

1 **Optical spatial filtering with plasmonic
2 directional image sensors**

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8 **Abstract:** Photonics provides a promising approach for image processing by spatial filtering,
9 with the advantage of faster speeds and lower power consumption compared to electronic
10 digital solutions. However, traditional optical spatial filters suffer from bulky form factors that
11 limit their portability. Here we present a new approach based on pixel arrays of plasmonic
12 directional image sensors, designed to selectively detect light incident along a small,
13 geometrically tunable set of directions. The resulting imaging systems can function as optical
14 spatial filters without any external filtering elements, leading to extreme size miniaturization.
15 Furthermore, they offer the distinct capability to perform multiple filtering operations at the
16 same time, through the use of sensor arrays partitioned into blocks of adjacent pixels with
17 different angular responses. To establish the image processing capabilities of these devices,
18 we present a rigorous theoretical model of their filter transfer function under both coherent and
19 incoherent illumination. Next, we use the measured angle-resolved responsivity of prototype
20 devices to demonstrate two examples of relevant functionalities: (1) the visualization of
21 otherwise invisible phase objects and (2) spatial differentiation with incoherent light. These
22 results are significant for a multitude of imaging applications ranging from microscopy in
23 biomedicine to object recognition for computer vision.

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26 **1. Introduction**

27 Spatial filtering operations, where different frequency components of an image are selectively
28 transmitted or blocked, play a key role in many high-impact applications in microscopy,
29 photography, and computer vision [1]. In particular, edge detection by high-pass filtering
30 allows for image sharpening as well as segmentation, to distill a highly compressed version of
31 the original image that is easier to store, transmit, and process. In fact, the latter idea is at the
32 core of the initial stage of the visual recognition process, where different filtered versions of
33 the original image are produced for subsequent analysis. The same principle is also observed
34 in the first layer of convolutional neural networks (CNNs), which have emerged as the leading
35 algorithmic approach for many demanding applications in visual data processing such as image
36 classification and object recognition [2]. These filtering operations can be readily implemented
37 in the electronic digital domain – at the expense, however, of substantial power consumption
38 and processing time. As a result, their adoption in many embedded and mobile edge-computing
39 applications remains a significant challenge (e.g., in autonomous vehicles, augmented reality
40 headsets, and robots, where power and bandwidth are highly constrained).

41 These considerations have created novel opportunities for optical computing solutions,
42 which in fact are currently enjoying a substantial resurgence of interest [3]. Photonics
43 intrinsically offers ultrafast processing bandwidths (essentially at the speed of light) and low
44 power consumption (only limited by optical propagation losses). Of particular relevance in the
45 context of spatial filtering are approaches based on Fourier optics [4], building on the well-
46 known two-lens $4f$ imaging system. In this setup, the first lens projects the Fourier transform

47 of the object field onto a pupil mask between the two lenses, where different spatial frequency
48 components are multiplied by different transmission coefficients before they are recombined
49 by the second lens to form a filtered image of the object. However, this system suffers from
50 large form factor and strict alignment requirements, which again limit its portability. In recent
51 years, several nanophotonic structures have been investigated as a means to provide similar
52 functionalities (particularly image differentiation for edge detection) with more compact
53 dimensions and enhanced design flexibility [5-18]. Specific examples include phase-shifted
54 Bragg reflectors [6], plasmonic filters [8], gradient metasurfaces [7, 12, 13, 16, 17], diffraction
55 gratings [10], and photonic crystal slabs [9, 11, 14, 15], all designed to introduce a sharp in-
56 plane-wavevector dependence in their free-space transmittance. The use of Fourier optical
57 filters in conjunction with neural networks is also being explored extensively [19-21].

58 In the present work, we introduce a different approach where optical spatial filtering is
59 achieved (on a pixel-by-pixel basis) with an image sensor array consisting of specially designed
60 directional photodetectors. Specifically, we employ devices coated with plasmonic
61 metasurfaces that only allow for the detection of light incident along a small set of directions
62 (determined by the metasurface design), whereas light incident along all other directions is
63 reflected. This novel capability has been demonstrated in recent work focused on a different
64 imaging application [22], i.e., planar lensless compound-eye vision with ultrawide field of
65 view. Similar devices can also be used as optical spatial filters, based on the notion that
66 different spatial-frequency components of an illuminated object correspond to plane waves
67 propagating from the object along different directions. Importantly, with this approach the
68 filter transfer function can be tailored through the design of the metasurface, and different
69 metasurfaces (i.e., different filters) can be applied on different adjacent pixels within the same
70 image sensor array. As a result, multiple filtering operations can be performed simultaneously
71 with the same pixel array. Furthermore, this approach does not require any external optical
72 components other than a standard imaging lens, and therefore is particularly convenient in
73 terms of system miniaturization and alignment simplicity.

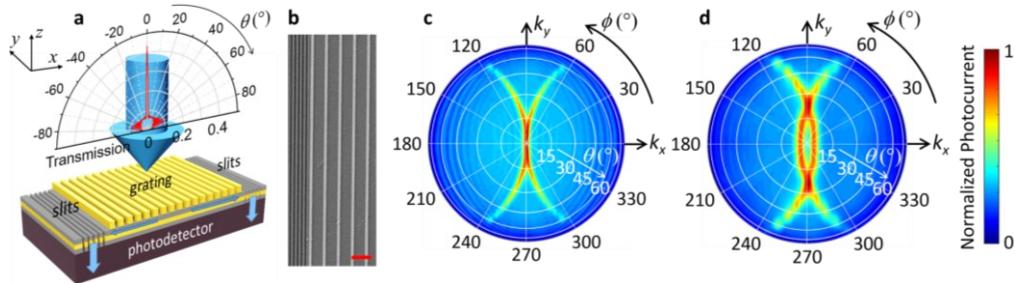
74 In the directional image sensors described below, light incident at the target detection angles
75 is selectively detected via resonant coupling to a guided plasmonic mode. As a result, sharp
76 responsivity peaks at geometrically tunable angles are obtained, which are particularly well
77 suited to engineer a wide range of transfer functions for high-contrast optical spatial filtering.
78 In contrast, other devices previously used for angle-sensitive vision [23-25] feature a more
79 gradual angular dependence and/or limited tunability. At the same time, the role of the guided
80 modes in our directional photodetectors complicates the conceptual analogy with standard
81 optical spatial filters. Therefore, in order to substantiate the image processing capabilities of
82 these devices, here we develop a rigorous theoretical model that quantifies and clarifies the
83 nature of their filter transfer function. Next, we combine this model with the experimental
84 angle-resolved responsivity of prototype samples to demonstrate two examples of relevant
85 spatial filtering functionalities.

86 In the first example, we use devices featuring high-pass filtering characteristics to visualize
87 a transparent phase-only object, which would otherwise be invisible to standard image sensor
88 arrays. Second, we address the task of spatial differentiation for edge detection of amplitude
89 objects with incoherent (i.e., natural) illumination. It is well established from Fourier optics
90 that, when the incident light is spatially incoherent, high-pass filtering is generally impossible
91 with a single filter [4], which represents a key limitation of optical-domain image processing.
92 A possible solution is to record two low-pass filtered images of the object of interest with
93 different cutoff frequencies, and then compute their difference [26-28]. Here we present a
94 particularly simple protocol to perform this task, based on a camera where low-pass-filtering
95 directional photodetectors are combined with standard pixels in a checkerboard pattern. The
96 same approach can be extended to implement more complex incoherent filtering operations that
97 similarly require suppression of the low-frequency components, e.g., for object recognition.
98

99 **2. Plasmonic directional image sensors**

100 The physical structure and principle of operation of our devices are illustrated in Fig. 1(a). In
 101 these devices, the illumination window of a photodetector is coated with a composite
 102 metasurface consisting of an optically thick metal film (Au) stacked with a periodic array of
 103 Au rectangular nanostripes (grating lines) and perforated with sub-wavelength slits. Two
 104 dielectric layers (SiO_2) are also introduced immediately below and above the metal film, to
 105 provide electrical insulation from the active layer and to optimize the film-grating coupling,
 106 respectively [22]. Light incident at the desired detection angle is diffracted by the grating into
 107 surface plasmon polaritons (SPPs) at the top surface of the metal film. These guided waves are
 108 then scattered by the slits into radiation propagating predominantly into the absorbing active
 109 layer, similar to the phenomenon of extraordinary optical transmission through sub-wavelength
 110 apertures in metal films [29, 30]. Correspondingly, a photocurrent signal is detected
 111 proportional to the SPP field intensity at the slit locations. Light incident along any other
 112 direction is instead either reflected or diffracted back into the air above.

