Relative energetics of CH_3CH_2O , CH_3CHOH , and CH_2CH_2OH radical products from ethanol dehydrogenation

Cite as: J. Chem. Phys. **155**, 114306 (2021); https://doi.org/10.1063/5.0062809 Submitted: 08 July 2021 . Accepted: 29 August 2021 . Published Online: 20 September 2021

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ABSTRACT

This study has examined the relative energetics of nine stationary points associated with the three different radical isomers generated by removing a H atom from ethanol at the O atom (ethoxy, CH_3CH_2O), the α C atom (CH_3CHOH), and the β C atom (CH_2CH_2OH). For the first time, CCSD(T) geometry optimizations and harmonic vibrational frequency computations with the cc-pVTZ and aug-cc-pVTZ basis sets have been carried out to characterize two unique minima for each isomer along with three transition state structures with C_s symmetry. Explicitly correlated CCSD(T) computations were also performed to estimate the relative energetics of these nine stationary points near the complete basis set limit. These benchmark results were used to assess the performance of various density functional theory (DFT) and wave function theory methods, and they will help guide method selection for future studies of alcohols and their radicals. The structures generated by abstracting H from the α C atom have significantly lower electronic energies (by at least 7 kcal mol⁻¹) than the CH₃CH₂O and CH₂CH₂OH radicals was found to be a transition state in this study with MP2, CCSD(T), and a number of DFT methods. An implicit solvation model used in conjunction with DFT and MP2 methods did not qualitatively change the relative energies of the isomers, but the results suggest that the local minima for the CH₃CHOH and CH₂CH₂OH radicals could become more energetically competitive in condensed phase environments, such as liquid water and ethanol.

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I. INTRODUCTION

Oxidative degradation of ethanol in the human liver leads to production of a series of reactive oxygen species as byproducts of lipid peroxidation. These include highly reactive C_2H_5O radicals, such as the hydroxyethyl radicals (HERs), CH₃CHOH (α) and CH₂CH₂OH (b), as well as the ethoxy radical, CH₃CH₂O (O).¹⁻⁴ Oxidation of ethanol to acetaldehyde is performed by three major classes of degradative enzymes, alcohol dehydrogenases, catalases, and cytochromes. Specifically, cytochrome P450 2E1 (CYP2E1), which is present in microsomes of hepatic cells, has been implicated in the formation of the harmful HERs under study here.^{5–7} Chronic ethanol exposure causes the upregulation of CYP2E1, which is responsible for the production of HERs *in vivo*, via abstraction of a hydrogen atom from ethanol.^{1–4,8,9} Increased concentrations of HERs in the blood are found to correlate with liver-related injuries as well as generating autoimmune responses that attack hepatocytes. Furthermore, increased HER levels are linked to lipid peroxidation, thus being a contributor to hepatic steatosis, or "fatty liver," which occurs frequently in users exhibiting heavy alcohol use.^{1,8,10}

This family of ethanol-derived radicals, denoted here as $C_2H_5O^{|\bullet}$, is also of keen interest regarding their applications in the combustion of ethanol, as well as atmospheric chemistry and pollution control.^{11–22} Bioethanol is a common gasoline additive. The autoignition and oxidation of ethanol occur through a variety of pathways, of which the reaction of ethanol with halogens or reaction of O + C₂H₅ and OH + C₂H₄ produces hydroxyethyl radicals as intermediates.^{16,23–29} The role of hydroxyalkyl radicals in ethanol combustion has been implicated by the discovery that the enol tautomers of aldehydes are significant intermediates in ethanol combustion.²⁹ A number of previous studies elucidated reaction mechanisms and kinetics of the autoignition of ethanol, involving the reactions of C₂H₅O^{|•} isomers with NO,³⁰ O₂,^{11,20} and H₂O.^{31,32}

Although CH₃CHOH is the lowest-energy isomer, the CH₂CH₂OH isomer exhibits a geometry favorable for internal abstraction, yielding the lowest dissociation pathway of the $C_2H_5O^{|\bullet|}$ isomers to produce OH and C_2H_4 as products.³²

