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Quasi–eclectic modular flavor symmetries

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ABSTRACT

Modular flavor symmetries provide us with a new, promising approach to the flavor problem. However, in their original formulation the kinetic terms of the standard model fields do not have a preferred form, thus introducing additional parameters, which limit the predictive power of this scheme. In this work, we introduce the scheme of quasi–eclectic flavor symmetries as a simple fix. These symmetries are the direct product of a modular and a traditional flavor symmetry, which are spontaneously broken to a diagonal modular flavor subgroup. This allows us to construct a version of Feruglio's model with the Kähler terms under control. At the same time, the starting point is reminiscent of what one obtains from explicit string models.

1. Introduction

Modular flavor symmetries [1–8] are an exciting new approach to the flavor problem. Very simple settings can, in principle, provide us with a surprisingly good fit to data while making a comparatively large number of nontrivial, testable predictions. We would like to refer the reader to [9] and references therein for more details and models.

What is the new ingredient of Feruglio's models [1] which appears to make the traditional A_4 models [10,11] even more compelling? The challenges in traditional models (see e.g. [12] for an extended list of examples and references) lie mainly in the flavon sector. More specifically, one has to align the flavons at some appropriate values, see e.g. [13] for a discussion and further references. However, often flavons naturally settle at symmetry–enhanced points (see e.g. [14]), which are typically not entirely realistic. As a consequence, the traditional flavor models often require an extended flavon sector, which introduces a number of free parameters, thus limiting the number of nontrivial predictions. Models with modular flavor symmetries evade these arguments because the flavons get replaced by multiplets of modular forms. One then faces the lesser challenge to find, and eventually justify, appropriate values of the half–period ratio τ which the modular forms depend on. The resulting models are very elegant and describe data surprisingly well [9].

However, there is a price one has to pay. The modular flavor symmetries, which we will review in some more detail in Section 2.1, are nonlinearly realized. As a consequence, the Kähler potential is not under control [15], i.e. there is no preferred field basis. This introduces additional parameters, thus limiting the predictive power of the construction. On the other hand, in the framework of traditional flavor symmetries the Kähler potential is under control. It is still subject to possibly important corrections [16,17], but one has at least a perturbative expansion in $\varepsilon = \langle\xi\rangle/\Lambda$, where $\langle\xi\rangle$ denotes the vacuum expectation value (VEV) of a so–called flavon and Λ is the cut–off scale.

The purpose of this study is to show that a hybrid scheme allows us to combine the advantages of both approaches while largely avoiding their limitations. The simplest models of this hybrid approach have a flavor symmetry of the form

$$G_{\text{flavor}} = G_{\text{traditional}} \times G_{\text{modular}} , \tag{1}$$

and a flavon χ, which is charged under both $G_{\text{traditional}}$ and G_{modular}. Once χ acquires a VEV, the flavor symmetry will be broken to its diagonal subgroup,

$$G_{\text{flavor}} = G_{\text{traditional}} \times G_{\text{modular}} \xrightarrow{\langle\chi\rangle} G_{\text{diagonal}} . \tag{2}$$

Matter fields are assumed to transform under $G_{\text{traditional}}$, which is why their Kähler potential is under control [16–18]. However, af-

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ter the breaking (2), their couplings will be effectively given by modular forms.

Our setup is heavily inspired by the scheme of "eclectic flavor groups" [19], which arise naturally in string models [18,20,21] and magnetized toroidal compactifications [22]. Generally in top–down models (cf. e.g. [23–26]) one can, at least in principle, compute the Kähler potential, but at this point it is probably also fair to say that this approach has not yet provided us with completely realistic predictive models. These groups are the result of combining nontrivially a traditional and a modular flavor group, such that G_{modular} is a subgroup of the outer automorphisms of $G_{\text{traditional}}$. Hence, eclectic groups represent a more complex hybrid scheme than Equation (1), sharing the feature of a controlled Kähler potential due to $G_{\text{traditional}}$. The purpose of the present work is to show how one can, in a bottom–up effective field theory (EFT) approach, combine modular flavor symmetries with perturbative control over the Kähler potential. We leave the question of an explicit stringy completion for future work.