113 Specifically, in the present work we use two devices where the grating is surrounded
 114 symmetrically by slits on both sides, leading to a symmetric angular response peaked either at
 115 normal incidence (device A) or at equal and opposite illumination angles (device B) depending
 116 on the grating period Λ . In passing, we note that the same design platform can also be used to
 117 produce an asymmetric angular response peaked at any desired off-axis angle, by replacing the
 118 slits on one side of the array with a suitable “reflector” unit [22]. While this configuration is
 119 not considered in the present work, it opens up additional filtering opportunities in the context
 120 of phase-contrast imaging, where asymmetric transfer functions are particularly beneficial [31].



121
 122 Fig. 1. Plasmonic directional image sensors. (a) Schematic illustration of the physical structure and principle
 123 of operation for a device designed to provide angle-sensitive photodetection peaked at normal incidence. The
 124 device is illuminated through its top surface, as shown by the arrow over the grating. The polar plot shows
 125 the calculated optical transmission coefficient through the metasurface for *p*-polarized light at $\lambda_0 = 1550$ nm
 126 versus angle of incidence θ on the x - z plane. (b) Scanning electron microscopy (SEM) image of an
 127 experimental sample (device A) showing a few periods of the grating and the adjacent slits. The scale bar is
 128 2 μm . (c) Measured responsivity of the same device versus polar θ and azimuthal ϕ illumination angles at λ_0
 129 = 1550 nm, summed over two orthogonal polarizations. In this device, the two SiO_2 layers have a nominal
 130 thickness of 60 nm, the metal film consists of 5 nm of Ti and 100 nm of Au, and each grating line consists of
 131 5 nm of Ti and 50 nm of Au with a width of 250 nm. The grating contains 15 lines with a period $\Lambda = 1485$
 132 nm. Each slit section contains 5 slits with 200-nm width and 400-nm center-to-center spacing. (d) Same as
 133 (c) for a different sample (device B) featuring a symmetric double-peaked angular response with maximum
 134 photocurrent at $\theta = \pm 3.8^\circ$. The geometrical parameters of this device are nominally the same as in the sample
 135 of (c), except for a larger array period $\Lambda = 1581$ nm.

136 The polar plot of Fig. 1(a) shows the *p*-polarized transmission coefficient at 1550-nm
 137 wavelength of the metasurface of device A, computed as a function of polar illumination angle
 138 θ on the x - z plane with finite difference time domain (FDTD) simulations. The device
 139 geometrical parameters are listed in the figure caption. A sharp transmission peak centered at
 140 $\theta = 0^\circ$ (normal incidence) is observed in this plot, with full width at half maximum as small as
 141 3° and peak value above 45%, originating from the excitation of SPPs propagating towards
 142 both sets of slits. For *s*-polarized incident light, the calculated transmission coefficient through

143 the same metasurface is isotropic and significantly smaller, < 0.2% at all angles, consistent with
144 the polarization properties of SPPs [22]. The angular response of Fig. 1(a), rescaled by a factor
145 of about 0.5, therefore also applies to unpolarized illumination. Because of its reliance on
146 diffraction, the device operation is also intrinsically wavelength dependent, and monochromatic
147 light at $\lambda_0 = 1550$ nm is considered throughout this work. In practice, the same behavior can
148 be obtained even under broadband illumination with the addition of a spectral filter on the top
149 surface of the image sensor array. Additionally, it should be noted that both polarization-
150 insensitive and achromatic broadband operation are also possible with the same general
151 platform, by replacing the periodic grating with a gradient metasurface [32, 33] or multilevel
152 diffractive elements [34] and leveraging the enhanced design flexibility of such systems.

153 Our experimental samples consist of metal-semiconductor-metal (MSM) Ge photo-
154 conductors, with the metasurfaces just described patterned in the region between two metal
155 contacts deposited on the top surface of a Ge substrate. Such photoconductors are particularly
156 simple to fabricate, and the same results in terms of angular response can be expected with any
157 other type of photodetectors (including image-sensor photodiodes). The metasurfaces were
158 developed with the multi-step fabrication process described in ref. 22, including electron-beam
159 lithography for the slits and nanostripes [Fig. 1(b)]. The completed devices were characterized
160 by measuring their photocurrent under laser light illumination at 1550-nm wavelength as a
161 function of polar θ and azimuthal ϕ angles of incidence. In order to simplify these angle-
162 resolved experiments (and to avoid the need for tightly focused incident light which would
163 degrade the measurement angular resolution), relatively large devices were used, with a lateral
164 dimension w of about 24 μm . In general, the value of this parameter controls the tradeoff
165 between spatial and angular resolution of the filtered images described below.

166 The experimental results [Figs. 1(c) and 1(d)] show highly directional response in good
167 agreement with theoretical expectations. In particular, the incident directions of high
168 responsivity form a rather narrow distribution within the full hemisphere, consisting of two C-
169 shaped regions of opposite curvature. The shape of this distribution is determined by the
170 diffractive coupling of the incident light into different SPP modes, and the two C-shaped
171 regions correspond to SPPs collected by the two slit sections surrounding the grating (see
172 Supplement 1, Section S1). In sample A [Fig. 1(c)], these two regions overlap at $\theta = 0^\circ$ so that
173 a single peak is produced in the horizontal line cut of the angular response. This device can
174 therefore provide low-pass spatial filtering along the x direction. In contrast, in sample B the
175 two C-shaped regions are slightly offset from one another around $\theta = 0^\circ$ [Fig. 1(d)], leading to
176 two symmetrically located response peaks at $\theta = \pm 3.8^\circ$. In conjunction with an imaging lens
177 of suitably small numerical aperture, the resulting transfer function corresponds to a high-pass
178 filter. The experimental responsivities of these and similar devices were also compared to
179 reference samples without any metasurface [22]. The results are generally consistent with the
180 calculated metasurface transmission penalty (about 45% and 23% for p-polarized and
181 unpolarized light, respectively, as discussed above), although large sample-to-sample
182 variations were observed (even in the reference samples) due to fabrication imperfections.

183 **3. Coherent transfer function and phase contrast imaging**

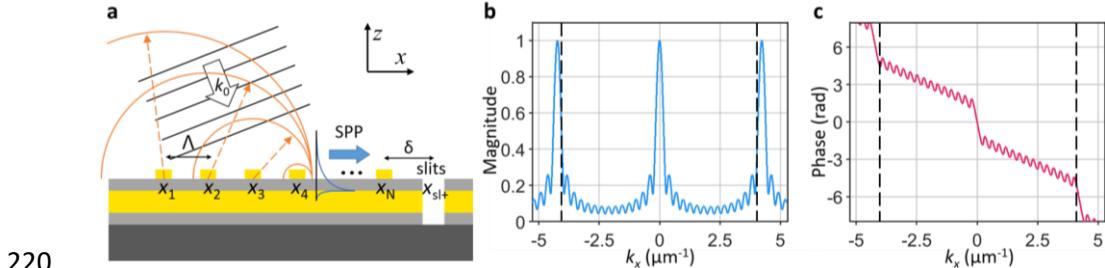
184 In order to establish a connection between the angular response maps of Figs. 1(c) and 1(d) and
185 the spatial filtering capabilities of the same devices, here we introduce and evaluate the
186 corresponding filter transfer function. In a standard optical spatial filter, such as a 4/ f system or
187 nanophotonic equivalent, the input and output signals are the optical field distributions $E_{\text{in}}(\mathbf{r})$
188 and $E_{\text{out}}(\mathbf{r})$ on the input and output planes, respectively, and the coherent transfer function
189 (CTF) is defined as the ratio of their Fourier transforms $t(\mathbf{k}) = E_{\text{out}}(\mathbf{k})/E_{\text{in}}(\mathbf{k})$. In contrast,
190 an array of directional plasmonic image sensors converts its incident optical field distribution
191 $E_{\text{in}}(\mathbf{r})$ into a plasmonic field distribution $E_{\text{SPP}}(\mathbf{r})$, which is then sampled at the slit locations
192 through the slit-scattering/photodetection process illustrated in Fig. 1(a). With this in mind, we
193 can take $E_{\text{SPP}}(\mathbf{r})$ as the output signal of interest $E_{\text{out}}(\mathbf{r})$, with the understanding that such signal

194 is only meaningfully defined at the slit locations $\mathbf{r} = \mathbf{r}_{\text{sl}}^n$, because it cannot be measured by the
 195 sensor array anywhere else (here $\mathbf{n} = \{n_x, n_y\}$ denotes a pair of integers n_x and n_y that label
 196 the different pixels in the array, and the spatial variable \mathbf{r}_{sl}^n indicates position along the slits in
 197 the \mathbf{n}^{th} pixel). A CTF can then again be defined as the ratio of the Fourier transforms of the
 198 (discrete-space) output and input signals $E_{\text{out}}(\mathbf{r}_{\text{sl}}^n) = E_{\text{SPP}}(\mathbf{r}_{\text{sl}}^n)$ and $E_{\text{in}}(\mathbf{r}_{\text{sl}}^n)$, and finally used to
 199 compute the image recorded by the sensor array.

