might represent numerical sizes without representing them as varying discretely. Arguably, this would be so if the only computations performed on the representations were well-defined on continuously varying magnitudes as well, such as comparison and addition/subtraction. (A system that also exhibited sensitivity to whether there is a one–one correspondence between two collections might be said to represent certain magnitudes as cardinal numbers; and one that displayed a sensitivity to the immediate successor relation might be taken to represent some magnitudes as natural numbers, if these are taken to be things related to zero by the ancestral of that relation.) Is this only a question of *how* the (numerical) magnitudes are represented?

In any case, it seems there is a difference between attributing the properties of *being eight in number* and *being **roughly** eight in number*: If the collection to which the property is attributed has nine items in it, the second attribution is correct, whereas the first is not. Therefore, this distinction would appear to concern what is represented, not how it is represented. Perhaps, Clarke and Beck will say this shows instead only that cognitive episodes involving the ANS have accuracy conditions, which admit of degrees, rather than veridicality or truth conditions, which do not – and that it is indeterminate what (i.e., which property) is represented by the ANS?

Financial support. This research was not supported by any dedicated funding.

Conflict of interest. There are no conflicts of interest to declare.

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Numerical cognition needs more and better distinctions, not fewer

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doi:10.1017/S0140525X21001163, e0

Abstract

We agree that the approximate number system (ANS) truly represents number. We endorse the authors' conclusions on the arguments from confounds, congruency, and imprecision, although we disagree with many claims along the way. Here, we discuss some complications with the meanings that undergird theories in numerical cognition, and with the language we use to communicate those theories.

We agree that the approximate number system (ANS) represents number and aim to clarify theoretical arguments that are entangled in questions about terminology. What do we mean by area, number, numerosity, and representation?

Although the authors are right that the "argument from congruency" and the "argument from confounds" ultimately fail, some evidence bolstering those arguments is shaky in the first place. When we use *physical* area to infer the relative contributions of continuous and discrete stimulus properties to quantity judgments, we're neglecting a history of psychophysical evidence that perceived and physical area differ (Barth, 2008). Empirical support for this idea came from three experiments in which cumulative area judgments were driven by perceived, not physical, area. Some "arguments from congruency" depend on interpretations based on physical area (e.g., Hurewitz, Gelman, & Schnitzer, 2006; Rousselle & Noël, 2008). Yet, quantity judgments can yield apparent congruency effects that disappear when perceived area is considered instead of physical area (Barth, 2008). Incorporating perceived area won't resolve controversies surrounding discrete versus continuous quantity (see Aulet & Lourenco, 2021; Savelkouls & Cordes, 2020; Yousif & Keil, 2020). Nevertheless, to identify processes underlying quantity judgments, *subjective* magnitudes should be explored as potential behavior cues. Otherwise, we'll get the wrong idea about whether number influences area or vice versa, or both, or neither.

We also have to be clear on the terms "number" and "numerosity." We were surprised at the authors' lengthy condemnation of "numerosity." In our usage, "numerosity" refers to a property of a stimulus, not a representation. An array of dots (or string of sounds) has a numerosity. That numerosity is larger when the elements are more numerous. If we've used the phrase "numerosity representation," we weren't referring to woolly "number-like properties" (Burge, 2010). We meant "mental representation of Q34
number that refers to the numerosity of a stimulus." It's not a hedge – it's shorthand.

Do other psychologists share our understanding of what "numerosity" means, in which case the target article is simply wrong that our language implies "an assumption that, strictly speaking, the ANS represents numerosities, not numbers" (Clarke & Beck, sect. 6)? Or, are psychologists' uses of "numerosity" inconsistent? We think Clarke and Beck (and Burge) are wrong about what "numerosity" means to researchers, but either way they've done a service in exposing this confusion, and the field had better get clear about what it does mean.

That said, dropping "numerosity" for "number" isn't the answer. "Number" is ambiguous, and ambiguity breeds confusion. "Number" can refer to number words and Arabic numerals (i.e., symbols for natural numbers) or a property of stimuli (i.e., numerosity) or mathematical entities. For psychologists, it is useful to have a term that unambiguously refers to the number of items in a stimulus. "Numerosity" allows psychologists to discuss discrete quantity without endorsing commitments about how it is represented in the mind.

