

Janus and RG-flow interfaces in three-dimensional gauged supergravity

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ABSTRACT: In this paper, we construct Janus-type solutions of three-dimensional gauged supergravity with sixteen supersymmetries. We find solutions which correspond to interfaces between the same CFT on both sides, as well as RG-flow interfaces between CFTs with different numbers of supersymmetries and central charges. The solutions are obtained by solving the flow equations derived from the supersymmetry variations, and they preserve some fraction of the supersymmetries of the AdS_3 vacua.

KEYWORDS: AdS-CFT Correspondence, Supergravity Models

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1 Introduction

Janus solutions provide holographic constructions of interface conformal field theories (CFTs). In many known examples, the solutions are constructed by considering an AdS_d slicing of a $(d+1)$ -dimensional space where the scalar fields depend non-trivially on the slicing coordinate. One of the most well-known examples is the Janus solution of [1], which is a deformation of the $\text{AdS}_5 \times S^5$ vacuum of type IIB and is given by an AdS_4 slicing where the dilation depends non-trivially on the slicing coordinate. The solution is dual to an interface of $N = 4$ super Yang-Mills (SYM) theory where the coupling g_{YM} jumps across a co-dimension one interface [2]. This solution breaks all the supersymmetries, but a general solution given by an $\text{AdS}_4 \times S^2 \times S^2$ space warped over a Riemann surface can preserve half the supersymmetries of the $\text{AdS}_5 \times S^5$ vacuum [3] and is dual to supersymmetric interface theories in $N = 4$ SYM [4–6]. For other examples of Janus solutions in type II and M-theory, see e.g. [7, 8, 10].

Instead of constructing solutions in ten or eleven dimensions, it is often useful to use lower-dimensional gauged supergravities since the ansatz and resulting equations are simpler. Often the resulting solutions can be uplifted to higher dimensions, but even if the uplift is not known the gauged supergravity solutions are useful for studying universal and qualitative features of interface solutions. For an incomplete list of such solutions in various dimensions, see e.g. [11–18].

A related construction is given by holographic RG-flows, which consider a Poincaré slicing instead of an AdS slicing. If the solutions asymptotically approach two AdS vacua with different cosmological constants, we can interpret this solution as an RG-flow from a CFT in the UV to a CFT in the IR which is triggered by turning on a relevant deformation in the UV [19–23]. On the other hand, for most examples of AdS-sliced holographic interface solutions the CFTs on both sides of the interface are the same and differ only by a marginal deformation (such as different values of g_{YM} in the example discussed above) or a position-dependent profile of the expectation value [8] or a source [9] of a relevant operator.

In [24, 25], the idea of a RG-interface in two-dimensional CFTs was proposed, where the two sides of the interface are CFTs related by a RG-flow. The goal of the current paper is to construct holographic solutions which realize this idea.¹ We consider three-dimensional $\mathcal{N} = 8$ gauged supergravity with $n = 4$ vector multiplets, first discussed in [29]. This theory has an AdS_3 vacuum with maximal $\mathcal{N} = (4, 4)$ supersymmetry as well as two families of AdS_3 vacua with $\mathcal{N} = (1, 1)$ supersymmetry [30]. The theory gauges a $\text{SO}(4) \times \text{SO}(4)$ symmetry of the $\text{SO}(8, 4)/\text{SO}(8) \times \text{SO}(4)$ coset. The gauging depends on a continuous parameter α where the superconformal algebra of the $\mathcal{N} = (4, 4)$ vacuum is given by the “large” superconformal algebra $D^1(2, 1; \alpha) \times D^1(2, 1; \alpha)$, and the three-dimensional supergravity is believed to be a truncation of M-theory on $\text{AdS}_3 \times S^3 \times S^3 \times S^1$ [31–34]. In this paper, we primarily focus on the case $\alpha = 1$ for which the expressions for the flow equations are the simplest.

In [30], the Poincaré-sliced holographic RG-flow solutions were constructed for the $\mathcal{N} = 8$ gauged supergravity, which interpolate between the $\mathcal{N} = (4, 4)$ and $\mathcal{N} = (1, 1)$ vacua. The goal of the present paper is to construct Janus solutions which realize the interface between the same CFT at different points in the moduli space as well as RG-flow interfaces between CFTs preserving different numbers of supersymmetries.

The structure of this paper is as follows. In section 2, we review the $\mathcal{N} = 8$ gauged supergravity theory we will be using in the paper and discuss the different supersymmetric AdS_3 vacua. In section 3, we derive the BPS equations for a Janus ansatz following from the vanishing of the gravitino and spin- $\frac{1}{2}$ supersymmetry variations. In section 4, we solve the BPS equations for various truncations which make the analysis of the flow equations manageable. For a Janus interface between the $\mathcal{N} = (4, 4)$ vacuum we find an analytic solution, whereas for the RG-flow interfaces the flow equations can only be solved numerically. We present the solutions and provide evidence for our interpretation of these solutions as RG-flow interfaces. In section 5, we close with a discussion and open questions. In appendix A, we present our conventions for $\text{SO}(8)$ gamma matrices.

¹See [14, 26–28] for work in this direction.

2 Three-dimensional $\mathcal{N} = 8$ gauged supergravity

In this section, we review the $\mathcal{N} = 8$ gauged supergravity first constructed in [29]. The theory is characterized by the number n of vector multiplets. The bosonic field content consists of a graviton $g_{\mu\nu}$, Chern-Simons gauge fields $B_\mu^{\mathcal{M}}$, and scalar fields living in a $G/H = \text{SO}(8, n)/\text{SO}(8) \times \text{SO}(n)$ coset, which has $8n$ degrees of freedom before gauging. The scalar fields can be parametrized by a G -valued matrix $L(x)$ in the vector representation, which transforms under H and the gauge group $G_0 \subseteq G$ by

$$L(x) \rightarrow g_0(x)L(x)h^{-1}(x) \quad (2.1)$$

for $g_0 \in G_0$ and $h \in H$. The Lagrangian is invariant under such transformations.

For future reference, we use the following index conventions:

- $I, J, \dots = 1, 2, \dots, 8$ for $\text{SO}(8)$.
- $r, s, \dots = 9, 10, \dots, n+8$ for $\text{SO}(n)$.
- $\bar{I}, \bar{J}, \dots = 1, 2, \dots, n+8$ for $\text{SO}(8, n)$.
- $\mathcal{M}, \mathcal{N}, \dots$ for generators of $\text{SO}(8, n)$.

Let the generators of G be $\{t^{\mathcal{M}}\} = \{t^{\bar{I}\bar{J}}\} = \{X^{IJ}, X^{rs}, Y^{Ir}\}$, where Y^{Ir} are the noncompact generators. Explicitly, the generators of the vector representation are given by

$$\left(t^{\bar{I}\bar{J}}\right)^{\bar{K}}_{\bar{L}} = \eta^{\bar{I}\bar{K}}\delta_{\bar{L}}^{\bar{J}} - \eta^{\bar{J}\bar{K}}\delta_{\bar{L}}^{\bar{I}} \quad (2.2)$$

where $\eta^{\bar{I}\bar{J}} = \text{diag}(+++++\dots)$ is the $\text{SO}(8, n)$ -invariant tensor. These generators satisfy the typical $\text{SO}(8, n)$ commutation relations,

$$\left[t^{\bar{I}\bar{J}}, t^{\bar{K}\bar{L}}\right] = 2\left(\eta^{\bar{I}[\bar{K}}t^{\bar{L}]\bar{J}} - \eta^{\bar{J}[\bar{K}}t^{\bar{L}]\bar{I}}\right) \quad (2.3)$$

