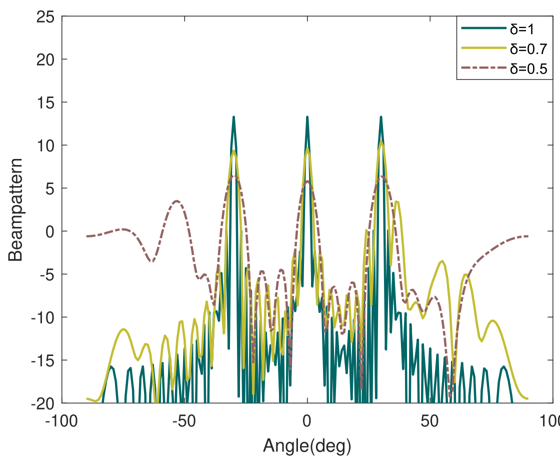


Fig. 8. Transmit beam-pattern with different weighting factor.

assumption of no CSI, and $\Delta\mathbf{H}$ denotes the estimation error matrix, following $\Delta h_{i,j} \sim \mathcal{CN}(0,1)$. For the performance of 128×16 MIMO system, we find that non-linear precoders achieve significant advantages in the case of $\varepsilon \leq 0.5$, and the proposed method slightly outperforms the existing methods in terms of BER when $\varepsilon \leq 0.2$. Note that the perfect CSI is always unavailable due to channel noisy [38]. In this line, the results validate that the proposed method is applicable in practical DFRC system.

E. Performance of 16QAM Modulation

The foregoing experiments are based on QPSK modulation, where we assume that all users adopt the same precoding factor to recover the transmitted symbol. In the case of constant-modulus constellation modulation (QPSK), we set that the receiver adopts symbol-wise nearest-neighbor decoding, where the assumption that the same precoding factor does not affect the decision performance ($\rho_u = \rho$), since the decision regions are circular sectors in the complex plane.

In this experiment, we investigate the performance of the proposed precoder in the view of BER and beam-pattern with 16QAM signaling. As for non-constant-modulus high-order

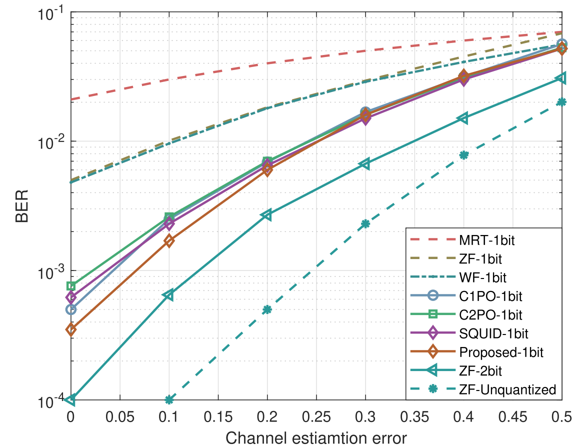


Fig. 9. BER of different precoders vs. channel estimation error with SNR=5dB.

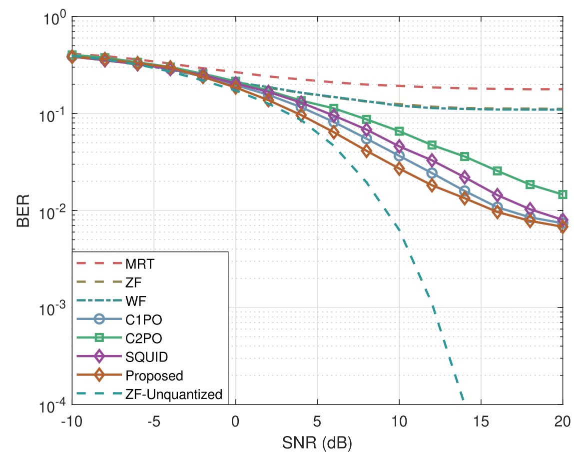


Fig. 10. BER of different precoders vs. SNR with 16QAM.

modulation, such as 16QAM considered in the numerical results, we introduce the diagonal matrix of precoding factor as $\mathbf{P} = diag\{\rho_1, \rho_2, \ldots, \rho_U\} \in \mathbb{R}^{B\times B}$. Hence, the problem (10) can be recast as