The target article itself suffers from terminological confusion, over "number" and, at times, "representation." Use of "number" when the authors appear to intend "natural number" frequently obscures their meaning. (We spent considerable time decoding what was meant by each instance of "number"!) And Clarke and Beck seem to answer claims about what is made explicit by a representational system with arguments about the contents of representations within that system. For example, Carey (2009) argued that the ANS *as a representational system* cannot grant natural number concepts to an organism. But, this is not a critique of the idea that ANS representations have true numerical content! Carey (2009) is clear that the ANS represents number: "that analog magnitude representations constitute one system of number representations deployed by human adults has been established beyond any reasonable doubt" (p. 131) and "analog magnitudes are explicit symbols of approximate cardinal values of sets" (p. 135). Carey's argument doesn't attempt "to undermine the hypothesis that the ANS represents number" (Clarke & Beck, sect. 5).

Furthermore, Clarke and Beck wave away the question of "modes of presentation," arguing that the same property under different modes of presentation is still the same property. Therefore, they argue, the word "number" should suffice to describe that property. For psychologists, however, mode of presentation is not an afterthought. How do different representations of identical aspects of the world map on to each other in the mind? Which modes of presentation subserve word learning, computational tasks, and behavior?

When we ask a question like "where do human number concepts come from," we see that the use of a single word like "number" elides questions of interest. The ANS as a representational system does not encode exactness or the successor function, essential components of natural number. This limitation is important in querying what roles the ANS can play in learning. We concur that ANS representations don't serve as the conceptual source of precise number concepts (Carey & Barner, 2020), and empirical evidence indicates that children don't learn number word meanings via mappings to ANS representations (Carey, Shusterman, Haward, & Distefano, 2017). The fact that the ANS encodes some aspects of number (e.g., its second-order character), but not others (e.g., exactness), highlights the importance of using more specific terminology to clarify which aspects of number, and which properties of relevant representations, are under discussion. The authors' push to use the term "number" promiscuously has a muddying effect rather than a clarifying one.

Eronen and Bringmann (2021) argue that theory development in psychology suffers, in part, from "the relative lack of robust phenomena that impose constraints on possible theories" and "problems of validity of psychological constructs." Numerical cognition is rich in robust phenomena, and construct validity is coming along. But, we have an enduring terminology tangle. Carey (2009) wrote: "It then becomes a merely terminological matter whether one wants to use the term 'number' only for natural number or for the integers or for the integers plus the rationals plus the reals (in which case there is no core cognition of number) and adopt some other term for the quantificational content of core cognition systems" (p. 297). Maybe it's not so "merely" after all.

Financial support. This work was supported by the National Science Foundation (HB: NSF DRL-1920445; AS: NSF DRK12-2010547)

Conflict of interest. None.

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The number sense does not represent numbers, but cardinality comparisons

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doi:10.1017/S0140525X21001072, e0

Abstract

Against Clarke and Beck's proposal that the approximate number system (ANS) represents natural and rational numbers, I suggest that the experimental evidence is better accommodated by the (much weaker) thesis that the ANS represents cardinality comparisons. Cardinality comparisons do not stand in arithmetical relations and being able to apply them does not involve basic arithmetical concepts and operations.

Clarke and Beck vigorously defend the thesis that the approximate number system (ANS) represents number, which they take to include the natural numbers and the rational numbers (fractions). Although they present compelling responses to some (but not all – see below) objections to their view, the evidence that they present seems consistent with the much weaker thesis that the ANS represents different types of cardinality comparisons. My challenge to Clarke and Beck is to explain why we need anything more than cardinality comparisons to account for the operation of the ANS.

To explain the simplest form of cardinality comparisons, we can begin with the concept of *equinumerosity*. Informally speaking, two sets are equinumerous when they have the same number of members. In mathematical logic, equinumerosity is standardly understood in terms of there being a 1:1 mapping (a *bijection*) between the two sets. This concept does not, of course, involve any reference to number or numbers, which is why it is the foundation for the influential approach to understanding numbers in the philosophy of mathematics known as *logicism*. But, there is no need for fancy mathematical machinery to put this concept to work – simply pairing each apple with exactly one orange and each orange with exactly one apple will establish that a set of apples and a set of oranges are equinumerous.

Much of the experimental evidence cited in support of an ANS takes the form of demonstrated sensitivity to situations where two