The gauging of the supergravity is characterized by an embedding tensor $\Theta_{\mathcal{MN}}$ (which has to satisfy various identities [35]) that determines which isometries are gauged, the coupling to the Chern-Simons fields, and additional terms in the supersymmetry transformations and action depending on the gauge coupling g . We will look at the particular case in [30] where $n \geq 4$ and the gauged subgroup is the $G_0 = \text{SO}(4) \times \text{SO}(4)$ subset of the $\text{SO}(8) \subset \text{SO}(8, n)$. The embedding tensor has the entries,²

$$\Theta_{\bar{I}\bar{J}, \bar{K}\bar{L}} = \begin{cases} \alpha\epsilon_{\bar{I}\bar{J}\bar{K}\bar{L}} & \text{if } \bar{I}, \bar{J}, \bar{K}, \bar{L} \in \{1, 2, 3, 4\} \\ \epsilon_{\bar{I}\bar{J}\bar{K}\bar{L}} & \text{if } \bar{I}, \bar{J}, \bar{K}, \bar{L} \in \{5, 6, 7, 8\} \\ 0 & \text{otherwise} \end{cases} \quad (2.4)$$

Note that the gauging depends on a real parameter α . As discussed in [30], the maximally supersymmetric AdS_3 vacuum has an isometry group,

$$D^1(2, 1; \alpha) \times D^1(2, 1; \alpha) \quad (2.5)$$

²We use the conventions $\epsilon_{1234} = \epsilon_{5678} = 1$.

which corresponds to the family of “large” superconformal algebras of the dual SCFT. In the following, we will consider the special case $\alpha = 1$ for which $D^1(2, 1; 1) = \text{OSp}(4|2)$ and the form of many quantities defined below are most compact. We expect that for generic values of α the qualitative behavior of the solutions will be similar.

From the embedding tensor, the G_0 -covariant currents can be obtained,

$$L^{-1} \left(\partial_\mu + g \Theta_{\mathcal{MN}} B_\mu^\mathcal{M} t^\mathcal{N} \right) L = \frac{1}{2} \mathcal{Q}_\mu^{IJ} X^{IJ} + \frac{1}{2} \mathcal{Q}_\mu^{rs} X^{rs} + \mathcal{P}_\mu^{Ir} Y^{Ir} \quad (2.6)$$

It is convenient to define the $\mathcal{V}^\mathcal{M}_\mathcal{A}$ tensors,

$$L^{-1} t^\mathcal{M} L = \mathcal{V}^\mathcal{M}_\mathcal{A} t^\mathcal{A} = \frac{1}{2} \mathcal{V}^\mathcal{M}_{IJ} X^{IJ} + \frac{1}{2} \mathcal{V}^\mathcal{M}_{rs} X^{rs} + \mathcal{V}^\mathcal{M}_{Ir} Y^{Ir} \quad (2.7)$$

and the T -tensor,

$$T_{A|B} = \Theta_{\mathcal{MN}} \mathcal{V}^\mathcal{M}_\mathcal{A} \mathcal{V}^\mathcal{N}_\mathcal{B} \quad (2.8)$$

The T -tensor is used to construct the tensors $A_{1,2,3}$ which will appear in the scalar potential and the supersymmetry transformations,

$$\begin{aligned} A_1^{AB} &= -\frac{1}{48} \Gamma_{AB}^{IJKL} T_{IJ|KL} \\ A_2^{A\dot{A}r} &= -\frac{1}{12} \Gamma_{A\dot{A}}^{IJK} T_{IJ|Kr} \\ A_3^{\dot{A}r\dot{B}s} &= \frac{1}{48} \delta^{rs} \Gamma_{\dot{A}\dot{B}}^{IJKL} T_{IJ|KL} + \frac{1}{2} \Gamma_{\dot{A}\dot{B}}^{IJ} T_{IJ|rs} \end{aligned} \quad (2.9)$$

where A, B and $\dot{A}, \dot{B}$ are $\text{SO}(8)$ -spinor indices. Our conventions for the $\text{SO}(8)$ Gamma matrices are presented in appendix A.

We take the spacetime signature $\eta^{ab} = \text{diag}(+ - -)$ to be mostly negative. The bosonic Lagrangian and scalar potential are

$$\begin{aligned} e^{-1} \mathcal{L}_{\text{bos}} &= -\frac{1}{4} R + \frac{1}{4} \mathcal{P}_\mu^{Ir} \mathcal{P}^{Ir\mu} + W - \frac{1}{4} e^{-1} \epsilon^{\mu\nu\rho} g \Theta_{\mathcal{MN}} B_\mu^\mathcal{M} \left(\partial_\nu B_\rho^\mathcal{N} + \frac{1}{3} g \Theta_{\mathcal{KL}} f^{\mathcal{N}\mathcal{K}}{}_\mathcal{P} B_\nu^\mathcal{L} B_\rho^\mathcal{P} \right) \\ W &= \frac{1}{4} g^2 \left(A_1^{AB} A_1^{AB} - \frac{1}{2} A_2^{A\dot{A}r} A_2^{A\dot{A}r} \right) \end{aligned} \quad (2.10)$$

The supersymmetry variations are

$$\begin{aligned} \delta \chi^{\dot{A}r} &= \frac{1}{2} i \Gamma_{A\dot{A}}^I \gamma^\mu \epsilon^A \mathcal{P}_\mu^{Ir} + g A_2^{A\dot{A}r} \epsilon^A \\ \delta \psi_\mu^A &= \left(\partial_\mu \epsilon^A + \frac{1}{4} \omega_\mu^{ab} \gamma_{ab} \epsilon^A + \frac{1}{4} \mathcal{Q}_\mu^{IJ} \Gamma_{AB}^{IJ} \epsilon^B \right) + i g A_1^{AB} \gamma_\mu \epsilon^B \end{aligned} \quad (2.11)$$

The Einstein equation of motion is

$$R_{\mu\nu} - \mathcal{P}_\mu^{Ir} \mathcal{P}_\nu^{Ir} - 4W g_{\mu\nu} = 0 \quad (2.12)$$

and the gauge field equation of motion is

$$e \mathcal{P}^{Ir\lambda} \Theta_{\mathcal{QM}} \mathcal{V}^\mathcal{M}_{Ir} = \epsilon^{\lambda\mu\nu} \left(\Theta_{\mathcal{QM}} \partial_\mu B_\nu^\mathcal{M} + \frac{1}{6} g B_\mu^\mathcal{M} B_\nu^\mathcal{K} \left(\Theta_{\mathcal{MN}} \Theta_{\mathcal{KL}} f^{\mathcal{N}\mathcal{L}}{}_\mathcal{Q} + 2 \Theta_{\mathcal{MN}} f^{\mathcal{L}\mathcal{N}}{}_\mathcal{K} \Theta_{\mathcal{LQ}} \right) \right) \quad (2.13)$$

2.1 The $n = 4$ case

Let us focus on the case of four vector multiplets, i.e. $n = 4$. The symmetries consist of a local $G_0 = \text{SO}(4) \times \text{SO}(4)$ and a global $\text{SO}(n = 4)$. Thus, the scalar potential only depends on $8 \cdot 4 - 3 \cdot 6 = 14$ parameters out of the original 32. Moreover, we will only consider a further consistent truncation outlined in [30] where the coset representative depends on eight of the fourteen scalars.

$$L = \begin{pmatrix} \cos A & \sin A \cosh B & \sin A \sinh B \\ -\sin A & \cos A \cosh B & \cos A \sinh B \\ 0 & \sinh B & \cosh B \end{pmatrix}$$

$$A = \text{diag}(p_1, p_2, p_3, p_4), \quad B = \text{diag}(q_1, q_2, q_3, q_4) \quad (2.14)$$