Curtis et al. previously investigated the energies of various $C_2H_5O^{\dagger}$ and $C_2H_5O^{\dagger}$ isomers.³³ They performed MP2/6-31G* geometry optimizations and G2 computations on seven conformations of the three $C_2H_5O^{|\bullet|}$ isomers, with the lowest energy corresponding to the CH₃CHOH isomer. Two conformations of CH₃CHOH were identified. They differed only in the orientation of the hydroxyl hydrogen (anti vs gauche), with anti-CH₃CHOH being slightly lower in energy (0.3 kcal mol⁻¹). Additionally, they reported three stationary points of CH₂CH₂OH, again differing in the orientation of the hydroxyl hydrogen (anti vs gauche), with a second anti-CH₂CH₂OH minimum identified exhibiting C_s symmetry. However, these structures were reported to be ~8 kcal mol⁻¹ higher in energy than the CH₃CHOH isomers. Finally, two minima were identified for the ethoxy isomer (CH₃CH₂O), one for the ${}^{2}A'$ electronic state and the other for the ${}^{2}A''$ electronic state. Both were determined to be $\sim 10 \text{ kcal mol}^{-1}$ higher in energy than the CH₃CHOH isomers.33 Similar results have been obtained for the relative energetics of the CH₃CHOH, CH₂CH₂OH, and/or CH₃CH₂O isomers from other theoretical studies examining the decomposition and isomerization of $C_2H_5O^{]\bullet}$ with *ab initio* electronic structure methods, such as CBS-QB3, CCSD, CCSD(T), and QCISD(T).^{28,32,34–38}

A total of nine $C_2H_5O^{|\bullet|}$ structures are characterized here with full geometry optimizations and harmonic vibrational frequency computations. Given the biochemical significance of HERs, this work examines the energetics of those $C_2H_5O^{|\bullet|}$ isomers in an aqueous environment with an implicit solvation model. CCSD(T) computations on the isolated radical species (*in vacuo*) are used to calibrate less demanding methods that are subsequently used to assess condensed phase effects on the energetics of this system.

II. THEORETICAL METHODS

Full geometry optimizations and corresponding harmonic vibrational frequency computations are performed on the C_s minimum energy structure of ethanol, CH₃CH₂OH, using density functional theory (DFT) and wave function theory (WFT) techniques, specifically the B3LYP, B3LYP-D3,³⁹⁻⁴² M06-2X, M06-2X, D3,^{40,41,43} ω -B97XD,^{44,45} ω -B97,⁴⁶ ω -B97X,⁴⁶ B2PLYP-D,^{47,48} DSD-PBEP86,^{49,50} PW6B95-D3,^{40,41,51} M06-L, M06-L-D3,^{40,41,52} MP2,⁵³ and CCSD(T)⁵⁴ methods. All computations for CH₃CH₂OH utilize a spin-restricted closed-shell reference function, denoted by the "r" prefix, and are performed with Dunning's correlation consistent triple- ζ basis sets with and without diffuse functions, aug-cc-pVTZ and cc-pVTZ, abbreviated here as aTZ and TZ.^{55,56}

Full geometry optimizations and corresponding harmonic vibrational frequency computations are performed on the nine $C_2H_5O^{|\bullet|}$ stationary points with the aforementioned methods and a spin-unrestricted reference function for the doublet electronic state of these radicals, denoted by the "u" prefix. All computations are performed with the aTZ and TZ basis sets. To examine potential effects from the choice of spin reference function, additional B3LYP, M06-2X, and ω -B97XD computations were performed with

a spin-restricted open-shell reference function (denoted by the "ro" prefix).

A series of single point energy computations were performed on the uCCSD(T) structures optimized with the TZ and aTZ basis sets. These spin unrestricted and partially spin restricted coupled-cluster energy computations employed a highspin restricted Hartree–Fock (RHF) reference wave function. The canonical CCSD(T) computations are denoted by RHF-uCCSD(T) and RHF-rCCSD(T), and they were carried out with the standard and augmented correlation consistent basis sets (cc-pVXZ and augcc-pVXZ, abbreviated here as XZ and aXZ, where X = D, T, Q). A suffix of either "–F12a" or "–F12b" is appended to denote the ansatz adopted for the corresponding explicitly correlated computations that employed the analogous cc-pVXZ-f12 basis sets, where X = D, T, and Q (abbreviated XZ-f12).⁵⁷ The spin-projected MP2 (pMP2)^{58–60} energies are also reported for the corresponding uMP2 optimized structures.

To investigate the potential of solvents to affect the molecular structure and energetics, the Polarized Continuum Model $(PCM)^{61,62}$ is employed with water, ethanol, and *n*-hexane. PCM is performed exclusively with DFT and MP2 methods employing the TZ and aTZ basis sets. The Gaussian 16 software package⁶³ is employed for the DFT and MP2 computations. The CCSD(T) optimizations and harmonic vibrational frequency computations are performed with the CFOUR software package64,65 using the available analytic gradients and Hessians. The canonical and explicitly correlated RHF-uCCSD(T) and RHF-rCCSD(T) single point energy computations were performed with the MOLPRO program.^{66,67} These energy computations utilized the default f12 auxiliary basis sets, and the triples contributions to the CCSD(T)-F12 energies were not scaled in this investigation. The numerical integration grids, convergence criteria, and PCM solvent parameters employed herein were the default options provided for DFT methods in Gaussian 16.

