Search for a Light Higgs Boson in Single-Photon Decays of $\Upsilon(1S)$ Using $\Upsilon(2S) \to \pi^+\pi^-\Upsilon(1S)$ Tagging Method

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We search for a light Higgs boson (A^0) decaying into a $\tau^+\tau^-$ or $\mu^+\mu^-$ pair in the radiative decays of $\Upsilon(1S)$. The production of $\Upsilon(1S)$ mesons is tagged by $\Upsilon(2S) \to \pi^+\pi^-\Upsilon(1S)$ transitions, using 158×10^6 $\Upsilon(2S)$ events accumulated with the Belle detector at the KEKB asymmetric energy electron-positron collider. No significant A^0 signals in the mass range from the $\tau^+\tau^-$ or $\mu^+\mu^-$ threshold to 9.2 GeV$/c^2$ are observed. We set the upper limits at 90% credibility level (C.L.) on the product branching fractions for $\Upsilon(1S) \to \gamma A^0$ and $A^0 \to \tau^+\tau^-$ varying from 3.8×10^{-6} to 1.5×10^{-4}. Our results represent an approximately twofold improvement on the current world best upper limits for the $\Upsilon(1S) \to \gamma A^0(\to \tau^+\tau^-)$ production. For $A^0 \to \mu^+\mu^-$, the upper limits on the product branching fractions for $\Upsilon(1S) \to \gamma A^0$ and $A^0 \to \mu^+\mu^-$ are at the same level as the world average limits, and vary from 3.1×10^{-7} to 1.6×10^{-5}. The upper limits at 90% credibility level on the Yukawa coupling $f_{\Upsilon(1S)}$ and mixing angle $\sin\theta_{A^0}$ are also given.

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In 2012, the last missing standard model (SM) particle, a Higgs boson, was discovered by ATLAS and CMS [1,2], demonstrating that the Higgs mechanism would break the electroweak symmetry and give rise to the masses of W and Z bosons as well as quarks and leptons [3,4]. Besides this massive Higgs boson, three CP-even, two CP-odd, and two charged Higgs bosons are predicted by the next-to minimal supersymmetric standard model (NMSSM) [5–9]. NMSSM adds an additional singlet chiral superfield to the minimal supersymmetric standard model [10] to address the so-called "little hierarchy problem" [11], in which the value of the supersymmetric Higgs mass parameter μ is many orders of magnitude below the Planck scale.

The lightest CP-odd Higgs boson, denoted as A^0, could have a mass smaller than twice the mass of the b quark, making it accessible via radiative $\Upsilon(nS) \to \gamma A^0$ ($n = 1$, 2, and 3) decays [5–9,12]. The coupling of the A^0 to $\tau^+\tau^-$ and $b\bar{b}$ is proportional to $\tan\beta\cos\theta_{A^0}$, where $\tan\beta$ is the ratio of vacuum expectation values for the two Higgs doublets, and θ_{A^0} is the mixing angle between doublet and singlet CP-odd Higgs bosons [7]. The branching fraction of $\Upsilon(nS) \to \gamma A^0$ could be as large as 10^{-4}, depending on the values of the A^0 mass, $\tan\beta$, and $\cos\theta_A$ [7]. For $2m_\tau < m_{A^0} < 2m_b$, the decay of $A^0 \to \tau^+\tau^-$ is expected to dominate [7,13]. For $m_{A^0} < 2m_\tau$, the $A^0 \to \mu^+\mu^-$ events can be copiously produced [13].

Identifying the origin and nature of dark matter (DM) is a longstanding unsolved problem in astronomy and particle physics. One type of DM, often called the weakly interacting massive particle (WIMP), is generally expected to be in the mass region ranging from $\mathcal{O}(1)$ MeV [14,15] to $\mathcal{O}(100)$ TeV [16–21]. An extensive experimental search program has been devoted to WIMPs with the electroweak mass, but no clear evidence has been found to date [22]. In recent years, the possibility that WIMPs have a mass at or below the GeV scale has gained much attraction. For example, the decay of $\Upsilon(nS) \to \gamma H$ followed by the H decaying into a lepton pair such as $\tau^+\tau^-$ and $\mu^+\mu^-$ is suggested to be searched for in the B factories [23–25], where H is the mediator having an interaction between the WIMP and SM particles.

