Running Head: COUNTING ERRORS

Acquisition of the counting principles during the subset-knower stages: Insights from

children's errors

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Research Highlights

- 1. We test the assumption that subset-knowers lack cardinal principle knowledge in a retrospective video analysis of children's counting errors on the Give-N task.
- 2. We coded children's adherence to or violation of each of the three counting principles: word-object correspondence, stable order, and cardinal principle.
- 3. The stable-order and word-object correspondence principles emerge early, yet children make errors of word-object correspondence throughout the learning process.
- 4. Children understand the cardinal principle before being classified as "cardinal-principle knowers", whose insight lies with coordinating all three counting principles simultaneously.

Abstract

Studies on children's understanding of counting examine when and how children acquire the cardinal principle: the idea that the last word in a counted set reflects the cardinal value of the set. Using Wynn's (1990) Give-N Task, researchers classify children who can count to generate large sets as having acquired the cardinal principle (cardinal-principle-knowers) and those who cannot as lacking knowledge of it (subset-knowers). However, recent studies have provided a more nuanced view of number word acquisition. Here, we explore this view by examining the developmental progression of the counting principles with an aim to elucidate the gradual elements that lead to children successfully generating sets and being classified as CP-knowers on the Give-N Task. Specifically, we test the claim that subset-knowers lack cardinal principle knowledge by separating children's understanding of the cardinal principle from their ability to apply and implement counting procedures. We also ask when knowledge of Gelman & Gallistel's (1978) other how-to-count principles emerge in development. We analyzed how often children violated the three how-to-count principles in a secondary analysis of Give-N data (N = 86). We found that children already have knowledge of the cardinal principle prior to becoming CP-knowers, and that understanding of the stable-order and word-object correspondence principles likely emerged earlier. These results suggest that gradual development may best characterize children's acquisition of the counting principles, and that learning to coordinate all three principles represents an additional step beyond learning them individually.

Keywords: counting, cardinal principle, counting principles, Give-N, preschool children

Numerous studies have shown that children can recite the count sequence early in development, but take 1 to 3 years to learn to use counting meaningfully to create sets of items (see Cheung & Ansari, 2020; Sarnecka, 2015, for reviews). This ability is related to the *cardinal principle*, or the knowledge that the last word of our count denotes the total quantity of the counted set (Gelman & Gallistel, 1978; Fuson, 1988; Wynn 1990, 1992). The standard assessment for testing cardinal principle understanding is the Give-N Task (Le Corre, Van de Walle, Brannon, & Carey, 2006; Sarnecka & Carey, 2008; Schaeffer, Eggleston, & Scott, 1974; Sella, Berteletti, Lucangeli, & Zorzi, 2017; Shusterman, Slusser, Halberda, & Odic, 2016; Slusser, Ribner, & Shusterman, 2019; Wynn, 1990, 1992). Using this task, researchers classify those who understand how to use counting to generate larger sets of objects (five or more) as cardinal-principle-knowers (CP-knowers for short), and those who fail to do so as subsetknowers, or more specifically as N-knowers (e.g., 1-knowers, 2-knowers, 3-knowers). Such findings have undergirded the influential 'conceptual change' account of number acquisition (Carey, 2009), which posits that CP-knowers have learned the cardinal principle, and subset-knowers lack such knowledge. However, an emerging body of literature has raised questions about this assumption, indicating both more knowledge among subset-knowers (Barner & Bachrach, 2009; Wagner, Chu, & Barner, 2018; Gunderson et al., 2015; O'Rear, McNeil, & Kirkland, 2020) and less knowledge to cardinal-principle knowers (Davidson, Eng, & Barner, 2012; Le Corre & Carey, 2007; Le Corre, 2014; Sella & Lucaneli, 2020) than was previously attributed to them in the canonical narrative. Here we explore this more nuanced view of children's number acquisition by carefully analyzing their spontaneous behaviors and errors, rather than the traditional measure of accuracy, on the Give-N task. In doing so, we highlight an element in analyzing children's number acquisition that has been neglected in previous studies, namely the coordination of the counting principles.

In their counting principles framework, Gelman and Gallistel (1978) proposed that children have to acquire three principles to learn how to count: the cardinal principle, the stableorder principle (using a count sequence consistently across all counting contexts), and the wordobject correspondence principle (applying one number word to one object). ¹ The cardinal principle is considered the most important principle of the three because it provides meaning for number words: to say there are "seven" items in a collection means that the collection of items has a cardinal value of seven. By contrast, the stable-order and word-object correspondence principles are often construed as foundational skills that are procedural in nature.

However, the implied hierarchy of the counting principles, with cardinality as the final step in a progression, is in some ways misleading. All three principles have to be followed for the counting algorithm to work correctly, and if the stable order or word-object correspondence principle is violated, the last word of a count does not reflect the cardinal value of a set. That is, the last word of a count denotes cardinality *if and only if* one follows the counting procedures (Fuson, 1988). The ability to generate a correct set of objects is used to distinguish subset-knowers from CP-knowers on the Give-N Task, with the tacit assumption that knowledge of the cardinal principle characterizes the critical difference between them. However, this assumption has not been well tested because previous studies that have adopted the knower-levels framework have not examined the developmental progression of the counting principles individually.