2. Modular and eclectic flavor symmetries

2.1. Modular flavor symmetries

The half–period ratio or modulus τ of a torus does not uniquely characterize a given torus. Rather, different τ related by transformations in the so–called modular group $\text{PSL}(2,\mathbb{Z})$ describe the same torus. Under an arbitrary element $\gamma \in \text{PSL}(2,\mathbb{Z})$, the modulus and matter superfields Φ_j transform as

$$\tau \xmapsto{\gamma} \gamma\,\tau := \frac{a\,\tau + b}{c\,\tau + d}\,, \tag{3a}$$

$$\Phi_j \xmapsto{\gamma} (c\,\tau + d)^{k_j}\,\rho_{\boldsymbol{r}_j}(\gamma)\,\Phi_j\,, \quad \text{where } \gamma := \begin{pmatrix} a & b \\ c & d \end{pmatrix} \tag{3b}$$

with $\det\gamma = 1$ and $a, b, c, d \in \mathbb{Z}$. Further, k_j denotes the so–called modular weight of the matter superfield Φ_j, which can build an r_j–dimensional representation of some finite modular group[2] Γ_N, $N = 2, 3, \ldots$. $\rho_{\boldsymbol{r}_j}(\gamma)$ corresponds to the $r_j \times r_j$ matrix representation of γ in the finite modular group. This transformation of the matter fields indicates that Γ_N can be regarded as a modular flavor symmetry [1,3], which is however nonlinearly realized, as is evident from Equation (3a). Finally, note that the action of the modular flavor symmetry is accompanied by $(c\tau + d)^{k_j}$, which is known as automorphy factor.

Note that, as a consequence of Equation (3a),

$$(-\mathrm{i}\tau + \mathrm{i}\bar{\tau})^k \xmapsto{\gamma} \big((c\,\tau + d)(c\,\bar{\tau} + d)\big)^{-k}(-\mathrm{i}\tau + \mathrm{i}\bar{\tau})^k\,, \tag{4}$$

for an arbitrary k. This implies that an invariant under the finite modular group is given by

$$(-\mathrm{i}\tau + \mathrm{i}\bar{\tau})^{k_j}\left(\bar{\Phi}_j\Phi_j\right)_{\mathbf{1}}\,, \tag{5}$$

where the subindex $\mathbf{1}$ refers to the trivial Γ_N singlet(s) resulting from tensoring the superfield Φ_j with its conjugate.

To complete a supersymmetric model based on modular flavor symmetries, we must specify its superpotential and Kähler potential. In terms of the matter fields Φ_j, the superpotential can be expressed as a polynomial of the form

$$\mathscr{W}(\Phi) = \sum_{i,j,k} \hat{Y}^{(k_Y)}_{\boldsymbol{s}}(\tau)\,\Phi_i\Phi_j\Phi_k + \text{higher order terms}\,, \tag{6}$$

where $\hat{Y}^{(k_Y)}_{\boldsymbol{s}}(\tau)$ are modular forms of level N and modular weights k_Y transforming as an s–dimensional representation of Γ_N. In general, the superpotential is constrained to transform according to

$$\mathscr{W}(\Phi) \xmapsto{\gamma} \mathscr{W}(\Phi') := (c\tau + d)^{k_{\mathscr{W}}}\,\mathscr{W}(\Phi)\,. \tag{7}$$

In our case, given our bottom-up approach, we choose the superpotential to be modular invariant, i.e., $k_{\mathscr{W}} = 0$. This amounts to demanding $\boldsymbol{s} \otimes \boldsymbol{r}_i \otimes \boldsymbol{r}_j \otimes \boldsymbol{r}_k \overset{!}{\supset} \mathbf{1}$ and $k_Y \overset{!}{=} -k_i - k_j - k_k$.

The Kähler potential of matter fields in models endowed with a modular flavor symmetry is typically assumed to take the canonical form

$$K(\Phi, \bar{\Phi}) \supset \sum_j (-\mathrm{i}\tau + \mathrm{i}\bar{\tau})^{k_j}\,|\Phi^j|^2\,, \tag{8}$$

as in [1]. However, the nonlinear realization of this symmetry implies that there are additional terms with free coefficients, which are at the same footing as the canonical terms, thus limiting the predictive power of the model [15].