III. RESULTS AND DISCUSSION

A. Optimized structures

The uCCSD(T)/TZ optimized molecular structures of the nine $C_2H_5O^{]\bullet}$ stationary points identified in this study are presented in Fig. 1. The uCCSD(T)/aTZ optimized structures are given in Fig. S1 of the supplementary material. Some redundant geometric parameters are reported for the C_s structures for consistency. The number of imaginary frequencies, n_i , for each structure is provided in the first row of Table I, and the corresponding uCCSD(T) harmonic vibrational frequencies can be found in the supplementary material. For comparison, the CCSD(T)/TZ and aTZ optimized structures and harmonic vibrational frequencies for ethanol, CH₃CH₂OH, are also provided in the supplementary material.

Abstraction of a hydrogen atom from ethanol at the α C (C1) produces two CH₃CHOH conformations, denoted here as *anti*-CH₃CHOH [Fig. 1(a)] and *gauche*-CH₃CHOH [Fig. 1(b)], which are predicted to be the lowest-energy structures by all of the DFT and WFT methods employed. Furthermore, the two α isomer conformations have rather similar electronic energies, with our best estimates of the CCSD(T) complete basis set (CBS) limit predicting *anti*-CH₃CHOH to be about 0.3 kcal mol⁻¹ lower in energy than *gauche*-CH₃CHOH. See Sec. III B for a detailed discussion of energetics. The changes in the C1–C2 and C1–O bond lengths, Δ R,

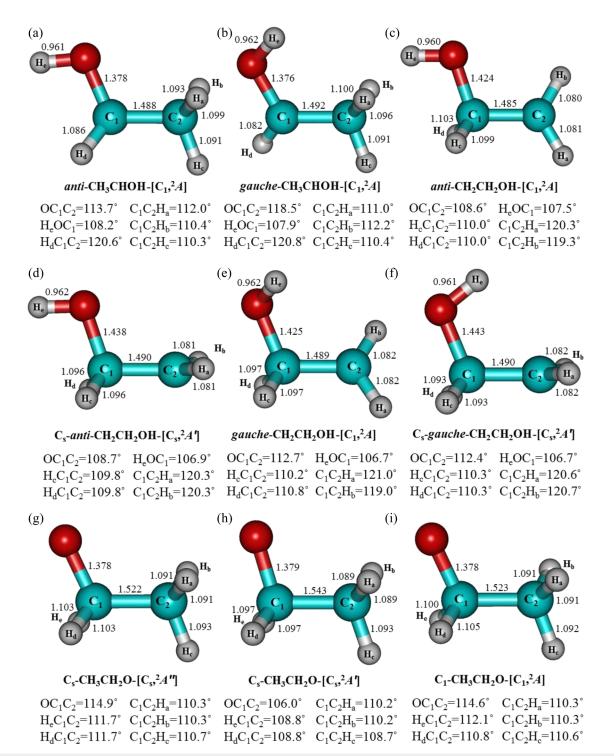


FIG. 1. uCCSD(T)/TZ optimized structures of (a) anti-CH₃CHOH, (b) gauche-CH₃CHOH, (c) anti-CH₂CH₂OH, (d) C_s-anti-CH₂CH₂OH, (e) gauche-CH₂CH₂OH, (f) C_s-gauche-CH₂CH₂OH, (g) C_s-CH₃CH₂O-²A'', (h) C_s-CH₃CH₂O-²A', and (i) C₁-CH₃CH₂O-²A. Bond lengths are presented in angstroms and bond angles in degrees.

	CH ₃ CHOH			CH	CH ₃ CH ₂ O				
Method	anti $[^{2}A]$	gauche [² A]	anti $[^{2}A]$	C_s -anti $[^2A']$	gauche [² A]	C_s -gauche [² A']	$C_{s}-[^{2}A^{\prime\prime}]$	C_s -[² A']	$C_1 - [^2A]$
n _i ^a	0	0	0	1	0	1	1	0	0
uCCSD(T)/TZ ^b	0	0.24	8.50	9.37	7.44	9.02	9.68	10.15	9.66
RHF-uCCSD(T)/TZ ^b	0	0.24	8.51	9.38	7.44	9.02	9.70	10.17	9.68
RHF-uCCSD(T)/QZ ^b	0	0.30	8.35	9.26	7.45	8.88	10.63	11.09	10.61
RHF-uCCSD(T)/aTZ ^b	0	0.32	8.18	9.01	7.33	8.63	10.13	10.55	10.11
RHF-uCCSD(T)/aQZ ^b	0	0.32	8.30	9.21	7.47	8.84	10.90	11.35	10.88
RHF-uCCSD(T)-F12b/DZ-f12 ^b	0	0.32	8.39	9.27	7.53	8.95	10.73	11.23	10.72
RHF-uCCSD(T)-F12b/TZ-f12 ^b	0	0.33	8.39	9.30	7.54	8.93	10.97	11.45	10.96
RHF-uCCSD(T)-F12b/QZ-f12 ^b	0	0.33	8.39	9.32	7.55	8.94	11.10	11.59	11.09
uCCSD(T)/aTZ ^c	0	0.32	8.17	8.98	7.32	8.61	10.12	10.54	10.11
RHF-uCCSD(T)/TZ ^c	0	0.24	8.52	9.39	7.45	9.04	9.69	10.16	9.67
RHF-uCCSD(T)/QZ ^c	0	0.30	8.35	9.26	7.45	8.88	10.63	11.09	10.62
RHF-uCCSD(T)/aTZ ^c	0	0.32	8.17	8.99	7.32	8.62	10.14	10.55	10.12
RHF-uCCSD(T)/aQZ ^c	0	0.32	8.30	9.20	7.47	8.83	10.91	11.36	10.89
RHF-uCCSD(T)-F12b/DZ-f12 ^c	0	0.32	8.39	9.26	7.53	8.94	10.74	11.23	10.72
RHF-uCCSD(T)-F12b/TZ-f12 ^c	0	0.32	8.39	9.30	7.54	8.93	10.98	11.46	10.96
RHF-uCCSD(T)-F12b/QZ-f12 ^c	0	0.32	8.39	9.32	7.55	8.94	11.11	11.60	11.10