200 To evaluate this CTF, we begin by considering the model structure shown in Fig. 2(a),
 201 which contains N nanostripes arranged periodically at positions x_l ($l = 1, 2, 3, \dots, N$) with period
 202 $\Lambda = x_l - x_{l-1}$, and one slit located to the right of the grating at $x_{\text{sl}+}$ (for simplicity in this discussion
 203 we omit the pixel-label superscript \mathbf{n}). The device is illuminated with a harmonic plane wave
 204 of in-plane wavevector component $k = (2\pi/\lambda_0)\sin\theta$ along the x direction, so that the incident
 205 field distribution on the grating is $E_{\text{in},k}(x) = E_{\text{in}}(k)e^{ikx}$. The incident light is scattered by all
 206 nanostripes, and the scattered waves can excite SPPs on the underlying metal film if the
 207 requirements of energy and momentum conservation are satisfied. The resulting SPP field at
 208 the slit position can then be expressed as

$$209 E_{\text{SPP},k}(x_{\text{sl}+}) = \sum_{l=1}^N E_{\text{in},k}(x_l) \int d\tilde{k} \eta_{\text{SPP}+}(k + \tilde{k}) e^{i(k + \tilde{k} + i\gamma)(x_{\text{sl}+} - x_l)}, \quad (1)$$

210 where each term in the sum is the contribution from a different nanostripe. In the integral,
 211 $\eta_{\text{SPP}+}(k + \tilde{k})$ is the probability amplitude that the light scattered by each nanostripe with in-
 212 plane wavevector $k + \tilde{k}$ excites a SPP that propagates in the $+x$ direction (i.e., towards the slit
 213 at $x_{\text{sl}+}$). The exponential factor accounts for the phase shift and attenuation experienced by this
 214 SPP as it travels from the nanostripe to the slit. The SPP propagation losses due to absorption
 215 and scattering are modeled with an attenuation coefficient $\gamma = 1/(2L_{\text{SPP}})$, where L_{SPP} is the
 216 SPP propagation length. The k dependence of $\eta_{\text{SPP}+}$ is determined by the phase matching
 217 condition including the SPP lifetime broadening, so that $|\eta_{\text{SPP}+}|^2$ can be expressed as a
 218 Lorentzian function of $k + \tilde{k}$ centered at k_{SPP} (the SPP wavenumber at the illumination
 219 wavelength λ_0) with full width at half maximum (FWHM) $1/L_{\text{SPP}}$.



220
 221 Fig. 2. Coherent transfer function of the plasmonic directional image sensors of Fig. 1. (a) Schematic
 222 illustration of the physical model used to evaluate the CTF contribution $t_+(k)$ from the slits on the right of the
 223 grating. The circles illustrate the phase relationship among the light waves scattered by different nanostripes
 224 into SPPs. (b), (c) Magnitude (normalized to unit peak value) (b) and phase (c) of $t_+(k_x, k_y=0)$ versus k_x ,
 225 computed using eqs. (3) and (4) with the parameter values of sample A. The dashed vertical lines indicate the
 226 range of k values (from $-2\pi/\lambda_0$ to $+2\pi/\lambda_0$) accessible with external illumination from air.

227 Equation (1) can be simplified by using $E_{\text{in},k}(x_l) = E_{\text{in}}(k)e^{ikx_l}$ and $x_{\text{sl}+} - x_l = (N - l)\Lambda + \delta$,
 228 where $\delta = x_{\text{sl}+} - x_N$ is the distance between the slit and its nearest nanostripe [see Fig. 2(a)].
 229 With these substitutions, we find that the SPP field at the slit can be expressed as $E_{\text{SPP},k}(x_{\text{sl}+}) =$
 230 $t_+(k)E_{\text{in},k}(x_{\text{sl}+})$, where

$$231 t_+(k) = \int d\tilde{k} \eta_{\text{SPP}+}(k + \tilde{k}) e^{i(\tilde{k} + i\gamma)\delta} f(\tilde{k}) \quad (2)$$

232 and

233
$$f(\tilde{k}) = \sum_{l=1}^N e^{i(\tilde{k}+iy)(N-l)\Lambda} = [1 - e^{i(\tilde{k}+iy)N\Lambda}] / [1 - e^{i(\tilde{k}+iy)\Lambda}]. \quad (3)$$

234 According to these equations, the output signal $E_{SPP,k}(x_{sl+})$ sampled by the plasmonic image
235 sensor under plane-wave illumination is linearly related to the input field $E_{in,k}(x_{sl+})$ at the same
236 location, as in a traditional optical spatial filter. The k -dependent proportionality factor $t_+(k)$ is
237 therefore the contribution to the device CTF from the slit at x_{sl+} . The same analysis can be
238 readily extended to evaluate the contribution $t_-(k)$ from a slit located symmetrically on the left-
239 hand side of the grating (at position $x_{sl-} = x_1 - \delta$), and to include a finite y component for the
240 in-plane wavevector \mathbf{k} of the incident light. The resulting expression for the CTF is (see
241 Supplement 1, Section S2)

242
$$t_{\pm}(\mathbf{k}) = \int d\tilde{k} \eta_{SPP\pm}(\mathbf{k} \pm \hat{x}\tilde{k}) e^{i(\tilde{k}+iy)\delta} f(\tilde{k}), \quad (4)$$

243 where η_{SPP+} and η_{SPP-} describe the excitation of SPPs propagating in the positive and negative
244 x directions, respectively, and therefore account for the two C-shaped regions of high
245 responsivity observed in the angular response maps of these devices [see Figs. 1(c) and 1(d)].
246 Multiple pairs of symmetrically positioned slits [as in the structure of Fig. 1(a)] can be modeled
247 in the same fashion, resulting in the same expression for $t_{\pm}(\mathbf{k})$ with slightly different values of
248 δ . However, as long as the separation between adjacent slits is small compared to the pixel
249 size, inclusion of these different values has negligible effect on the overall frequency response.

250 The CTF $t_{\pm}(k_x, k_y=0)$ of eqs. (3) and (4) consists of a series of identical peaks centered at k_x
251 $= k_{SPP} - mg$, where $g = 2\pi/\Lambda$, m is an integer, and each peak corresponds to a different order of
252 diffraction. Figures 2(b) and 2(c) show, respectively, the magnitude (normalized to unit peak
253 value) and phase of $t_+(k_x, k_y=0)$ computed with these equations. The corresponding plots for t_-
254 (\mathbf{k}) can be inferred directly from these traces using the relation $t_-(\mathbf{k}) = t_+(-\mathbf{k})$, which follows
255 from eq. (4) (see Supplement 1, Section S2) and is consistent with the symmetric device
256 geometry under study. These calculations are based on the parameter values of sample A,
257 including $\Lambda = \delta = 1485$ nm, $N = 15$, $\lambda_0 = 1550$ nm, and $k_{SPP} = 2\pi/\Lambda$ so that the $m=1$ peak is
258 centered at $k_x = 0$ as in Fig. 1(c). For the SPP propagation length we use $L_{SPP} = 80$ μm , selected
259 with a numerical fit so that the peaks of $|t_{\pm}(k_x, k_y=0)|^2$ have the same linewidth as in our measured
260 responsivity data [the horizontal line cut of the color map in Fig. 1(c)]. The dashed vertical
261 lines in Figs. 2(b) and 2(c) indicate the range of k values (from $-2\pi/\lambda_0$ to $+2\pi/\lambda_0$) accessible
262 with external illumination from the air above the grating. The experimental data of Fig. 1(c)
263 are well reproduced by the calculation results plotted in Fig. 2(b), including the fringes around
264 the main peak which originate from incomplete cancellation of the scattered waves away from
265 the Bragg condition in the presence of a finite number of grating lines.