With this truncation, the scalar potential is³

$$g^{-2}W = 1 + \prod_{i=1}^4 \cosh q_i + \frac{1}{4} \sum_{i=1}^4 \sinh^2 q_i - \frac{1}{4} \sum_{i < j < k} (x_i^2 x_j^2 x_k^2 + y_i^2 y_j^2 y_k^2) - \frac{1}{2} \left(\prod_{i=1}^4 x_i + \prod_{i=1}^4 y_i \right)^2$$

$$x_i = \cos p_i \sinh q_i, \quad y_i = \sin p_i \sinh q_i \quad (2.15)$$

The $\mathcal{Q}_\mu$ and $\mathcal{P}_\mu$ currents, excluding the $g\Theta_{\mathcal{MN}}B_\mu^{\mathcal{M}}\mathcal{V}^{\mathcal{N}}_{\mathcal{A}}$ term, are

$$\mathcal{Q}_\mu^{IJ} = \begin{pmatrix} 0 & 0 & 0 & 0 & \cosh q_1 \partial_\mu p_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \cosh q_2 \partial_\mu p_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cosh q_3 \partial_\mu p_3 & 0 \\ -\cosh q_1 \partial_\mu p_1 & 0 & 0 & 0 & 0 & 0 & 0 & \cosh q_4 \partial_\mu p_4 \\ 0 & -\cosh q_2 \partial_\mu p_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\cosh q_3 \partial_\mu p_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\cosh q_4 \partial_\mu p_4 & 0 & 0 & 0 & 0 \end{pmatrix}_{IJ}$$

$$\mathcal{Q}_\mu^{rs} = 0$$

$$\mathcal{P}_\mu^{Ir} = \begin{pmatrix} \sinh q_1 \partial_\mu p_1 & 0 & 0 & 0 \\ 0 & \sinh q_2 \partial_\mu p_2 & 0 & 0 \\ 0 & 0 & \sinh q_3 \partial_\mu p_3 & 0 \\ 0 & 0 & 0 & \sinh q_4 \partial_\mu p_4 \\ \partial_\mu q_1 & 0 & 0 & 0 \\ 0 & \partial_\mu q_2 & 0 & 0 \\ 0 & 0 & \partial_\mu q_3 & 0 \\ 0 & 0 & 0 & \partial_\mu q_4 \end{pmatrix}_{Ir} \quad (2.16)$$

Using these matrices, we can check that the combination $\mathcal{P}_\mu^{Ir} \mathcal{V}^{JK}_{Ir}$ vanishes whenever the indices $J, K \in \{1, 2, 3, 4\}$ or $J, K \in \{5, 6, 7, 8\}$. This implies that there is no source for $B_\mu^{\mathcal{M}}$ in the gauge field equation of motion (2.13), so it is consistent to set $B_\mu^{\mathcal{M}} = 0$. We will make this choice from now on.

2.2 Truncations and supersymmetric AdS₃ vacua

In order to make our analysis more tractable, we make further truncations to reduce the number of independent scalar fields.⁴ Below we consider three truncations, which together explore the AdS₃ vacua with $\mathcal{N} = (4, 4)$ and $\mathcal{N} = (1, 1)$ supersymmetry.

³We correct a small typo in the potential given in [30].

⁴These truncations are consistent due to internal rotational symmetry as shown in [30].

2.2.1 Truncation 1

The first truncation is given by calling $q_1 = q$, $p_1 = p$ and setting all remaining $q_i = p_i = 0$ for $i = 2, 3, 4$. The scalar potential is

$$W = \frac{g^2}{2} \cosh^2 \frac{q}{2} (3 + \cosh q) \quad (2.17)$$

The $\mathcal{N} = (4, 4)$ vacuum is given by setting $q = 0$ and the vacuum potential is $W_0 = 2g^2$.

2.2.2 Truncation 2

The second truncation is given by setting all the q s and p s equal, i.e. $q_i = q$, $p_i = p$ for $i = 1, 2, 3, 4$. The scalar potential is

$$W = \frac{g^2}{8192} \left(8103 + 6856 \cosh 2q + 1452 \cosh 4q - 8 \cosh 6q - 19 \cosh 8q \right. \\ \left. - 768(3 + \cosh 2q) \cos 4p \sinh^6 q - 128 \cos 8p \sinh^8 q \right) \quad (2.18)$$

The $\mathcal{N} = (4, 4)$ vacuum is given by $q = 0$ as before, and $\mathcal{N} = (1, 1)$ vacua are given by $q = \pm \sinh^{-1} \sqrt{2}$ and $p = \pi(\mathbb{Z}/2 + 1/4)$ which have a vacuum potential of $W_0 = 8g^2$.

2.2.3 Truncation 3

The third truncation is given by setting the first three q s and p s equal, i.e. $q_i = q$, $p_i = p$ for $i = 1, 2, 3$, and setting the remaining $q_4 = p_4 = 0$. The scalar potential is

$$W = \frac{g^2}{1024} \left(690 + 768 \cosh q + 309 \cosh 2q + 256 \cosh 3q \right. \\ \left. + 30 \cosh 4q - 5 \cosh 6q - 96 \cos 4p \sinh^6 q \right) \quad (2.19)$$

The $\mathcal{N} = (4, 4)$ vacuum is given by $q = 0$ as before, and $\mathcal{N} = (1, 1)$ vacua are given by $q = \pm \sinh^{-1} \sqrt{2 + 2\sqrt{2}}$ and $p = \pi(\mathbb{Z}/2 + 1/4)$ which have a vacuum potential of $W_0 = 2(1 + \sqrt{2})^2 g^2$.

3 Janus flow equations

In this section, we present the equations of motion and supersymmetry variations for a Janus ansatz where the three-dimensional metric is written as an AdS_2 slicing and the scalar fields only depend on the slicing coordinate. We will also set the Chern-Simons gauge $B_\mu^{\mathcal{M}}$ fields to zero, which is consistent as argued in section 2.1. Hence, the Janus ansatz is given by

$$\text{d}s^2 = e^{2B(u)} \left(\frac{\text{d}t^2 - \text{d}z^2}{z^2} \right) - \text{d}u^2, \quad B_\mu^{\mathcal{M}} = 0 \\ q_i = q_i(u), \quad p_i = p_i(u) \quad (3.1)$$

The Ricci tensor has the non-zero components,

$$\begin{aligned} R_{tt} &= -R_{zz} = z^{-2} \left(1 + e^{2B} (2B'^2 + B'') \right) \\ R_{uu} &= -2(B'^2 + B'') \end{aligned} \quad (3.2)$$

The prime ' denotes a derivative with respect to the slicing coordinate u . The gravitino supersymmetry variation $\delta\psi_\mu^A = 0$ is

$$\begin{aligned} 0 &= \partial_t \epsilon + \frac{1}{2z} \gamma_0 \left(\gamma_1 - B' e^B \gamma_2 + 2ig e^B A_1 \right) \epsilon \\ 0 &= \partial_z \epsilon + \frac{1}{2z} \gamma_1 \left(-B' e^B \gamma_2 + 2ig e^B A_1 \right) \epsilon \\ 0 &= \partial_u \epsilon + \frac{1}{4} \mathcal{Q}_u^{IJ} \Gamma^{IJ} \epsilon + ig \gamma_2 A_1 \epsilon \end{aligned} \quad (3.3)$$

where we have suppressed the $\text{SO}(8)$ -spinor indices of ϵ^A and A_1^{AB} . There are two integrability conditions which can be derived from the gravitino variations (3.3) in the t, z and z, u directions respectively

$$\begin{aligned} 0 &= \left(1 - (2ge^B A_1)^2 + (B' e^B)^2 \right) \epsilon \\ 0 &= 2ig e^B \left(A_1' - \frac{1}{4} [A_1, \mathcal{Q}_u^{IJ} \Gamma^{IJ}] \right) \epsilon + \left(-\frac{d}{du} (B' e^B) + (2ge^B A_1)^2 e^{-B} \right) \gamma_2 \epsilon \end{aligned} \quad (3.4)$$

We can use the first integrability condition to express the second one as

$$2ig \left(A_1' - \frac{1}{4} [A_1, \mathcal{Q}_u^{IJ} \Gamma^{IJ}] \right) \epsilon + \left(-B'' + e^{-2B} \right) \gamma_2 \epsilon = 0 \quad (3.5)$$

The spin- $\frac{1}{2}$ variation $\delta\chi^{\dot{A}} = 0$ is

$$\left(-\frac{i}{2} \Gamma^I \mathcal{P}_u^{Ir} \gamma_2 + g A_2^r \right)_{A\dot{A}} \epsilon^{\dot{A}} = 0, \quad r = 9, 10, \dots, 8+n \quad (3.6)$$

3.1 Eigenvectors of A_1

It follows from (2.9) that A_1 is a 8×8 matrix which has eigenvectors

$$A_1^{AB} n_\pm^{(i)B} = \pm w_i n_\pm^{(i)A}, \quad i = 1, 2, 3, 4 \quad (3.7)$$