TABLE I. Relative electronic energies, ΔE (in kcal mol⁻¹), predicted with both canonical and explicitly correlated CCSD(T) energy computations for each CCSD(T)/TZ optimized energetic minimum. CCSD(T)/TZ optimized relative energies are included for reference.

^aNumber of imaginary vibrational frequencies from the analytic Hessians available for the uMP2, uCCSD(T), and various spin-unrestricted DFT methods with the TZ and aTZ basis sets. All frequencies are reported in the supplementary material for readers interested in the different values of n_i obtained with the default numerical integration grids and convergence criteria for certain functionals and/or basis sets.

^buCCSD(T)/TZ optimized geometry.

^cuCCSD(T)/aTZ optimized geometry.

induced by dehydrogenation for the lowest-energy conformation of each isomer class are provided in Tables S4 and S5, i.e., deviations relative to ethanol. All of the DFT and WFT methods predict both the C1–C2 and C1–O bond lengths to contract by 0.031 ± 0.003 and 0.050 ± 0.004 Å, respectively, when H is removed from the α C of ethanol to form *anti*-CH₃CHOH. Interestingly, the lowest-energy C₂H₅O^{]•} isomer, *anti*-CH₃CHOH, exhibits the greatest overall geometric changes from ethanol (large contractions in both the C1–C2 and C1–O bonds).

Four CH_2CH_2OH structures [Figs. 1(c)-1(f)] were found upon abstraction of a hydrogen atom from ethanol at β C (C2), all ~8 kcal mol⁻¹ higher in energy than anti-CH₃CHOH. All DFT and WFT computations indicate that the lowest-energy conformation of the β isomer corresponds to gauche-CH₂CH₂OH [Fig. 1(e)]. Our uCCSD(T) harmonic vibrational frequency computations with the TZ and aTZ basis sets show that two of the β structures characterized here correspond to transition states, Cs-anti-CH2CH2OH and Cs-gauche-CH2CH2OH, as shown in Figs. 1(d) and 1(f), respectively. Although uMP2/6-31G* frequencies indicated that the former was a minimum $(n_i = 0)$,³³ our characterization of that stationary point is consistent with the detailed study of the CH₂CH₂OH radical by Karpichev et al., indicating that this particular structure is a transition state possessing a single imaginary vibrational mode $(n_i = 1)$ at the CCSD/6-311+G(d,p) level of theory.³⁴ Both Cs transition state structures of the CH₂CH₂OH radical connect equivalent forms of the corresponding minima (mirror images of anti- and gauche-CH₂CH₂OH). The electronic barrier heights

associated with this conformational transition states are on the order of 1 kcal mol⁻¹. Additionally, Liu *et al.* identified a conformational transition state (TS4/4') between the *anti*- and *gauche*-CH₂CH₂OH minima, lying ~0.6 kcal mol⁻¹ above the former based on QCISD(T)/6-311G(2*df*,*p*) single point energy computations with B3LYP/6-311G(*d*,*p*) optimized structures. All of the DFT and WFT methods predict the C1–C2 bond length to contract by 0.031 ± 0.004 Å when H is removed from the β C of ethanol to form *gauche*-CH₂CH₂OH. Unlike the α isomer, the C1–O bond length in the β isomer remains practically unchanged from that of ethanol, depending on the method, at 0.000 ± 0.001 Å (Tables S4 and S5).