A series of experiments by Wynn (1990, 1992) and Le Corre et al (2006) help to illustrate characteristic differences between subset and CP-knowers. First, when asked to provide a puppet

¹ We use the term word-object correspondence principle (Briars & Siegler, 1984; LeFevre et al., 2006) rather than one-to-one correspondence principle (Gelman & Gallistel, 1978) to highlight that we are referring to the counting procedure of tagging one object with one number word. We do not make claims about whether children understand that two sets have the same numerosity if they are in 1-1 correspondence with each other, also known as Hume's principle.

with six objects on the Give-N task, CP-knowers can correctly count out six items, but subsetknowers fail to give large sets of items such as "five" or "six" because they do not use counting to give N items (Le Corre et al., 2006; Wynn, 1990, 1992). Many other studies have shown a similar pattern, in which subset-knowers "grab" a number of items, rather than counting them, to meet the experimenter's request (Dowker, 2008; Sarnecka & Carey, 2008). Second, in the "What's on this Card" task (WOC), Le Corre and colleagues showed children a picture of stickers, ranging in quantity from one to eight, and asked "What's on this card?" (Gelman, 1993). They tested whether children understand the cardinal principle by looking at trials in which children counted a set correctly and asked how often they provided a cardinal response that matched the last word of a count. For example, when shown five apples, they measured how likely subset- and CP-knowers counted "one-two-three-four-five" followed by a cardinal response, "five". They found that CP-knowers are more likely to provide a correct cardinal response that matched the last word of a count relative to subset-knowers. Subset-knowers, on the other hand, either counted but did not provide a cardinal response, or provided a cardinal response without counting. These studies suggest that subset-knowers do not understand the significance of the last word of a count, but CP-knowers do, and that the acquisition of the cardinal principle is a radical shift from the prior conceptual state (Carey, 2009).

One challenge with the framing of cardinality acquisition as a radical conceptual change is that the focus on the discontinuity obscures more gradual elements of the learning process. Indeed, recent studies that have administered both WOC and Give-N to preschoolers have shown that approximately 20 to 30% of children can be classified as subset-knowers on one task but CPknowers on another (O'Rear & McNeil, 2020; Le Corre et al., 2006; Marchand and Barner, 2020). Consistent with this, Le Corre et al. (2006) found that together with CP-knowers, 3- and 4knowers as a group ended their count on the requested number word when generating a set on the Give-N Task, but 1- and 2-knowers never did. They also found that among the 3- and 4-knowers who counted correctly on WOC, a majority of them also produced a cardinal response when asked "What's on this card?" after counting the set. Similar findings have also been reported on the How Many Task in which children become increasingly likely to provide last word responses when asked how many as they approach the CP-knower stage (Sarnecka & Carey, 2008). These findings are in line with a more nuanced view of number word acquisition and highlight a need to examine the gradual elements that lead to children's success in generating large sets on the Give-N Task. Importantly, they suggest that children may have acquired the cardinal principle prior to becoming "CP-knowers" as assessed by Give-N, and motivate the current study on the developmental progression of the counting principles. By systematically examining the acquisition of counting principles, our study provides insights into the gradual elements that may reflect important learning mechanisms integral to large conceptual leaps.

The present study

In this study, we examine children's knowledge of each of the three counting principles by analyzing their counting behavior, and specifically their counting errors, when they have a prompted opportunity to count.

We used a retrospective video analysis of previously collected Give-N data to examine children's behaviors related to counting and cardinality. We conducted secondary data analysis and analyzed a part of the Give-N task where all children were asked to count. On the Give-N task, there are two moments when children can count. First, when the experimenter asks children to give N, some children count to give N, and some do not. Previous researchers have termed those who count as "counters" and those who do not as "grabbers" (Le Corre et al., 2006; Wynn, 1990, 1992). Second, *after* giving some items, the experimenter can ask children to count to

check if they have provided N.² Previous studies tend to focus on the first episode and ask whether children spontaneously count to generate a correct set of items, or whether their final responses are correct. Those analyses address whether children realize that they can use counting to give N and whether they do so correctly. In the current study, we focus on the second episode in the Give-N Task, in which we asked all children to count on every trial to 'count and check' if they had given N. Each child thus had an equal opportunity to demonstrate their counting ability. Importantly, in the dataset used for this analysis, the procedure asked children to count and check their responses on every trial greater than one, whether or not their initial response was correct; this helped to slow children down so that they could consider their responses and prevented them from interpreting a request to check their response as feedback that they were wrong. Furthermore, while children were asked to "count and check" whether the cardinality of their set matched their response, they were never asked "how many" items were in the set; the request to "check" arguably invited children to engage in meaningful rather than rote counting. We coded children's adherence to or violation of each of the three counting principles: word-object correspondence, stable order, and cardinality. Thus, unlike previous studies, we did not focus on spontaneous counts nor overall count accuracy, both of which require knowledge or behavior beyond the three counting principles. This approach afforded a window onto children's knowledge of the counting principles separate from their ability to apply and implement counting procedures.

² Not all previous studies included this procedure in the Give-N task (Mussolin, Nys, Leybaert, & Content, 2012; Sella, Bertellei, Lucangeli, & Zorzi, 2017; see Krajcsi, 2021, for discussion), and some studies only asked children to count to check when they had provided an incorrect number (Le Corre et al., 2006). Here, we used a version of the Give-N Task in which all children were asked to count to check for N on all trials greater than "one" (Le Corre & Carey, 2007; Shusterman, et al., 2016; Shusterman et al., 2017; Wagner, Chu, & Barner, 2018).