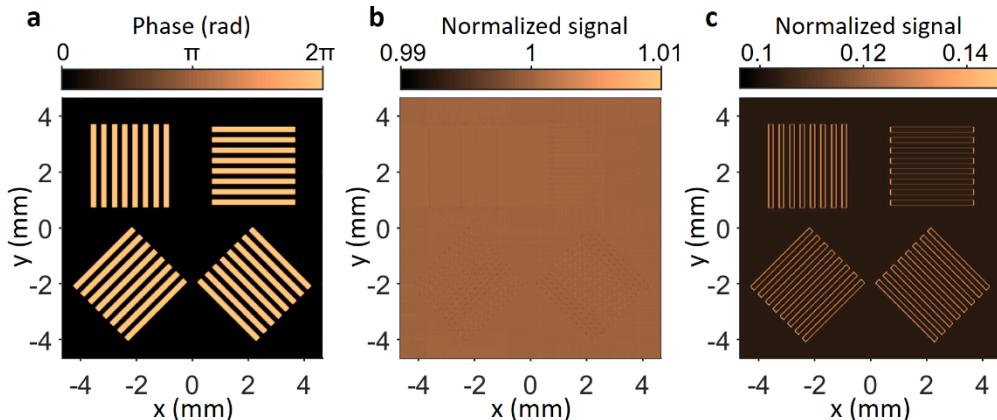
266 As shown in Fig. 2(c), the phase response $\phi_+(\mathbf{k}) = \arg\{t_+(\mathbf{k})\}$ exhibits a linear dependence
267 on k_x with negative slope $d\phi_+(\mathbf{k})/dk_x = -\alpha$ across the entire linewidth of the peak at $\mathbf{k} = 0$, i.e.,
268 for all accessible values of \mathbf{k} for which $|t_+(\mathbf{k})|$ is non-negligible. The value of the slope
269 parameter α inferred from this plot is 9.8 μm , which is relatively close to the distance between
270 the slit and the center of the grating $x_{sl+} - x_c = 11.9$ μm in device A. In fact, as shown in
271 Supplement 1, Section S3, α becomes exactly equal to $x_{sl+} - x_c$ in the limit of large L_{SPP} .
272 Detailed FDTD simulations (also presented in Supplement 1, Section S3) similarly indicate a
273 linear phase profile for $t_+(k_x, k_y=0)$ near $k_x = 0$ with comparable slope parameter $\alpha = 12.2$ μm .
274 By the shifting property of Fourier transforms, this linear phase profile corresponds to a
275 displacement in real space by the amount α in the negative x direction. We can therefore
276 conclude that the SPP signal sampled by the slit at x_{sl+} is an amplitude-filtered version of the
277 light incident on the device at position $x_{sl+} - \alpha$, close to the center of the grating. Similar
278 considerations apply to the phase of $t_-(\mathbf{k})$ with positive slope α , so that the slit at x_{sl-} filters the
279 input signal at $x_{sl-} + \alpha$, also close to the center of the device. It should be noted that this

280 sampling behavior is fundamentally different from the operation of standard photodetectors,
 281 which instead average the incident light across their entire illumination window.

282 When the same plasmonic devices are illuminated with an arbitrary incident field $E_{in}(\mathbf{r}) =$
 283 $\int d\mathbf{k} E_{in}(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{r}}$, the measured photocurrent is proportional to the sum of the intensities of the
 284 corresponding SPP fields detected by the two slits $E_{SPP}(\mathbf{r}_{sl\pm}) = \int d\mathbf{k} t_{\pm}(\mathbf{k}) E_{in}(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{r}_{sl\pm}}$, each
 285 averaged over the slit length along the y direction. This photocurrent signal can therefore be
 286 computed from the CTF $t_{\pm}(\mathbf{k})$. To evaluate the filtering capabilities of our experimental
 287 samples, in the calculations presented below this CTF is expressed as

288
$$t_{\pm}(\mathbf{k}) \propto e^{\mp i\alpha k_x} \sqrt{R_{\pm}(\mathbf{k})}, \quad (5)$$

289 where $R_{\pm}(\mathbf{k})$ is the contribution to the measured angle-resolved responsivity from the slits at $x = x_{sl\pm}$. This formula follows from the observation that $R_{\pm}(\mathbf{k})$ is proportional to the magnitude
 290 squared of the SPP field at the slit locations, with a \mathbf{k} -independent proportionality factor
 291 determined by the efficiency of the SPP slit-scattering process and the quantum efficiency of
 292 the photodetector active layer. For the phase slope parameter α , we use the value of $9.8 \mu\text{m}$
 293 obtained from the analytical model above fitted to the experimental data. Furthermore, the
 294 Fourier transform of the incident light $E_{in}(\mathbf{k})$ can be related to that of the object $E_{obj}(\mathbf{k})$ according
 295 to $E_{in}(\mathbf{k}) = t_{lens}(\mathbf{k}) E_{obj}(\mathbf{k})$, where $t_{lens}(\mathbf{k})$ is the transfer function of the imaging lens in front
 296 of the sensor array. For a circular lens, $t_{lens}(\mathbf{k})$ is a cylindrical step function with cutoff
 297 frequency $k_c = \pi/(\lambda_0 F)$, where F is the lens F number (see Supplement 1, Section S4). With
 298 these prescriptions, we can compute the image of any object produced by any array of
 299 plasmonic directional sensors under coherent illumination.
 300



301

302 Fig. 3. Phase imaging simulation results. (a) Phase distribution of the object. (b) Image of the object of (a)
 303 computed for an array of 392x392 uncoated pixels [i.e., with \mathbf{k} -independent CTF] combined with an NA=0.13
 304 imaging lens. (c) Image of the same object computed for an otherwise identical camera where every pixel is
 305 coated with the metasurface of device B [modeled using the experimental data of Fig. 1(d)]. The signal
 306 intensity in (b) and (c) is normalized to that of the uncoated devices when illuminated with the same plane
 307 wave incident on the object.

308 As an example, we simulate the “phase” object shown in Fig. 3(a) (e.g., a phase-grating set
 309 in a transparent glass plate of refractive index n_{glass} and variable thickness h , where the phase
 310 value ϕ displayed in the figure is given by $\phi = 2\pi(n_{glass} - n_{air})h/\lambda$). In this case, the object field
 311 $E_{obj}(\mathbf{r})$ has uniform amplitude across the entire field of view, and therefore the object could not
 312 be visualized using a standard imaging system with \mathbf{k} -independent response, except for
 313 negligibly small diffraction fringes [Fig. 3(b)]. At the same time, as light is transmitted through
 314 the object, its local direction of propagation is deflected by an angle proportional to the local
 315 phase gradient. As a result, if the response of each pixel varies with angle of incidence, the
 316 recorded photocurrent signals acquire a dependence on the object phase gradient. Spatially

317 varying features of the phase object (in this case, the edges of the grating lines) can therefore
 318 be resolved. The same behavior can also be described in the spatial frequency domain as edge
 319 enhancement caused by the CTF \mathbf{k} dependence. These ideas have been explored extensively
 320 with different types of optical spatial filters [35-38], for applications ranging from label-free
 321 imaging of biological samples [39] to semiconductor wafer inspection [40]. By virtue of their
 322 intrinsic angular sensitivity, the directional image sensors under study can provide the same
 323 functionality without any external spatial filtering elements.