Finally, abstraction of the hydrogen atom from the oxygen atom in ethanol leads to the highest-energy isomer, CH₃CH₂O, with ΔE around 11 kcal mol⁻¹ near the CCSD(T) CBS limit. When the CH₃CH₂O isomer has C_s symmetry, the electronic energies reported in Table I indicate that the ²A' electronic state lies only ~0.5 kcal mol⁻¹ above the ²A'' state. Although the higher-energy ²A' stationary point is a minimum, the lower-energy ²A'' optimized structure has one imaginary frequency according to analytic Hessians computed with the uMP2, uCCSD(T), and several DFT methods using both the TZ and aTZ basis sets. Interestingly, QCISD/6-311++G(*d*,*p*) frequencies obtained by finite difference procedures did yield a single imaginary vibrational mode for the ²A'' C_s structure of the ethoxy radical.²⁸ However, B3LYP/6-311++G(*d*,*p*) frequencies computed analytically in that same study indicated that it was a minimum ($n_i = 0$). When the symmetry of the ²A'' structure is relaxed, a nearly isoenergetic C₁ minimum is identified from

subsequent uMP2, uCCSD(T), and other geometry optimizations that deviates slightly from the C_s symmetry with an O–C1–C2–H_c torsional angle near 178°. All methods consistently predict a slight elongation of the C1–C2 bond length (0.008 ± 0.002 Å) and the largest observed contraction overall in the C1–O bond length (0.056 ± 0.009 Å) when moving from ethanol to the lowest-energy O electronic state, C1-²A (Tables S4 and S5). This is opposite to the behavior exhibited by the β isomer, which exhibits large contractions in the C1–C2 bond length and virtually no change in the C1–O bond length.

B. Energetics

Table I provides the relative electronic energies, ΔE , for a selection of the canonical and explicitly correlated uCCSD(T) energy computations performed on the uCCSD(T)/TZ and uCCSD(T)/aTZ optimized structures. The full set of relative energies predicted by the canonical and explicitly correlated CCSD(T) energy computations is presented in the supplementary material, Tables S6 and S7. Use of an unrestricted vs restricted open-shell reference function with the DFT and MP2 methods results in virtually the same ΔE for the C₂H₅O^{|•} isomers. Similarly, the maximum absolute deviation (MaxAD) for ΔE between RHF-uCCSD(T) and RHF-rCCSD(T) is 0.11 kcal mol⁻¹ for both canonical and explicitly correlated results. The differences between the results obtained with the a vs b ansatz for the F12 energy computations were even smaller, with a MaxAD in ΔE of 0.04 kcal mol⁻¹ when employing the TZ-f12 and QZ-f12 basis sets. There is very little basis set dependence for ΔE with H abstraction from the C atoms. The canonical and F12 results are within ~0.01 kcal mol⁻¹ for ΔE obtained with any quadruple- ζ basis set. ΔE for the O isomer conformations is more sensitive to the choice of basis set, as the same metric grows to ~ 0.5 kcal mol⁻¹.

The QZ-f12 results in Table I are near our best estimates of the CCSD(T) CBS limit and are thus used to discuss ΔE for the $C_2H_5O^{]\bullet}$ isomers, and those ΔE values do not change by more than 0.01 kcal mol⁻¹ when the uCCSD(T)/aTZ optimized structures are used instead. Clearly, the *anti*-CH₃CHOH conformation of the α isomer has the lowest energy, residing 7.55 kcal mol⁻¹ below *gauche*-CH₂CH₂OH and 11.10 kcal mol⁻¹ below the C₁-²A O isomer. The other conformation of the α isomer, *gauche*-CH₃CHOH, is predicted to lie only 0.33 kcal mol⁻¹ higher in energy than *anti*-CH₃CHOH. The *gauche* conformation of the β isomer is confirmed to be the lowest-energy, with the *anti*-local minimum only 0.84 kcal mol⁻¹ higher in energy. The QZ-f12 results in Table I indicate that ΔE is \geq 11.10 kcal mol⁻¹ for all O isomer structures and that the ²A' minimum is only 0.50 kcal mol⁻¹ above the C₁ minimum on the ground state surface of CH₃CH₂O.

A detailed analysis of the CH₂CH₂OH isomer by Karpichev et al. also provided relative energies near the CCSD(T) CBS limit from TZ/QZ extrapolations that are virtually identical (within 0.04 kcal mol⁻¹) to those obtained here for that subset of structures with Ref. 34. Our results presented in Table I are also consistent with published relative energetics from CCSD(T), QCISD(T), and other computations for select structures of these radical isomers,^{28,32,35-38} many of which are tabulated in the supplementary material for convenience.

The RHF-uCCSD(T)-F12b/QZ-f12 relative electronic energies in Table I are employed as the reference to assess the accuracy of a series of DFT and WFT methods. Table II presents the deviations from those reference ΔE values for each optimized stationary point in kcal mol⁻¹ computed with the selected methods and the TZ basis set. The average absolute deviation (AvgAD) and maximum absolute deviation (MaxAD) from the reference values are also presented. The full table of ΔE computed with the selected DFT and WFT methods and the TZ and aTZ basis sets is provided in the supplementary material, Table S6. The AvgADs and MaxADs computed with the aTZ basis set are also provided in Table S7. Although zero-point vibrational energies are not discussed here, they can be assessed from the harmonic vibrational frequencies reported in the supplementary material.