The goal of the current study was twofold. First, we examined whether acquisition of the cardinal principle happens right around the moment when children become CP-knowers as assessed by Give-N, or whether it emerges more gradually. If the analysis of errors suggests that some subset-knowers obey the cardinal principle in their counting behavior, then this would provide evidence for the more gradualist view of cardinal principle acquisition, and would invite some theoretical revision about the nature of the conceptual change. Second, we examined the relative timing of children's knowledge of the word-object correspondence and stable-order principles within the knower-level framework, testing whether children progress through a clear order of acquiring stable order, then word-object correspondence, then cardinality. Specifically, children can typically count up to 10 in a count elicitation task before they become CP-knowers, and that most subset-knowers can count higher than the highest number requested on the Give-N Task (e.g., Le Corre et al., 2006; Shusterman et al., 2016; Van Rinsveld, et al., 2020). Therefore, the stable order of the count list is likely established prior to the other two principles. Less is known about when children acquire the word-object correspondence principle under the knowerlevel framework, and understanding the timing of word-object correspondence relative to acquisition of cardinality should illuminate the nature of the conceptual change. Together, these analyses help to triangulate the development of children's counting knowledge using a new source of data: children's counting errors.

Methods

Participants.

We conducted secondary data analysis on a sample of 86 preschool-aged children (M = 3 years 4 months; Range = 2;10 to 4;2; 54 females). These children were part of a larger study that

examines number word learning (BLINDED).³ They were recruited in BLINDED, and were fluent English speakers. No information about race or socio-economic background was collected, but the sample was drawn from a database of families representative of the surrounding area: White non-Hispanic (84%), Black and African American (5%), Hispanic (6%), and Asian (3%), with a median per capita income of \$42,647 and 7.3% of households below the federal poverty line (US Census Bureau, 2019).

Tasks.

Count Elicitation. Children were asked to count a set of 20 fish that was lined up linearly, and the highest correct count was assessed. This task assessed children's count lists and knowledge of stable order.

Give-N Initial Response. Children were presented with 15 toy fish and a large blue bowl, which served as a pond. On each trial, children were asked to put N fish in the pond (e.g., "Can you put *one* fish in the pond?"). The experimenter then asked, "Is that N?" to ensure that the child was done putting fish into the bowl.

Give-N Count-and-check. The experimenter took the fish out of the bowl, lined them up in front of the child, and asked him/her to count. After the child was done counting, the experimenter asked, "Is that N?", referring to the number initially asked for. If the child answered no, the experimenter gave the child an opportunity to fix it to be N. The experimenter repeated this process until the child answered yes.

³ All children first completed count elicitation and Give-N, and depending on their knower-level, they also completed a novel word generalization task. Only data from the first two tasks were reported here.

The first trial asked for *one* fish. If children successfully gave 1 when asked for *one*, the next request was N + 1; if they incorrectly responded to the request for N, the next request was N - 1. The highest numeral requested for all children was "six".⁴

Children were called 'N-knowers' (e.g., '1-knowers') if they (1) correctly gave N fish two out of three times when asked for N, (2) correctly avoided giving N fish for requests larger than N; and (3) failed to give the correct number two out of three times for N + 1. Children who failed to give one fish when asked for "one" were classified as 'non-knowers'. Children who gave the correct number of fish for all numerals asked for up to 'six' were called cardinal-principle (CP)knowers. Children were classified using the same criteria as the original study (BLINDED).

Coding.

We initially approached the coding of children's errors using a data-driven grounded theory framework (Glaser & Strauss, 1967; Birks & Mills, 2015), in which coding categories are derived bottom-up from the data rather than top-down from a theoretical model. Two researchers created an initial codebook characterizing children's errors in each count-and-check trial. They then used several subsequent coding passes to refine the initial codes into a smaller, more abstract set of categories of errors. The final categories almost perfectly matched the theoretical description of counting principles laid out by Gelman & Gallistel (1978), and we ultimately opted to use those principles straightforwardly as the guiding theoretical framework for coding.

On each trial (i.e., each time a child was asked to count to check if they provided N), we coded separately whether children made at least one violation of the cardinal principle and the word-object correspondence principle. Importantly, the two principles were coded independently from each other.

⁴ One child was asked to give "seven" rather than "six", and correctly provided 7 objects.

For the cardinal principle, we coded whether children used the last word of a count to denote the cardinality of a set *regardless of whether the count was correct or not*. We were not able to code children's violations of the stable-order principle because the count lists produced by children on the Give-N Task were very short due to the nature of the task. Instead, we analyzed knowledge of the stable-order principle using the Count Elicitation task. Table 1 presents examples that illustrate the three types of children's counting errors on the Give-N Task and the Count Elicitation Task.

Consider two illustrative scenarios; in the typical coding of Give-N, the first would be marked simply as "incorrect" and the second would be marked "correct". In our coding scheme, children could be marked as correct or incorrect on the cardinal principle in either scenario, depending on their attention to the last word.

- A child was asked for "three" and gave 4 objects. When asked to check if she had given three, the child placed 4 objects and correctly counted one-two-three-four (thus following the stable-order and word-object correspondence principles). After the child had counted, the experimenter asked "Is that three?" If the child agreed that there were "three," when her last word ended on "four", this child would be coded as violating the cardinal principle. However, if the child replied that no, she had not given "three", then this trial would not be coded as a cardinal principle violation.
- 2. A child was asked for "two" and correctly gave 2 objects. When asked to check if she had given two, the child incorrectly counted one-two-three for a set of 2 objects. The child was then asked "is that two?" If the child agreed that it was two, when his last word was "three", it would be coded as a cardinal principle violation. However, if he said that it was *not* two, his response would not be coded

as cardinal principle violation, because his last word in the count was indeed not "two". Either way, the child failed to tag one object with one number word, and thus violated the word-object correspondence principle.

In other cases, children might be marked as correct on following the cardinal principle, if the last word matched the requested number, but made other errors in their count. Table 1 describes children's behaviors that illustrate each of the three types of errors.

Table 1. Examples of counting errors.