BaBar and Belle Collaborations have searched for A^0 decaying into a pair of low mass dark matter with the invisible final states in $\Upsilon(1S)$ radiative decays [26,27]. Searches for A^0 decaying into $\tau^+\tau^-$ and $\mu^+\mu^-$ have been also performed in $\Upsilon(1S, 2S, 3S)$ radiative decays by CLEO [28] and *BaBar* [29–32]. No significant signals were found.

The upper limits at 90% credibility level (C.L.) on the product of branching fractions $\mathcal{B}(\Upsilon(nS) \to \gamma A^0)\mathcal{B}(A^0 \to \tau^+\tau^-/\mu^+\mu^-)$ ($n = 1$, 2, and 3) have been set at levels of 10^{-6} and 10^{-5}. In particular, for $\Upsilon(1S)$ decays, more stringent upper limits are obtained by *BaBar* [29,30].

In this Letter, we conduct a search for the light CP-odd Higgs boson A^0 in $\Upsilon(1S)$ radiative decays with $A^0 \to \tau^+\tau^-$ and $A^0 \to \mu^+\mu^-$. This search is based on an $\Upsilon(2S)$ data sample with the integrated luminosity of 24.91 fb^{-1}, corresponding to $(158 \pm 4) \times 10^6$ $\Upsilon(2S)$ events, collected by the Belle detector [33] at the KEKB asymmetric-energy e^+e^- collider [34]. A detailed description of the Belle detector can be found in Refs. [33]. The $\Upsilon(1S)$ mesons are selected via the $\Upsilon(2S) \to \pi^+\pi^-\Upsilon(1S)$ transitions. In this case one must trigger and reconstruct final states in which two extra low momentum pions are identified in the detector, trying to avoid collecting too many background events and at the same time maintaining a high trigger efficiency. We assume that the width of A^0 can be neglected compared to the experimental resolution and the lifetime of A^0 is short enough [35].

We use EVTGEN [36] to generate signal Monte Carlo (MC) events to determine signal line shapes and efficiencies, and optimize selection criteria. The VVPIPI model [36] is used to generate the decay $\Upsilon(2S) \to \pi^+\pi^-\Upsilon(1S)$. The angle of the radiative photon in the $\Upsilon(1S)$ frame (θ_γ) is distributed according to $1 + \cos^2\theta_\gamma$ for $\Upsilon(1S) \to \gamma A^0$. The effect of final-state radiation (FSR) is taken into account in the simulation using the PHOTOS package [37]. The simulated events are processed with a detector simulation based on GEANT3 [38]. Multiple A^0 masses are generated: 3.6(0.22) GeV$/c^2$ to 9.2 GeV$/c^2$ in steps of 0.5 GeV$/c^2$ or less for $A^0 \to \tau^+\tau^-(\mu^+\mu^-)$. Inclusive MC samples of $\Upsilon(2S)$ decays with four times the luminosity as the real data are produced to check possible peaking backgrounds from $\Upsilon(2S)$ decays [39].

The entire decay channel can be written as $\Upsilon(2S) \to \pi^+\pi^-\Upsilon(1S)$, $\Upsilon(1S) \to \gamma A^0$, and $A^0 \to \tau^+\tau^-/\mu^+\mu^-$. In selecting $A^0 \to \tau^+\tau^-$ candidates, at least one tau lepton decays leptonically, resulting in five different combinations: $\tau\tau \to ee$, $\mu\mu$, $e\mu$, $e\pi$, and $\mu\pi$, writing with neutrinos omitted. Note that $\tau^- \to \pi^-\nu_\tau$, $\tau^- \to \pi^-\nu_\tau + n\pi^0$ ($n \geq 1$), are all included in $\tau \to \pi$. Events in which both tau leptons decay hadronically ($\tau\tau \to \pi\pi$) suffer from significantly larger and poorly modeled backgrounds than in the leptonic channels, and therefore this mode is excluded.

The charged tracks and particle identifications for the pions and leptons are performed using the same method as in Ref. [40]. An electromagnetic calorimeter cluster is treated as a photon candidate if it is isolated from the projected path of charged tracks in the central drift chamber. The energy of photons is required to be larger than 50 MeV. The most energetic photon is regarded as the $\Upsilon(1S)$ radiative photon.