The eigenvalues w_i determine whether a supersymmetric AdS_3 vacuum can exist. In the following we denote the positive supersymmetric eigenvalue w , which can be determined as follows: for the AdS_2 -sliced metric given in (3.1), the AdS_3 vacuum solution with potential W_0 is given by

$$B_{\text{vac}}(u) = \ln \frac{\cosh(\sqrt{2W_0}u)}{\sqrt{2W_0}} \quad (3.8)$$

which satisfies

$$B_{\text{vac}}'^2 + e^{-2B_{\text{vac}}} - 2W_0 = 0 \quad (3.9)$$

When we expand the spinors ϵ^A in terms of the eigenvectors of A_1 , the first equation in (3.4) implies for the spinor component associated with the eigenvalue w that

$$B'^2 + e^{-2B} - 4g^2w^2 = 0 \quad (3.10)$$

For the AdS_3 vacuum solution (3.8), this condition relates the eigenvalue evaluated at the vacuum w_{vac} to the potential W_0 via

$$w_{\text{vac}}^2 = \frac{W_0}{2g^2} \quad (3.11)$$

As discussed in section 4 for truncation 1, A_1 has eight eigenvectors $n_{\pm}^{(i)}$ for $i = 1, 2, 3, 4$ all with the same with eigenvalue $\pm w$ that satisfy the supersymmetry condition (3.11) for the AdS_3 vacuum with $q = 0$. Hence, this vacuum preserves $\mathcal{N} = (4, 4)$ supersymmetry. On the other hand for truncations 2 and 3, there are only two eigenvectors $n_{\pm}$ with an eigenvalue $\pm w$ that satisfy (3.11) for the AdS_3 vacua with non-trivial values for the scalars. Consequently, these vacua only preserve $\mathcal{N} = (1, 1)$ supersymmetry.

For the RG-flow solutions which interpolate between the different vacua, we expand the spinors in the basis of the eigenspinors that correspond to the supersymmetric vacuum when the scalars take their vacuum values. This implies that (3.10) can be solved for B' ,

$$\begin{aligned} B' &= \pm \sqrt{4g^2w^2 - e^{-2B}} \\ &= \pm 2gw\gamma \end{aligned} \quad (3.12)$$

where we defined the convenient combinations,

$$\gamma(u) = \sqrt{1 - \frac{e^{-2B}}{4g^2w^2}}, \quad \sqrt{1 - \gamma^2(u)} = \frac{e^{-B}}{2gw} \quad (3.13)$$

which will be useful later on. The two signs in (3.12) are two branches of solutions which for Janus solutions will be patched together — the numerical evolution usually breaks down at $B' = 0$ and this is the location where the two branches will be glued together.

3.2 AdS_2 Killing spinors

The Killing spinors for a unit radius AdS_2 with metric,

$$ds_{\text{AdS}_2}^2 = \frac{dt^2 - dz^2}{z^2} \quad (3.14)$$

satisfy the following equation,

$$D_{\mu}\zeta_{\eta} = i\frac{\eta}{2}\gamma_{\mu}\zeta_{\eta}, \quad \mu = t, z \quad (3.15)$$

with $\eta = \pm 1$. The covariant derivatives on AdS_2 take the form,

$$D_t = \partial_t \epsilon + \frac{1}{2z}\gamma_0\gamma_1, \quad D_z = \partial_z \quad (3.16)$$

Since the general spinor in AdS_2 is a two-component spinor, the $\zeta_{\pm}$ form a basis of two-component spinors. Since spinors in three dimensions are also two-component spinors, the $\zeta_{\pm}$ are also a basis of the spinors in three dimensions. Note that $\gamma_2 = i\gamma_{\#}$ where $\gamma_{\#}^2 = 1$ and

$$i\gamma_2\zeta_{\eta} = \zeta_{-\eta}, \quad \eta = \pm 1 \quad (3.17)$$

The general ansatz for ϵ^A is given by

$$\epsilon^A = \sum_i \left(f_+^{(i)} n_+^{(i)A} + f_-^{(i)} n_-^{(i)A} \right) \zeta_+ + \left(g_+^{(i)} n_+^{(i)A} + g_-^{(i)} n_-^{(i)A} \right) \zeta_- \quad (3.18)$$

For truncation 1 we have $i = 1, 2, 3, 4$, which label four eigenvectors of A_1 , whereas in truncations 2 and 3 the index i is dropped.

3.3 First projector

With this ansatz for the spinors ϵ^A , the first two components of the gravitino variation,

$$\begin{aligned} 0 &= \partial_t \epsilon + \frac{1}{2z} \gamma_0 \left(\gamma_1 - B' e^B \gamma_2 + 2ig e^B A_1 \right) \epsilon \\ 0 &= \partial_z \epsilon + \frac{1}{2z} \gamma_1 \left(-B' e^B \gamma_2 + 2ig e^B A_1 \right) \epsilon \end{aligned} \quad (3.19)$$

can be expressed as follows by using the properties of the AdS_2 Killing spinors,

$$0 = i \left\{ \left(f_+^{(i)} n_+^{(i)A} + f_-^{(i)} n_-^{(i)A} \right) \zeta_+ - \left(g_+^{(i)} n_+^{(i)A} + g_-^{(i)} n_-^{(i)A} \right) \zeta_- \right\} \quad (3.20)$$

$$\begin{aligned} &+ iB' e^{-B} i\gamma_2 \left\{ \left(f_+^{(i)} n_+^{(i)A} + f_-^{(i)} n_-^{(i)A} \right) \zeta_+ + \left(g_+^{(i)} n_+^{(i)A} + g_-^{(i)} n_-^{(i)A} \right) \zeta_- \right\} \\ &+ 2igwe^B \left\{ \left(f_+^{(i)} n_+^{(i)A} - f_-^{(i)} n_-^{(i)A} \right) \zeta_+ + \left(g_+^{(i)} n_+^{(i)A} - g_-^{(i)} n_-^{(i)A} \right) \zeta_- \right\} \end{aligned} \quad (3.21)$$

Using $i\gamma_2\zeta_{\eta} = \zeta_{-\eta}$ and the linear independence of the $n_{\pm}^{(i)}$ and $\zeta_{\pm}$, one obtains a set of equations,

$$\begin{aligned} f_+ + B' e^B g_+ + 2gwe^B f_+ &= 0 \\ -g_+ + B' e^B f_+ + 2gwe^B g_+ &= 0 \\ f_- + B' e^B g_- - 2gwe^B f_- &= 0 \\ -g_- + B' e^B f_- - 2gwe^B g_- &= 0 \end{aligned} \quad (3.22)$$

which are consistent if the integrability condition (3.10) holds. In terms of the $\gamma(u)$ defined in (3.13) we have

$$f_+ = \frac{\sqrt{1-\gamma^2}-1}{\gamma} g_+, \quad f_- = \frac{\sqrt{1-\gamma^2}+1}{\gamma} g_- \quad (3.23)$$

3.4 Second projector

The spin- $\frac{1}{2}$ variation (3.6) can be rewritten in the following form

$$\left[-\frac{1}{2g} \left((A_2^r)^T \right)^{-1} \left(\Gamma^I \mathcal{P}_u^{Ir} \right)^T i\gamma_2 + 1 \right]^{AB} \epsilon^B = 0, \quad r = 9, 10, 11, 12 \quad (3.24)$$