Type of Error	Counting Behavior
Violation of word-object	• A child gave 5 objects when asked for 'six', double-
correspondence principle	counting the last object.
	• A child gave 4 objects when asked for 'three', counted
	1-2-3 but did not touch each object as he counted.
Violation of cardinal	• A child gave 3 objects when asked for 'two', counted 1-
principle	2-3, and responded affirmatively when asked "Is that two?"
Violation of stable-order	• A child who skipped or mixed the order of number
principle (from the Count	words: 1-2-3-4-5-6-7-8-9-10-13. The child followed a
Elicitation Task)	stable order of the count list up to "ten".

Results

Give-N. The number of children and the mean age in each knower-level group are

	N	Mean Age (SD in months)	Age Range
1-knowers	12	3;1 (2.3)	2;10-3;7
2-knowers	33	3;3 (3.2)	2;10-3;11
3-knowers	14	3;5 (3.0)	2;11 – 3;11
4-/5-knowers ⁵	7	3;5 (2.4)	3;2-3;9
CP-knowers	20	3;6 (4.0)	2;11-4;2

Table 2. Age and number of children at each knower-level.

Acquisition of the Counting Principles

Count Elicitation.

On average, the length of children's count list was 13 (*Median* = 12; *Range* = 3 to 23). A total of 81 children provided an audible count sequence, and 75 of them counted up to 10. Data were missing from five children because their count list was inaudible (N = 2) or they did not respond (N = 3). Thus, most 3-year-olds (92.6%) in our sample had already acquired a stable count sequence up to 10.

Counting Errors on Give-N Task.

⁵ We found four 4-knowers and two 5-knowers, and they were collapsed into the 4-knower group. Their patterns of performance were very similar on all critical analyses.

We asked whether children followed the word-object correspondence principle and the cardinal principle before becoming CP-knowers using the Give-N Task. For each child and for each of the two counting principles, we computed the average proportion of trials in which they made at least one error. Given that previous studies have identified 3- and 4-knowers as potentially different from other subset-knowers on cardinal principle knowledge (e.g., Le Corre et al., 2006), we formed three knower-level groups for the analyses: 1- and 2-knowers (n = 45), 3- and 4-knowers (n = 21), and CP-knowers (n = 20).

Counting Errors on All Trials. First, we examined the overall developmental trajectories of the counting principles by analyzing the frequency of counting errors across all trials. We conducted a linear mixed effects model predicting the proportion of counting errors, with Error Type (2 levels: Word-Object, Cardinal Principle) and Knower-Level Groups (3 levels: 1- and 2-knowers vs. 3- and 4-knowers vs. CP-knowers) as predictors, with random intercepts for participants and error type.⁶ We compared this model against one with age in months that were mean-centered as an additional predictor, and found that age did not significantly improve model fit ($\chi^2(1) = 1.23$, p = .27). We thus reported results from the model without age below. Coefficient estimates are shown in Table 3. Fixed effects were contrast coded.

There was a significant main effect of Knower-Level Groups and no effect of Error Type on the frequency of children's counting errors. Importantly, there was a significant interaction between Knower-Level Groups and Error Type. Post-hoc comparisons with Holm-Bonferroni corrections revealed that 1- and 2-knowers were significantly less likely to follow the two

⁶ We first fitted a model with error type as random slope, with random intercepts for participants, followed by a model with random intercepts for error type and participants, but neither of them converged due to insufficient data.

counting principles than CP-knowers, t's > -2.58, p's < .013, d's > -2.14. Three- and 4-knowers differed from CP-knowers on how likely they were to follow the cardinal principle, t(31.04) = -3.55, p = .0013, d = -1.10, but the two groups did not differ on the word-object correspondence principle, t(39.0) = -1.91, p = .063, d = -.060. We also found that 3- and 4-knowers were more likely than 1- and 2-knowers to follow the cardinal principle, t(47.7) = -4.03, p < .001, d = -1.02, but the two groups did not differ for word-object correspondence principle, t(60.0) < 1, p > .42, d = -0.20. Word-object correspondence errors remained relatively stable across the knower-levels, but children made noticeably fewer cardinal principle errors as they progressed through the knower-level stages (Figure 1).

Table 3. Coefficient estimates from linear mixed effects models predicting the likelihood of children following the two counting principles. CP-knowers were the reference category. Tests reaching statistical significance at the .05 criterion are in bold.

	Coefficient	SE	p(z)
1- and 2-knowers	-0.33	0.063	<.001
3- and 4-knowers	-0.16	0.073	.028
Error Type	-0.086	0.054	.12
1-2-knowers: Error Type	0.33	0.065	<.001
3-4-knowers: Error Type	0.098	0.076	.20

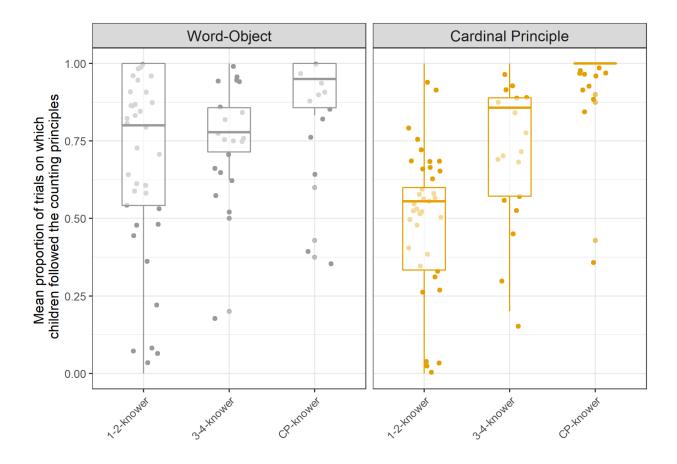


Figure 1. Mean proportion of trials in which children followed the word-object correspondence and cardinal principles, by knower-level groups. Each small dot represents one child.