324 To illustrate, we have computed the image of the phase gratings of Fig. 3(a) for an array of
 325 392×392 pixels described by the experimental angular response map of device B [Fig. 1(d)],
 326 combined with an F/3.8 imaging lens (corresponding to a numerical aperture $NA = 0.13$ and a
 327 field of view of 15°). Following the prescriptions above, this image was obtained by summing
 328 the contributions from the two slit sections governed by the CTFs $t_{\pm}(\mathbf{k})$ (see Supplement 1,
 329 Sections S5 and S6 for more details). The resulting plot is shown in Fig. 3(c), where the signal
 330 measured by each plasmonic pixel is normalized to that of an identical uncoated device under
 331 the same illumination conditions. The y-oriented edges of the grating lines are clearly
 332 visualized in this image, consistent with the strong k_x -dependence of the responsivity $R(\mathbf{k})$ of
 333 device B at small angles of incidence. The x-oriented edges can also be discerned, but with
 334 significantly lower contrast, due to the weaker variations of $R(\mathbf{k})$ with k_y (mostly related to the
 335 C shapes of the responsivity peaks). For comparison, the grating lines are essentially invisible
 336 in the image computed for an otherwise identical camera of standard pixels with \mathbf{k} -independent
 337 CTF [Fig. 3(b)]. It should also be noted that isotropic phase imaging could similarly be
 338 achieved with this approach, using alternative metasurface designs featuring rotationally-
 339 invariant angular response, e.g., based on circular grating lines and slits.

340 4. Optical transfer function and incoherent edge enhancement

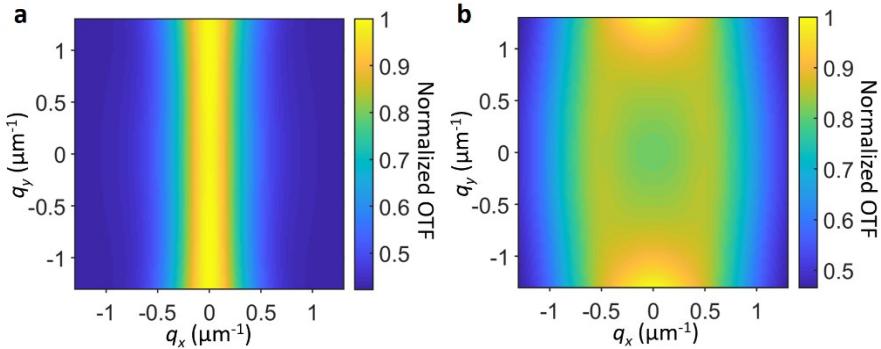
341 Next, we consider the frequency response of the same devices under natural, and therefore
 342 spatially incoherent, illumination. In this case, the incident light features a highly localized
 343 correlation function, which can be modeled as $\langle E_{in}^*(\mathbf{r} - \frac{\delta\mathbf{r}}{2})E_{in}(\mathbf{r} + \frac{\delta\mathbf{r}}{2}) \rangle \propto I_{in}(\mathbf{r})\exp(-\frac{\delta\mathbf{r}^2}{4\Delta^2})/\Delta$,
 344 where the brackets $\langle \dots \rangle$ indicate an ensemble average and the transverse coherence length Δ is
 345 small compared to the size of the image. Under these conditions, the operation of any optical
 346 spatial filter is governed by its optical transfer function (OTF) $T(\mathbf{q}) = I_{out}(\mathbf{q})/I_{in}(\mathbf{q})$, where
 347 $I_{in}(\mathbf{q})$ and $I_{out}(\mathbf{q})$ are the Fourier transforms of the input and output field-intensity distributions,
 348 respectively [4]. This function can be computed by expressing the output intensity $I_{out}(\mathbf{r}) \propto$
 349 $\langle E_{out}^*(\mathbf{r})E_{out}(\mathbf{r}) \rangle$ in terms of the Fourier transform of the output field $E_{out}(\mathbf{k}) = t(\mathbf{k})E_{in}(\mathbf{k})$,
 350 and then evaluating the ensemble average in the spatial-frequency domain. In the directional
 351 image-sensor arrays of interest in this work, $I_{out}(\mathbf{r})$ is again only accessible at the slit locations
 352 $\mathbf{r}_{sl\pm}^n$, but otherwise the OTF can be defined and computed in the same fashion. As detailed in
 353 Supplement 1, Section S7, the resulting expression is

$$354 T_{\pm}(\mathbf{q}) \propto \int d\mathbf{k} t_{\pm}^*(\mathbf{k})t_{\pm}(\mathbf{k} + \mathbf{q})\exp(-\frac{\Delta^2|2\mathbf{k} + \mathbf{q}|^2}{4}), \quad (6)$$

355 with $T_{-}(\mathbf{q}) = T_{+}^*(\mathbf{q}) = T_{+}(-\mathbf{q})$ in a symmetric device. In the limit where $\Delta \rightarrow 0$ (i.e., for
 356 completely incoherent light), $T_{\pm}(\mathbf{q})$ is simply equal to the autocorrelation function of the CTF
 357 $t_{\pm}(\mathbf{k})$, which is maximum for $\mathbf{q} = 0$ regardless of the detailed wavevector dependence of $t_{\pm}(\mathbf{k})$.
 358 This observation confirms the aforementioned statement that, under incoherent illumination, an
 359 optical spatial filter cannot be used to perform any operation that requires suppression of the
 360 DC components of the image, such as edge detection by spatial differentiation.

361 Figures 4(a) and 4(b) show the calculated magnitude of the OTF $T_{+}(\mathbf{q})$ of devices A and B,
 362 respectively. In these calculations, the parameter Δ is evaluated based on the Van Cittert –
 363 Zernike theorem [4]. Specifically, we use $\Delta = 6.7 \mu\text{m}$, which corresponds to a fully incoherent
 364 object at a representative distance of 10 times its lateral size. The corresponding radius of
 365 coherence r_c on the sensor array is $18.9 \mu\text{m}$ (see Supplement 1, Section S8), which is sufficiently

366 large (i.e., larger than the half-size of the grating) to ensure that the SPPs scattered by all the
 367 grating lines can properly interfere with one another. This value of $18.9 \mu\text{m}$ should also be
 368 regarded as a lower bound for r_c , because it neglects the finite coherence of the light at the
 369 object. The CTF $t_{\pm}(\mathbf{k})$ in eq. (6) is computed using eq. (5), with the responsivity contributions
 370 from the two slits $R_{\pm}(\mathbf{k})$ obtained from the experimental data of Figs. 1(c) and 1(d) as
 371 described in Supplement 1, Section S5. As expected, both transfer functions plotted in Fig. 4
 372 are nonzero and near-maximum at $\mathbf{q} = 0$, which again originates from the autocorrelation nature
 373 of eq. (6). At the same time, the detailed shape of these OTFs is determined by the \mathbf{k} -
 374 dependence of the corresponding CTFs (which in turn can be tailored through the metasurface
 375 design), combined with the windowing action of the exponential term in eq. (6). More complex
 376 OTFs can therefore be envisioned, for example involving additional peaks at finite \mathbf{q} values,
 377 with metasurfaces designed to produce multiple peaks in $t_+(\mathbf{k})$ and $t_-(\mathbf{k})$ individually.



378

379 Fig. 4. Optical transfer function of the plasmonic directional image sensors of Fig. 1. Panels (a) and (b) show
 380 the magnitude of $T_{\pm}(\mathbf{q})$ for devices A and B, respectively, computed using their measured responsivity maps
 381 and normalized to unit peak value.