2.2. Eclectic flavor symmetries

The so–called eclectic flavor symmetries [19] arise naturally in string models [18,20,21] and magnetized toroidal compactifications [22]. They are given by group–theoretic unions of a traditional (flavor) symmetry, $G_{\text{traditional}}$, and a modular symmetry, G_{modular},

$$G_{\text{eclectic}} = G_{\text{traditional}} \cup G_{\text{modular}}\,, \tag{9}$$

such that the modular symmetry is built out of outer automorphisms of $G_{\text{traditional}}$, $G_{\text{modular}} \subset \text{Out}(G_{\text{traditional}})$. The union "∪" in Equation (9) is to be understood as the multiplicative closure of the groups.

Crucially, G_{eclectic} has representations which transform nontrivially under both $G_{\text{traditional}}$ and G_{modular}. This means that, by giving a VEV to a representation of that kind, we can break G_{eclectic} to a diagonal subgroup which inherits properties from $G_{\text{traditional}}$ as well as G_{modular}.

Even though eclectic groups can also be built from a bottom–up perspective [19], in this work we refrain from working out an explicit eclectic model. Rather, in what follows we will analyze the somewhat simpler situation in which the union "∪" in Equation (9) gets replaced by a direct product, i.e. $G_{\text{quasi-eclectic}} = G_{\text{traditional}} \times G_{\text{modular}}$. As we shall see, the emerging scheme is still simple enough to be analyzed and at the same time illustrates how the desirable properties of $G_{\text{traditional}}$ and G_{modular} get inherited by the diagonal group.

3. A simple quasi–eclectic example

3.1. Symmetries and representations

To illustrate the main points of our quasi–eclectic scheme, let us consider a model by Feruglio [1], but with a slight twist. We will take the original flavor symmetry to be

$$G_{\text{flavor}} = A_4^{\text{traditional}} \times \Gamma_3\,, \tag{10}$$

where Γ_3 can be thought of as a modular version of A_4. The quantum numbers of the states are listed in Table 1. We take the superpotential to have modular weight $k_{\mathscr{W}} = 0$.

[2] We restrict here to Γ_N finite modular groups, but our discussion can be readily extended to their double cover Γ'_N and metaplectic extensions (cf. [5,7,27–30,26]).

Table 1
Variation of model 1 of [1]. $E_i^{\mathcal{C}}$, L, H_u and H_d are the superfields of the charged leptons, left–handed doublets, up–type Higgs and down–type Higgs, respectively. S_χ and S_φ are part of the VEV alignment, see Appendix A. In our notation, $A_4 \cong \Gamma_3$ has the representations $\mathbf{3}, \mathbf{1}_0, \mathbf{1}_1$ and $\mathbf{1}_2$, whose tensor products are given e.g. in [1, Appendix C].

	$(E_1^{\mathcal{C}}, E_2^{\mathcal{C}}, E_3^{\mathcal{C}})$	L	H_d	H_u	χ	φ	S_χ	S_φ	Y
$\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_Y$	$\mathbf{1}_1$	$\mathbf{2}_{-1/2}$	$\mathbf{2}_{-1/2}$	$\mathbf{2}_{1/2}$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{1}_0$
$A_4^{\text{traditional}}$	$(\mathbf{1}_0, \mathbf{1}_2, \mathbf{1}_1)$	$\mathbf{3}$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{1}_0$
$\mathbb{Z}_3^\chi$	0	0	0	1	1	0	1	0	0
$\mathbb{Z}_3^\varphi$	1	0	1	0	0	1	0	1	0
Γ_3	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{3}$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{3}$
k	$(k_{E_1}, k_{E_2}, k_{E_3})$	k_L	k_{H_d}	k_{H_u}	k_χ	k_φ	k_S	k_S	k_Y
modular weights	$(1, 1, 1)$	-1	0	0	0	0	0	0	2