To assess whether some subset-knowers have knowledge of the cardinal principle, we conducted 1-sample t-tests against chance. We used a data-driven approach to define chance. Chance was determined using a group of children whom we hypothesized to lack cardinal principle knowledge – i.e., non-knowers. We applied the same coding scheme as the other subset-knowers and found that non-knowers (n = 20 trials) followed the cardinal principle (i.e., matched the last word of their count to the requested set) 30% of the time. We took this to be our chance level. We used Holm-Bonferroni corrections to adjust for multiple comparisons. Both knower-level groups performed significantly above chance on the cardinal principle: 1- and 2-knowers, *M*

= .47, SD = .30, t(44) = 3.84, p < .001, d = 0.57; 3- and 4-knowers, M = .75, SD = .24, t(20) = 8.51, p < .001, d = 1.86.

We conducted a similar analysis comparing subset-knowers' tendencies to follow the word-object correspondence principle to non-knowers, who followed this principle 59% of the time. Both knower-level groups were significantly more likely than non-knowers in following the word-object correspondence principle: 1- and 2-knowers, M = .71, SD = .33, t(43) = 2.35, p = .023, d = 0.35; 3- and 4-knowers, M = .76, SD = .19, t(20) = 4.01, p < .001, d = 0.87.

Counting Errors on Unknown Number Trials. Given that children at higher knowerlevels have acquired more number word meanings than those at the lower levels, an analysis of all trials may overestimate performance for children at the higher levels and thus obscure the developmental trajectories for the counting principles. For example, a 4-knower can correctly generate sets of up to four objects, and thus has more trials with positive results than a 1-knower. To address this, we focused on unknown number trials (e.g., N + 1, N + 2, N + 3, etc.), because by definition, subset-knowers have not yet acquired meanings for numbers beyond their knowerlevel. Given that the analysis focused on unknown number trials, we removed CP-knowers from this analysis. For this analysis, a request for "six" was coded as the N + 1 trial for the two 5knowers, who were combined with the 4-knowers.

As in our previous analysis, the word-object correspondence principle remained stable across both of the knower-level groups, but 3- and 4-knowers were much more likely to follow the cardinal principle than those at the lower knower-levels (Figure 2). This observation was borne out in post-hoc t-tests. Three- and 4-knowers were significantly more likely than 1- and 2knowers to follow the cardinal principle on unknown trials, t(29.3) = -3.49, p = .0015, d = 0.99, but the two groups did not differ in following the word-object correspondence principle, t(35.6) < 1, *p* > .32, *d* = -0.27.

Next, we asked whether subset-knowers were above chance on following the two principles, by conducting 1-sample t-tests using the same chance level and Holm-Bonferroni corrections as the earlier analysis. We found that 3- and 4-knowers were more likely than chance to follow the cardinal principle on unknown number trials, M = .62, SD = .40, t(19) = 3.60, p =.0019, d = 0.80, but 1- and 2-knowers were at chance, M = .27, SD = .30, t(43) < 1, p > .53, d = -0.10. Both knower-level groups did not significantly differ from non-knowers in following the word-object correspondence principle for unknown numbers, $M_{1k-2k} = .51$, SD = .40; $M_{3k-4k} = .61$, SD = .39, both t's < 1, p's > .38.⁷ These results suggest that children's cardinal principle understanding develops rapidly throughout the subset-knower stages even for unknown numbers. They also highlight a lack of development in the procedural skill of tagging one object per word.

⁷ Children at the lower knower-levels might have had a higher likelihood to violate the counting principles given that they were tested on more unknown number trials than children at higher knower-levels. Thus, we also conducted this analysis on N + 1 trials only. Across knower-levels, children received on average 2 to 3 N + 1 trials ($M_{1k} = 3.0$ trials, $M_{2k} = 2.8$ trials, $M_{3k} = 2.6$ trials, $M_{4k} = 2.0$ trials). The same pattern of findings emerged when we analyzed just N + 1 trials as when we analyzed all unknown number trials for both principles. For the cardinal principle, 1- and 2-knowers were at chance, t < 1, p = .51 (M = .34, SD = .37), and 3- and 4-knowers were significantly above chance, t = 3.65, p = .0017 (M = .63, SD = .40). For the word-object correspondence principle, neither group of subset knowers performed significantly better than non-knowers: M_{1k-2k} = .60, SD = .41, M_{3k-4k} = .58, SD = .42, both t's < 1, p's > .81.

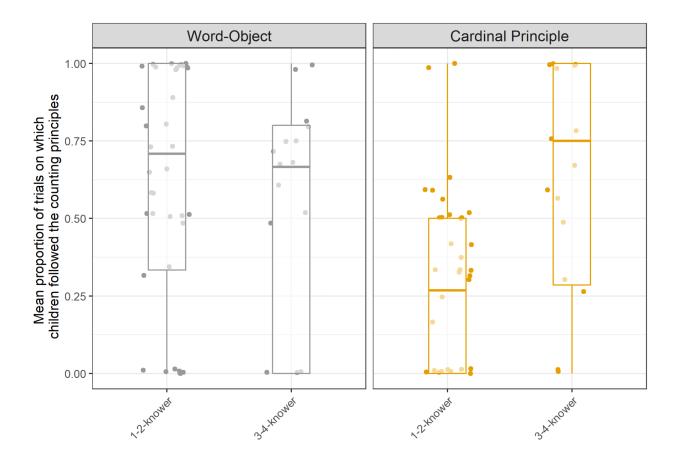


Figure 2. The likelihood of subset-knowers following the word-object and cardinal principle on unknown number trials. Each dot represents one child.