Overall, deviations from the CCSD(T) CBS limit for the DFT methods are somewhat smaller when the basis set includes diffuse functions (aTZ vs TZ); however, this trend reverses for the uMP2 and pMP2 ab initio methods. Based on the results in Table II, it is clear that most methods struggle to quantitatively reproduce the relative energies of the three different isomers (CH₃CHOH, CH₂CH₂OH, and CH₃H₂O). The double-hybrid functional, DSD-PBEP86, outperforms all other methods when compared to the RHF-uCCSD(T)-F12b/QZ-f12 energies with an AvgAD of 0.39 kcal mol⁻¹ and MaxAD of 0.58 kcal mol⁻¹. The second double-hybrid functional B2PLYP-D also performs well compared to the other selected methods, confirming B2PLYP-D's previously described good performance in computing the energetics of small carboncentered radicals at a reduced cost.68 The Minnesota hybrid functional, M06-2X, and each of its variants are also among the most accurate, with uM06-2X providing an AvgAD of 0.51 kcal mol⁻¹ and MaxAD of 0.91 kcal mol⁻¹. The two methods based on secondorder Møller-Plesset perturbation theory provide accurate energetics when considering the CH₃CHOH and CH₂CH₂OH isomers but begin to break down when investigating the varying electronic states of the CH₃CH₂O isomer. In fact, uMP2/TZ exhibits one of the highest MaxAD's of 3.70 kcal mol⁻¹. Likewise, the pure functionals, M06-L and M06-L-D3, performed worse in computing the relative energies of the $C_2H_5O^{]\bullet}$ isomers, with an AvgAD of 2.23 kcal mol⁻¹ and MaxAD of 4.51 kcal mol^{-1} .

C. Implicit solvation effects

In an effort to determine how solvation affects the energetics of the $C_2H_5O^{]\bullet}$ isomers, implicit solvation computations were performed using the Polarized Continuum Model (PCM) and three different solvents, ranging in their polarity and dielectric constants: water, ethanol, and *n*-hexane. Overall, these solvent models induced fairly small changes to the relative energies of the nine different $C_2H_5O^{]\bullet}$ structures characterized in this investigation, generally on the order of a few tenths of a kcal mol⁻¹.

The H₂O solvent model induces the largest changes to the relative energies (Table III), ranging from ~-0.4 to +0.8 kcal mol⁻¹, but the effects from ethanol are quite similar. Hexane consistently produced the smallest changes to ΔE , typically less than ±0.1 kcal mol⁻¹ but reaching ~±0.2 kcal mol⁻¹ for CH₃CH₂O. The values obtained with the TZ basis set tend to be slightly larger in magnitude than those computed with the aTZ basis set. The changes to ΔE for the ethanol and hexane solvent models are tabulated in the supplementary material along with the aTZ results for water.

	CH ₃ CHOH		CH ₂ CH ₂ OH				CH ₃ CH ₂ O					
Method	anti $[^{2}A]$	gauche [² A]	anti $[^{2}A]$	C_s -anti $[^2A']$	gauche [² A]	C _s -gauche [² A']	C_s - $[^2A'']$	C_s -[² A']	$C_1 - [^2A]$	AvgAD	MaxAD	
uM06-L	0.00	+0.13	+1.10	+1.36	+1.16	+1.37	-4.51	-3.73	-4.49	2.23	4.51	
uM06-L-D3	0.00	+0.13	+1.11	+1.37	+1.17	+1.39	-4.50	-3.71	-4.48	2.23	4.50	
uB3LYP	0.00	-0.01	+1.33	+1.59	+1.10	+1.32	-2.52	-2.00	-2.51	1.55	2.52	
roB3LYP	0.00	-0.01	+1.43	+1.64	+1.16	+1.42	-2.18	-1.69	-2.16	1.46	2.18	
uB3LYP-D3	0.00	-0.11	+1.17	+1.47	+0.89	+1.16	-2.71	-2.31	-2.70	1.56	2.71	
uM06-2X	0.00	-0.11	+0.43	+0.40	+0.18	+0.35	-0.91	-0.80	-0.90	0.51	0.91	
roM06-2X	0.00	-0.11	+0.52	+0.46	+0.25	+0.48	-0.79	-0.71	-0.78	0.51	0.79	
uM06-2X-D3	0.00	-0.10	+0.43	+0.40	+0.19	+0.36	-0.90	-0.78	-0.89	0.51	0.90	
uωB97XD	0.00	-0.11	+1.44	+1.79	+1.28	+1.59	-2.07	-1.55	-2.05	1.49	2.07	
roωB97XD	0.00	-0.11	+1.55	+1.86	+1.36	+1.73	-1.75	-1.23	-1.73	1.42	1.86	
uωB97	0.00	-0.03	+1.34	+1.72	+1.28	+1.56	-2.61	-2.08	-2.60	1.65	2.61	
uωB97X	0.00	-0.02	+1.37	+1.77	+1.28	+1.60	-2.34	-1.77	-2.33	1.56	2.34	
uPW6B95-D3	0.00	-0.11	+1.87	+2.31	+1.62	+1.93	-2.59	-2.20	-2.58	1.90	2.59	
uB2PLYP-D	0.00	-0.09	+0.85	+0.98	+0.63	+0.92	-1.15	-1.01	-1.15	0.85	1.15	
uDSD-PBEP86	0.00	-0.06	+0.38	+0.43	+0.20	+0.47	-0.58	-0.44	-0.58	0.39	0.58	
uMP2	0.00	-0.01	+0.14	+0.07	-0.01	+0.28	+3.70	+3.64	+3.69	1.44	3.70	
pMP2	0.00	-0.02	-0.01	-0.03	-0.12	+0.09	+3.46	+3.41	+3.44	1.32	3.46	
uCCSD(T)	0.00	-0.09	+0.10	+0.05	-0.11	+0.07	-1.43	-1.44	-1.43	0.59	1.44	
RHF-uCCSD(T)-F12b	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00			