3.2. Diagonal breaking

Let us now *assume* that the flavon χ attains a "diagonal" VEV, i.e. in the real basis

$$\langle \chi_i^a \rangle = v_1 \mathbb{1}_3 \,. \tag{11}$$

In the complex basis, this diagonal VEV has the shape[3]

$$\langle \chi_i^a \rangle = v_1 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} . \tag{12}$$

We discuss the alignment of the flavon in Appendix A. Similarly to Feruglio, we introduce a flavon φ (as in [1]). Here, a is a Γ_3 index and i an $A_4^{\text{traditional}}$ index. The VEV (12) breaks $A_4^{\text{traditional}} \times \Gamma_3$ to $\Gamma_3^{\text{diagonal}}$. Both Γ_3 and $\Gamma_3^{\text{diagonal}}$ are nonlinearly realized.

3.3. Charged lepton Yukawa couplings

The charged fermion masses are obtained just like in [1]. Since we assigned them the $\mathbf{1}_0$, $\mathbf{1}_1$ and $\mathbf{1}_2$ under $A_4^{\text{traditional}}$, respectively, we can write down superpotential terms[4]

$$\mathscr{W}_e = \frac{\widetilde{y}_e}{\Lambda} H_d (L\varphi E_1^{\mathcal{C}})_{\mathbf{1}_0} + \frac{\widetilde{y}_\tau}{\Lambda} H_d (L\varphi E_2^{\mathcal{C}})_{\mathbf{1}_0} + \frac{\widetilde{y}_\mu}{\Lambda} H_d (L\varphi E_3^{\mathcal{C}})_{\mathbf{1}_0} \,, \tag{13}$$

which involve the three free parameters $\widetilde{y}_e$, $\widetilde{y}_\mu$ and $\widetilde{y}_\tau$, and the cut–off scale Λ of the model. Here, a $\mathbf{1}_0$ subscript indicates a contraction to a G_{flavor} singlet. In order to get a diagonal charged lepton Yukawa coupling matrix, we will take the VEV of φ to be

$$\langle \varphi_i \rangle = v_2 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{14}$$

in the complex basis and

$$\langle \varphi_i \rangle = \frac{v_2}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{15}$$

in the real basis, similarly to Feruglio's model [1]. This choice will be justified in Appendix A. Equation (13) along with Equation (14) gives the charged lepton mass matrix

$$m_e = v_d \frac{v_2}{\Lambda} \operatorname{diag}\left(\widetilde{y}_e, \widetilde{y}_\tau, \widetilde{y}_\mu\right) , \tag{16}$$

where v_d is the VEV of H_d, as usual. Like in [1], we introduced three parameters, $\widetilde{y}_e$, $\widetilde{y}_\mu$ and $\widetilde{y}_\tau$. These parameters can be used to reproduce the observed charged lepton masses. In order to reproduce the observed τ lepton mass, $\varepsilon_2 := v_2/\Lambda$ cannot become too small. This sector does not really contain any novel ingredients, nor does it by itself make nontrivial predictions. Some ideas to address the question of fermion mass hierarchies can be found in e.g. [31–33].

3.4. Weinberg operator

Like in Feruglio's model [1] the new ingredients are in the Weinberg operator, which emerges from the superpotential couplings

$$\mathscr{W}_\nu = \frac{1}{\Lambda^2} \left[(H_u \cdot L)\ \chi\ (H_u \cdot L)\ Y \right]_{\mathbf{1}_0} \,. \tag{17}$$

To construct the couplings at the component level, we first contract $Y\chi$ to Γ_3 singlets. Since χ consists of three Γ_3 $\mathbf{3}$–plets, we obtain an $A_4^{\text{traditional}}$ triplet

$$[(Y\chi)_{(\mathbf{3},\mathbf{1}_0)}]_i = Y_1 \chi_i^{\,1} + Y_2 \chi_i^{\,3} + Y_3 \chi_i^{\,2} \,, \tag{18}$$

where i is an $A_4^{\text{traditional}}$ index. Here, $(\boldsymbol{r}, \boldsymbol{r'})$ means that the contraction transforms as $(\boldsymbol{r}, \boldsymbol{r'})$ under $A_4^{\text{traditional}} \times \Gamma_3$. This $A_4^{\text{traditional}}$ triplet can be contracted with the unique $A_4^{\text{traditional}}$ triplet that emerges from combining the $A_4^{\text{traditional}}$ triplet L with itself,

$$(LL)_{(\mathbf{3},\mathbf{1}_0)} = \frac{2}{\sqrt{3}} \begin{pmatrix} L_1^2 - L_2 L_3 \\ L_3^2 - L_1 L_2 \\ L_2^2 - L_1 L_3 \end{pmatrix} . \tag{19}$$