For $A^0 \to \tau^+\tau^-$, the missing energy in the laboratory frame is required to be greater than 2 GeV to suppress non-τ decays and ISR backgrounds. The dominant backgrounds come from $\Upsilon(2S) \to \pi^+\pi^-\Upsilon(1S)[\to \ell^+\ell^-(\gamma)]$ ($\ell = e$, μ, or τ) decays, which have an event topology similar to that of the signal. The backgrounds from π^0 decays are also large, where photons from π^0 decays are misidentified as $\Upsilon(1S)$ radiative photons, especially when the energy of $\Upsilon(1S)$ radiative photon is low. To reduce such backgrounds, a likelihood function is employed to distinguish isolated photons from π^0 daughters using the invariant mass of the photon pair, photon energy in the laboratory frame, and the angle with respect to the beam direction in the laboratory frame [41]. We combine the signal photon candidate with any other photon and then reject both photons of a pair whose π^0 likelihood is larger than 0.3. To further suppress π^0 backgrounds in $\rho^\pm \to \pi^\pm\pi^0$, we require $\cos\theta(\gamma\pi^\pm) < 0.4$, where $\cos\theta(\gamma\pi^\pm)$ is the cosine of the angle between the photon from $\Upsilon(1S)$ decays and $\pi^\pm$ from $\tau^\pm$ decays in the laboratory frame. We impose requirements of $\cos\theta(\gamma e) < 0.95$ and $\cos\theta(\gamma\mu) < 0.8$ to remove FSR and $\Upsilon(1S) \to \mu^+\mu^-(\gamma)/e^+e^-(\gamma)$ backgrounds, where $\cos\theta(\gamma e)$ and $\cos\theta(\gamma\mu)$ are the cosine of the angle between the $\Upsilon(1S)$ radiative photon and e and μ from τ decays in the laboratory frame. All of the above selection criteria have been optimized by maximizing $\mathrm{FOM} = N_{\mathrm{sig}}/\sqrt{N_{\mathrm{sig}} + N_{\mathrm{bkg}}}$, where N_{sig} is the expected signal yield from signal MC samples assuming $\mathcal{B}(\Upsilon(1S) \to \gamma A^0)\mathcal{B}(A^0 \to \tau^+\tau^-) = 10^{-5}$ [28,29], and N_{bkg} is the number of normalized background events from inclusive MC samples.

For $A^0 \to \mu^+\mu^-$, a four-constraint ($4C$) kinematic fit constraining the four momenta of the final-state particles to the initial e^+e^- collision system is performed to suppress backgrounds with multiple photons and improve mass resolutions. The χ^2/n.d.o.f. of the $4C$ fit is required to be less than 12.5, where the number of degrees of freedom (n.d.o.f.) is four. The cosine of the angle between the $\Upsilon(1S)$ radiative photon and μ is required to be less than 0.8 to suppress FSR and $\Upsilon(1S) \to \mu^+\mu^-(\gamma)$ backgrounds. These requirements have also been optimized using the FOM method assuming $\mathcal{B}[\Upsilon(1S) \to \gamma A^0]\mathcal{B}(A^0 \to \mu^+\mu^-) = 10^{-6}$ [28,30].

The $\Upsilon(1S)$ is tagged by the requirement on the mass recoiling against a pion pair (recoil mass). The best candidate is chosen by selecting the recoil mass of dipion closest to the $\Upsilon(1S)$ nominal mass [42].

Considering τ decays with undetected neutrinos, we identify the A^0 signal using the photon energy in the $\Upsilon(1S)$ rest frame [$E^*(\gamma)$], which can be converted to $M(\tau^+\tau^-)$ via $M^2(\tau^+\tau^-) = m^2_{\Upsilon(1S)} - 2m_{\Upsilon(1S)}E^*(\gamma)$, where $m_{\Upsilon(1S)}$ is the nominal mass of $\Upsilon(1S)$ [42]. Hereinafter, M represents a measured invariant mass. For $A^0 \to \mu^+\mu^-$, we identify the A^0 signal using the invariant mass distribution of $\mu^+\mu^-$

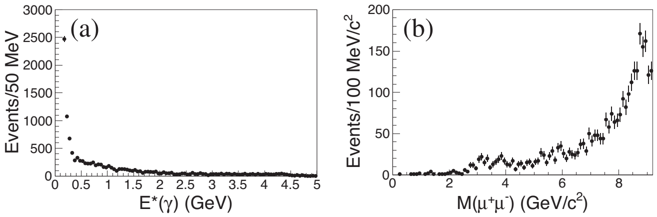

FIG. 1. The (a) $E^*(\gamma)$ and (b) $M(\mu^+\mu^-)$ distributions from the $\Upsilon(2S)$ data sample.