We grouped the main analyses above in order to have adequate statistical power, in line with previous studies that have identified differences between 3- and 4-knowers and lower knower-levels (Le Corre, et al., 2006). To take a closer look at developmental patterns, we also explored performance across individual knower-levels, presented in Table 3. The overall finding that children have knowledge of the cardinal principle before becoming CP-knowers holds. The six 4-knowers were statistically above chance on unknown number trials (M = .97, SD = .08, t(5)= 20, p < .001, d = 8.88). Three-knowers, whose counting behavior was more variable, were not above chance as a group (M = .47, SD = .39, t(13) = 1.66, p = .12, d = .44), likely due to a lack of power. These results show that as children approach the CP-knower stage, they become increasingly consistent at following the cardinal principle. Interestingly, at all knower-levels, children made errors of word-object correspondence. As shown in Table 3, 4-knowers were highly *unlikely* to make cardinal principle errors in counting, but they continued to make word-object correspondence errors at rates similar to other subset-knowers.

Table 3. Mean proportion (standard deviation) of unknown number trials on which children followed the Word-Object Correspondence Principle and the Cardinal Principle. (CP-knowers are not presented since there are no 'unknown numbers' for those children.)

	Word-Object Correspondence	Cardinal Principle
1-knowers	0.63 (0.42)	0.21 (0.27)
2-knowers	0.61 (0.38)	0.29 (0.31)
3-knowers	0.47 (0.39)	0.47 (0.39)
4-knowers	0.60 (0.43)	0.97 (0.08)

Individual Patterns of Acquiring Cardinal Principle Knowledge. The analyses on all trials and on unknown number trials show that children at the advanced knower-level stages begin to have knowledge of the cardinal principle. However, group analyses obscure whether individual children tend to follow or violate the cardinal principle. We thus asked how many individual subset-knowers followed the cardinal principle on unknown number trials 67% of the time. We chose 67% as our criterion because (1) subset-knowers in this study had, on average, 3.5 unknown number trials (SD = 2.1 trials), and (2) children on the Give-N task are generally

classified as an N-knower if they succeed on 2 out of 3 trials on N, and fail on 2 out of 3 trials on N + 1. We thus adopted a similar criterion to classify whether children consistently followed the cardinal principle on the unknown number trials. Only 6.8% of 1- and 2-knowers passed the 67% criterion, and approximately half (55%) of the 3- and 4-knowers did. Consistent with the group analysis, these results suggest that 1- and 2-knowers likely lack cardinal principle knowledge. Despite the fact that only half of the 3- and 4-knowers reliably demonstrated knowledge of the cardinal principle, they were seven times more likely to follow the cardinal principle in counting than the 1- and 2-knowers.

Discussion

Knowledge of the cardinal principle has been highlighted as the critical difference between subset-knowers and CP-knowers in the knower-levels framework. However, this claim has not been well tested because previous studies using this framework have not examined the developmental progression of the counting principles individually. Our results challenge the view that acquisition of the cardinal principle is the driving force that divides subset-knowers from CPknowers in four ways. First, we show that some subset-knowers follow the cardinal principle prior to being classified as CP-knowers on the Give-N task: as a group, 3- and 4-knowers were more likely than chance to follow the cardinal principle, both when we examined all trials and when we restricted our analysis to unknown number trials. Second, in both individual and group level analyses, 3- and 4-knowers were far more likely than 1- and 2-knowers to follow the cardinal principle, suggesting that subset-knowers are not a homogeneous group, and that children's ability to follow the cardinal principle develops gradually rather than suddenly. Third, we found that approximately half of the 3- and 4-knowers showed reliable cardinal principle knowledge, even on unknown number trials. Although the proportion of children who showed such knowledge is far from ceiling, this finding forces a reconsideration of the claim that subsetknowers lack the cardinal principle. Fourth, in an exploratory descriptive analysis on individual knower-levels, we found that 4-knowers almost always followed the cardinal principle, but were far less likely to do so for the word-object correspondence principle (approximately 50 to 60% of the time). Thus, at the higher knower-levels, children's errors are often not ones of cardinality, but rather ones of word-object correspondence. In short, these results show that cardinal principle knowledge emerges in subset-knowers prior to them being classified as "CP-knowers" on the Give-N task. Critically, these new data show that cardinal principle knowledge develops gradually rather than suddenly, and that violations of various counting principles, not just cardinality, characterize children's errors in the late subset-knower stages. These findings provide a more nuanced look at the gradual elements that lead to children's success in generating large sets on the Give-N Task, and lay the groundwork to update the conceptual change account.

The nature of cardinal principle knowledge

We turn our focus first to the cardinal principle and then to the other counting principles and the issue of coordination. As laid out in the introduction, there are discrepant findings in the literature regarding whether or not subset knowers have any knowledge of the cardinal principle. Here, we used a novel methodological approach, retrospectively analyzing behavior and errors from a particular segment of the Give-N Task in which all children were asked to count to check whether they had given N. In doing so, all children were given an opportunity to demonstrate knowledge of the counting principles in a relatively ecologically valid task, where they were operating on sets that they had constructed. Traditionally, in the Give-N task, children are given credit for correct responses if the set size they provide exactly matches the request. Thus, in addition to assessing knowledge of number word meanings, the Give-N task requires children to spontaneously use counting to give N and follow all of the principles simultaneously. In contrast, our method allowed us to examine children's knowledge of cardinality separately from their spontaneous decisions to count, and separately from knowledge of the other counting principles. It also contrasts with the What's-on-this-card (WOC) task, which also tests whether children spontaneously count and respond with the last word of a count (e.g., Sarnecka & Carey, 2008; Sarnecka et al., 2007; Le Corre et al., 2006). These differences in methodology, separating children's decision to count (i.e., when creating a set) from their interpretation of the last word in their own counts (i.e., when checking the set), may explain why we found evidence for cardinal principle knowledge in subset-knowers where previous studies did not.