After inserting the "diagonal" VEV (12), the effective superpotential coincides, up to an irrelevant prefactor, with the one proposed in [1],

$$\mathscr{W}_\nu = \frac{v_1}{\Lambda^2} \left[(H_u \cdot L)\ Y\ (H_u \cdot L) \right]_{\mathbf{1}_0} \,. \tag{20}$$

In particular, the matrix structure of the Weinberg operator is identical to the one in [1]. That is, the neutrino mass matrix is given by

$$m_\nu = \frac{v_u^2 \varepsilon_1}{\sqrt{3}\Lambda} \begin{pmatrix} 2Y_1(\tau) & -Y_3(\tau) & -Y_2(\tau) \\ -Y_3(\tau) & 2Y_2(\tau) & -Y_1(\tau) \\ -Y_2(\tau) & -Y_1(\tau) & 2Y_3(\tau) \end{pmatrix} , \tag{21}$$

where $\varepsilon_1 = v_1/\Lambda$ and v_u is the VEV of H_u. Then this matrix has only three free real parameters: Λ, $\mathrm{Re}\,\tau$ and $\mathrm{Im}\,\tau$.

3.5. Kinetic terms

Before χ and φ attain VEVs, the Kähler potential of the charged leptons is diagonal because of the presence of $A_4^{\text{traditional}}$. Therefore, the Kähler potential is under control. After the breaking to the diagonal flavor symmetry,

$$K_L \;=\; L^\dagger L + \mathcal{O}(\varepsilon_1^2) + \mathcal{O}(\varepsilon_2^2) \,. \tag{22}$$

This is because the corrections to the Kähler potential come from terms involving[5] χ and φ. A priori these terms are not known. In this work we ask how much we can limit the effects of these terms in a bottom–up approach.

[3] The relation between these bases is explained in Appendix B.

[4] Following Feruglio's model (cf. [1, discussion between Equations (39) and (40)]), we exchange here $\widetilde{y}_\mu$ and $\widetilde{y}_\tau$ to best fit data.

[5] Terms including the fields L and the modular forms Y (see e.g. [15, Equation (12)]) are restricted by $A_4^{\text{traditional}}$ to be just the product of the $L^\dagger L$ and $Y^\dagger Y$ trivial singlets of $A_4^{\text{traditional}}$, and are hence diagonal too.

Let us first turn our attention to χ. χ enters the leptonic superpotential only through the Weinberg operator, yielding Equation (21). Therefore, we cannot place a stringent lower bound on the size of v_1.

On the other hand, the magnitude of the VEV of φ, v_2, is bounded from below by the requirement to reproduce a realistic τ Yukawa coupling, y_τ. The value y_τ depends on the Higgs VEV ratio $\tan\beta$, $y_\tau = \sqrt{1+\tan^2\beta}\, m_\tau / v_{\rm EW} \sim 10^{-2}\sqrt{1+\tan^2\beta}$ (at tree level), where $v_{\rm EW}$ denotes the electroweak VEV. In [3], the best fits to data are obtained for small $\tan\beta$, in which case y_τ is suppressed, and the lower bound on v_2 is less stringent.