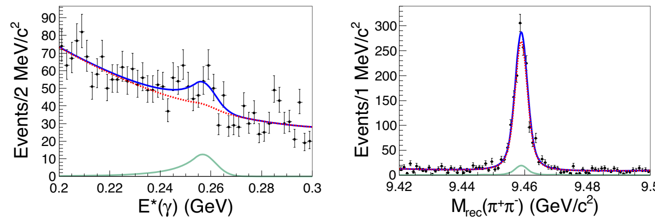

FIG. 2. The fitted result corresponding to the maximum local significance of 3.5σ with A^0 mass fixed at 9.2 GeV/c^2 for $A^0 \to \tau^+\tau^-$. The blue solid curves show the best fitted result, and the red dashed curves show the fitted total backgrounds. The green curves show the signal component.

[$M(\mu^+\mu^-)$]. After requiring the events within the $\Upsilon(1S)$ signal region of [9.45, 9.47] GeV/c^2 and the application of the above requirements, the $E^*(\gamma)$ and $M(\mu^+\mu^-)$ distributions from the $\Upsilon(2S)$ data sample are as shown in Fig. 1. No significant signals are seen.

For $A^0 \to \tau^+\tau^-$, we perform a series of two-dimensional (2D) unbinned maximum-likelihood fits to $E^*(\gamma)$ and $M_{\rm rec}(\pi^+\pi^-)$ distributions to extract the $\Upsilon(1S) \to \gamma A^0(\to \tau^+\tau^-)$ signal yields. The 2D fitting function $f(E, M)$ is expressed as

$$f(E,M) = N^{\rm sig} s_1(E) s_2(M) + N^{\rm bg}_{\rm sb} s_1(E) b_2(M) + N^{\rm bg}_{\rm bs} b_1(E) s_2(M) + N^{\rm bg}_{\rm bb} b_1(E) b_2(M), \quad (1)$$

where $s_1(E)$ and $b_1(E)$ are the signal and background probability density functions (PDFs) for the $E^*(\gamma)$ distributions, and $s_2(M)$ and $b_2(M)$ are the corresponding PDFs for the $M_{\rm rec}(\pi^+\pi^-)$ distributions. Here, $N^{\rm bg}_{\rm sb}$ and $N^{\rm bg}_{\rm bs}$ denote the numbers of peaking background events in the $E^*(\gamma)$ and $M_{\rm rec}(\pi^+\pi^-)$ distributions, respectively, and $N^{\rm bg}_{\rm bb}$ is the number of combinatorial backgrounds in both A^0 and $\Upsilon(1S)$ candidates. For $A^0 \to \mu^+\mu^-$, similar 2D unbinned maximum-likelihood fits to the $M(\mu^+\mu^-)$ and $M_{\rm rec}(\pi^+\pi^-)$ distributions are performed.

In each 2D unbinned fit, the A^0 signal in the $E^*(\gamma)$ distribution is described by a crystal ball function [43], and that in the $M(\mu^+\mu^-)$ distribution by a double Gaussian function. The $\Upsilon(1S)$ signal in the $M_{\rm rec}(\pi^+\pi^-)$ distribution is described by a double Gaussian function. The values of the signal parameters are fixed to those obtained from the fits to the corresponding signal MC distributions. The background shapes are described by a polynomial function. All parameters are floated in the fits. We choose the order of the polynomial to minimize the Akaike information test [44], and find that the first-order polynomial for $M(\mu^+\mu^-)$ and second-order polynomials for $E^*(\gamma)$ and $M_{\rm rec}(\pi^+\pi^-)$ are suitable. The fitting step is approximately half of the resolution in $E^*(\gamma)$ or $M(\mu^+\mu^-)$, resulting in total of 724 and 2671 points for $A^0 \to \tau^+\tau^-$ and $A^0 \to \mu^+\mu^-$, respectively. From the $\tau^+\tau^-(\mu^+\mu^-)$ threshold [3.6 (0.22) GeV/c^2] to 9.2 GeV/c^2, the resolution of the $E^*(\gamma)$ distribution decreases from 5.5 to 0.5 MeV, and the mass resolution of the $M(\mu^+\mu^-)$ distribution increases from 1.4 to 10.0 MeV/c^2. For each 2D unbinned fit in $A^0 \to \mu^+\mu^-$ ($m_{A^0} > 3.0$ GeV/c^2) and $A^0 \to \tau^+\tau^-$, the fitting range covers a $\pm 10\sigma$ region. Since the number of selected signal candidate events in the $\mu^+\mu^-$ mode with $m_{A^0} < 3.0$ GeV/c^2 is small, we select the following fitting intervals for different A^0 masses: $2m_\mu \le M(\mu^+\mu^-) \le 2.2$ GeV/c^2 for 0.22 GeV/$c^2 \le m_{A^0} \le 2.0$ GeV/c^2, and 1.8 GeV/$c^2 \le M(\mu^+\mu^-) \le 3.2$ GeV/c^2 for 2.0 GeV/$c^2 < m_{A^0} \le 3.0$ GeV/c^2.

