Students’ Written Homework Responses using Digital Games in Inquiry-Oriented Linear Algebra

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In this report, we characterize seven of twenty-five students’ responses to a single written homework assignment from the Spring 2021 academic semester. The homework was designed to incorporate the Vector Unknown 2D digital game to investigate how students answered questions about span and linear independence after playing various levels of the game. We present our modification of the roles and characteristics framework of Zandieh et al. (2019), our identification of students’ grammatical use of game language and math language, as well as the results of analyzing students’ homework responses using our framework.

Keywords: Linear Algebra, Inquiry-Oriented Instruction, Game-Based Learning

The teaching and learning of linear algebra are important due to their prevalence in many STEM disciplines. One of the goals of our team was to develop a sequence of game-based written homework assignments that paralleled the course progression of the Inquiry-Oriented Linear Algebra (IOLA) curriculum (Wawro, Zandieh, et al., 2013) for use in any linear algebra classroom. Here we build on past research stemming from the IOLA curriculum that uses Realistic Mathematics Education (RME) constructs to inform the design of curricular materials by iterating research, design, and implementation (Gravemeijer, 1999; Wawro, Rasmussen, et al., 2012).

Figure 1. Two screenshots of the Vector Unknown game for Level 3 (left) and Level 4 (right).

Vector Unknown (VU) introduces novice linear algebra learners to beginning topics such as linear combinations of vectors and span. Players move a bunny through a level to collect baskets and keys by dragging vectors and setting scalar values to create linear combination equations as seen in Figure 1. VU draws heavily on the use of the travel metaphor that is leveraged in the IOLA curriculum during the Magic Carpet Ride (MCR) task sequence (Plaxco & Wawro, 2015; Wawro et al., 2012). We extend the use of VU by augmenting its instructional use with a set of homework assignments. Since this was our first attempt at designing this set of game-based homework, we wanted to understand how our students thought about span and linear (in)dependence after playing several levels of VU. To accomplish our goal, we analyzed students’ responses and gameplay screenshots from their second written homework assignment.

Research question: How did students use the Vector Unknown 2D game in their responses to questions about span and linear independence in their written homework?

Background Literature and Theoretical Perspective

Stewart, Andrews-Larsen, and Zandieh (2019) provided a thorough overview of the current state of the research literature on the teaching and learning of Linear Algebra ideas. Stewart and colleagues synthesized 54 research articles across various themes, including span and linear (in)dependence, that are relevant concepts within most Linear Algebra curricula. We use the literature review of Stewart et al. to inform our own.

Span and Linear (In)dependence

Plaxco and Wawro (2015) determined that travel is a consistent metaphor that students use to reason about span in the context of the IOLA curriculum. Adiredja and Zandieh investigated students' everyday examples of basis using an anti-deficit perspective (Adiredja & Zandieh, 2017; 2020). Extending their initial framework for students' example generation, Zandieh et al. (2019) determined that travel was a ubiquitous metaphor for the notion of basis even though the students had never encountered the IOLA curriculum. There have been several studies that analyzed how students think about linear (in)dependence: Ertekin et al. (2010) discussed student thinking about linear independence and dependence in geometric contexts, Hannah et al. (2013; 2016) and Stewart and Thompson (2010) made connections between students' embodied ways of reasoning about linear independence and dependence even when the students were unable to produce a formal definition. Rasmussen et al. (2015) analyzed video data to investigate students' progressive mathematization of examples of linearly (in)dependent vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$.

Most relevant to our work is a similar study by Dogan-Dunlap (2010) who analyzed students' homework responses with an online module to learn more about the various ways students think about linear (in)dependence. When using the geometric web module, students' responses were characterized by Dogan-Dunlap across 17 categories, 11 of which were geometric. It could be the case that using manipulatives and educational video games support students in forming geometric interpretations of span and linear (in)dependence. However, no study has considered how students think about span and linear independence by analyzing students' written homework responses after playing a research-designed video game.