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The transfer function of eq. (6) can be used to model the incoherent imaging capabilities of arrays of these plasmonic devices. To that purpose, we begin by noting that the photocurrent measured by the \mathbf{n}^{th} pixel is proportional to $I_{\text{meas}}(\mathbf{n}) = I_{\text{meas+}}(\mathbf{n}) + I_{\text{meas-}}(\mathbf{n})$, where

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$$I_{\text{meas}\pm}(\mathbf{n}) = \int_{-w/2}^{w/2} \frac{dy}{w} I_{\text{out}\pm}(x_c^n \pm d, y_c^n + y) \quad (7)$$

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is the intensity detected by each slit, averaged over the slit length w along the y direction. Here, as before, $\mathbf{n} = \{n_x, n_y\}$ indicates a pair of integers n_x and n_y that label the different pixels, \mathbf{r}_c^n is the center position of the \mathbf{n}^{th} pixel, and $d = x_{\text{sl+}}^n - x_c^n = x_c^n - x_{\text{sl-}}^n$ is the slit-to-center distance. Next, we express $I_{\text{out}\pm}(\mathbf{r})$ in eq. (7) in terms of its Fourier transform $I_{\text{out}\pm}(\mathbf{q}) = T_{\pm}(\mathbf{q})I_{\text{in}}(\mathbf{q})$, evaluate the integral over y , and finally extract the Fourier transform of $I_{\text{meas}}(\mathbf{n}) = \int d\mathbf{q} I_{\text{meas}}(\mathbf{q}) e^{i\mathbf{q} \cdot \mathbf{r}_c^n}$. With this procedure [and using $T_{-}(\mathbf{q}) = T_{+}^*(\mathbf{q})$] we find that $I_{\text{meas}}(\mathbf{q}) = T_{\text{meas}}(\mathbf{q})I_{\text{in}}(\mathbf{q})$, where

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$$T_{\text{meas}}(\mathbf{q}) = \text{sinc}\left(\frac{q_y w}{2\pi}\right) \text{Re}\left\{e^{i\mathbf{q} \cdot \mathbf{r}_c^n} T_{+}(\mathbf{q})\right\} \quad (8)$$

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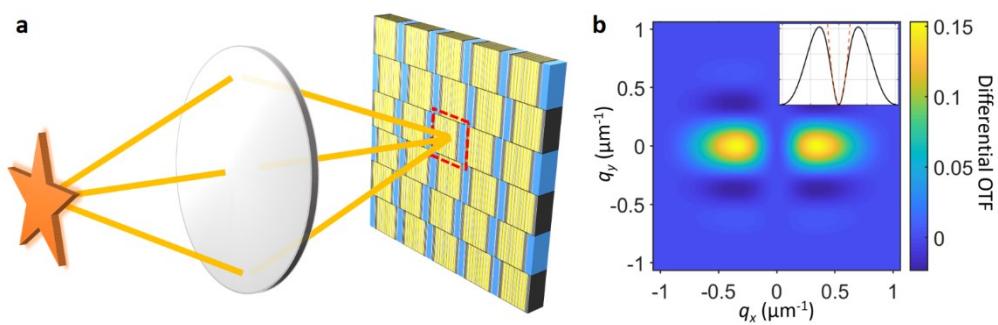
is an effective OTF that describes the measurement of the recorded image by the pixel array. In particular, pixelation effects are also included in this expression through its w and d dependence. Finally, the Fourier transform of the incident intensity $I_{\text{in}}(\mathbf{q})$ can be related to that of the object $I_{\text{obj}}(\mathbf{q})$ according to $I_{\text{in}}(\mathbf{q}) = T_{\text{lens}}(\mathbf{q})I_{\text{obj}}(\mathbf{q})$, where $T_{\text{lens}}(\mathbf{q})$ is the OTF of the imaging lens. As described in Supplement 1, Section S4, under incoherent illumination $T_{\text{lens}}(\mathbf{q})$ decreases almost linearly from 1 to 0 as q varies from 0 to $2\pi/(\lambda_0 F)$.

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The geometrical tunability of the transfer function of these devices is particularly significant for use in sensor arrays partitioned into identical blocks of multiple adjacent pixels, each coated with a different metasurface. With this arrangement, a single camera could produce multiple

403 filtered images of a same object simultaneously, which could then be exploited to perform
 404 specific visual processing tasks (e.g., object recognition) with reduced electronic computational
 405 cost. Within this framework, even the constraint that $T_{\text{meas}}(\mathbf{q}=0) \neq 0$ under incoherent
 406 illumination can be effectively circumvented by subtracting the signals of different adjacent
 407 pixels within each block, to produce an overall response equal to the difference of their
 408 respective OTFs. In fact, a similar idea has already been explored to enable incoherent edge
 409 detection with a $4f$ system, where two masks of different cutoff frequencies are inserted
 410 sequentially at the Fourier plane and the resulting images are subtracted from one another [26].
 411 Such setup, however, is particularly bulky and rather impractical. More recently, nanophotonic
 412 implementations have also been proposed based on wavelength or polarization multiplexing
 413 [27, 28]. By virtue of their ability to enable multiple filtering operations simultaneously on a
 414 pixel-by-pixel basis, the directional image sensors under study are ideally well suited to
 415 implement this general approach for incoherent image processing.

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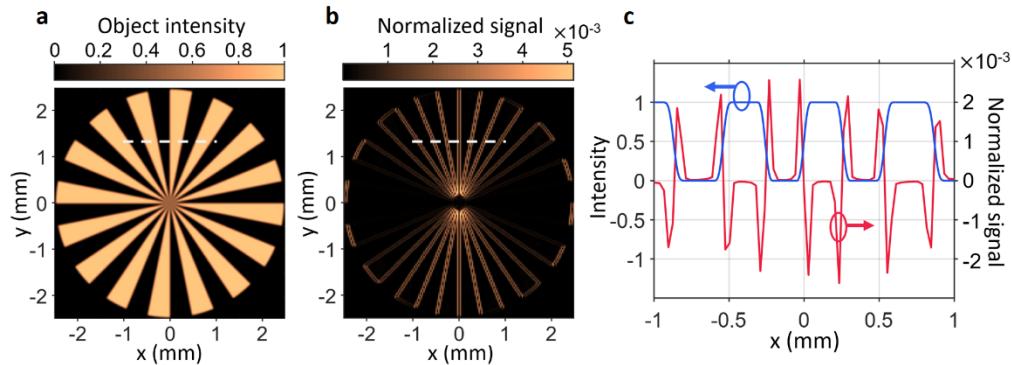
418 Fig. 5. Incoherent edge detection protocol. (a) Schematic illustration of the envisioned sensor array, which
 419 consists of pixels based on the plasmonic directional photodetectors of Fig. 1(a) (yellow squares) combined
 420 with bare pixels (blue rectangles) in a checkerboard pattern. The edge-enhanced image is obtained by
 421 subtracting the signals of neighboring pixels. (b) Differential OTF $\Delta T_{\text{tot}}(\mathbf{q}) = \Delta T_{\text{meas}}(\mathbf{q})T_{\text{lens}}(\mathbf{q})$ of the resulting
 422 imaging system with an NA=0.13 lens, computed by subtracting the cumulative OTFs of the two types of
 423 pixels. Here $\Delta T_{\text{tot}}(\mathbf{q})$ is normalized to the cumulative OTF of the bare pixels at $\mathbf{q} = 0$. The inset shows the \mathbf{q}_y
 424 = 0 line cut of $\Delta T_{\text{tot}}(\mathbf{q})$ (solid line), together with a numerical fit of the low- \mathbf{q}_x portion of this trace to a quadratic
 function of \mathbf{q}_x (dashed line).

425

426 As a particularly simple illustration, we consider the configuration shown schematically in
 427 Fig. 5(a). Here, the low-pass-filter metasurface of device A is fabricated on every other pixel
 428 of an image sensor array in a checkerboard pattern, while all the other devices are left uncoated.
 429 The photocurrent measured by each plasmonic sensor is then subtracted from that of its adjacent
 430 uncoated pixel to produce a null at $\mathbf{q} = 0$, and therefore high-pass filtering. The resulting image
 431 is related to the object according to $I_{\text{meas}}(\mathbf{q}) = \Delta T_{\text{tot}}(\mathbf{q})I_{\text{obj}}(\mathbf{q}) = \Delta T_{\text{meas}}(\mathbf{q})T_{\text{lens}}(\mathbf{q})I_{\text{obj}}(\mathbf{q})$,
 432 where $\Delta T_{\text{meas}}(\mathbf{q})$ is the difference between the measurement OTFs of the two neighboring
 433 pixels. In the uncoated reference pixels, the responsivity $R(\mathbf{k})$ is essentially constant with \mathbf{k} ,
 434 and the frequency dependence of the measurement OTF is mostly determined by pixelation
 435 effects. At the same time, their photocurrent signal at normal incidence ($\mathbf{q} = 0$) is larger than
 436 in the plasmonic devices due to the aforementioned transmission penalty of the metasurfaces
 437 (about 23% for unpolarized light). This difference can be normalized out in the digital data
 438 processing before the subtraction step. Alternatively, it could be handled in the optical domain
 439 by reducing the size of the reference pixels along the x direction by the same factor of 23%,
 440 e.g., using the checkerboard pattern with alternating square and rectangular pixels shown in
 441 Fig. 5(a). In fact, this approach provides several other important advantages. First, it reduces
 442 the size of each super-pixel (i.e., each block of adjacent coated and uncoated devices), which
 443 is favorable to increase the spatial resolution of the high-pass-filtered images. Second, it
 444 produces a flatter frequency response for the reference pixels across the full bandwidth of
 $T_{\text{lens}}(\mathbf{q})$, and therefore increases the frequency range over which $\Delta T_{\text{tot}}(\mathbf{q})$ can be tailored through

445 the plasmonic pixel design. Finally, it can also result in improved noise cancellation upon
 446 signal subtraction.