TABLE II. Deviations (in kcal mol⁻¹) from the RHF-uCCSD(T)-F12b/QZ-f12 relative electronic energy (ΔE) values for the selected DFT and WFT methods with the TZ basis set.

For the *gauche* structures of both the CH₃CHOH and CH₂CH₂OH isomers, none of the solvent models induce changes of more than ± 0.1 kcal mol⁻¹ at any level of theory with the TZ basis set. In contrast, the ΔE of the *anti*-conformation of CH₂CH₂OH consistently decreases by ~0.4 kcal mol⁻¹ with the water and ethanol solvent models. The largest energetic effects of implicit solvation

are always associated with H abstraction from the O atom, with ΔE increasing by ~0.5 to 0.7 kcal mol⁻¹ when using the methods that most reliably reproduced the CCSD(T) energetics as discussed in Sec. III B (e.g., M06-2X and DSD-PBEP86).

These results can be used to estimate the relative energetics of the isomers near the CCSD(T) CBS limit in an aqueous environment

TABLE III. Shifts of the relative electronic energies (in kcal mol⁻¹) for each optimized stationary point from isolation to an implicit aqueous environment, computed with PCM and the TZ basis set.

PCM-water	CH ₃	СНОН		СН	CH ₃ CH ₂ O				
Method	anti $[^{2}A]$	gauche [² A]	anti $[^{2}A]$	C_s -anti $[^2A']$	gauche [² A]	C_s -gauche [² A']	$\overline{\mathrm{C}_{\mathrm{s}}\text{-}[^{2}A^{\prime\prime}]}$	C_s -[² A']	$C_1 - [^2A]$
uM06-L	0.00	-0.10	-0.36	-0.35	+0.06	+0.02	+0.19	+0.38	+0.19
uM06-L-D3	0.00	-0.10	-0.36	-0.35	+0.06	+0.02	+0.19	+0.38	+0.19
uB3LYP	0.00	-0.09	-0.39	-0.41	+0.05	0.00	+0.32	+0.41	+0.31
uB3LYP-D3	0.00	-0.09	-0.39	-0.41	+0.02	-0.01	+0.31	+0.41	+0.31
uM06-2X	0.00	-0.08	-0.41	-0.42	+0.06	0.00	+0.55	+0.65	+0.55
uM06-2X-D3	0.00	-0.08	-0.41	-0.42	+0.06	0.00	+0.55	+0.65	+0.55
uωB97XD	0.00	-0.08	-0.37	-0.40	+0.06	+0.04	+0.43	+0.55	+0.42
u ω B97	0.00	-0.07	-0.37	-0.43	+0.02	0.00	+0.48	+0.61	+0.48
u <i>ω</i> B97X	0.00	-0.08	-0.37	-0.42	+0.04	+0.02	+0.47	+0.60	+0.47
uPW6B95-D3	0.00	-0.07	-0.38	-0.40	+0.07	+0.05	+0.35	+0.45	+0.35
uB2PLYP-D	0.00	-0.10	-0.40	-0.43	+0.01	-0.04	+0.48	+0.55	+0.48
uDSD-PBEP86	0.00	-0.09	-0.39	-0.41	+0.03	-0.03	+0.62	+0.70	+0.62
uMP2	0.00	-0.10	-0.42	-0.43	-0.01	-0.08	+0.69	+0.76	+0.69
pMP2	0.00	-0.10	-0.39	-0.44	0.00	-0.06	+0.68	+0.75	+0.68
Average	0.00	-0.09	-0.39	-0.41	+0.04	0.00	+0.45	+0.56	+0.45
Maximum	0.00	-0.10	-0.42	-0.44	+0.07	-0.08	+0.69	+0.76	+0.69
Minimum	0.00	-0.07	-0.36	-0.35	0.00	0.00	+0.19	+0.38	+0.19
Range	0.00	0.03	0.06	0.09	0.07	0.07	0.50	0.39	0.50