Game-Based Learning

The research team that developed the game VU incorporated Game-based learning (GBL) research as a lens to better understand the instructional affordances and limitations of using video games for teaching mathematical ideas (Coleman & Money, 2020; Gresalfi & Barnes, 2016). There has not been a lot of research leveraging games within higher-level mathematics courses. As such, our research adds to the burgeoning area of GBL within the IOLA curriculum (Mauntel et al., 2021). Digital Game-Based Learning (DGBL) has gained popularity in the K-12 literature over the past 20 years, "...DGBL in mathematics education can be regarded as the use of digital games within the context of learning mathematics," (Byun & Joung, 2018, p. 114). Other researchers within the IOLA research group have investigated the development of students' gameplay strategies using VU (Mauntel, Levine, et al., 2020; Mauntel et al., 2021). We agree that playing a well-designed video game does not imply students are mathematizing anything, "...a young child counting blocks, may...be experiencing mathematical play; whereas a middle school student who is being forced to play a very well-designed mathematics game in the classroom may not," (Williams-Pierce, 2019, p. 592). We view our design of written homework as a structured set of questions intended to support students in forming and testing conjectures about the game to engage in mathematical play while playing VU.

Roles and Characteristics of Basis Vectors Framework

We view the framework of Zandieh, Adiredja, and Knapp (2019) as viable to analyze the spontaneous game examples given by students in their written homework. Zandieh et al. built upon their previous framework of *roles* and *characteristics* codes to understand how students thought about basis through everyday example generation. Zandieh and colleagues used *roles* codes to describe the role that a set of basis vectors has relative to the ambient vector space. Zandieh and colleagues' *characteristics* codes described important conditions of a set of vectors. We did not explicitly ask students to generate examples using VU on their written homework. However, students' spontaneous use of game examples included descriptions of the roles and characteristics of the list of vectors given to the player in VU. We later elaborate on our alteration of the roles and characteristics of basis vectors framework of Zandieh, Adiredja, and Knapp (2019).

Methods

We designed a single written homework assignment to be used in the third author's IOLA class during the Fall 2020 semester. After reviewing student responses and meeting weekly, we refined the single homework assignment and wrote four new assignments during the Spring 2021 semester. Out of the forty-two students enrolled in the third author's IOLA class during Spring 2021, we collected the written homework responses of about twenty-five students (not every student completed each assignment). We analyzed the responses of seven out of the twenty-five students that submitted the second homework assignment due to their use of spontaneous game examples from VU in their response to the first question pictured in Fig. 2: Adrian (chose not to identify), Kieran (chose not to identify), Brooks (White man), Michael (White man), Jing (Asian/Asian American man), Tanner (White man), and Aahan (Asian/Asian American man).

The second homework was written with two mathematical concepts in mind: span and linear (in)dependence. In both the first and second written homework assignment, we explicitly used the word span, though we did not explicitly mention linear (in)dependence in either assignment. Students had encountered the IOLA lessons covering both ideas (Magic Carpet Task and Return to Home Task) by the time they received the second written homework assignment. Students were asked to play Levels 3, 4, and 5 on the hard difficulty of VU multiple times. Level 3 of VU includes four game objects (collect three virtual keys then travel to the location of the lock), Level 4 includes three collinear baskets where the line passes through the origin, and Level 5 includes three collinear baskets where the line of the baskets does not pass through the origin.

5.) (2 points) If you remove one of the vectors from your list in number 3) would the span of this new list be the same as what you marked in number 4)? Explain why or why not. Include in your answer whether it matters which vector you removed.
7.) (Extra Credit) Reflect on the similarities and differences between your answers in the Level 4 column and the Level 5 column. Make connections between as many of the boxes on your answer sheet as you want to receive up to 5 possible extra credit points.