447 For the pixel configuration of Fig. 5(a), this procedure leads to the differential OTF $\Delta T_{\text{tot}}(\mathbf{q})$
 448 plotted in Fig. 5(b) (see Supplement 1, Section S9 for more details). As expected, this transfer
 449 function is zero at $\mathbf{q} = 0$, and features two pronounced peaks at symmetric locations around the
 450 origin along the q_x direction. The imaging system of Fig. 5(a) can therefore be used to enhance
 451 rapidly varying features of the object (i.e., edges) along the x direction. In particular, the $q_y =$
 452 0 line cut of $\Delta T_{\text{tot}}(\mathbf{q})$ [shown by the solid line in the inset of Fig. 5(b)] is well approximated by
 453 a quadratic function of q_x (dashed line) over a broad portion of the accessible spatial-frequency
 454 range. Since multiplication by q_x^2 in the frequency domain is equivalent to taking the second-
 455 order derivative with respect to x in the space domain, the pixel array of Fig. 5 can provide
 456 (directional) spatial differentiation, leading to edge enhancement. Importantly, the
 457 computational cost of this protocol (associated with the pixel subtraction steps) is significantly
 458 smaller than that of the standard digital-electronics approach for second-order differentiation
 459 (based on a Laplacian of Gaussian filter), by an estimated factor of about 5 or larger [27].



460
 461 Fig. 6. Incoherent edge detection simulation results. (a), (b) Object (a) and absolute value of its edge-enhanced
 462 image (b) computed for an array of 171×210 super-pixels based on the configuration of Fig. 5 [with the
 463 plasmonic pixels modeled using their experimental data of Fig. 1(c)] combined with an $\text{NA} = 0.13$ lens.
 464 (c) Red trace: line cut of the image along the dashed line of (b). Blue trace: line cut of the object along the same
 465 line. In (b) and (c), the differential signal produced by each super-pixel is normalized to the signal of the
 466 uncoated device under maximum illumination from the object.

467 The expected filtering behavior is illustrated in Figs. 6(a) and 6(b), where we plot a simple
 468 amplitude object and the absolute value of its detected image, for an array of 171×210 super-
 469 pixels again combined with a lens of $\text{NA} = 0.13$. Clear edge enhancement is observed with
 470 maximum contrast for the edges oriented along the y direction, whereas x -oriented edges are
 471 not resolved. In Fig. 6(c), the red and blue traces show, respectively, the line cut of the filtered
 472 image along the dashed line of Fig. 6(b) and the corresponding object. The comparison between
 473 the two traces clearly demonstrates the second-order derivative nature of this optical spatial
 474 filter along the x direction, which produces the two peaks per edge observed in the image. The
 475 magnitude of the detected signals in Fig. 6(b) is limited by the differential nature of the
 476 underlying data acquisition, combined with pixelation effects. In any case, the resulting image
 477 can be fully resolved with the signal-to-noise ratio (SNR) levels accessible with near-infrared
 478 photodetectors of similar dimensions [41] (see Supplement 1, Section S10). These results
 479 therefore demonstrate the feasibility of incoherent high-pass filtering with the plasmonic image
 480 sensors under study. Additional filtering operations (e.g., edge enhancement along different
 481 orientations and/or with different transfer functions) could similarly be produced at the same
 482 time with the same sensor array, using different metasurface designs and pixel arrangements.

483 **5. Conclusion**

484 We have introduced a new approach for optical spatial filtering based on pixel arrays of
485 plasmonic directional image sensors with tailored angular response. To establish the image
486 processing capabilities of these devices, we have developed a rigorous theoretical model of
487 their filter transfer function under both coherent and incoherent illumination. The effectiveness
488 of this approach for phase imaging and incoherent edge detection has also been demonstrated
489 through imaging simulations based on the measured angle-resolved responsivity of prototype
490 samples. High-quality filtered images were correspondingly obtained, showing that these
491 experimental samples can provide the required angular selectivity and contrast for the
492 envisioned image processing functionalities.

493 These results are promising for a wide range of application areas including microscopy
494 (e.g., for the visualization of transparent biological cells) and computer vision (e.g., for object
495 recognition). Compared to more traditional optical spatial filters based on Fourier optics, our
496 approach does not require any external spatial filtering elements, and therefore can provide
497 extreme size miniaturization and improved ease of alignment, which are beneficial for
498 embedded and mobile applications. Furthermore, this approach allows controlling the filter
499 transfer function on a pixel-by-pixel basis, so that multiple filtered images of a same object can
500 be produced simultaneously, similar to the output of the first layer of a CNN. These images
501 could then be fed into the subsequent CNN layers to perform various visual recognition tasks.
502 Compared to fully electronic solutions, this ability to synthesize multiple filtered images in the
503 optical domain can provide significant savings in power consumption, estimated at about
504 tenfold in prior studies of hybrid optoelectronic CNN configurations [19, 42].

505 The devices developed in the present work rely on a diffractive plasmonic metasurface to
506 produce the required angular sensitivity, and as a result are limited to narrow-band operation at
507 infrared wavelengths. Specific applications of near-infrared light that would directly benefit
508 from their unique capabilities include, for example, imaging through biological tissue,
509 navigation and surveillance under low-visibility conditions, and facial/gesture recognition.
510 Furthermore, more advanced metasurfaces, including dielectric systems and angle-sensitive
511 meta-units [43], can be envisioned to extend the bandwidth and accessible wavelength range,
512 as well as enable more complex filter transfer functions. We believe that the results presented
513 in this work will provide a strong motivation, as well as theoretical guidance, for the further
514 development of this new family of flat-optics devices.
515

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519 **Disclosures.** The authors declare no conflicts of interest.

520 **Data availability.** Data underlying the results presented in this paper are not publicly available at this time but may
521 be obtained from the authors upon reasonable request.

522 **Supplemental document.** See Supplement 1 for supporting content.

523 **References**

- 524 1. R. Gonzales and R. Woods, *Digital Image Processing*, 3rd ed. (Prentice Hall, 2008).
- 525 2. A. Krizhevsky, I. Sutskever, and G. E. Hinton, “Imagenet classification with deep convolutional neural
526 networks,” *Adv. Neural Inf. Process. Syst.* **25**, 1097-1105 (2012).
- 527 3. G. Wetzstein, A. Ozcan, S. Gigan, S. Fan, D. Englund, M. Soljačić, C. Denz, D. A. B. Miller, and D. Psaltis,
528 “Inference in artificial intelligence with deep optics and photonics,” *Nature* **588**, 39–47 (2020).
- 529 4. J. W. Goodman, *Introduction to Fourier Optics* (Roberts and Company Publishers, 2005).
- 530 5. A. Silva, F. Monticone, G. Castaldi, V. Galdi, A. Alú, and N. Engheta, “Performing mathematical operations
531 with metamaterials,” *Science* **343**, 160–163 (2014).
- 532 6. D. A. Bykov, L. L. Doskolovich, E. A. Bezus, and V. A. Soifer, “Optical computation of the Laplace operator
533 using phase-shifted Bragg grating,” *Opt. Express* **22**, 25084–25092 (2014).
- 534 7. A. Pors, M. G. Nielsen, and S. I. Bozhevolnyi, “Analog computing using reflective plasmonic metasurfaces,”
535 *Nano Lett.* **15**, 791–797 (2015).