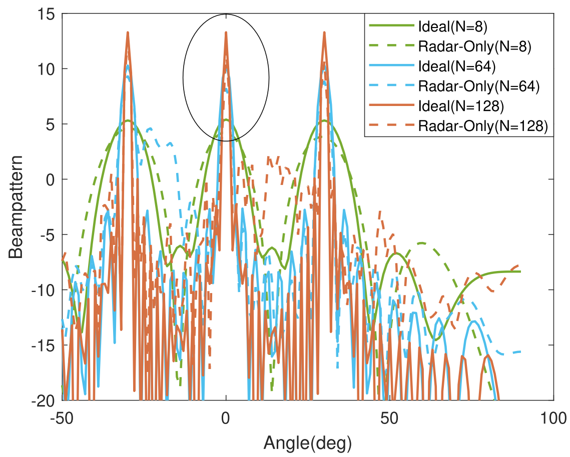
$$\begin{aligned}\min_{\mathbf{X},\mathbf{X_R},\mathbf{P}} \quad & (1-\delta)\left(\|\mathbf{S}-\mathbf{PHX}\|_F^2 + \|\mathbf{P}\|_F^2 ULN_0\right) \\ & + \delta\|\mathbf{X}-\mathbf{X_R}\|_F^2 \\ \text{s.t.} \quad & x_{i,j} \in \mathcal{X}_{\text{DAC}}, \quad \forall i \in \mathcal{I}, \ \forall j \in \mathcal{J}, \\ & \frac{1}{L}\mathbf{X_R}\mathbf{X_R}^\dagger = \mathbf{C}. \end{aligned} \tag{30}$$

Therefore, the subproblem of solving precoding matrix $\mathbf{P}$ can be given as

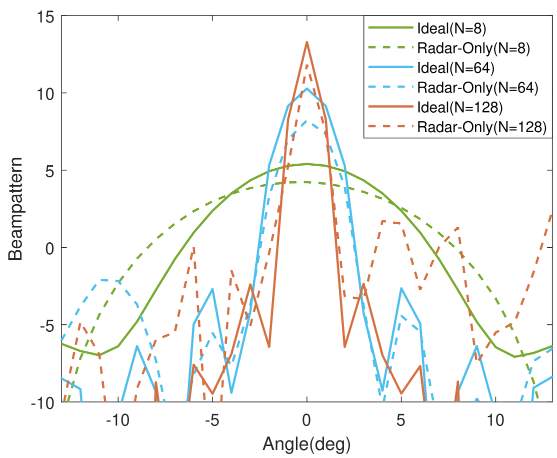
$$\mathbf{P} = \left(\mathbf{S}\mathbf{X}^\dagger\mathbf{H}^\dagger\right)\left(\mathbf{H}\mathbf{X}\mathbf{X}^\dagger\mathbf{H}^\dagger + \mathbf{I}_U LNU\right)^{-1} \tag{31}$$

Then we use the proposed algorithm to obtain the transmitted signal $\mathbf{X}$ and the radar matrix $\mathbf{X_R}$ alternately until the convergence condition is satisfied.

In Fig. 10, we compare the BER performance of various precoders with $B = 8$ antennas and $U = 2$ users for 16QAM modulation. The results show that all the schemes have lower performance due to high order modulation (16QAM), while the nonlinear methods can well support stable transmission.



(a)



(b)

Fig. 11. Transmit beam-pattern with 16QAM.

In the case of high SNR, the performance of linear precoders tends to saturation and no longer decreases significantly, the BER of nonlinear precoders decreases obviously with the increase of SNR. Moreover, we observe that the communication performance of the proposed scheme is promising for small-scale MIMO systems with 16QAM modulation, since the proposed method is superior to other methods.

Furthermore, Fig. 11(a) presents the associated radar beam-patterns under various antennas for 16QAM modulation. Fig. 11(b) is a locally enlarged graph of Fig. 11(a). We set the angles of the three targets tracked by beams that are fixed as $[\theta_1 = -30^\circ, \theta_2 = 0^\circ, \theta_3 = 30^\circ]$. It can be observed in Fig. 11(b) that as the number of antennas increases, the radar beam-pattern performance of the proposed precoder improves. Especially for large MIMO systems ($B = 128$), the corresponding radar beam-patterns only have a slight performance loss, e.g., the error between the designed radar beam and the ideal radar beam becomes marginal. By further comparing the radar performance under QPSK modulation and 16QAM modulation, we find out that, although the proposed algorithm has performance loss under high-order modulation, it can provide reliable transmission with a large number of antennas.

VI. Conclusion

In this paper, we have developed a non-linear precoding approaching for the MIMO DFRC downlink system, with the aim at addressing the joint optimization problem of dual functions between radar and communication when employing one-bit DACs. We formulate a weighted optimization between the performance of communication and radar via minimizing the mean value of the power of the error at downlink and approaching ideal radar beam-pattern. To resolve this nonconvex problem, we propose the MVAM framework that consists of three steps at each iteration, corresponding to the iterative solution of multiple variables. Besides, we propose the BPSO-SA algorithm ensuring to achieve optimal or near-optimal solutions to obtain transmit signal at each iteration. The feasibility and reliability of the proposed method are numerically demonstrated via simulation results under various test scenarios.

We will extend this work to time and frequency dual-selective channel in the future research, and consider the system model with coarse quantilization both at the transmitter as well as the receiver. Furthermore, in the actual environment, out of band spectrum regeneration may lead to poor performance of quantization system. In the future we will investigate how to reduce out of band distortion and analyze the cost and power consumption of the proposed DFRC system.

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