Overall, our results are consistent with recent studies that argue for a more nuanced and gradual view of number word acquisition (O'Rear et al., 2020; Wagner et al., 2019). Although CP-knowers are more likely than subset-knowers to spontaneously count to give N (Le Corre et al., 2006; Sarnecka & Carey, 2008), Posid and Cordes (2018) found that subset-knowers can be trained within 5 minutes of a simple counting game to use counting to generate sets on the Give-N Task. Thus, subset-knowers may have implicit knowledge about the role of counting in generating cardinalities, and spontaneous counts may not provide a complete picture of children's counting knowledge. Relatedly, recent studies indicate emerging knowledge in some subset-knowers. Wagner and colleagues (2019) reanalyzed Give-N data using medians of performance, rather than means, and found that subset-knowers ("N"-knowers) sometimes responded correctly to requests for N + 1 and N + 2. In a secondary analysis of previous studies, O'Rear and colleagues (2020) also found that approximately 20 to 30% of subset-knowers were accurate on N + 1 (see also Barner & Bachrach, 2009).

Our findings contribute to this growing body of literature on partial number understanding. The fact that children produce occasional instances of above-chance performance in following each of the counting principles can help explain the isolated instances of subsetknowers' ability to give N + 1 on the Give-N Task in the previous studies. That is, subsetknowers may be able to correctly generate numbers larger than their "known" number *because* they can sometimes access each of the three counting principles. This may also explain why subset-knowers on the Give-N Task are occasionally classified as CP-knowers on another task (Marchand & Barner, 2020; Le Corre et al., 2006).

If some subset-knowers do understand the cardinal principle, why don't they achieve the level of CP-knowers on the Give-N Task? One possibility is that these children do not know how to fix an incorrect set upon recognizing that they did not provide N. Indeed, in an exploratory analysis, we examined children's attempts to fix an incorrect set, and found that CP-knowers were more likely to fix correctly, with 61.9% of incorrect sets being fixed correctly (out of 21 instances), compared to 44% for 1- and 2-knowers (out of 9 instances) and 37.5% for 3- and 4knowers (out of 8 instances). An anecdote from a recently tested child (not included in this study) nicely illustrates children's difficulty with fixing errors. The child was asked to give "eight". On previous trials, when he counted fish carefully into the bowl, he was successful (including on some requests for "six" and "seven").⁸ When asked for "eight", however, he dumped 12 fish into the bowl. The experimenter laid out the fish in a line to count and check. When the child got to the eighth one and said "eight", he looked puzzled, took the eighth object, and moved it to the end of the row. He started counting again, got to the 8th object, again looked puzzled, and repeated his correction. If anything, this child was doing his utmost to follow the cardinal principle, making it so that the label of "eight" would appear at the end of the counted row. One might say that he was committed to a kind of word-object correspondence, having glued the label

⁸ This child responded correctly to two requests for "six", both after fixing incorrect responses, and to one request for "seven" by counting carefully. He was unable to fix initially incorrect responses on one trial asking for "seven" and two trials asking for "eight". Thus, he is a highly unusual case and possibly right on the transition to becoming a CP-knower.

"eight" to a specific object. Yet, despite following the counting principles, this type of fixing would never get him to the right answer. Like this child, it seems that more advanced subsetknowers may technically understand the cardinal principle, but have not yet integrated it into a more sophisticated logic of counting, coordinating all three principles simultaneously within the placeholder structure of the count list.

Relatedly, children have to grasp not only that the last word represents the cardinality of the set, but also why the last word represents the cardinality of the set. To do so, they must also grasp that the procedure only works when no other rules are violated. Violations of other counting principles would unravel the causal structure that leads the last word to represent cardinality. It is possible that before becoming CP-knowers, children might have some idea about the cardinal principle, but not a complete causal theory. If we consider the child described above, his error in moving the eighth object indicates an aspect of procedural knowledge about cardinality: that the count should end on the target number word. His behavior was ambiguous with respect to knowledge that the word "eight" captured the value of the whole set, not the identity of a single item. It is ambiguous because it is not possible to tell why he wanted the object labeled "eight" to be at the end of the row: because he knew, procedurally, that the item tagged "eight" should be at the end (i.e., the "last word rule"), or because he knew, conceptually, that ending on "eight" would mean that he had the correct number of items in the set (i.e., the cardinal principle). But, either way, he clearly did not understand the causal structure that connects counting to cardinal values; he didn't understand that the last item had to be in the eighth position for the count to produce a cardinality of eight. It might be that understanding this causal structure is critical for children's ability to establish sets on Give-N. Furthermore, our preliminary results on children's ability to fix their errors suggest that, when children become CP-knowers, they have also acquired some knowledge of *relations* between numbers, which

allows them to employ appropriate addition or subtraction strategies to fix an incorrect set, despite the fact that this relational knowledge may only apply to a small range of numbers initially (Carey & Barner, 2019; Cheung et al., 2017). Future studies should examine children's attempts to fix incorrect sets to shed light into children's understanding of counting and relations between number words and quantities.