Figures 2 and 3 show the fitted results when the A^0 masses are fixed at 9.2 GeV/c^2 and 8.51 GeV/c^2 for $A^0 \to \tau^+\tau^-$ and $A^0 \to \mu^+\mu^-$, respectively, where we find the maximum local signal significances for possible A^0 peaks. We define the local signal significance as ${\rm sign}(N_{\rm sig})\sqrt{-2\ln(\mathcal{L}_0/\mathcal{L}_{\rm max})}$ [45], where $\mathcal{L}_0$ and $\mathcal{L}_{\rm max}$ are the maximized likelihoods without and with the A^0 signal, respectively. The signal yields are 116.5 ± 33.4 and 22.6 ± 8.2 with statistical significances of 3.5σ and 3.0σ, respectively. The global significances are obtained to be 2.2σ and 2.0σ with look-elsewhere-effect included by extending the searched mass ranges to be 0.15–0.4 GeV in the $E^*(\gamma)$ distribution for $A^0 \to \tau^+\tau^-$ and 8.3–8.7 GeV/c^2 in the $M(\mu^+\mu^-)$ distribution for $A^0 \to \mu^+\mu^-$, respectively [46]. The statistical signal significances as a function of A^0 mass for $A^0 \to \tau^+\tau^-$ and $A^0 \to \mu^+\mu^-$ are shown in Figs. 4(a) and 4(b).

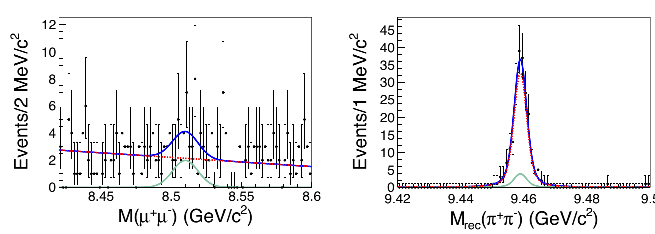

FIG. 3. The fitted result corresponding to the maximum local significance of 3.0σ with A^0 mass fixed at 8.51 GeV/c^2 for $A^0 \to \mu^+\mu^-$. The blue solid curves show the best fitted result, and the red dashed curves show the fitted total backgrounds. The green curves show the signal component.