Since $\mathcal{P}_u^{Ir}$ contains the first derivatives of the scalar fields, the flow equations for the scalars can be derived from the condition of vanishing of this supersymmetry variation. The projectors for $r = 9, 10, 11, 12$ take the form

$$(M^{AB}i\gamma_2 + \delta^{AB})\epsilon^B = 0 \quad (3.25)$$

For consistency, the matrix must satisfy $M^{AB}M^{BC} = \delta^{AC}$. Plugging in the ansatz (3.18) for the spinors ϵ^A , we get

$$\begin{aligned} 0 = & \left(f_+^{(i)}n_+^{(i)A} + f_-^{(i)}n_-^{(i)A}\right)\zeta_+ + \left(g_+^{(i)}n_+^{(i)A} + g_-^{(i)}n_-^{(i)A}\right)\zeta_- \\ & + M^{AB}i\gamma_2 \left\{ \left(f_+^{(i)}n_+^{(i)B} + f_-^{(i)}n_-^{(i)B}\right)\zeta_+ + \left(g_+^{(i)}n_+^{(i)B} + g_-^{(i)}n_-^{(i)B}\right)\zeta_- \right\} \end{aligned} \quad (3.26)$$

Using the fact that the eigenvectors can be orthonormalized,

$$n_+^{(i)A}n_+^{(j)A} = \delta^{ij}, \quad n_-^{(i)A}n_-^{(j)A} = \delta^{ij}, \quad n_+^{(i)A}n_-^{(j)A} = 0 \quad (3.27)$$

and projecting onto $n_\pm^{(i)}$ gives

$$\begin{aligned} f_+ + M_{++}g_+ + M_{+-}g_- &= 0 \\ g_+ + M_{++}f_+ + M_{+-}f_- &= 0 \\ f_- + M_{+-}g_+ + M_{--}g_- &= 0 \\ g_- + M_{+-}f_+ + M_{--}f_- &= 0 \end{aligned} \quad (3.28)$$

where we define

$$M_{++} = n_+^A M^{AB} n_+^B, \quad M_{--} = n_-^A M^{AB} n_-^B, \quad M_{+-} = M_{-+} = n_+^A M^{AB} n_-^B \quad (3.29)$$

If there is more than one $n_\pm$ (as in truncation 1) the $M_{\pm\pm}, M_{\pm\mp}$ have to take the same form for all $n_\pm^{(i)}$, which is a consistency condition. Using (3.23) it can be shown that equations (3.28) can only⁵ be satisfied if we have

$$M_{++} = \gamma, \quad M_{--} = -\gamma, \quad M_{+-} = M_{-+} = \sqrt{1 - \gamma^2} \quad (3.30)$$

The relations (3.30) for the matrix M^{AB} given in (3.24) provide first-order flow equations for the scalar fields. Note that the second integrability condition for the gravitino variation in (3.4) is also the form of (3.25). For all the solutions which we find, this condition is automatically satisfied and does not constrain the flow further.

4 Janus and RG-flow solutions

In this section we obtain the flow equations and solve them. Only for truncation 1 are we able to solve the system analytically. For truncations 2 and 3 we solve the resulting flow equations numerically.

⁵We can also have $M_{+-} = M_{-+} = -\sqrt{1 - \gamma^2}$, which gives a similar solution. For example, in section 4.1 for truncation 1, this sends $p(u) \rightarrow p(-u)$. This resolves an ambiguity in the definition of our eigenvectors, as we can freely send $n_+ \rightarrow -n_+$ or $n_- \rightarrow -n_-$.

4.1 Truncation 1

For the truncation to a single scalar described in section 2.2.1, the matrix A_1 takes the form,

$$A_1 = \begin{pmatrix} 0 & 0 & 0 & \cos p \cosh^2 \frac{q}{2} & 0 & 0 & \sin p \cosh^2 \frac{q}{2} & 0 \\ 0 & 0 & -\cos p \cosh^2 \frac{q}{2} & 0 & 0 & 0 & 0 & \sin p \cosh^2 \frac{q}{2} \\ 0 & -\cos p \cosh^2 \frac{q}{2} & 0 & 0 & -\sin p \cosh^2 \frac{q}{2} & 0 & 0 & 0 \\ \cos p \cosh^2 \frac{q}{2} & 0 & 0 & 0 & 0 & -\sin p \cosh^2 \frac{q}{2} & 0 & 0 \\ 0 & 0 & -\sin p \cosh^2 \frac{q}{2} & 0 & 0 & 0 & 0 & -\cos p \cosh^2 \frac{q}{2} \\ 0 & 0 & 0 & -\sin p \cosh^2 \frac{q}{2} & 0 & 0 & \cos p \cosh^2 \frac{q}{2} & 0 \\ \sin p \cosh^2 \frac{q}{2} & 0 & 0 & 0 & 0 & \cos p \cosh^2 \frac{q}{2} & 0 & 0 \\ 0 & \sin p \cosh^2 \frac{q}{2} & 0 & 0 & -\cos p \cosh^2 \frac{q}{2} & 0 & 0 & 0 \end{pmatrix} \quad (4.1)$$

We have four pairs of eigenvectors $n_{\pm}^{(i)}$ for $i = 1, 2, 3, 4$ with the same eigenvalues $\pm w$, where

$$w = \cosh^2 \frac{q}{2} \quad (4.2)$$

so the supersymmetry condition (3.11) is satisfied for the vacuum where $q = 0$. The pairs of eigenvectors are

$$\begin{aligned} n_+^{(1)} &= \left\{ \frac{1}{\sqrt{2}}, 0, 0, \frac{\cos p}{\sqrt{2}}, 0, 0, \frac{\sin p}{\sqrt{2}}, 0 \right\}, & n_-^{(1)} &= \left\{ 0, 0, 0, -\frac{\sin p}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}, \frac{\cos p}{\sqrt{2}}, 0 \right\} \\ n_+^{(2)} &= \left\{ 0, \frac{1}{\sqrt{2}}, -\frac{\cos p}{\sqrt{2}}, 0, 0, 0, 0, \frac{\sin p}{\sqrt{2}} \right\}, & n_-^{(2)} &= \left\{ 0, 0, \frac{\sin p}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{\cos p}{\sqrt{2}} \right\} \\ n_+^{(3)} &= \left\{ 0, 0, -\frac{\sin p}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}, 0, 0, -\frac{\cos p}{\sqrt{2}} \right\}, & n_-^{(3)} &= \left\{ 0, -\frac{1}{\sqrt{2}}, -\frac{\cos p}{\sqrt{2}}, 0, 0, 0, 0, \frac{\sin p}{\sqrt{2}} \right\} \\ n_+^{(4)} &= \left\{ 0, 0, 0, -\frac{\sin p}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}, \frac{\cos p}{\sqrt{2}}, 0 \right\}, & n_-^{(4)} &= \left\{ \frac{1}{\sqrt{2}}, 0, 0, -\frac{\cos p}{\sqrt{2}}, 0, 0, -\frac{\sin p}{\sqrt{2}}, 0 \right\} \end{aligned} \quad (4.3)$$

Given the eigenvectors, we can compute the M_{++} and M_{+-} matrix elements for the matrix in (3.24) for any pair of $n_{\pm}^{(i)}$. Note that for this truncation, only the flow equation for index $r = 9$ is nontrivial while the others are identically vanishing. Then (3.30) gives us flow equations for the scalars q and p . The remaining flow equation for the metric factor B comes from (3.12). The flow equations are

$$q' = -g\gamma \sinh q, \quad p' = g\sqrt{1 - \gamma^2}, \quad B' = \pm 2g\gamma \cosh^2 \frac{q}{2} \quad (4.4)$$

which solve the equations of motion. We can solve for a flow where $p(0) = p_0$, $q(0) = q_0$ are arbitrary and $B'(0) = 0$, which is equivalent to $\gamma(0) = 0$. This first-order system can be rewritten using the function γ in lieu of B , in which case the third equation above is replaced with

$$\gamma' = 2g(1 - \gamma^2) \quad (4.5)$$

The solution is

$$\begin{aligned} \gamma(u) &= \tanh(2gu) \\ \tanh \frac{q(u)}{2} &= \sqrt{\text{sech}(2gu)} \tanh \frac{q_0}{2} \\ \tan[p(u) - p_0] &= \tanh(gu) \end{aligned} \quad (4.6)$$