536 8. T. Zhu, Y. Zhou, Y. Lou, H. Ye, M. Qiu, Z. Ruan, and S. Fan, "Plasmonic computing of spatial differentiation,"
537 Nat. Commun. **8**, 15391 (2017).
538 9. C. Guo, M. Xiao, M. Minkov, Y. Shi, and S. Fan, "Photonic crystal slab Laplace operator for image
539 differentiation," Optica **5**, 251–256 (2018).
540 10. Y. Fang and Z. Ruan, "Optical spatial differentiator for a synthetic three-dimensional optical field," Opt. Lett.
541 **43**, 5893–5896 (2018).
542 11. A. Cordaro, H. Kwon, D. Sounas, A. F. Koenderink, A. Alú, and A. Polman, "High-index dielectric
543 metasurfaces performing mathematical operations," Nano Lett. **19**, 8418–8423 (2019).
544 12. T. J. Davis, F. Eftekhari, D. E. Gómez, and A. Roberts, "Metasurfaces with asymmetric optical transfer
545 functions for optical signal processing," Phys. Rev. Lett. **123**, 013901 (2019).
546 13. J. Zhou, H. Qian, C.-F. Chen, J. Zhao, G. Li, Q. Wu, H. Luo, S. Wen, and Z. Liu, "Optical edge detection based
547 on high-efficiency dielectric metasurface," Proc. Natl. Acad. Sci. U. S. A. **116**, 11137–11140 (2019).
548 14. A. Saba, M. R. Tavakol, P. Karimi-Khoozani, A. Khavasi, "Two-dimensional edge detection by guided mode
549 resonant metasurface," IEEE Photonics Technol. Lett. **30**, 853–856 (2018).
550 15. Y. Zhou, H. Zheng, I. I. Kravchenko, and J. Valentine, "Flat optics for image differentiation," Nat. Photon. **14**,
551 316–323 (2020).
552 16. J. Zhou, S. Liu, H. Qian, Y. Li, H. Luo, S. Wen, Z. Zhou, G. Guo, B. Shi, and Z. Liu, "Metasurface enabled
553 quantum edge detection," Sci. Adv. **6**, eabc4385 (2020).
554 17. P. Huo, C. Zhang, W. Zhu, M. Liu, S. Zhang, S. Zhang, L. Chen, H. J. Lezec, A. Agrawal, Y. Lu, and T. Xu,
555 "Photonic spin-multiplexing metasurface for switchable spiral phase contrast imaging," Nano Lett. **20**,
556 2791–2798 (2020).
557 18. L. Wesemann, T. J. Davis, and A. Roberts, "Meta-optical and thin film devices for all-optical information
558 processing", Appl. Phys. Rev. **8**, 031309 (2021).
559 19. J. Chang, V. Sitzmann, X. Dun, W. Heidrich, and G. Wetzstein, "Hybrid optical-electronic convolutional neural
560 networks with optimized diffractive optics for image classification," Sci. Rep. **8**, 12324 (2018).
561 20. S. Colburn, Y. Chu, E. Shilzerman, and A. Majumdar, "Optical frontend for a convolutional neural network,"
562 Appl. Opt. **58**, 3179–3186 (2019).
563 21. B. Muminov and L. T. Vuong, "Fourier optical preprocessing in lieu of deep learning," Optica **7**, 1079–1088
564 (2020).
565 22. L. Kogos, Y. Li, J. Liu, Y. Li, L. Tian, and R. Paiella, "Plasmonic ommatidia for lensless compound-eye
566 vision," Nat. Commun. **11**, 1637 (2020).
567 23. J. Duparré, P. Dannberg, P. Schreiber, A. Bräuer, and A. Tünnermann, "Artificial apposition compound eye
568 fabricated by micro-optics technology," Appl. Opt. **43**, 4303–4309 (2004).
569 24. P. R. Gill, C. Lee, D.-G. Lee, A. Wang, and A. Molnar, "A microscale camera using direct Fourier-domain
570 scene capture," Opt. Lett. **36**, 2949–2951 (2011).
571 25. S. Yi, J. Xiang, M. Zhou, Z. Wu, L. Yang, and Z. Yu, "Angle-based wavefront sensing enabled by the near
572 fields of flat optics," Nat. Commun. **12**, 6002 (2021).
573 26. W. T. Rhodes, "Incoherent spatial filtering," Opt. Eng. **19**, 323–330 (1980).
574 27. H. Wang, C. Guo, Z. Zhao, and S. Fan, "Compact incoherent image differentiation with nanophotonic
575 structures," ACS Photon. **7**, 338–343 (2020).
576 28. D. S. Hazineh, Q. Guo, Z. Shi, Y.-W. Huang, T. Zickler, and F. Capasso, "Compact incoherent spatial
577 frequency filtering enabled by metasurface engineering," 2021 IEEE Conference on Lasers and Electro-Optics
578 (CLEO), FTu2M.1 (2021).
579 29. T. W. Ebbesen, H. J. Lezec, H. F. Ghaemi, T. Thio, and P. A. Wolff, "Extraordinary optical transmission
580 through sub-wavelength hole arrays," Nature **391**, 667–669 (1998).
581 30. H. J. Lezec, A. Degiron, E. Devaux, R. A. Linke, L. Martin-Moreno, F. J. Garcia-Vidal, and T. W. Ebbesen,
582 "Beaming light from a subwavelength aperture" Science **297**, 820–822 (2002).
583 31. L. Tian, and L. Waller, "Quantitative differential phase contrast imaging in an LED array microscope," Opt.
584 Express **23**, 11394–11403 (2015).
585 32. F. Ding, A. Pors, and S. I. Bozhevolnyi, "Gradient metasurfaces: a review of fundamentals and applications,"
586 Rep. Prog. Phys. **81**, 026401 (2018).
587 33. W. T. Chen, A. Y. Zhu, and F. Capasso, "Flat optics with dispersion-engineered metasurfaces," Nat. Rev.
588 Mater. **5**, 604–620 (2020).
589 34. S. Banerji, M. Meem, A. Majumder, F. G. Vasquez, B. Sensale-Rodriguez, and R. Menon, "Imaging with flat
590 optics: metalenses or diffractive lenses?" Optica **6**, 805–810 (2019).
591 35. F. Zernike, "Phase contrast, a new method for the microscopic observation of transparent objects," Physica **9**,
592 686–698 (1942).
593 36. T. Kim, S. Sridharan, and G. Popescu, "Gradient field microscopy of unstained specimens," Opt. Express **20**,
594 6737–6745 (2012).
595 37. S. Fürhapter, A. Jesacher, S. Bernet, and M. Ritsch-Marte, "Spiral phase contrast imaging in microscopy," Opt.
596 Express **13**, 689–694 (2005).
597 38. H. Kwon, E. Arbabi, S. M. Kamali, M. Faraji-Dana, and A. Faraon, "Single-shot quantitative phase gradient
598 microscopy using a system of multifunctional metasurfaces," Nat. Photon. **14**, 109–114 (2020).
599 39. Y. K. Park, C. Depeursinge, and G. Popescu, "Quantitative phase imaging in biomedicine," Nat. Photon. **12**,
600 578–589 (2018).

601 40. R. Zhou, C. Edwards, A. Arbabi, G. Popescu, and L. L. Goddard, “Detecting 20 nm wide defects in large area
602 nanopatterns using optical interferometric microscopy,” *Nano Lett.* **13**, 3716-3721 (2013).

603 41. M. Murata, R. Kuroda, Y. Fujihara, Y. Otsuka, H. Shibata, T. Shibaguchi, Y. Kamata, N. Miura, N. Kuriyama,
604 and S. Sugawa, “A high near-infrared sensitivity over 70-dB SNR CMOS image sensor with lateral overflow
605 integration trench capacitor,” *IEEE Trans. Electron Devices* **67**, 1653-1659 (2020).

606 42. H. G. Chen, S. Jayasuriya, J. Yang, J. Stephen, S. Sivaramakrishnan, A. Veeraraghavan, and A. Molnar, “ASP
607 vision: Optically computing the first layer of convolutional neural networks using angle sensitive pixels,”
608 *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, 903–912 (2016).

609 43. S. M. Kamali, E. Arbabi, A. Arbabi, Y. Horie, M. S. Faraji-Dana, and A. Faraon, “Angle-multiplexed
610 metasurfaces: encoding independent wavefronts in a single metasurface under different illumination angles,”
611 *Phys. Rev. X* **7**, 041056 (2017).