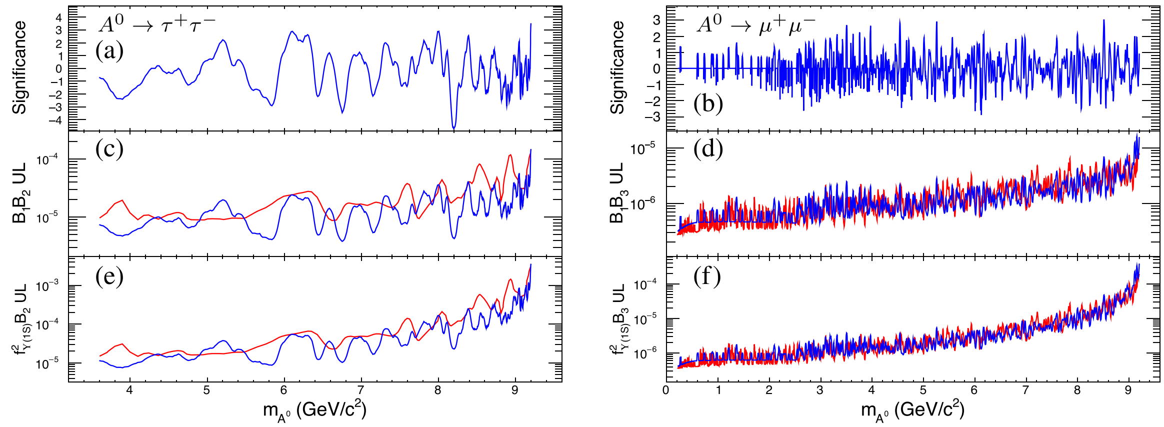

FIG. 4. The (a),(b) statistical significances, (c),(d) upper limits at 90% C.L. on $\mathcal{B}[\Upsilon(1S) \to \gamma A^0]\mathcal{B}(A^0 \to \tau^+\tau^-)$ (B_1B_2) and $\mathcal{B}[\Upsilon(1S) \to \gamma A^0]\mathcal{B}(A^0 \to \mu^+\mu^-)$ (B_1B_3), and (e),(f) upper limits at 90% C.L. on $f^2_{\Upsilon(1S)}\mathcal{B}(A^0 \to \tau^+\tau^-)$ $(f^2_{\Upsilon(1S)}B_2)$ and $f^2_{\Upsilon(1S)}\mathcal{B}(A^0 \to \mu^+\mu^-)$ $(f^2_{\Upsilon(1S)}B_3)$ as a function of m_{A^0}. The blue curves show the Belle results, and the red curves show the *BaBar* results [29,30].

The sources of systematic uncertainties in the measurements of upper limits on $\mathcal{B}(\Upsilon(1S) \to \gamma A^0)\mathcal{B}(A^0 \to \tau^+\tau^-/\mu^+\mu^-)$ include detection efficiency, MC statistics, trigger simulation, branching fractions of intermediate states, signal parametrization, background parametrization, and total number of $\Upsilon(2S)$ events. The detection efficiency uncertainties include those for tracking efficiency (0.35%/track), particle identification efficiency (1.1%/pion, 1.2%/electron, and 2.8%/muon), and photon reconstruction efficiency (2.0%/photon). The above individual uncertainties from different $\tau^+\tau^-$ decay modes are added linearly, weighted by the product of the detection efficiency and all secondary branching fractions. Assuming these uncertainties are independent and adding them in quadrature, the final uncertainty related to the detection efficiency is 6.4% for $A^0 \to \tau^+\tau^-$. For $A^0 \to \mu^+\mu^-$, the total uncertainty of detection efficiency is obtained by adding all sources in quadrature; it is 6.5%. The statistical uncertainty in the determination of efficiency from signal MC samples is 1.0%. We include uncertainties of 1.5% and 1.3% from trigger simulations for $A^0 \to \tau^+\tau^-$ and $A^0 \to \mu^+\mu^-$, respectively. The uncertainty of 1.5% from $\mathcal{B}[\Upsilon(2S) \to \pi^+\pi^-\Upsilon(1S)]$ is included [42]. The uncertainties of the branching fractions of τ decays can be neglected [42].

Using the control sample of $\pi^0/\eta \to \gamma\gamma$, the maximum energy bias and fudge factor for the radiative photon are 1.004 and 1.05 [47], respectively. Thus, in the fitting to the $E^*(\gamma)$ spectrum for $A^0 \to \tau^+\tau^-$, we change the central value by 0.4% and energy resolution by 5% for each A^0 mass point to recalculate the 90% C.L. upper limit, and the difference compared to the previous result is taken as the uncertainty of signal parametrization. For $A^0 \to \mu^+\mu^-$, the systematic uncertainty in the mass resolution is estimated by comparing the upper limit when the mass resolution is changed by 10% for each A^0 mass point. By comparing the upper limits in different fit ranges and using higher-order polynomial functions, the systematic uncertainty attributed to the background parametrization can be estimated. The uncertainties on the total number of $\Upsilon(2S)$ events is 2.3%. All the uncertainties are summarized in Table I and, assuming all the sources are independent, summed in quadrature for the total systematic uncertainties.