At first glance, one may suspect to find linear contributions to the Kähler metric,

$$K \supset (\varphi L L^\dagger)_{\mathbf{1}_0} \quad \text{and/or} \quad \left(\varphi\, E_i^{\mathcal{C}} (E_i^{\mathcal{C}})^\dagger\right)_{\mathbf{1}_0} . \tag{23}$$

However, the terms (23) are forbidden due to the symmetry $\mathbb{Z}_3^\varphi$ (cf. Table 1). Thus, the first nontrivial flavon–dependent contributions to the Kähler metric are given by $(L\varphi)^\dagger(L\varphi)$ and $(\varphi\varphi^\dagger)(EE^\dagger)$, which we will call ΔK_L and ΔK_R, respectively. Let us first focus on the L contribution. Considering the discrete charges of L and φ, we identify seven $A_4^{\rm traditional}$ invariant terms from the product $(\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{3})$. After inserting on the VEV of φ (14), these are reduced to only three nonvanishing invariant contributions to ΔK_L, which are associated with three independent coefficients C_i. The resulting contribution to the Kähler metric, in the complex basis, is

$$\Delta K_L = \frac{v_2^2}{3\Lambda^2}\begin{pmatrix} 3C_1+4C_2 & 0 & 0 \\ 0 & 3C_1-2C_2+2\sqrt{3}C_3 & 0 \\ 0 & 0 & 3C_1-2C_2-2\sqrt{3}C_3 \end{pmatrix}, \tag{24}$$

which can be decomposed as

$$\Delta K_L = \varepsilon_2^2\left(C_1\,\mathbb{1}_3 + \frac{2C_2}{3}\,\mathrm{diag}\,(2,-1,-1) + \frac{2C_3}{\sqrt{3}}\,\mathrm{diag}\,(0,1,-1)\right). \tag{25}$$

In the case of the R contribution, after evaluating in Equation (14), we get nine invariant terms from which only three are nonvanishing. The resulting contribution, in both complex and real basis, is

$$\Delta K_R \;=\; \varepsilon_2^2\,\mathrm{diag}(D_1, D_2, D_3)\,, \tag{26}$$

where D_i is defined similarly as in Equation (24).

The impact of these corrections can be estimated using the discussion in [16,17]. We see that the corrections of the mixing angles come from ΔK_L only. Generically, the solar angle θ_{12} is the most sensitive angle in a scheme with inverted mass ordering, its correction gets enhanced by a factor $m_1^2/\Delta m^2_{\rm sol}$, which is about 34 in the Feruglio model. The corrections are also proportional to $\varepsilon_2^2 = v_2^2/\Lambda^2 \gtrsim y_\tau^2$. Furthermore since the unperturbed theory has diagonal kinetic terms, the coefficients of the Kähler corrections are also not arbitrarily large. For corrections associated with the coefficient C_i of the Kähler metric ΔK_L in Equation (25), we find

$$\Delta\theta_{12} \simeq C_i \left(\frac{\varepsilon_2}{0.03}\right)^2 \cdot \begin{cases} 0\,, & \text{if } i=1\,, \\ -0.05\,, & \text{if } i=2\,, \\ 0.01\,, & \text{if } i=3\,. \end{cases} \tag{27}$$

While an exact computation of the coefficient C_i would require a UV completion of the model (cf. e.g. [34,35]), we make the EFT assumption that the coefficients are at most of the order unity. Equation (27) shows that, if the correction is proportional to the unit matrix, θ_{12} does not change, as expected. For small $\tan\beta$, $\varepsilon_2 \sim 0.03$ is possible, and the Kähler corrections are comparable to the experimental uncertainties. However, for large $\tan\beta$, the model we discuss here requires additional ingredients to allow us to make precise predictions.

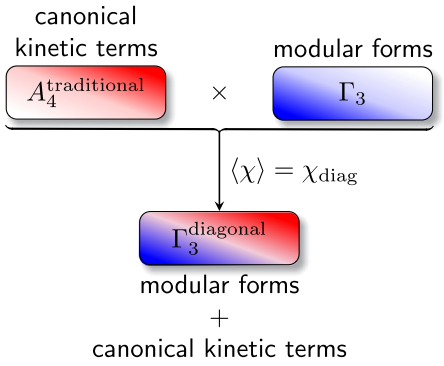

Fig. 1. Diagonal breaking of traditional and modular flavor symmetries.

Altogether we see that the Kähler corrections are controlled by ε_2, which also governs the charged lepton Yukawa couplings. In this regard this bottom–up analysis is somewhat reminiscent of minimal avor violation (MFV) [36–38]. We can hence conclude that the quasi–eclectic scheme presented here allows us to construct predictive bottom–up models with modular flavor symmetries.