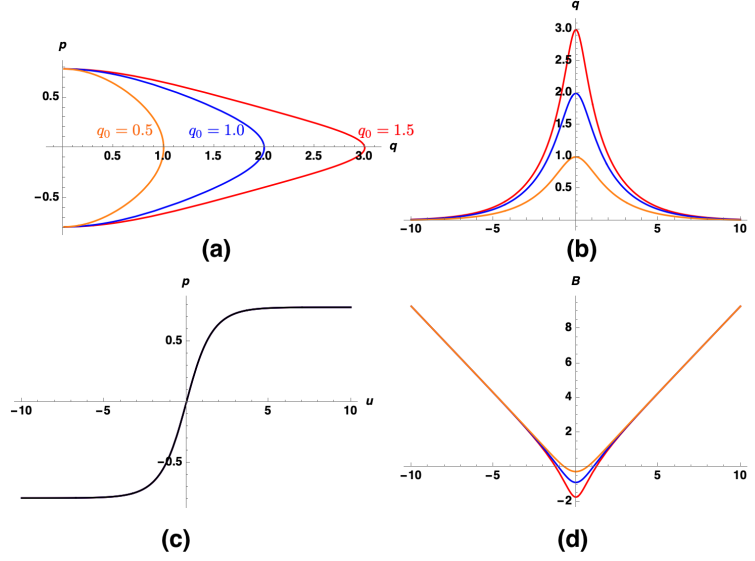


Figure 1. (a) pq parametric plot, (b) plot of q , (c) plot of p , (d) plot of the metric function B as functions of the AdS_2 slicing coordinate u for truncation 1. The colors denote three different values for q_0 . $p_0 = 0$ for all three plots. The behavior of p is the same for all three examples.

which gives the metric factor

$$e^{B(u)} = \frac{\cosh(2gu)}{2g} \text{sech}^2 \frac{q(u)}{2} \quad (4.7)$$

This is a Janus solution which approaches the $\mathcal{N} = (4, 4)$ vacuum at the two endpoints $u \rightarrow \pm\infty$. We note that the flow equation is the same for each pair of eigenvectors $n_{\pm}^{(i)}$ and hence the solution preserves eight of the sixteen supersymmetries of the $\mathcal{N} = (4, 4)$ vacuum. We present plots for three choices of the parameters $q_0 = 0.5, 1.0, 1.5$ in figure 1 and set $p_0 = 0$ for all three. The qualitative behavior is very similar for all three choices and corresponds to a Janus interface which interpolates between different values of $p(u)$ as $u \rightarrow \pm\infty$.

4.2 Truncation 2

Recall that the truncation presented in section 2.2.2 sets all the q_i equal and p_i equal. The matrix A_1 takes the form,

$$A_1 = \begin{pmatrix} 0 & 0 & 0 & a_1 + a_3 & 0 & 0 & 0 & 0 \\ 0 & -2a_3 & -a_1 + a_3 & 0 & a_2 & 0 & 0 & a_2 \\ 0 & -a_1 + a_3 & -2a_3 & 0 & -a_2 & 0 & 0 & -a_2 \\ a_1 + a_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_2 & -a_2 & 0 & 2a_3 & 0 & 0 & -a_1 + a_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & a_1 + a_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_1 + a_3 & 0 & 0 \\ 0 & a_2 & -a_2 & 0 & -a_1 + a_3 & 0 & 0 & 2a_3 \end{pmatrix} \quad (4.8)$$

where we define, for this truncation,

$$\begin{aligned} a_1 &= \frac{1}{8}(3 + \cos 4p)(1 + \cosh^4 q) \\ a_2 &= \frac{1}{8}(3 + \cosh 2q) \cosh q \sin 4p \\ a_3 &= 2 \cos^2 p \sin^2 p \cosh^2 q \end{aligned} \quad (4.9)$$

The eigenvectors and eigenvalues can be obtained by considering the matrix $(A_1)^2$ first. There are two eigenvalues, the first is $(a_1 + a_3)^2$ which is six-fold degenerate but does not satisfy the condition (3.11) for the $\mathcal{N} = (1, 1)$ vacuum. The second eigenvalue is $4a_2^2 + (a_1 - 3a_3)^2$ which is two-fold degenerate and does satisfy (3.11) for the $\mathcal{N} = (1, 1)$ vacua. The corresponding eigenvectors of $(A_1)^2$ take the form

$$v_1 = \left\{0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0\right\}, \quad v_2 = \left\{0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0, \frac{1}{\sqrt{2}}\right\} \quad (4.10)$$

Let

$$w = \sqrt{4a_2^2 + (a_1 - 3a_3)^2} \quad (4.11)$$

be the positive eigenvalue of A_1 . One can reduce the A_1 matrix on the subspace spanned by v_1, v_2 and find that the (not yet normalized) eigenvectors $v_{\pm}$ of A_1 with eigenvalue $\pm w$ are given by

$$v_{\pm} = 2a_2 v_1 + (-a_1 + 3a_3 \pm w) v_2 \quad (4.12)$$

The flow equations take the form of a first-order system of ordinary differential equations for the functions $p(u)$, $q(u)$, and $B(u)$. These equations do not take a simple form and are too unwieldy to be presented here. Using Mathematica, we have checked that the flow equations imply that the equations of motion are satisfied as well as the second integrability condition of the gravitino variation (3.5).

The flow equations can be numerically integrated.⁶ In figure 2 we present some examples for the numerical solutions of the flow equations. By fine-tuning initial conditions we can produce flows that (i) look like the Janus solutions in truncation 1 plotted, in red in figure 2, (ii) connect $\mathcal{N} = (4, 4)$ and $\mathcal{N} = (1, 1)$ vacua, plotted in blue in figure 2, and (iii) connect two $\mathcal{N} = (1, 1)$ vacua which are related by flipping signs of p , plotted in orange in figure 2. In the pq parametric plot in figure 2(a), the locations of the $\mathcal{N} = (1, 1)$ vacua $p = \pm \frac{\pi}{4}$, $q = \sinh^{-1} \sqrt{2}$ are denoted by black dots. Note that for the $\mathcal{N} = (4, 4)$ vacuum where $q = 0$, the asymptotic value p is a modulus which can take any value, hence the flow towards the $\mathcal{N} = (4, 4)$ can end at any point on the p -axis at $q = 0$.

In the numerical integration, the $\mathcal{N} = (1, 1)$ points are repulsive and require fine-tuning in order to obtain flows that approach these points. This can be explained as follows. The choice $g = 1/2$ sets the $\mathcal{N} = (4, 4)$ vacuum potential to $W_0 = 1/2$ and the AdS_3 length scale

⁶We use the method described in [14]: we choose the location $p(0), q(0)$ of a turning point where $B' = 0$ and use the flow equations to determine $p'(0), q'(0)$. These values provide the initial conditions for the second order equations of motion to give $p(u), q(u)$ and $B(u)$.

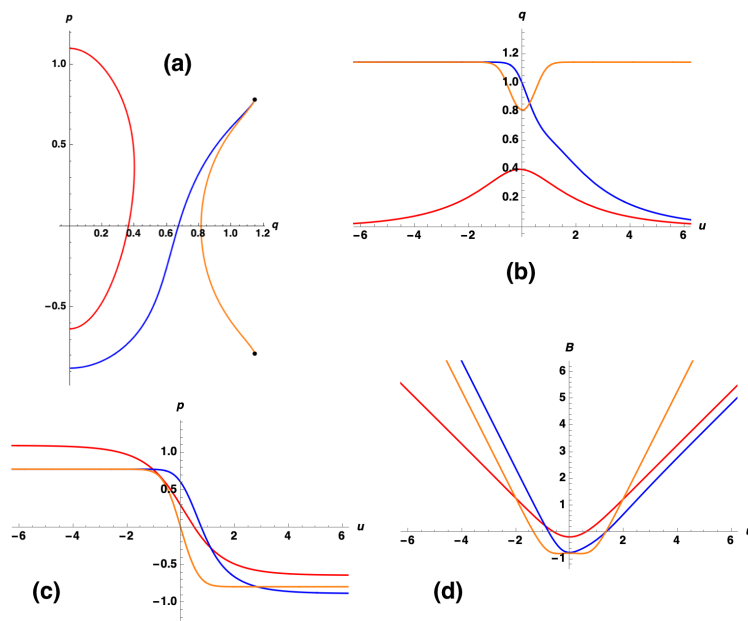


Figure 2. (a) pq parametric plot, (b) plot of q , (c) plot of p , (d) plot of the metric function B as functions of the AdS_2 slicing coordinate u .