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by combining the QZ-f12 results from Table I with the DSD-PBEP86 PCM shifts from Table III. The *anti*-conformation of CH₃CHOH still has the lowest energy of the nine structures, but the *gauche* form is only about 0.2 kcal mol⁻¹ higher in energy in an aqueous environment. ΔE for the *gauche* minimum of the CH₂CH₂OH isomer remains +7.6 kcal mol⁻¹, whereas the relative energy for the *anti*minimum decreases slightly in the water solvent model to +8.0 kcal mol⁻¹ near the CCSD(T) CBS limit. The analogous estimate for the C₁ minimum of the CH₃CH₂O isomer still yields a large relative energy of +11.7 kcal mol⁻¹ in an aqueous environment.

IV. CONCLUSIONS

Nine conformations of the $C_2H_5O^{\dagger \bullet}$ isomers generated by the abstraction of a hydrogen atom from ethanol at the α carbon (C1), β carbon (C2), or oxygen (O) atom have been characterized in isolation (*in vacuo*) with a range of DFT and WFT methods. Two minima have been identified for each isomer from CCSD(T) geometry optimizations and harmonic vibrational frequency computations with the TZ and aTZ basis sets, along with three transition state structures having the C_s symmetry.

RHF-uCCSD(T)-F12b/QZ-f12 computations have confirmed that the CH₃CHOH radical isomer has the lowest energy, with the *gauche* local minimum having an electronic energy roughly 0.3 kcal mol⁻¹ higher than *anti*-CH₃CHOH. Four stationary points have been identified for the CH₂CH₂OH radical isomer. The *gauche*-CH₂CH₂OH structure has the lowest electronic energy of these four conformations, all of which lie at least 7.5 kcal mol⁻¹ higher than *anti*-CH₃CHOH according to CCSD(T) electronic energies near the CBS limit. Three CH₃CH₂O isomer structures have been reported, each with $\Delta E \ge +11$ kcal mol⁻¹ according to our RHF-uCCSD(T)-F12b/QZ-f12 single point energy computations. The ²A'' C₈ structure of the CH₃CH₂O radical was found to be a transition state with the MP2, CCSD(T), and most DFT methods, which led to the identification of a C₁ minimum on the doublet ground state surface that deviates only slightly from the ²A'' transition state structure.

Analysis of the accuracy of a selection of DFT and WFT methods has revealed that M06-2X and the double-hybrid functionals (uB2PLYP-D and uDSD-PBEP86) outperform the other DFT and WFT methods employed here in reliably computing relative energetics of the $C_2H_5O^{\bullet}$ isomers when compared to the benchmark ΔE values. The effects of solvation on the energetics and molecular geometries of the C2H5O^{|•} isomers have also been investigated with PCM computations employing the default solvent parameters for water, ethanol, and hexane. Although no large, qualitative changes were observed in the $C_2H_5O^{\dagger}$ energetics when moving from isolation to an implicit solvent model, the local minima of the CH₃CHOH and CH₂CH₂OH isomers became more energetically competitive in water and ethanol. A subsequent study is planned to investigate this effect in greater detail by explicitly solvating these radical isomers with water and ethanol molecules to form a diverse set of clusters spanning a wide range of sizes and hydrogen bonding topologies.

SUPPLEMENTARY MATERIAL

The relative energies obtained with the DFT methods mentioned above and the TZ and aTZ basis sets can be found in the supplementary material along with the harmonic vibrational frequencies computed with all levels of theory. The deviations in energies and geometries induced by implicit solvation models in ethanol and hexane are presented to support our conclusion. The Cartesian coordinates for the nine $C_2H_5O^{]\bullet}$ structures examined in this study are also provided.

AUTHORS' CONTRIBUTIONS

All authors contributed equally to this work.

ACKNOWLEDGMENTS

This work was supported, in part, by the National Science Foundation (Grant No. CHE-1664998). The Mississippi Center for Supercomputing Research (MCSR) is also acknowledged for allocation of time on their computational resources.

The authors have no conflicts to disclose.

DATA AVAILABILITY

Data from the geometry optimizations and corresponding harmonic vibrational frequency computations for each $C_2H_5O^{]\bullet}$ isomer conformation are available from the corresponding author upon reasonable request.

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