4. Summary and outlook

4.1. Summary

Motivated by the great success of Feruglio's models, we have proposed a simple way to fix the kinetic terms in this and related bottom–up scenarios. To this end, we started with a larger flavor symmetry, $G_{\rm flavor} = A_4^{\rm traditional} \times \Gamma_3$, and broke it to its diagonal subgroup, which is given by the finite modular group Γ_3 in our main example (cf. Fig. 1). In the limit of an exact $G_{\rm flavor}$, the Kähler metric is proportional to the unit matrix because of $A_4^{\rm traditional}$. Therefore, the deviations from canonical kinetic terms are parametrized by a flavon VEV, which also induces the charged lepton masses, somewhat similarly to the MFV scheme. In particular, we can perform an EFT analysis to assess the impact of the corrections to the Kähler potential. We refer to $G_{\rm flavor}$ as a quasi–eclectic modular flavor symmetry, since this bottom–up hybrid scheme shares some of the features of top–down eclectic flavor groups.

We have commented on how to align the VEV in such a way that it yields the desired breaking. Since the corresponding configuration has enhanced symmetries, it is rather straightforward to achieve this without affecting the lepton parameters, thus leaving the number of nontrivial predictions unchanged.

The corrections to the Kähler potential can still be relevant, but are under control. Apart from that, they correspond to error bars in our predictions, yet crucially we are able to specify these error bars, which is, to the best of our knowledge, not possible if one starts from the nonlinearly realized modular flavor symmetries alone.

4.2. Outlook

We have shown that, by adding ingredients reminiscent to what one finds in top–down constructions, one can coin predictive bottom–up models with modular flavor symmetries. Yet it is clear that the toy model presented in this work leaves some questions unanswered. For instance, we are able to assign the modular

Table 2
Flavon sector.

field	χ	φ	S_χ	S_φ
$A_4^{\text{traditional}}$	**3**	**3**	$\mathbf{1}_0$	$\mathbf{1}_0$
Γ_3	**3**	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{1}_0$
$\mathbb{Z}_3^\chi$	1	0	1	0
$\mathbb{Z}_3^\varphi$	0	1	0	1

weights and representations at will whereas in top–down models they derive from the underlying geometry. Also, while the flavon alignment works, it does not appear to be the final word on this story. One may also envisage flavon potentials in which the coefficients are modular forms, such that the VEVs inherit the pattern from the modular forms. In this case, the kinetic terms may be under control similarly to what we found in our toy model. It therefore appears worthwhile to explore similar top–down motivated ingredients to address the flavor puzzle.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Flavon VEV alignment[6]

We use the flavon VEVs (12) and (14) (cf. Section 3.3), which in the so–called real basis (cf. [1, Appendix C]) are given by

$$\langle \chi_i^a \rangle = v_1 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad \langle \varphi_i \rangle = \frac{v_2}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} . \tag{28}$$

We assume that the $A_4^{\text{traditional}}$ representation matrices act from the left, and the Γ_3 matrices act from the right. Then the VEV $\langle \chi_i^a \rangle$ is the unique VEV which is invariant simultaneous S and T transformations from both groups,

$$S \langle \chi \rangle S^T = T \langle \chi \rangle T^T = \langle \chi \rangle . \tag{29}$$

So it is a symmetry–enhanced point, which suggests that it should not be too difficult to obtain such VEVs [14].

One can make this more explicit. Let us consider the most general renormalizable superpotentials involving the flavons χ, φ, S_χ and S_φ (see Table 2 for their charges),

$$\mathscr{W} = \mathscr{W}_\chi + \mathscr{W}_\varphi , \tag{30}$$

where

$$\mathscr{W}_\chi = \frac{\kappa_\chi}{2} S_\chi (\chi\chi)_{\mathbf{1}_0} - \frac{\lambda_1}{3} (\chi\chi\chi)^{(1)}_{\mathbf{1}_0} - \frac{\lambda_2}{3} (\chi\chi\chi)^{(2)}_{\mathbf{1}_0} + \kappa_1 S_\chi^3 , \tag{31a}$$

$$\mathscr{W}_\varphi = \frac{\kappa_\varphi}{2} S_\varphi (\varphi\varphi)_{\mathbf{1}_0} - \frac{\lambda_3}{3} (\varphi\varphi\varphi)_{\mathbf{1}_0} + \kappa_2 S_\varphi^3 . \tag{31b}$$