to unity. For the $\mathcal{N} = (1, 1)$ point, the vacuum potential becomes $W_0 = 2$ and the AdS_3 length scale is $L = 1/2$. Taking the linear expansion around the $\mathcal{N} = (1, 1)$ point,

$$\begin{aligned} p(u) &= \frac{\pi}{4} + \delta p(u) + \mathcal{O}(\delta p^2) \\ q(u) &= \sinh^{-1} \sqrt{2} + \delta q(u) + \mathcal{O}(\delta q^2) \end{aligned} \quad (4.13)$$

the mass-squares for the δp and δq fluctuations are

$$m_p^2 L^2 = \frac{5}{4}, \quad m_q^2 L^2 = \frac{21}{4} \quad (4.14)$$

Using $\Delta(\Delta - 2) = m^2 L^2$, the scaling dimensions of the corresponding dual operators are

$$\Delta_p = \frac{5}{2}, \quad \Delta_q = \frac{7}{2} \quad (4.15)$$

In our AdS -sliced coordinates, the boundary is given by the two AdS_2 components at $u = \pm\infty$, which are joined together at the $z = 0$ interface. The coordinates (z, u) can be mapped to Fefferman-Graham coordinates (ρ, x) where the boundary is located at $\rho = 0$.⁷

⁷Recall that the AdS_3 metric in Poincaré coordinates,

$$ds^2 = \frac{-d\rho^2 + dt^2 - dx^2}{\rho^2}$$

is related to an AdS_2 -sliced metric by the coordinate change,

$$z = \sqrt{x^2 + \rho^2} \quad \sinh u = x/\rho.$$

Let us consider the boundary at $u \rightarrow +\infty$. In the (ρ, x) coordinates, the metric factor B has the expansion $e^B = \rho^{-1} + \mathcal{O}(\rho^0)$ near the boundary. But in the (z, u) coordinates, from (3.8) the expansion near the boundary is $B = u/L + \dots$. Therefore, the asymptotic form of the coordinate change $(\rho, x) \mapsto (z, u)$ takes the form,

$$e^u = \rho^{-L} + \dots \quad (4.16)$$

The linearized flow equations around the $\mathcal{N} = (1, 1)$ point are

$$\begin{aligned} \delta p' &= -5\delta p + \dots \\ \delta q' &= 3\delta q + \dots \end{aligned} \quad (4.17)$$

which are solved by $\delta p \sim C_p e^{-5u}$ and $\delta q \sim C_q e^{3u}$, or in terms of ρ ,

$$\delta p \sim C_p \rho^{5/2} \quad \delta q \sim C_q \rho^{-3/2} \quad (4.18)$$

These asymptotic forms are consistent with the scaling dimensions in (4.15), as we either have solutions that scale as ρ^Δ or $\rho^{2-\Delta}$. We see that the q scalar diverges as we approach the $\mathcal{N} = (1, 1)$ point as $\rho \rightarrow 0$ unless we fine-tune the coefficient C_q to zero. This is identified with turning off the source for an operator with scaling dimension larger than 2 on the boundary CFT.

A similar counting as before shows that the flow preserves two of the four supersymmetries of the $\mathcal{N} = (1, 1)$ vacuum. Therefore, we have RG-flow interfaces between a CFT with central charge $c^{(4,4)}$ and a CFT with central charge $c^{(1,1)}$, where [36, 37]

$$\frac{c^{(1,1)}}{c^{(4,4)}} = \sqrt{\frac{W_0^{(4,4)}}{W_0^{(1,1)}}} = \frac{1}{2} \quad (4.19)$$

4.3 Truncation 3

The analysis of the flow equations and their solutions for truncation 3 proceeds very similarly to the one for truncation 2, presented in the previous section. The matrix A_1 takes the form,

$$A_1 = \begin{pmatrix} 0 & 0 & 0 & (a_1 - a_2 + b_2) \cos p & 0 & (-a_1 + a_2 + b_1) \sin p & 0 & 0 \\ 0 & -2a_1 \cos p & (a_1 + a_2 - b_2) \cos p & 0 & (a_1 + a_2 - b_1) \sin p & 0 & 0 & 2a_2 \sin p \\ 0 & (a_1 + a_2 - b_2) \cos p & -2a_1 \cos p & 0 & -2a_2 \sin p & 0 & 0 & (-a_1 - a_2 + b_1) \sin p \\ (a_1 - a_2 + b_2) \cos p & 0 & 0 & 0 & 0 & 0 & (a_1 - a_2 - b_1) \sin p & 0 \\ 0 & (a_1 + a_2 - b_1) \sin p & -2a_2 \sin p & 0 & 2a_1 \cos p & 0 & 0 & (a_1 + a_2 - b_2) \cos p \\ (-a_1 + a_2 + b_1) \sin p & 0 & 0 & 0 & 0 & 0 & (a_1 - a_2 + b_2) \cos p & 0 \\ 0 & 0 & 0 & (a_1 - a_2 - b_1) \sin p & 0 & (a_1 - a_2 + b_2) \cos p & 0 & 0 \\ 0 & 2a_2 \sin p & (-a_1 - a_2 + b_1) \sin p & 0 & (a_1 + a_2 - b_2) \cos p & 0 & 0 & 2a_1 \cos p \end{pmatrix} \quad (4.20)$$

where we define, for this truncation,

$$\begin{aligned} a_1 &= \frac{1}{2}(1 + \cosh q) \cosh q \sin^2 p, & a_2 &= \frac{1}{2}(1 + \cosh q) \cosh q \cos^2 p \\ b_1 &= \frac{1}{2}(1 + \cosh q)(1 + \cosh^2 q) \sin^2 p, & b_2 &= \frac{1}{2}(1 + \cosh q)(1 + \cosh^2 q) \cos^2 p \end{aligned} \quad (4.21)$$

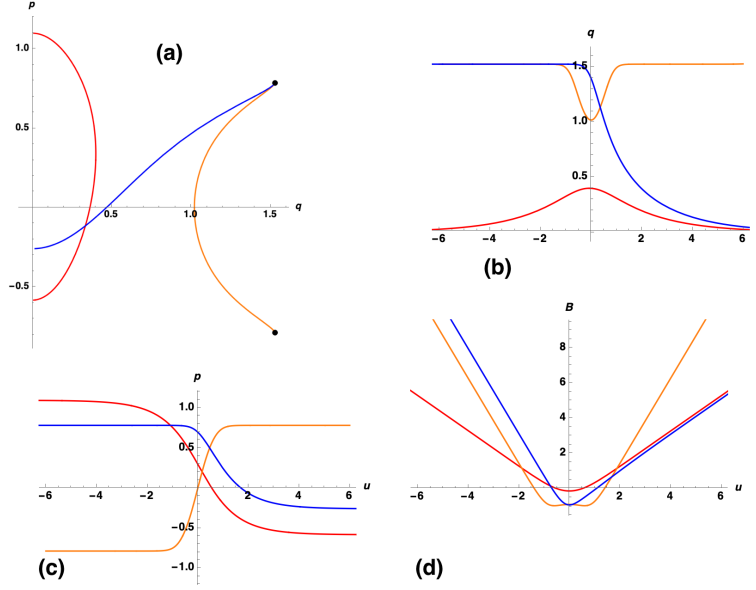


Figure 3. (a) pq parametric plot, (b) plot of q , (c) plot of p , (d) plot of the metric function B as functions of the AdS_2 slicing coordinate u for truncation 3.