Figure 2. Statement of the problems Q1 and Q2 from the second written homework

We conducted our first pass at data analysis of the second homework by open-coding every student's responses to each question to characterize the nature of the data and identify trends and themes (Strauss & Corbin, 1990). During this initial pass, we noted some students appeared to make connections between their understanding of span, linear independence and the explanations of their gameplay. While there were some selected response questions in the homework, we focused on the free response questions to better characterize students' connections. The framework of Zandieh et al. (2019) proved useful once we determined the subset of seven students who provided explanations using gameplay examples. To focus on how the students were using the game to reason about Linear Algebra concepts, we used the code *game language* to focus on pieces of the data where this might occur. An excerpt was originally coded as *game language* if the excerpt mentioned the student's actual gameplay, mechanics of the game, features of the game, or limitations of the game. For example, Adrian discussed increasing the distance traveled by the bunny in game language as opposed to saying something about increasing the value of the scalar multiple of said vector:

> "Therefore, it would likely only affect the first **movement** by increasing the distance **traveled** by the **bunny** in those two directions," -Adrian.

After determining the seven students that used game language to respond to Level 3 Question 5 (Q1), we used our modified framework of Zandieh et al. (2019) to code the seven students' responses to Q1 and Level 4 and 5 Question 7 (Q2) using our span and independence codes framework pictured in Figure 3. We will now focus our discussion primarily on the responses of the seven students who used game language in their responses to Q1.

Findings

Our findings include (1) students' mixed use of game and math language when responding to Q1 and Q2, (2) our modification of the Zandieh et al. (2019) basis framework, and (3) the results of using our frameworks to analyze students' responses to Q1 and Q2. In Figure 3, we present our span and independence framework. For each code, a portion of a students' response is given as an example to the reader. In Figure 4, we present the results of coding each student's response to Q1 and Q2 using our modified framework. In the column for students' responses to Q2, we separate the codes using semicolons if the student broke their response into paragraphs or sections.

Students' use of Game Language and Math Language After we determined some students responded to the homework using spontaneous examples from their gameplay, we wanted to know how some students used game examples in their open-ended homework responses to Q1 (25 responses) and Q2 (19 responses). We created various lists of words that would fall under *game language* usage (e.g. "bunny", "baskets", "route") and *math language* usage (e.g. "span", "plane", "magnitude"). We added a *mixed language* category for words that could be considered relevant to the game, but weren't always used to refer to one of the elements of the game (e.g. "vector", "line", "scalar"). We coded the second homework assignment itself and every students' responses to Q1 and Q2 at a word-for-word grain of analysis using our lists. We wanted to know if any students used *game language* when responding to Q1. We thought it would be interesting if a student responded primarily in game language to Q1, which was phrased primarily in mixed and math language. While many students' responses to Q1 involved math and mixed language use (69 instances of math language across 23 responses; 86 instances of mixed language across 22 responses), our seven research subjects were the only ones who responded to Q1 using game language. Out of the twenty-five responses to Q1, there were only 18 instances of game language across 7 out of the 25 responses. The data presented here is from the written responses to Q1 and Q2 of these seven students.

The Span and Independence Framework Zandieh et al. (2019) used *roles* codes to describe students' examples relative to how the basis vectors relate to the ambient vector space. In our modification of their framework, we instead use *span* codes to characterize students' descriptions of how vectors or objects relate to the span of said vector set. Further, Zandieh and colleagues used *characteristics* codes to describe students' examples based on the conditions that a set of basis vectors has. We instead use *independence* codes to characterize students' descriptions of the conditions of a set of vectors or objects. We view our use of *span* and *independence* codes as highly overlapping with the use of *roles* and *characteristics* codes. We present a description of the *span* and *independence* codes in Figure 3 with some short examples. Figure 3 is altered from the framework used by Zandieh et al. (2019) with the addition of our own restricting code, modification of three codes, and exclusion of two codes. There are some codes in Figure 3 whose names have been changed to better describe our data (i.e., restricting, redundancy, and sameness). If we did not find evidence of a particular code in our students' written responses, we did not include it in Figure 3 (i.e., systematic and structuring).