Here, the subscript "$\mathbf{1}_0$" indicates the contraction to a singlet. There are two independent such contractions of three χ fields,

$$(\chi\chi\chi)^{(1)}_{\mathbf{1}_0} = \chi_1^1 \chi_3^2 \chi_2^3 + \chi_2^1 \chi_1^2 \chi_3^3 + \chi_3^1 \chi_2^2 \chi_1^3 , \tag{32a}$$

$$(\chi\chi\chi)^{(2)}_{\mathbf{1}_0} = \chi_1^1 \chi_2^2 \chi_3^3 + \chi_2^1 \chi_3^2 \chi_1^3 + \chi_3^1 \chi_1^2 \chi_2^3 . \tag{32b}$$

We assume that S_χ and S_φ acquire VEVs $\langle S_\chi \rangle \ll \Lambda$ and $\langle S_\varphi \rangle \ll \Lambda$. This is plausible since in string–derived models often the VEVs get fixed by D–terms [39,40]. In fact, in the heterotic orbifold models, which underlie the eclectic scheme, the Fayet–Iliopoulos (FI) D–terms drive the flavons to nonzero VEVs [41], which has been verified in many explicitly constructed models (cf. e.g. [42]). One can verify that there is a nontrivial solution to the F–term equations,[7] where the VEVs are given by Equation (28) with

$$v_1 = \frac{\kappa_\chi \langle S_\chi \rangle}{\lambda_2} \quad \text{and} \quad v_2 = \frac{\kappa_\varphi \langle S_\varphi \rangle}{\lambda_3} . \tag{33}$$

Of course, there is another solution at which all VEVs vanish, and there are solutions in which only one of the VEVs vanish. Technically, in supergravity the above solution is the deepest minimum of the scalar potential, but addressing the vacuum energy is beyond the scope of this study. Furthermore, Equation (31) exhibits two accidental R symmetries which get spontaneously broken by the flavons. As a consequence, there are, at this level, two flat directions. However, these flat directions parametrize the overall magnitudes of the VEVs but do not alter their shapes given by Equations (28) and (33). The stabilization of the flat directions is beyond the scope of this study as the magnitudes of the VEVs are input parameters and only the shapes of the VEVs are important for our scenario. We also note that at higher orders there are additional terms that can alter the above solution slightly. Especially cross terms between χ and φ can shift the VEVs. However, these terms appear at much higher order, and are thus suppressed against the Kähler corrections which we discuss and tame in the main text. Altogether we find that, in a bottom–up EFT theory approach we can successfully align the VEVs to provide us with a scenario of diagonal breaking $A_4^{\text{traditional}} \times \Gamma_3 \to \Gamma_3^{\text{diagonal}}$.

Appendix B. Basis change

Considering the three-dimensional representation of A_4, the group generators can be expressed in the *complex basis*,

$$S_{\mathbf{3}}^C = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} , \qquad T_{\mathbf{3}}^C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix} , \tag{34}$$

where $\omega = \exp(2\pi i/3)$. However, one might find it useful to express these generators in the *real basis*, as we do in Appendix A, where they adopt the form

$$S_{\mathbf{3}}^R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} , \qquad T_{\mathbf{3}}^R = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} . \tag{35}$$

These bases are related by the unitary transformation

[6] The computations of this Appendix can be checked in the attached supplementary Mathematica notebook which makes use of the Discrete package.

[7] Note that, since we have assumed that S_χ and S_φ acquire VEVs by a different mechanism, their dynamics is not relevant in the F–term equations.

$$S_{\mathbf{3}}^{R} = U S_{\mathbf{3}}^{C} U^{\dagger} \quad \text{and} \quad T_{\mathbf{3}}^{R} = U T_{\mathbf{3}}^{C} U^{\dagger}\,, \tag{36}$$

where U is a unitary matrix, given by

$$U = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}\,. \tag{37}$$

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