As with truncation 2, there are two eigenvalues of $(A_1)^2$: one with six-fold degeneracy that does not satisfy (3.11), and one with two-fold degeneracy that does. This eigenvalue is

$$w^2 = \frac{1}{64} \cosh^4 \frac{q}{2} \left(175 - 224 \cosh q + 140 \cosh 2q - 32 \cosh 3q + 5 \cosh 4q + 24 \cos 4p \sinh^4 q \right) \quad (4.22)$$

The corresponding eigenvectors of A_1 with eigenvalue $\pm w$ are

$$v_{\pm} = (-(3a_1 + a_2 - b_2) \cos p \pm w) v_1 + (a_1 + 3a_2 - b_1) \sin p v_2 \quad (4.23)$$

where v_1, v_2 are defined as before in (4.10). The flow equations once again do not take a simple form and must be solved numerically. In figure 3 we present some examples for the numerical solutions of the flow equations for truncation 3, which exhibit very similar features to the solutions of the flow equations for truncation 2. By fine-tuning initial conditions we can produce flows that (i) look like the Janus solutions in truncation 1, plotted in red in figure 3, (ii) connect $\mathcal{N} = (4, 4)$ and $\mathcal{N} = (1, 1)$ vacua, plotted in blue in figure 3, and (iii) connect two $\mathcal{N} = (1, 1)$ vacua which are related by flipping signs of p , plotted in orange in figure 3. In the pq parametric plot given in figure 3(a), the location of the $\mathcal{N} = (1, 1)$ vacua $p = \pm \frac{\pi}{4}$, $q = \sinh^{-1} \sqrt{2 + 2\sqrt{2}}$ are denoted by black dots.

The $\mathcal{N} = (1, 1)$ points are again repulsive. With $g = 1/2$, the $\mathcal{N} = (1, 1)$ vacuum potential is $W_0 = (1 + \sqrt{2})^2/2$ and the AdS_3 length scale is $L = \sqrt{2} - 1$. The linear expansion around the vacuum yields the following mass-squares for δp and δq fluctuations,

$$m_p^2 L^2 = 1, \quad m_q^2 L^2 = 2 + 2\sqrt{2} \quad (4.24)$$

which correspond to the scaling dimensions,

$$\Delta_p = 1 + \sqrt{2}, \quad \Delta_q = 2 + \sqrt{2} \quad (4.25)$$

The linearized flow equations around the $\mathcal{N} = (1, 1)$ point are

$$\begin{aligned} \delta p' &= -(3 + 2\sqrt{2})\delta p + \dots \\ \delta q' &= (2 + \sqrt{2})\delta q + \dots \end{aligned} \quad (4.26)$$

which are solved by $\delta p \sim C_p e^{-(3+2\sqrt{2})u}$ and $\delta q \sim C_q e^{(2+\sqrt{2})u}$, or in terms of ρ by substituting $e^u \sim \rho^{-L}$,

$$\delta p \sim C_p \rho^{1+\sqrt{2}} \quad \delta q \sim C_q \rho^{-\sqrt{2}} \quad (4.27)$$

These asymptotic forms are consistent with the scaling dimensions in (4.25). Again, we see that the q scalar diverges as $\rho \rightarrow 0$ unless we fine-tune the coefficient C_q to zero, which turns off the source for an operator with scaling dimension larger than 2 on the boundary CFT.

A similar counting as before shows that the flow preserves two of the four supersymmetries of the $\mathcal{N} = (1, 1)$ vacuum. Therefore, we have RG-flow interfaces between a CFT with central charge $c^{(4,4)}$ and a CFT with central charge $c^{(1,1)}$, where

$$\frac{c^{(1,1)}}{c^{(4,4)}} = \sqrt{\frac{W_0^{(4,4)}}{W_0^{(1,1)}}} = \sqrt{2} - 1 \quad (4.28)$$

5 Discussion

In the paper, we constructed new solutions of three-dimensional gauged supergravity. The solutions produced describe interface CFTs holographically. We considered three different truncations of the scalar fields which are associated with three different supersymmetric AdS₃ vacua with $\mathcal{N} = (4, 4)$ and $\mathcal{N} = (1, 1)$ supersymmetry. The solution in the first truncation is a Janus solution that is very similar to the one found in [38] for a simpler three-dimensional gauged supergravity. The CFTs on both sides of the interface are deformations of the $\mathcal{N} = (4, 4)$ vacuum with a source for a marginal operator turned on as well as a position-dependent expectation value for a relevant operator. The interface preserves half of the sixteen supersymmetries of the $\mathcal{N} = (4, 4)$ vacuum.

The solutions for the other two truncations represent RG-flow interfaces in the sense that the solutions we find have different CFTs on each side of the interface. For example, we find solutions where the interface connects the $\mathcal{N} = (4, 4)$ CFT (with a marginal operator and relevant expectation value) and the $\mathcal{N} = (1, 1)$ CFT (with an irrelevant source). Note that there is no clear distinction between the UV and the IR in the RG-flow Janus solutions, since both sides put together form the boundary of the asymptotically AdS space. This is to be contrasted with a Poincaré-sliced RG-flow solution, where the AdS boundary with the larger curvature radius (or central charge) is viewed as describing the UV CFT. A irrelevant source is turned on near the $\mathcal{N} = (1, 1)$ asymptotic AdS, which means that, from the perspective of the flow, this constitutes a repulsive direction. To find a flow that comes

very close to the $\mathcal{N} = (1, 1)$ vacuum, we have to fine-tune our initial conditions, which corresponds to fine-tuning the source of the irrelevant operator.

We have worked in truncations where the dynamics of the eight scalar fields q_i, p_i for $i = 1, 2, 3, 4$ are reduced to the dynamics of two scalars q, p , where in all three cases $q = 0$ corresponds to the $\mathcal{N} = (4, 4)$ vacuum. Hence, we could find interface CFTs of the $\mathcal{N} = (4, 4)$ CFT with one of the $\mathcal{N} = (1, 1)$ CFTs. In this truncation, we cannot find an interface solution connecting the two distinct $\mathcal{N} = (1, 1)$ vacua. For such a solution, we would have to consider the flow equations with at least four independent scalars. The fine-tuning of the initial conditions to produce the interface solution would also be more challenging.

The $\text{SO}(4) \times \text{SO}(4)$ gauging depends on a real parameter α and in this paper we have only considered the case $\alpha = 1$ which simplifies the expression of the A_i matrices and the scalar potential. We expect that the solutions for other choices of α behave qualitatively the same, since the supersymmetric vacua exist for other values of α . Nevertheless, it would be interesting to check this expectation. It would also be interesting to consider holographic observables such as the entanglement entropy around the interface or correlation functions. We plan to investigate some of these questions in the future.

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A $\text{SO}(8)$ Gamma matrices

We are working with 8×8 Gamma matrices Γ_{AA}^I and their transposes Γ_{AA}^I , which satisfy the Clifford algebra,

$$\Gamma_{AA}^I \Gamma_{AB}^J + \Gamma_{AA}^J \Gamma_{AB}^I = 2\delta^{IJ} \delta_{AB} \quad (\text{A.1})$$

Explicitly, we use the basis in Green-Schwarz-Witten [39],

$$\begin{aligned} \Gamma_{AA}^8 &= 1 \otimes 1 \otimes 1, & \Gamma_{AA}^1 &= i\sigma_2 \otimes i\sigma_2 \otimes i\sigma_2 \\ \Gamma_{AA}^2 &= 1 \otimes \sigma_1 \otimes i\sigma_2, & \Gamma_{AA}^3 &= 1 \otimes \sigma_3 \otimes i\sigma_2 \\ \Gamma_{AA}^4 &= \sigma_1 \otimes i\sigma_2 \otimes 1, & \Gamma_{AA}^5 &= \sigma_3 \otimes i\sigma_2 \otimes 1 \\ \Gamma_{AA}^6 &= i\sigma_2 \otimes 1 \otimes \sigma_1, & \Gamma_{AA}^7 &= i\sigma_2 \otimes 1 \otimes \sigma_3 \end{aligned} \quad (\text{A.2})$$

The matrices Γ_{AB}^{IJ} , Γ_{AB}^{IJ} and similar are defined as antisymmetrized products of Γ s with the appropriate indices contracted. For instance,

$$\Gamma_{AB}^{IJ} \equiv \frac{1}{2}(\Gamma_{AA}^I \Gamma_{AB}^J - \Gamma_{AA}^J \Gamma_{AB}^I) \quad (\text{A.3})$$

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