Span Codes	Information	Independence	Information
Generating (Gen)	Creating a line or plane through combining vectors or game objects. E.g., The spans being used **form** a plane; you could **form** a plane using the 4 vectors provided	Minimal (Min)	The set being the smallest or having the least number of vectors to fulfill the role in space. E.g., to accomplish the task with **smallest number of vectors**
Covering (Cov)	Filling or encompassing the space. E.g., The span must be a plane because all vector combinations **lie within** a certain range	Redundancy (Redun)	Having extraneous objects in the set. E.g., If one of the two vectors was **removed**, it likely **wouldn't have made much of a difference**
Traveling (Trav)	Moving (themselves or the bunny) through the game space or along a path. E.g., you can **reach** anywhere within R2 with no problem.	Sameness (Same)	Vectors in the set or objects related to the vectors (direction, parallel lines, etc.) are the same. E.g., The vectors <-1,5> and <-3,15> **represent the same line**
Restricting (Restr)	Certain conditions keep the span of a set of vectors from having a higher or lower dimension. E.g., There is also no z point in the vectors allowing 3D traversal, **limiting** it to a plane	Essential (Ess)	The critical importance of including a vector in the set or using a vector while playing the game. E.g., Two [vectors] **were needed** instead of one

Figure 3. Table altered from Zandieh et al. (2019)

Results from Span and Independence Coding We used our modified *span* and *independence* codes as a codebook to produce the set of codes for each student's response to Q1 and Q2 as shown in Figure 4. Looking across columns it can be shown that Q1 and Q2 elicited different sorts of responses from our research subjects. For example, there are far more uses of the essential code for Q2 than Q1. Looking across the rows of Figure 4, it can be seen that students emphasized different aspects of the roles or characteristics of the set of travel vectors provided in Levels 3 through 5 in VU.

Student Name	Q1: Level 3 Question 5 Response	Q2: Level 4/5 Question 7 Response
Adrian	Redun, Same, Trav	Ess, Gen, Ess, Trav; Trav, Ess, Trav; Trav
Kieran	Restr, Trav, Redun	Gen; Restr, Cov; Cov, Ess
Brooks	Cov, Trav, Restr	Trav, Min; Ess, Trav, Redun, Ess, Restr
Michael	Trav, Cov	Ess, Gen; Ess, Trav, Cov
Jing	Redun, Trav, Same	Ess, Cov, Ess, Same; Min, Gen, Ess, Trav; Gen, Cov, Same, Trav; Gen, Ess, Trav, Ess, Gen, Min, Trav, Gen, Same
Tanner	Restr, Trav, Gen	Gen; Gen, Ess
Aahan	Same, Trav, Redun, Trav	No response

Figure 4. The results of coding students' responses to Q1 and Q2

To demonstrate how we distinguished between students' responses using game language, consider the following portion of Adrian's response. For clarity, game language will be *italicized*, math language will be underlined, and mixed language will be *both* italicized and underlined.

"Therefore, in order to reach all of the *goals*, the *bunny* needed to use a *vector* to get into *position* before using a second *vector* to *follow* the *linear* *path* towards each of the *goals*," -Adrian.

This excerpt was chosen to show a response that primarily used *game language*. Adrian claimed that an additional vector was needed from the perspective of the bunny, which was an indicator of *game language*. Further, to reach the goals (i.e., the baskets), Adrian claimed the bunny needed to first "get into position" to collect the baskets in Level 5. This need to "get into position" was related to the setup for Level 5 of VU (i.e., the line of the baskets did not pass through the origin).

To demonstrate how we used our *span* and *independence* codes, consider the following portion of Jing's response. For clarity, each indication of a *span* or *independence* code will be **bolded**. We coded Jing's response as Trav, Ess, Gen:

"In level 5, I need to **reach** (Trav) the point of the line and that will **need an extra vector** (Ess) to make it happen, and it will **form** (Gen) a plane," -Jing.

This excerpt was chosen to show a common pairing of codes, *essential* and *traveling* as well as a response given primarily in *math language*. The reader may note the similarity in this portion of Adrian and Jing's responses to Q2 about their Level 5 gameplay in that both excerpts consist of an *essential* and *traveling* pairing of codes.

Discussion and Conclusion

We begin our discussion by first comparing students' span and independence codes across their responses to Q1 and Q2, pictured in Figure 4. We will also discuss trends in pairings of codes that emerged in students' responses similarly to the work of Zandieh et al. (2019).