Our findings also raise the possibility that if advanced subset-knowers are provided with feedback on the Give-N Task, they might then perform more like CP-knowers. For example, if the child correctly rejects that they have given N if their count ends on a different number, the experimenter could highlight that it is indeed not N, but M. Future studies can examine this by modifying the Give-N task procedure by adding feedback.

Our findings corroborate those of Sarnecka and Carey (2008), who reported that even subset-knowers frequently gave the last word in a count as the response to "how many?", and that their knowledge of a 'last-word rule' increased with age. Sarnecka & Carey distinguished knowledge of the "last word rule" from knowledge of the cardinal principle. They identified a difference between the Give-N task and the How-Many task as a potential source of the difference in children's apparent appreciation of the cardinal principle, namely that the How-Many task explicitly asks "how many" whereas the Give-N task does not use those words. Here, we did not ask children "how many", yet some subset-knowers still attended to the last word in the count. Thus, children's understanding of the importance of the last word in a count is more flexible than simply identifying the last word as the answer to a "how many?" question. Children's attention to the last word may in fact represent an emerging concept of the cardinal principle, rather than a shallow or rote response.

Nevertheless, we do not claim that subset-knowers have full conceptual understanding of the cardinal principle. We suggest that they procedurally match the last word of a count to a

target amount, and that they likely understand that the last number denotes the value of the set. Recall that in our study, children could get credit for having cardinal principle knowledge even if they made errors on their response to create a set of N. Children were credited as having knowledge of the cardinal principle if they accepted that they had given N when their last word of the count ended on N, or if they rejected that they had given N when their last word ended on not-N, regardless of whether they correctly generate a set of N. Thus, we specifically tested whether children apply the last word of a count to denote the cardinality of a set they have created, even in the context of other errors. Critically, our analysis suggests that children sometimes uphold the cardinal principle while abandoning the other counting principles, leading to an overall incorrect response. Thus, an important aspect of counting that CP-knowers have, but subset-knowers may not, is the ability to *coordinate all three counting principles at once* to correctly give sets of items. After all, the cardinal principle--that the last word in a count denotes the cardinal value of the set-only holds if and only if the other how-to-count principles are obeyed as well. Recent research has highlighted a need to tease apart the procedural and conceptual understanding of the cardinal principle (Carey & Barner, 2019; Cheung et al., 2017; Davidson et al., 2012). The current study contributes to this literature by showing that, at the very least, the procedural application of the cardinal principle develops gradually as children acquire number word meanings, and that the application of this principle is well in place before the transition to the CPknower stage in the knower-levels framework.

Our claims are consistent with the conceptual change account theorized by Carey (2009), but shift the proposed locus of the conceptual change. Specifically, we suggest that children understand the cardinal principle prior to generalizing their understanding of higher numbers on the Give-N task. The major conceptual shift is therefore not about acquisition of the cardinal principle per se, but rather the coordination and integration of the cardinal principle with the other counting principles, the ordinal structure of the count list, and the causal relations that link counting to cardinality.

Developmental trajectory of the stable-order and word-object correspondence principles.

Finally, consistent with previous studies, we found that most children can recite the count sequence up to "ten" at age 3, and thus stable order may be the first how-to-count principle to be acquired by children. Our data also show that knowledge of the word-object correspondence shows little developmental change during the subset-knower period, unlike cardinal principle knowledge. Even at the earliest stages of number word learning, children can already apply the word-object correspondence principles, suggesting that the two foundational skills for procedural counting are in place before children acquire procedural knowledge of the cardinal principle. Interestingly, though their performance is well above chance from early on, children continue to make errors of word-object correspondence at every subset-knower level. Indeed, we observed that 4-knowers almost always followed the cardinal principle, yet they made relatively more word-object correspondence errors. Such a finding supports the argument that CP-knowers' central insight lies with coordinating the three counting principles and perhaps not the cardinal principle per se.

These results have implications for interventions that aim to train children's understanding of counting, and highlight the need to focus on teaching young children the meaning of the last word of a count *and* the coordination of the three principles when creating sets of items (Mix, Sandhofer, Moore, & Russell, 2012; Paliwal & Braoody, 2018; O'Rear & McNeil, 2017). In particular, once children reach the higher knower-levels, allowing them to engage in the act of counting so they can coordinate the three counting principles may constitute more effective training than highlighting that the last word of a count tells how many.

Previous studies have suggested that even CP-knowers or older children struggle with the

word-object correspondence principle when counting larger sets of objects (Frye, Braisby, Lowe, Maroudas, & Nicholls, 1989; Barclay, Cheung, & Shusterman, 2017), and our results echo those findings. The current study suggests that prior to becoming CP-knowers, children recognize the need to tag one object with one number word, but they may yet have trouble *applying* this principle to larger sets. It is not clear whether these challenges are due to higher procedural demands of coordinating a longer verbal count list with the pointing gesture when counting large sets, or to a more profound failure to extend a conceptual understanding of the word-object correspondence principle to larger sets. Further research would be useful to illuminate whether procedural or working memory demands fully explain children's difficulties with coordinating counts in the context of larger sets.

Summary

The current study examined children's knowledge of the counting principles prior to becoming CP-knowers, using a retrospective video analysis of the Give-N Task. We assessed knowledge of the cardinal principle independent of children's ability to provide correct responses on the Give-N Task, and found evidence that advanced subset-knowers understand that the last word of a count reflects the cardinal value of a set. We also found that subset-knowers have acquired the stable-order and word-object correspondence principles, though not always perfectly. Our findings suggest that children have knowledge of all three counting principles before learning how to coordinate them to generate sets of items, and that the coordination of all three principles may itself represent an important conceptual step.

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