We compute 90% C.L. upper limits x^{UL} on the signal yields and the products of branching fractions by solving the equation $\int_0^{x^{\mathrm{UL}}} \mathcal{L}(x)dx / \int_0^{+\infty} \mathcal{L}(x)dx = 0.90$, where x is the assumed signal yield or product of branching fractions, and $\mathcal{L}(x)$ is the corresponding maximized likelihood of the fit to the assumption. To take into account systematic uncertainties, the above likelihood is convolved with a Gaussian function whose width equals the total systematic uncertainty. The upper limits at 90% C.L. on the product branching fractions of $\Upsilon(1S) \to \gamma A^0$ and $A^0 \to \tau^+\tau^-/\mu^+\mu^-$ are calculated using

$$\mathcal{B}^{\mathrm{UL}}[\Upsilon(1S) \to \gamma A^0]\mathcal{B}(A^0 \to \tau^+\tau^-/\mu^+\mu^-) = \frac{N^{\mathrm{UL}}}{N^{\mathrm{total}}_{\Upsilon(2S)} \times \varepsilon}, \tag{2}$$

where N^{UL} is the upper limit at 90% C.L. on the signal yield, $N^{\mathrm{total}}_{\Upsilon(2S)} = 1.58 \times 10^8$ is the number of $\Upsilon(2S)$ events,

TABLE I. Relative systematic uncertainties (%) in the measurements of upper limits for $A^0 \to \tau^+\tau^-$ and $A^0 \to \mu^+\mu^-$.

Sources	$A^0 \to \tau^+\tau^-$	$A^0 \to \mu^+\mu^-$
Detection efficiency	6.4	6.5
MC statistics	1.0	1.0
Trigger	1.5	1.3
Branching fractions	1.5	1.5
Signal parametrization	0.1–24.4	0.1–19.4
Background parametrization	0.1–19.6	0.1–17.2
Total number of $\Upsilon(2S)$ events	2.3	2.3
Sum	7.2–32.2	7.3–26.9

and ε is the reconstruction efficiency with the branching fractions of $\Upsilon(2S) \to \pi^+\pi^-\Upsilon(1S)$ and τ decays included. For $A^0 \to \tau^+\tau^-$, the reconstruction efficiency decreases from 2.1% to 0.7% with the increased A^0 mass, and for $A^0 \to \mu^+\mu^-$ the reconstruction efficiency decreases from 4.7% to 0.6% in the studied mass range from the $\mu^+\mu^-$ threshold to 9.2 GeV$/c^2$.

The upper limits at 90% C.L. on the product branching fractions of $\Upsilon(1S) \to \gamma A^0$ and $A^0 \to \tau^+\tau^-/\mu^+\mu^-$ are shown by the blue curves in Figs. 4(c) and 4(d), where the B_1, B_2, and B_3 represent $\mathcal{B}[\Upsilon(1S) \to \gamma A^0]$, $\mathcal{B}(A^0 \to \tau^+\tau^-)$, and $\mathcal{B}(A^0 \to \mu^+\mu^-)$, respectively. Note that the systematic uncertainties have been taken into account. The corresponding results from *BaBar* [29] are also shown by the red curves. For $A^0 \to \tau^+\tau^-$, in most A^0 mass points, our limits are lower than those from *BaBar* [29]. The most stringent upper limit can reach 4×10^{-6} from Belle. While from *BaBar*, the typical upper limit is at the level of 10^{-5}. More stringent constraints on $A^0 \to \tau^+\tau^-$ production in radiative $\Upsilon(1S)$ decays are given. For $A^0 \to \mu^+\mu^-$, the upper limits at Belle are almost at the same level as those from *BaBar* [30].

The upper limit at 90% C.L. on the product branching fractions can be converted to the Yukawa coupling $f_{\Upsilon(1S)}$ directly via [12,48,49]

$$\frac{\mathcal{B}[\Upsilon(1S) \to \gamma A^0]}{\mathcal{B}[\Upsilon(1S) \to \ell^+\ell^-]} = \frac{f^2_{\Upsilon(1S)}}{\sqrt{2}\pi\alpha}\left(1 - \frac{m^2_{A^0}}{m^2_{\Upsilon(1S)}}\right), \qquad (3)$$

where $\ell = e$ or μ and α is the fine structure constant. The upper limits at 90% C.L. on the $f^2_{\Upsilon(1S)}\mathcal{B}(A^0 \to \tau^+\tau^-/\mu^+\mu^-)$ as a function of A^0 mass are shown by blue curves in Figs. 4(e) and 4(f). The results from *BaBar* [29] are also shown by red curves.