In Figure 4, we found that we had assigned more students the *generating* code in Q2 than Q1. It may be the case that asking students to compare the span of their vector list in Level 4 and Level 5 led to more examples indicative of creating and forming lines and planes. This makes sense based on the way we asked Q1. Our emphasis was on having students mentally compare the span of their vector list in Level 3 and then comparing that to the span of a new vector list with one fewer vector. Students were assigned the *redundancy* code more times in their responses to Q1 than Q2. Since students were thinking about whether the span of their initial vector set was different than the span of the new vector set (with one fewer vector), this may have contributed to students' responses being characteristic of having extraneous objects in their imagined vector set. We see far fewer *sameness* and *redundancy* codes in general across students' responses to Q2, which also makes some sense. Instead of being asked about the effects of adding or removing a vector from a particular set, students were instead asked about whether the span of the set of vectors they used to win the level was a line (Level 4) or a plane (Level 5).

The *essential* and *traveling* code pairing was the most common pairing in students' responses to the Q2 which asked the students to compare Level 4 and 5. The major difference in these two levels is whether the baskets lie along a line that goes through the origin (Level 4) or not (Level 5). In the *essential/traveling* pairing, students were describing the need for an extra vector to "get to" the line or "detour" to the line containing the baskets in Level 5. The inclusion of the "offset" vector in Level 5 changed the span of the set of vectors needed to win Level 4 from a line to a plane (five out of the seven correctly answered this question). Students' use of gameplay language to compare Levels 4 and 5 may be indicative of the students' explicating an intuitive need for an offset vector, while still working towards more formal mathematization.

It is interesting that many students' intuitive notion about traveling on the offset vector, as indicated by the prevalence of the essential and traveling code pairing, did not appear to translate to more formal mathematization when writing the parametric equation for the line of the baskets in Level 5. It may be that thinking about "traveling to the line of the baskets" is a useful travel metaphor to support students' in transitioning towards more formal ways of reasoning. For example, we view the need to first travel to the line of baskets before collecting them as an informal way of reasoning that foreshadows a transition towards symbolizing the parametric vector equation for the line of the baskets in Level 5. Students were asked to write the vector equation for the line of the baskets on their homework, but only Michael, out of our seven research subjects, produced an equation of the form we had in mind (i.e., $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} + a \begin{bmatrix} 1 \\ 2 \end{bmatrix}$).

Our results add to the body of knowledge about student thinking about span and linear (in)dependence in the context of the IOLA curriculum. Specifically, our contribution is through incorporating the use of a research-designed game and written homework assignments to support students in thinking about the notions of span and linear independence while playing the game. The results of this study demonstrate the ability of the VU game, when accompanied with a written homework assignment, to evoke different ways of thinking about span and linear independence in an experientially real context. We see this as an extension of some of the other work using VU in the context of IOLA (Mauntel et al., 2021).

Further, some students adopted the perspective of the bunny when responding to the homework questions. Students adopting the perspective of the bunny seems to be related to Serbin, Wawro, and Storms' notions of *physics-centered* and *physicist-centered* grammatical constructions (Serbin, Wawro, & Storms, 2021). For example, a student may give an explanation that includes a description of a person within the span of a set of vectors (*physicist-centered*). We conjecture that students' use of *game language* may be connected to *physicist-centered* language, and *math language* use may be extremely similar to the notion of *physics-centered* grammar. Future work may investigate the nature of students' use of *game language* in conjunction with students' use of "bunny-centered" language when discussing their responses to the written homework.

Since we collected all five written homework assignments as data from every student in the third author's IOLA classroom during the Spring 2021 semester, we intend to expand our current analysis across all five homework assignments. Further, we intend to use the results of this study to motivate further revision of the game-based written homework assignments in the context of the IOLA curriculum.

Limitations and Acknowledgements

We acknowledge that this data was collected from a single IOLA course at a large University in the United States. This likely impacted the data we collected. The hope would be to eventually scale these homework assignments to more institutions using the IOLA curriculum as motivation to incorporate the Vector Unknown 2D and 3D game into their classroom.

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