The limit on the A^0 production in $\Upsilon(1S)$ radiative decays is related to the mixing angle ($\sin\theta_{A^0}$), which can be compared with those from other experiments. The mixing angle is defined as [25]

$$\frac{\mathcal{B}[\Upsilon(1S) \to \gamma A^0]\mathcal{B}(A^0 \to \text{hadrons})}{\mathcal{B}[\Upsilon(1S) \to \ell^+\ell^-]} = \sin^2\theta_{A^0}\frac{G_F m_b^2}{\sqrt{2}\pi\alpha}\sqrt{\left(1 - \frac{m^2_{A^0}}{m^2_{\Upsilon(1S)}}\right)}, \qquad (4)$$

where G_F is the Fermi constant and m_b is the mass of bottom quark [42]. When the mass of A^0 is smaller than $\tau^+\tau^-$ threshold, upper limits from $A^0 \to \mu^+\mu^-$ are used to calculate the $\sin\theta_{A^0}$; on the contrary, upper limits from $A^0 \to \tau^+\tau^-$ are used. The ratios of $\mathcal{B}(A^0 \to \mu^+\mu^-)/\mathcal{B}(A^0 \to \text{hadrons})$ and $\mathcal{B}(A^0 \to \tau^+\tau^-)/\mathcal{B}(A^0 \to \text{hadrons})$ are taken from Ref. [13]; they are changed from 0.08 to 0.28 and 0.7 to 1.0 for $A^0 \to \mu^+\mu^-$ and $A^0 \to \tau^+\tau^-$, respectively.

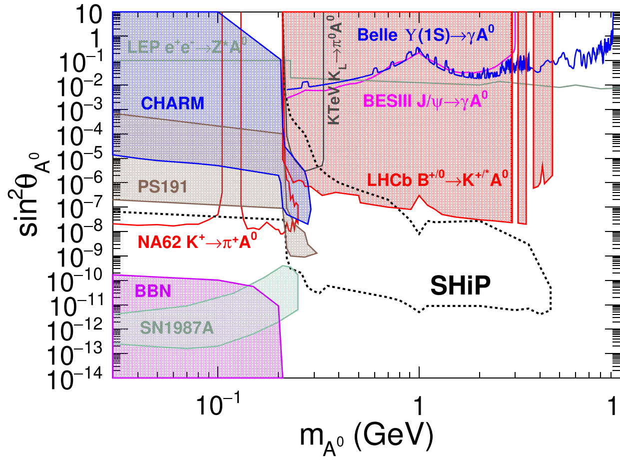

FIG. 5. The surviving parameter space on the plane of $\sin\theta_{A^0}$ and m_{A^0}. The constraints from LEP [50] (direct production of Higgs), BESIII [51] (J/ψ decay), Belle ($\Upsilon(1S)$ decay), LHCb [52,53] ($B^{+/0}$ decay), NA62 [54–56] (K^+ decay), KTeV [13,57,58] (K_L decay), CHARM [13,59–62] (beam dump), PS191 [63] (beam dump), SN1987A [64], BBN [65], and the prospect of future SHiP [13,62] (beam dump) are shown.

The surviving parameter space on the plane of $\sin\theta_{A^0}$ and m_{A^0} (the same as m_ϕ and m_H in Refs. [13] and [25]) from different processes are shown in Fig. 5.

To conclude, we have searched for the light *CP*-odd Higgs boson in $\Upsilon(1S) \to \gamma A^0$ with $\Upsilon(2S) \to \pi^+\pi^-\Upsilon(1S)$ tagging method using the largest data sample of $\Upsilon(2S)$ at Belle. The upper limits at 90% C.L. on the product branching fractions for $\Upsilon(1S) \to \gamma A^0$ and $A^0 \to \tau^+\tau^-/\mu^+\mu^-$ are set. In comparisons with previous studies [28–30], our results can further constrain the parameter space in NMSSM models [6,7] for $\Upsilon(1S) \to \gamma A^0(\to \tau^+\tau^-)$ and have the same restrictions for $\Upsilon(1S) \to \gamma A^0(\to \mu^+\mu^-)$. Our limits are applicable to any light scalar or pseudoscalar boson and dark matter, which arises in various extensions of SM. We have used the branching fraction limits to set limits on the Yukawa coupling $f_{\Upsilon(1S)}$ and mixing angle $\sin\theta_{A^0}$. For the latter, different processes from diffferenct experiments are compared to it.

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